Achievable Rates for the Three User Cooperative Multiple Access Channel

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***Abstract*— For a three user Gaussian multiple access channel (MAC), we propose a new superposition block Markov encoding based cooperation scheme. Our scheme allows the three users to simultaneously cooperate both in pairs, and collectively, by dividing the transmitted messages into sub-messages intended for each cooperating partner. The proposed encoding and decoding at the transmitters take into account the relative qualities of the cooperation links between the transmitters. We obtain and evaluate the achievable rate region based on our encoding strategy, and compare it with the achievable rates for the two user cooperative MAC. We demonstrate that the added diversity by the presence of the third user improves the region of achievable rates, and this improvement is especially significant as far as the sum rate of the system is concerned.**

I. Introduction

The growing demand for high data rate mobile applications challenges researchers to develop wireless communication systems which are able to accommodate a higher number of concurrent users, communicating reliably at improved rates. Although an increase in the number of users in a system seems to cause more interference and hence worse performance, this interference may actually be viewed as free side information, which is distributed to all communicating parties thanks to the propagative nature of the wireless communication channel. Therefore, if the users are allowed to make use of the free side information and cooperate in sending each other's messages, the diversity provided to the participating users will increase with increasing number of users, potentially leading to higher rates.

The idea of user cooperation diversity in wireless networks can be traced back to the pioneering works of Carleial [1] and Willems et al. [2] on multiple access channels with generalized feedback (MAC-GF). The MAC-GFs in [1] and [2] are both modelled by $(\mathcal{X}_1 \times \mathcal{X}_2, P(y, y_1, y_2|x_1, x_2), \mathcal{Y} \times \mathcal{Y}_1 \times \mathcal{Y}_2)$, where the receiver observes the channel output Y, and the transmitters have access to separate channel outputs Y_1 and Y_2 through feedback links. The transmitters are then able to encode their messages based on these feedback signals, and can achieve higher rates than the traditional MAC. Carleial and Willems et al. used different versions of block Markov superposition encoding, as well as different decoding policies (sliding window decoding versus backwards decoding respectively), to obtain achievable rate regions for the MAC-GF. The rate region obtained in [2] is in general simpler and larger than that of [1], however it incurs significantly more decoding delay due to the use of backwards decoding.

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More recently, MAC-GF was realized to be a very suitable model for wireless channels as it takes into account the overheard information by the transmitters. Sendonaris et al. [3] applied the results of [2] to obtain the achievable rates for a cooperative Gaussian MAC in the presence of fading. They demonstrated that the user cooperation diversity provides a significant improvement in the achievable rates, over the traditional MAC with non-cooperating transmitters. At this point, we would like to mention that, there has been a tremendous amount of work on several aspects of user cooperation diversity within the last decade, and the surge still continues. Therefore, instead of trying to provide an exhaustive list of many other relevant works in the area, we find it useful to refer the interested reader to a relatively recent paper by Kramer et al. [4], which provides an extensive set of references on cooperative systems.

The efforts in obtaining achievable rates for multi-terminal cooperative communication are certainly not limited to two transmitter scenarios. One major setting involving multiple transmitters is the multiple relay channel, in which many relays, which do not have their own messages, help a single transmitter to send its message to an intended receiver. This type of channel has recently been of particular interest, and resulted in many interesting results on the capacity of such systems [5], [6], [7], [8], [9], [10], [11]. A second major setting is the case of multiple-access relay channel, which is composed of an M-user MAC, and one additional relay whose sole task is to assist the MAC users in their transmission [12], [13]. However, not much has been done for the case where all users participating in the cooperative communication have their own messages to be transmitted, which is our problem at hand in this paper.

In this paper, we focus on the three user Gaussian cooperative MAC. This channel model is of great interest, since it not only provides increased diversity to all participating users, but also it contains as special cases the multiple relay channel and the multiple access relay channel. However, immediately upon moving from the two user MAC-GF to its three user counterpart, various types and structures of encoding strategies to choose from become available, and the block Markov superposition encoding strategy becomes more complicated, as there arise many new cooperation signals. Among many possible cooperation strategies, we choose one which makes

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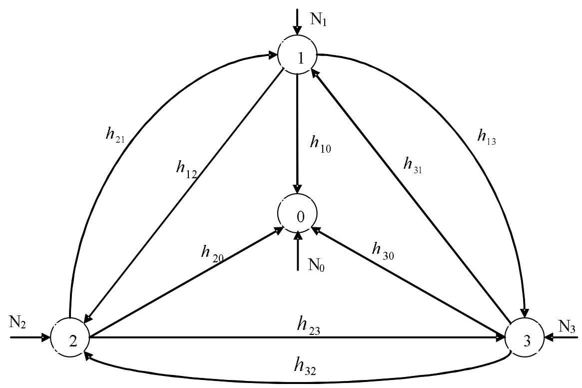

Fig. 1. Three user cooperative channel model

use of the relative qualities of the channel states between the users, in order to adapt the encoding scheme. We first propose a specific decoding strategy at the transmitters for building up the cooperative information, and then we extend the block Markov superposition encoding for the 2 user MAC-GF to three users. We obtain an achievable rate region for our encoding strategy, and we demonstrate by simulations that the presence of a third cooperating user may indeed expand the region of achievable rates significantly, when compared to the rates achievable by two user cooperation.

II. System Model

We consider a three user fading Gaussian MAC, where both the receiver and the transmitters receive noisy versions of the transmitted messages, as illustrated in Figure 1. The transmitters are assumed to be operating in the full duplex mode. The system is modelled by,

$$Y_0 = \sqrt{h_{10}}X_1 + \sqrt{h_{20}}X_2 + \sqrt{h_{30}}X_3 + N_0 \quad (1)$$

$$Y_1 = \sqrt{h_{21}}X_2 + \sqrt{h_{31}}X_3 + N_1 \quad (2)$$

$$Y_2 = \sqrt{h_{12}}X_1 + \sqrt{h_{32}}X_3 + N_2 \quad (3)$$

$$Y_3 = \sqrt{h_{13}}X_1 + \sqrt{h_{23}}X_2 + N_3 \quad (4)$$

where X_i is the symbol transmitted by node i, Y_i is the symbol received at node i, and the receiver is denoted by $i = 0$; N_i is the zero-mean additive white Gaussian noise at node i, having variance σ_i^2, and $\sqrt{h_{ij}}$ are the random fading coefficients, the instantaneous realizations of which are assumed to be known by both the transmitters and the receiver. We further define the normalized fading coefficients $s_{ij} = \frac{h_{ij}}{\sigma_j^2}$, for the simplicity of our discussions.

Throughout this paper, we assume that the normalized channel gains satisfy $s_{ij} > s_{i0}, \ \forall i, j \in \{1, 2, 3\}, \ i \neq j$; that is, the inter-user cooperation links are uniformly stronger than the direct links. This particular case is of practical interest since the cooperating transmitters are likely to be closely located with less number of scatterers and obstructions on the paths connecting them, when compared to their paths to the receiver, and thus have better channel conditions among each other.

III. An Extension of Block Markov Encoding Strategy

Moving from the two user MAC with generalized feedback to its three user counterpart, the block Markov encoding strategy is not trivially generalized, as the presence of the extra user presents a choice among a multitude of new and more complicated cooperation strategies. In this case, the following questions need to be answered: how should the users form their codewords, so as to allow for cooperation among each other? Will the users cooperate in pairs, all together, or using a mixture of both? What kind of cooperation signals will be used, and which signal is to be decoded by which terminal? There are many answers to each of these questions, and in this paper we only focus on one seemingly logical approach in which we design the encoding and decoding strategies based on the knowledge of channel states among the users.

Following the development in the case of two user MAC-GF, we divide the messages of the users into sub-messages intended for each receiver, i.e., $w_1 = \{w_{10}, w_{12}, w_{13}\}$, $w_2 = \{w_{20}, w_{21}, w_{23}\}$, $w_3 = \{w_{30}, w_{31}, w_{32}\}$, where w_{ij} denotes the message of user i intended for user j in each block of transmission. These sub-messages will then be used in the next block to create common cooperation signals, which will be sent to the ultimate receiver (0). It becomes immediately obvious upon crowding the list of these sub-messages that, except for the ultimate receiver, each receiver has two sub-messages intended for it, and six sub-messages which will cause interference to it, should the codewords that will be used to transmit these sub-messages be treated as noise, as it is done in [3]. Moreover, as will become clearer from our oncoming exposition, the pairwise cooperation signals, that shall be generated after the messages are decoded at their intended receivers, will cause additional interference in the upcoming block. This is because of the fact that not all sub-messages will be known to all users, unlike the two user MAC-GF, where both cooperating nodes get to know each other's cooperation signals after each block.

In order to avoid the added interference at the transmitters, we instead propose a modified block Markov encoding strategy, in which the users try to decode as many messages as they can, before forming their cooperation signals. To this end, we first start by assuming without loss of generality that the normalized inter-user link gains s_{ij} are distributed so as to satisfy

$$s_{12} > s_{13}, \quad s_{21} > s_{23}, \quad s_{32} > s_{31} \quad (5)$$

We will make use of this particular ordering in deciding which sub-messages are to be decoded by which users. Although our encoding strategy does depend on this ordering, it can be easily modified to fit any other ordering of the channel gains. Our proposed encoding and decoding strategy is inspired by the capacity achieving encoding/decoding for Gaussian broadcast channels, where the stronger receiver decodes not only its own message, but also the weaker users' messages.

It is evident from (5) that user 2 is the stronger receiver for transmissions from both user 1 and user 3. For one moment

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TABLE I
DECODING STRATEGY AT THE TRANSMITTERS, BEFORE FORMING THE COMMON COOPERATION SIGNALS

User	Decoded Messages	Own Messages
1	w_{21}, w_{31}, w_{23}	w_{12}, w_{13}
2	w_{12}, w_{32}, w_{13}, w_{31}	w_{21}, w_{23}
3	w_{13}, w_{23}	w_{31}, w_{32}

let us assume that user 1 was broadcasting alone: user 2 would be able to decode correctly not only its intended message w_{12}, but also the message w_{13} intended for user 3, provided the message w_{13} were sent at a rate that is supported by user 3. A similar argument would apply to the message transmitted by user 3, as user 2 would be able to decode w_{31}, in addition to its intended message w_{32}. Likewise, since user 1 is in a stronger position than user 3 when user 2 is transmitting alone, it would be able to resolve the message w_{23}, as long as user 3 itself can resolve this same message.

Motivated by the above argument, we propose a decoding strategy for the transmitters, which is summarized in Table I. This table provides a list of messages known to each user after common information is established in each block. The rate requirements for reliable decoding of each message at the corresponding receiver will be given in the next section. Looking at Table I, it is easy to observe that the messages w_{13}, w_{23} and w_{31} are known to all transmitters, the messages w_{12}, w_{21} are only known to the transmitters 1 and 2, and the message w_{32} is only known to the transmitters 2 and 3. This grouping of common information immediately suggests a way to form the cooperation signals: we shall use one cooperation signal common to all users, and two other cooperation signals common to pairs {1,2} and {2,3} respectively. Following the notation in [2], [3], and suitably extending the codebook generation process described therein, the signals transmitted by each user can be generated by block Markov superposition encoding as follows:

$$\begin{aligned} X_1 =& \sqrt{P_{10}}X_{10} + \sqrt{P_{12}}X_{12} + \sqrt{P_{13}}X_{13} \\ &+ \sqrt{P_{1U_1}}U_1 + \sqrt{P_{1U}}U \end{aligned} \tag{6}$$

$$\begin{aligned} X_2 =& \sqrt{P_{20}}X_{20} + \sqrt{P_{21}}X_{21} + \sqrt{P_{23}}X_{23} \\ &+ \sqrt{P_{2U_1}}U_1 + \sqrt{P_{2U_3}}U_3 + \sqrt{P_{1U}}U \end{aligned} \tag{7}$$

$$\begin{aligned} X_3 =& \sqrt{P_{30}}X_{30} + \sqrt{P_{31}}X_{31} + \sqrt{P_{32}}X_{32} \\ &+ \sqrt{P_{3U_3}}U_3 + \sqrt{P_{3U}}U \end{aligned} \tag{8}$$

Here, the signals X_{i0} carry the fresh information intended for the receiver, X_{ij} carry the information intended for transmitter j for cooperation in the next block, and U, U_1, U_3 are the common information sent by groups of three, two and two transmitters respectively for the resolution of the remaining uncertainty from the previous block, all chosen from unit-power Gaussian distributions. The transmit power is thus captured by the powers associated with each component, which

TABLE II
BLOCK MARKOV ENCODING FOR THREE USERS

User	Transmitted Codeword
1	$X_{10}(w_{10}, X_{12}, X_{13}, U_1, U)$, $X_{12}(w_{12}, U_1, U)$, $X_{13}(w_{13}, U)$, $U_1(w'_{12}, w'_{21}, U)$, $U(w'_{13}, w'_{23}, w'_{31})$
2	$X_{20}(w_{20}, X_{21}, X_{23}, U_1, U_3, U)$, $X_{21}(w_{21}, U_1, U)$, $X_{23}(w_{23}, U_3, U)$, $U_1(w'_{12}, w'_{21}, U)$, $U_3(w'_{32}, U)$, $U(w'_{13}, w'_{23}, w'_{31})$
3	$X_{30}(w_{30}, X_{31}, X_{32}, U_3, U)$, $X_{31}(w_{31}, U)$, $X_{32}(w_{32}, U_3, U)$, $U_3(w'_{32}, U)$, $U(w'_{13}, w'_{23}, w'_{31})$

are required to satisfy the average power constraints,

$$\begin{aligned} P_{10} + P_{12} + P_{13} + P_{1U_1} + P_{1U} &\leq P_1 \\ P_{20} + P_{21} + P_{23} + P_{2U_1} + P_{2U_3} + P_{2U} &\leq P_2 \\ P_{30} + P_{31} + P_{32} + P_{3U_3} + P_{3U} &\leq P_3 \end{aligned} \tag{9}$$

The encoding strategy, and the dependency of the transmitted codewords on the messages are depicted in more detail in Table II. In Table II, the sub-messages w'_{ij} stand for the messages received in the previous block: the cooperation signals depend on the messages received in previous block, and new information is also encoded into codewords X_{ij}, taking into account the messages received in the previous block.

Once all information blocks are transmitted using the modified block Markov superposition encoding, the receiver decodes the messages of all users starting from the cooperation signals in the last block, using backwards decoding, as in [2], [3]. The conditions on the rates of each sub-message for reliable decoding both at the transmitters and at the receiver is obtained in the next section.

IV. THE ACHIEVABLE RATE REGION

Before proceeding to characterize the achievable rate region, we would like to make a final simplification in our encoding scheme. For a two user cooperative MAC where channel state information is available to the transmitters, it has recently been shown in [14] that, when the inter-user cooperation links are uniformly stronger than the direct links to the receiver, to maximize the achievable rates the signals X_{i0} should never be transmitted and all the available power should be allocated to cooperative signals. In order to prove a similar statement for the three user MAC in question here, the achievable rate region first needs to be characterized, and then optimized over the transmit powers. However, in view of our assumption about the strength of inter-user links when compared to user-destination links, and the results in the two user case [14] we simply choose to drop the signals X_{i0} from our encoding rule, so that the achievable rate region expressions are more tractable, and more easily evaluated using simulations.

The cooperative communication proceeds reliably if the rates at which we transmit each of the sub-messages are supported both on the inter-user links while building up common information, and on the user-to-ultimate-receiver links, where the users' messages are decoded with the help of cooperation signals. The rate at which a message w_{ij} is transmitted is denoted by R_{ij}.

For notational convenience, we first define the following variables which will be used to simplify the rate expressions throughout this section.

$$\begin{aligned} A =& s_{21}P_{2U_3} + s_{31}(P_{32} + P_{3U_3}) + 2\sqrt{s_{21}s_{31}P_{2U_3}P_{3U_3}} + 1 \\ B =& s_{13}(P_{12} + P_{1U_1}) + s_{23}(P_{21} + P_{2U_1}) \\ &+ 2\sqrt{s_{13}s_{23}P_{1U_1}P_{2U_1}} + 1 \\ C =& 2\sqrt{s_{10}s_{20}P_{1U_1}P_{2U_1}} \\ D =& 2\sqrt{s_{20}s_{30}P_{2U_3}P_{3U_3}} \\ E =& 2(\sqrt{s_{10}s_{20}P_{1U}P_{2U}} + \sqrt{s_{10}s_{30}P_{1U}P_{3U}} \\ &+ \sqrt{s_{20}s_{30}P_{2U}P_{3U}}) \end{aligned} \tag{10}$$

Now, let us focus on the decoding of the messages at the transmitters. From Table I, it is easy to see that user 2 simultaneously decodes all messages (those remaining after dropping direct messages X_{i0}) in the system. Therefore, the rates of the messages w_{12}, w_{13}, w_{31}, w_{32} should satisfy the traditional MAC constraints

$$R_{12} < E\left[\log\left(1 + s_{12}P_{12}\right)\right] \tag{11}$$
$$R_{13} < E\left[\log\left(1 + s_{12}P_{13}\right)\right] \tag{12}$$
$$R_{31} < E\left[\log\left(1 + s_{32}P_{31}\right)\right] \tag{13}$$
$$R_{32} < E\left[\log\left(1 + s_{32}P_{32}\right)\right] \tag{14}$$
$$R_{1} < E\left[\log\left(1 + s_{12}(P_{12} + P_{13})\right)\right] \tag{15}$$
$$R_{12} + R_{31} < E\left[\log\left(1 + s_{12}P_{12} + s_{32}P_{31}\right)\right] \tag{16}$$
$$R_{12} + R_{32} < E\left[\log\left(1 + s_{12}P_{12} + s_{32}P_{32}\right)\right] \tag{17}$$
$$R_{13} + R_{31} < E\left[\log\left(1 + s_{12}P_{13} + s_{32}P_{31}\right)\right] \tag{18}$$
$$R_{13} + R_{32} < E\left[\log\left(1 + s_{12}P_{13} + s_{32}P_{32}\right)\right] \tag{19}$$
$$R_{3} < E\left[\log\left(1 + s_{32}(P_{31} + P_{32})\right)\right] \tag{20}$$
$$R_{1} + R_{31} < E\left[\log\left(1 + s_{12}(P_{12} + P_{13}) + s_{32}P_{31}\right)\right] \tag{21}$$
$$R_{1} + R_{32} < E\left[\log\left(1 + s_{12}(P_{12} + P_{13}) + s_{32}P_{32}\right)\right] \tag{22}$$
$$R_{12} + R_{3} < E\left[\log\left(1 + s_{12}P_{12} + s_{32}(P_{31} + P_{32})\right)\right] \tag{23}$$
$$R_{13} + R_{3} < E\left[\log\left(1 + s_{12}P_{13} + s_{32}(P_{31} + P_{32})\right)\right] \tag{24}$$
$$R_{1} + R_{3} < E\left[\log\left(1 + s_{12}(P_{12} + P_{13}) + s_{32}(P_{31} + P_{32})\right)\right] \tag{25}$$

User 1 does not intend to decode the message w_{32}, and therefore treats it as noise. Moreover, unlike the two user cooperative MAC, the cooperation signal U_3 is also unknown to the user 1, as it involves the message w_{32} not affiliated with this user. Therefore, the coherently combined version of the cooperation signals U_3 from users two and three should also be treated as noise at user 1. Then, the reliable decoding of all other messages is possible if

$$R_{21} < E\left[\log\left(1 + s_{21}P_{21}/A\right)\right] \tag{26}$$
$$R_{23} < E\left[\log\left(1 + s_{21}P_{23}/A\right)\right] \tag{27}$$
$$R_{31} < E\left[\log\left(1 + s_{31}P_{31}/A\right)\right] \tag{28}$$
$$R_{2} < E\left[\log\left(1 + s_{21}(P_{21} + P_{23})/A\right)\right] \tag{29}$$
$$R_{21} + R_{31} < E\left[\log\left(1 + (s_{21}P_{21} + s_{31}P_{31})/A\right)\right] \tag{30}$$
$$R_{23} + R_{31} < E\left[\log\left(1 + (s_{21}P_{23} + s_{31}P_{31})/A\right)\right] \tag{31}$$
$$R_{2} + R_{31} < E\left[\log\left(1 + (s_{21}(P_{21} + P_{23}) + s_{31}P_{31})/A\right)\right] \tag{32}$$

Lastly, since user 3 is only interested in decoding the two messages directly intended for itself, we only require the rates of these messages to satisfy the two user MAC capacity bound, where all other signals, including the cooperation signal U_1, are treated as noise.

$$R_{13} < E\left[\log\left(1 + s_{13}P_{13}/B\right)\right] \tag{33}$$
$$R_{23} < E\left[\log\left(1 + s_{23}P_{23}/B\right)\right] \tag{34}$$
$$R_{13} + R_{23} < E\left[\log\left(1 + (s_{13}P_{13} + s_{23}P_{23})/B\right)\right] \tag{35}$$

Once the common information is reliably established at the transmitters, it remains to make sure that the transmitted messages are also reliably decoded at the ultimate receiver. Note that, as we backwards decode the cooperative signals using joint typicality decoding at the receiver, the message groups $\{w_{12}, w_{21}\}$, w_{32} and $\{w_{13}, w_{23}, w_{31}\}$ appear jointly in cooperative codewords, and they will be decoded jointly as if each group is a single message. Having this in mind, traditional arguments on MAC capacity can be used to obtain the set of constraints on the rates that should be satisfied for achievability at the ultimate receiver. These constraints are given in equations (36)-(42) at the bottom of this page. Note that, the constraints (38), (40), (41) are dominated by the tighter constraint (42), which has the same right hand side but bounds more rate components. Therefore, the inequalities (38), (40), (41) can be omitted from the final solution.

Finally, the set of achievable rate triplets $\{R_1, R_2, R_3\}$ can be obtained by the convex hull of all rate points computed by $R_1 = R_{12} + R_{13}$, $R_2 = R_{21} + R_{23}$, and $R_3 = R_{31} + R_{32}$, where $\{R_{12}, R_{13}, R_{21}, R_{23}, R_{31}, R_{32}\}$ satisfy the constraints (11)-(42).

$$R_{32} < E\left[\log\left(1 + s_{20}P_{2U_3} + s_{30}(P_{32} + P_{3U_3}) + D\right)\right] \tag{36}$$
$$R_{12} + R_{21} < E\left[\log\left(1 + s_{10}(P_{12} + P_{1U_1}) + s_{20}(P_{21} + P_{2U_1}) + C\right)\right] \tag{37}$$
$$R_{13} + R_{23} + R_{31} < E\left[\log\left(1 + s_{10}P_1 + s_{20}P_2 + s_{30}P_3 + C + D + E\right)\right] \tag{38}$$
$$R_{12} + R_{21} + R_{32} < E\left[\log\left(1 + s_{10}(P_{12} + P_{1U_1}) + s_{20}(P_{21} + P_{2U_1} + P_{2U_3}) + s_{30}(P_{32} + P_{3U_3}) + C + D\right)\right] \tag{39}$$
$$R_{13} + R_{23} + R_{3} < E\left[\log\left(1 + s_{10}P_1 + s_{20}P_2 + s_{30}P_3 + C + D + E\right)\right] \tag{40}$$
$$R_{1} + R_{2} + R_{31} < E\left[\log\left(1 + s_{10}P_1 + s_{20}P_2 + s_{30}P_3 + C + D + E\right)\right] \tag{41}$$
$$R_{1} + R_{2} + R_{3} < E\left[\log\left(1 + s_{10}P_1 + s_{20}P_2 + s_{30}P_3 + C + D + E\right)\right] \tag{42}$$

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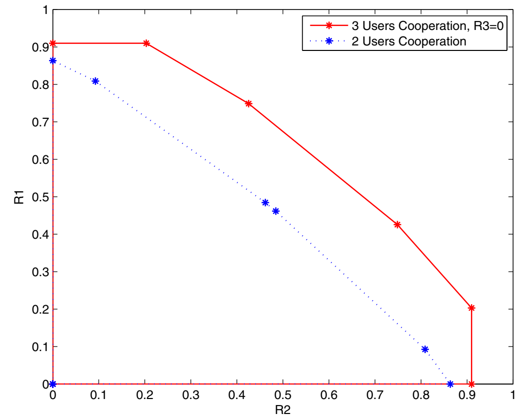
V. Simulation Results

In this section we numerically evaluate the achievable rate region described by the equations (11)-(42) for the three user cooperative MAC. Since the achievable rate region obtained as a result of the three dimensional convex hull operation turns out to be hard to visualize, we simply plot its cross-sections on $R_1 - R_2$, $R_1 - R_3$ and $R_2 - R_3$ planes. This enables us to compare the three user achievable rate region with the corresponding two user cooperation strategies, obtained by the encoding/decoding structure in [3].

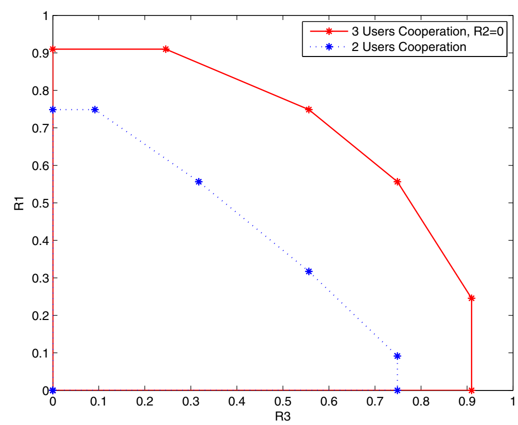
The achievable rate region is generated for two sets of channel state distributions, each of which is chosen uniformly to satisfy the assumption in (5), as well as the assumption about the cooperative links being stronger than the direct links: $s_{10},\ s_{20},\ s_{30}$ are i.i.d uniform random variables taking the values from the set $\{0.1 : 0.2 : 0.9\}$, $s_{13},\ s_{23},\ s_{31}$ are i.i.d taking values from $\{1.1 : 0.2 : 1.9\}$ and $s_{12},\ s_{21},\ s_{32}$ are also i.i.d with values $\{2.1 : 0.2 : 2.9\}$. The average transmit power for each user is chosen to be 1. The resulting sets of achievable rate pairs are plotted in Figures 2(a)-2(c), along with the two user cooperation strategy of Sendonaris et al. in [3]. We see in all three figures that the existence of a third user improves the set of achievable rates significantly, especially for rate tuples near the sum rate. When we search for the active constraints for the points on the axes in all figures, we see that each single user rate is bounded by the rate constraint coming from the inter-user links, rather than the direct links for this selection of fading coefficients. As an example, let us consider Figure 2(a). The main advantage of having a third user in the system, as far as the maximum achievable rate R_1 is concerned, is that user 1 does not have to allocate any part of its power to a cooperation signal; it is able to use its power solely to establish common information, while users 2 and 3 send only cooperation signals to establish a coherent combining gain. This way, the rate constraint on the direct link becomes loose, and the rate R_1 can be pushed all the way to the rate on the inter-user link. Another interesting observation is from Figure 2(c): although the rate region is asymmetric for the two user cooperative MAC, it is symmetric for the three user MAC, with the same maximum achievable rate for all users. In this case, the channel coefficients s_{32} are better than the coefficients s_{23}, therefore it is expected that the two user cooperation will yield higher rates for user 3, which has a better outgoing link. However, the presence of a third user creates additional diversity by making a better channel condition, namely s_{21}, available to user 2, which is no longer constrained to cooperate solely with user 3, and the resulting achievable rates are increased.

Note that, fixing one of the rate components in our region is simply equivalent to considering a multiple access relay channel. Meanwhile, the points where our rate regions intersect the axes correspond to the case of two parallel relays.

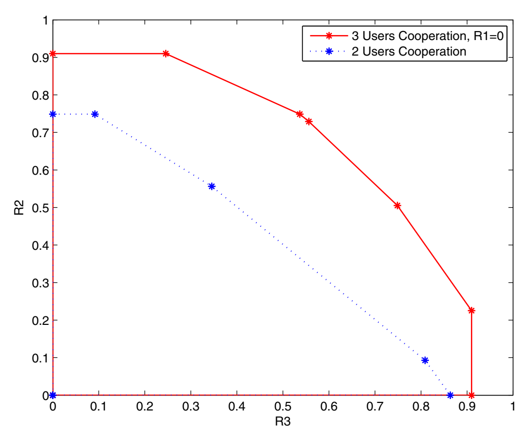
The second set of uniform fading distributions; s_{10}, s_{20}, s_{30} i.i.d from $\{0.5 : 0.05 : 0.7\}$, s_{13}, s_{23}, s_{31} i.i.d from $\{0.8 : 0.05 : 1\}$ and s_{12}, s_{21}, s_{32} i.i.d from $\{1.1 : 0.05 : 1.3\}$, yield a more interesting set of achievable rate regions, depicted in Figures 3(a)-3(c). Namely, it can be seen in Figure 3(a) that the two and three user cooperation strategies give the same maximum individual rates. This is because of the fact that it



(a) Rate region for users 1 and 2, with user 3 acting as a relay.



(b) Rate region for users 1 and 3, with user 2 acting as a relay.



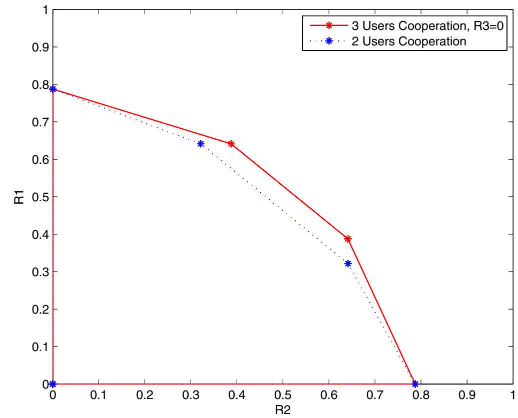
(c) Rate region for users 2 and 3, with user 1 acting as a relay.

Fig. 2. Projection of the 3-D rate region onto 2-D rate planes for the three user cooperative strategy, first set of fading coefficients.

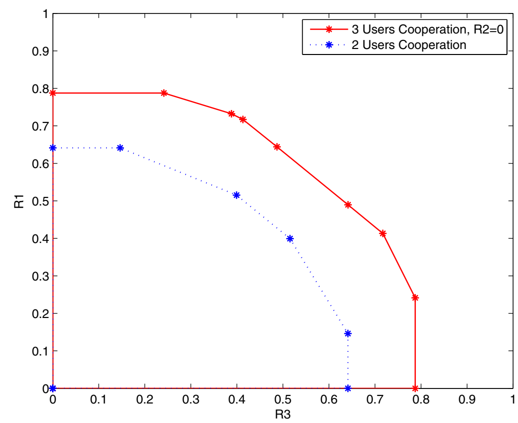
is no longer profitable to use the cooperation links as far as the individual rates are concerned. However as we get closer to the sum rate point, the common cooperation signal becomes useful, as it is a function of many sub-messages coming from all users. At the points where the two and three user cooperative rate regions coincide, the employed power distribution for both strategies are the same, and the extra user is treated as if it is not present in the system. Lastly, since the two user cooperation strategy is simply a subset of its three user counterpart, we always expect to have the achievable rate region of the former to be also a subset of the latter.

VI. Conclusions

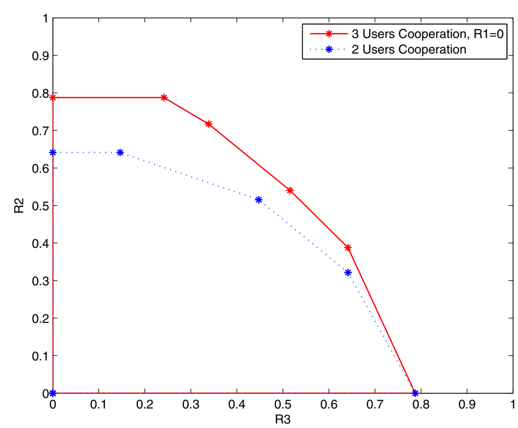
In this paper we introduced a three user cooperative MAC model, and we proposed encoding and decoding policies that rely on a non-trivial extension of the well known block Markov superposition coding. We characterized, and evaluated the rate region achievable by our proposed encoding-decoding techniques. We demonstrated that the added diversity due to the presence of an additional user may translate into significant rate gains, especially near the sum rate point. It has to be noted that our propositions and derivations here are only preliminary results on a wide open and relatively untouched problem, and many variations to the encoding policy can be developed.



(a) Rate region for users 1 and 2, with user 3 acting as a relay.



(b) Rate region for users 1 and 3, with user 2 acting as a relay.



(c) Rate region for users 2 and 3, with user 1 acting as a relay.

Fig. 3. Projection of the 3-D rate region onto 2-D rate planes for the three user cooperative strategy, second set of facing coefficients.

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