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Phase Damping Destroys Quantum Fisher Information of W states

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We study the quantum Fisher information (QFI) of W states in the basic decoherence channels. We show that, as decoherence starts and increases, under i) depolarizing, QFI smoothly decays; ii) amplitude damping, QFI first exhibits a sudden drop to the shot noise level, then decreases to zero and finally increases back to the shot noise level; *iii)* phase damping, QFI is zero for all non-zero decoherence. We also find that on the contrary to GHZ states, QFI of W states in x and y directions are equal to each other and zero in z direction.

Keywords: *Quantum Fisher information, multipartite entanglement, W states, decoherence channels*

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Quantum Fisher information (QFI) is the natural extension of Fisher information in the quantum regime and QFI of a parameter quantifies the sensitivity of a state with respect to changes of the parameter [1–3]. The limit on the variance of the estimation of a parameter ϕ of a general density matrix $\rho(\phi)$ is given by the quantum Cramer-Rao bound [2, 3]

$$\Delta\hat{\phi} \geq \Delta\phi_{QCB} \equiv \frac{1}{\sqrt{N_m F}} \tag{1}$$

where N_m is the number of experiments, F is the quantum Fisher information and the estimator $\hat{\phi}$ satisfies $\langle\hat{\phi}\rangle = \phi$. In particular, QFI characterizes the phase sensitivity of a state with respect to SU(2) rotations, i.e. consider that the parameter ϕ is acquired by an SU(2) rotation $\rho(\phi) = U_\phi \rho U_\phi^\dagger$, where $U_\phi = e^{i\phi J_{\vec{n}}}$ with the angular momentum operators on each particle in each direction, i.e.

$$J_{\vec{n}} = \sum_{\alpha=x,y,z} \frac{1}{2} n_\alpha \sigma_\alpha, \tag{2}$$

σ_α being the Pauli matrices. Considering $N_m = 1$, for separable states of N particles, $F \leq N$, where equality holds for coherent spin states, for example. Therefore the precision limit of the estimation with the best separable states is $1/\sqrt{N}$, which is called the shot-noise limit. On the other hand, quantum Fisher information of an entangled state, such as a pure GHZ state can reach N^2, implying the fundamental limit $1/N$, which also called the Heisenberg limit. It was shown that QFI provides a sufficient condition to recognize multipartite entanglement: If QFI of a state surpasses the shot-noise limit, then it is multipartite entangled and it is called a "useful" state [4]. A basic property of multipartite entangled states is that they fall into inequivalent classes such as GHZ, W and Dicke states, and in general, a state in one class cannot be converted to a state in another class via local operations and classical communication (LOCC) [5], and for several tasks a specific multipartite entangled state is strictly required [6]. Together with the discovery that not all multipartite entangled states exceeds the shot-noise limit -even when they are free of any decoherence [7–9], this property makes exploring QFI and the usefulness of each generic state a crucial step for quantum information science, especially when the state is subjected to decoherence due to natural effects. Recently, quantum metrology has been studied in non-markovian environments [10, 11] and in dissipative environments [12–15]. It was shown that the superpositions of pure Dicke states achieves larger QFI than pure Dicke states themselves [16]. We have studied the behavior of QFI of pure states in the superposition of GHZ and W states of several particles [17, 18], QFI of Bell states under decoherence [19] and proposing an LOCC optimization, analyzed QFI with entanglement measured [20]. QFI of NOON states in relativistic channels [21] and QFI of GHZ states in the basic decoherence channels have been studied [22]. In the latter, it was found that in all three channels, there appears a competition between the phase sensitivities in each direction. Therefore QFI exhibits sudden change points. Also QFI of decohered GHZ states exhibit a smooth and continues decay starting from the QFI of the pure GHZ state.

In this work, we study the QFI of W states in the three basic decoherence channels, i.e. depolarizing, amplitude damping and phase damping. On the contrary to GHZ states [22], we first show that, no matter being pure or decohered, W states do not provide phase sensitivity in z direction and the phase sensitivities in x and y directions are equal to each other, which implies no sudden change points due to competition between directions. More interestingly we show that phase sensitivity of W states under decoherence exhibits discontinuities such as a sudden drop in amplitude damping channel and even sudden death in phase damping channel but exhibits a smooth and continues decay in depolarizing channel.

A general W state of N particles can be written as $|W_N\rangle = \frac{1}{\sqrt{N}}(|0^{\otimes(N-1)}\rangle|1\rangle + \sqrt{N-1}|W_{N-1}\rangle|0\rangle)$ for $N >$

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2, where $|W_2\rangle$ corresponds to the Bell pair in the form $\frac{1}{\sqrt{2}}(|01\rangle+|10\rangle)$ and a W state of 3 particles to the state $|W_3\rangle=\frac{1}{\sqrt{3}}(|001\rangle+|010\rangle+|100\rangle)$.

The maximal mean quantum Fisher information (QFI), i.e. the quantum Fisher information F per qubit maximized over directions, $\bar{F}_{max}=max\{\bar{F}_x,\bar{F}_y,\bar{F}_z\}$ of a possibly mixed state ρ of N qubits is given in [1, 22] as

$$\bar{F}_{max}(\rho)=\frac{c_{max}}{N} \tag{3}$$

where c_{max} is the largest eigenvalue of the matrix $\mathbf{C}$ of which elements are given as

$$\mathbf{C}_{kl}=\sum_{i\neq j}\frac{(\lambda_i-\lambda_j)^2}{\lambda_i+\lambda_j}[\langle V_i|J_k|V_j\rangle\langle V_j|J_l|V_i\rangle + \langle V_i|J_l|V_j\rangle\langle V_j|J_k|V_i\rangle] \tag{4}$$

where, $\lambda_{i,j}$ and $|V_{i,j}\rangle$ are the eigenvalues and the associated eigenvectors of the density matrix of the state ρ respectively. For a pure state QFI is simplified to maximum of $4(\Delta J_{\vec{n}})^2/N$ over spin directions x, y and z, where $(\Delta J_{\vec{n}})^2$ is the variance of the operator $J_{\vec{n}}$, being $(\Delta J_{\vec{n}})^2=\langle J_{\vec{n}}^2\rangle-\langle J_{\vec{n}}\rangle^2$. For a pure GHZ state of N particles, $4(\Delta J_x)^2=4(\Delta J_y)^2=0$ and $4(\Delta J_z)^2=N^2$; whereas for a pure W state, $4(\Delta J_x)^2=4(\Delta J_y)^2=3N-2$ and $4(\Delta J_z)^2=0$. In the case of decoherence, matrix $\mathbf{C}$ for W states appears as $diag\{C_{xx},C_{yy},C_{zz}\}$ where $C_{xx}=C_{yy}$, $C_{zz}=0$ and the element C_{kk} represents the phase sensitivity in k direction. In other words, because of the form of the $J_{\vec{n}}$ operator that imparts the parameter to be estimated, pure or decohered W states have no phase sensitivity in z direction and sensitivities in x and y directions are equal to each other.

Decoherence channels for a density matrix ρ can be given in Kraus representation as [23, 24]

$$\varepsilon(\rho)=\sum_{\mu}E_\mu\rho E_\mu^\dagger \tag{5}$$

where the Kraus operators E_μ satisfy the completeness relation

$$\sum_{\mu}E_\mu^\dagger E_\mu=\mathbb{1}. \tag{6}$$

and $\mathbb{1}$ is the $2x2$ identity matrix.

Below we will study QFI of W states in the basic decoherence channels. For each decoherence channel, as in [22] we assume that each particle of the state is subjected to the same decoherence effect. We find that the general behavior of QFI of W states does not depend on the number of particles N, and for the sake of simplicity we present some of the results for W_3, i.e. a W state of 3 qubits. Following the work of Ref.[25] we also relate the QFI of W states under decoherence to the geometric representation with the Bloch sphere.

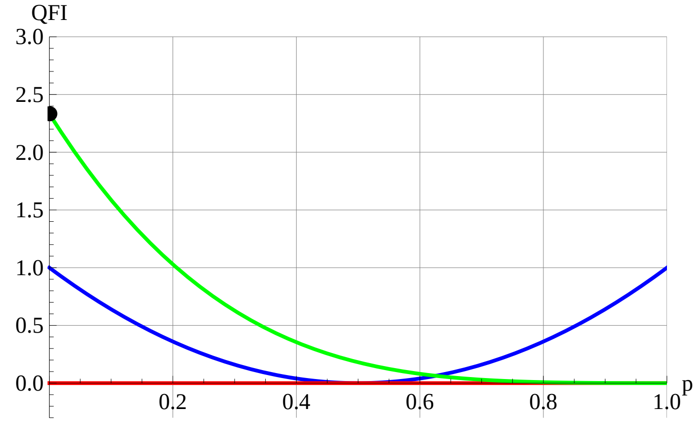

FIG. 1: (Color online). QFI of W_3 in decoherence channels with respect to the decoherence strength p. Black dot at 7/3 is for a pure $|W_3\rangle$ state, i.e. $p=0$. Green, blue and red curves are for a W_3 state in depolarizing, amplitude damping and phase damping channels, respectively.

I. DEPOLARIZING CHANNEL

A d-level quantum system under depolarizing channel is depolarized with some probability, i.e. replaced by the maximally mixed state $\mathbb{1}/d$ with some probability or left untouched. The Kraus operators of depolarizing channel for a single qubit are given by

$$E_0=\sqrt{1-\frac{3}{4}p}\mathbb{1},\quad E_1=\sqrt{\frac{p}{4}}\sigma_x, \\ E_2=\sqrt{\frac{p}{4}}\sigma_y,\quad E_3=\sqrt{\frac{p}{4}}\sigma_z. \tag{7}$$

The eigenvalues of W_3 in the depolarizing channel appear as $\lambda_1=\frac{1}{8}(-2+p)^2p$; $\lambda_2=-\frac{1}{8}(-2+p)p^2$; $\lambda_3=\lambda_4=\frac{1}{24}p(8-6p+p^2)$; $\lambda_5=\frac{1}{24}p(16-24p+11p^2)$; $\lambda_6=\frac{1}{24}(24-52p+42p^2-11p^3)$ and $\lambda_7=\lambda_8=\frac{1}{24}(4p-p^3)$, with the associated eigenvectors,

$$\begin{aligned}|V_1\rangle&=[1,0,0,0,0,0,0,0]^\dagger,\\ |V_2\rangle&=[0,0,0,0,0,0,0,1]^\dagger,\\ |V_3\rangle&=[0,-\tfrac{1}{\sqrt{3}},0,0,\tfrac{1}{\sqrt{3}},0,0,0]^\dagger,\\ |V_4\rangle&=[0,-\tfrac{1}{\sqrt{3}},\tfrac{1}{\sqrt{3}},0,0,0,0,0]^\dagger,\\ |V_5\rangle&=[0,0,0,\tfrac{1}{\sqrt{3}},0,\tfrac{1}{\sqrt{3}},\tfrac{1}{\sqrt{3}},0]^\dagger,\\ |V_6\rangle&=[0,\tfrac{1}{\sqrt{3}},\tfrac{1}{\sqrt{3}},0,\tfrac{1}{\sqrt{3}},0,0,0]^\dagger,\\ |V_7\rangle&=[0,0,0,-\tfrac{1}{\sqrt{3}},0,0,\tfrac{1}{\sqrt{3}},0]^\dagger,\\ |V_8\rangle&=[0,0,0,-\tfrac{1}{\sqrt{3}},0,\tfrac{1}{\sqrt{3}},0,0]^\dagger.\end{aligned}$$

Using Eq.(4) it is straightforward to find that $\bar{F}_{max}=((-1+p)^2(435456+(-2+p)p(1187088+5(-2+p)p(233008+(-2+p)p(96379+13872(-2+p)p)))))/(972(4+3(-2+p)p)(8+p(-9+4p))(6+p(-7+4p)))$. That is, the maximal mean QFI of W_3 in depolarizing channel, starting from the value of QFI of a pure W_3 state, which is 7/3, exhibits a smooth decrease with respect to the depolarization strength and vanishes when the depolarization strength is maximum (see the green curve in Fig.1). QFI of a general W_N in depolarizing channel, exhibiting the same behavior, starts decaying from $3-\frac{2}{N}$ at $p=0$; vanishes at $p=1$ and only the steepness of the decrease of the QFI depends on the number of particles. Behavior of QFI of W states in depolarizing channel is similar to that of GHZ states [22]. The smooth decay of QFI of both GHZ and W states under depolarization can be linked to the fact that the radius of the Bloch sphere of a qubit in depolarization channel is reduced uniformly in each direction, such that the phase sensitivity of the state is reduced in each direction uniformly.

II. AMPLITUDE DAMPING CHANNEL

A quantum system dissipating energy to (or receiving energy from) its environment -such as an atom loosing (or receiving) a photon- can be modelled as a damping (or an amplification) in its amplitude. The Kraus operators of the amplitude damping channel for a single qubit are given by

$$E_0 = |0\rangle\langle 0| + \sqrt{1-p}|1\rangle\langle 1|, \quad E_1 = \sqrt{p}|0\rangle\langle 1| \tag{8}$$

where p is the probability of decay from upper level $|1\rangle$ to the lower level $|0\rangle$ with the damping rate γ i.e. $1-p = e^{-\gamma t/2}$. We find the eigenvalues of the density matrix of a W_3 as $\lambda_1 = 1-p$ and $\lambda_2 = p$, with the associated eigenvectors,

$$\begin{aligned} |V_1\rangle &= [0, \tfrac{1}{\sqrt{3}}, \tfrac{1}{\sqrt{3}}, 0, \tfrac{1}{\sqrt{3}}, 0, 0, 0]^\dagger, \\ |V_2\rangle &= [1, 0, 0, 0, 0, 0, 0, 0]^\dagger. \end{aligned}$$

For $|W_N\rangle$, the eigenvectors are the $|W_N\rangle$ itself and $|0^{\otimes N}\rangle$ with the same eigenvalues. Using Eq.(4) we construct the $\mathbf{C}$ matrix and find the largest eigenvalue of $\mathbf{C}$ matrix as $c_{max} = N(1-2p)^2$, therefore $\bar{F}_{max}$ is independent of N. Therefore the maximal mean quantum Fisher information of a W state of N particles in amplitude damping channel with decoherence strength p is

$$\bar{F}_{max} = \begin{cases} 3-\frac{2}{N}, & p=0, \\ (1-2p)^2, & 0<p\leq 1. \end{cases} \tag{9}$$

This result, as plotted in Fig.1 (the blue curve) shows that, QFI of W states exhibit a sudden drop to shot-noise level, when subjected to amplitude damping noise, and as the strength increases, QFI first vanishes and then increases back to shot-noise level. The reason of this unexpected revival of QFI of W states for $0.5 \geq p \geq 1$ is that, any entangled state decoheres towards the separable state $|000..0\rangle$ (or to the state $|111..1\rangle$) in amplitude damping (or amplifying) channel -where the entire Bloch sphere shrinks to north (or south) pole- and the states on the poles provide the shot noise limit. This sudden change to $(1-2p)^2$ for $p>0$ is observed in the case of GHZ states as well, in a sense that when a pure GHZ state starts to decohere, its $\bar{F}_x$ and $\bar{F}_y$ exhibit a sudden jump from 0 to 1. However, since $\bar{F}_z$ is dominant until $p=0.5$ and it decays smoothly from N, QFI (i.e. the maximum of F in each direction) of GHZ states do not exhibit the sudden change in the final picture. Regarding W states, $\bar{F}_z$ is zero and dominant directions are x and y. Therefore QFI of W states exhibits sudden drop from $3-N/2$ to 1 in amplitude damping channel.

III. PHASE DAMPING CHANNEL

Quantum systems decohere not only due to energy loss as in amplitude damping but also due to the loss of the quantum information without loosing energy. This type of noise is modelled as phase damping channel since the relative phase between the energy eigenstates of the system is lost, decaying the off-diagonal elements of the density matrix of the system. The Kraus operators for the phase damping channel for a single qubit system are given by

$$E_0 = \sqrt{1-p}\mathbb{1}, \quad E_1 = \sqrt{p}|0\rangle\langle 0|, \quad E_2 = \sqrt{p}|1\rangle\langle 1|. \tag{10}$$

Phase damping shrinks the Bloch sphere in x and y directions and has no effect on z direction. In the phase damping channel, the eigenvalues of W_3 state appear as $\lambda_1 = \frac{1}{3}(3-4p+2p^2)$ and $\lambda_2 = \lambda_3 = \frac{1}{3}(2p-p^2)$; with the associated eigenvectors,

$$\begin{aligned} |V_1\rangle &= [0, \tfrac{1}{\sqrt{3}}, \tfrac{1}{\sqrt{3}}, 0, \tfrac{1}{\sqrt{3}}, 0, 0, 0]^\dagger, \\ |V_2\rangle &= [0, \tfrac{1}{\sqrt{2}}, -\tfrac{1}{\sqrt{2}}, 0, 0, 0, 0, 0]^\dagger, \\ |V_3\rangle &= [0, -s, -s, 0, 2s, 0, 0, 0]^\dagger. \end{aligned}$$

where $s = 0.408248$. It is easy to see that $\langle V_i|J_k|V_j\rangle = 0$ for any $i,j = 1,2,3, i \neq j$ and $k = x,y,z$. Therefore via Eq.(4) we find that at any non-zero strength of phase damping, QFI vanishes, i.e. for a W state of N qubits in phase damping channel,

$$\bar{F}_{max} = \begin{cases} 3-\frac{2}{N}, & p=0, \\ 0, & 0<p\leq 1, \end{cases} \tag{11}$$

which shows that W states in phase damping channel do not provide phase sensitivity in any directions. We also calculated the quantum Fisher information of the state $|W_3(\gamma,\theta)\rangle = \frac{1}{\sqrt{3}}(|100\rangle + e^{i\gamma}|010\rangle + e^{i\theta}|001\rangle)$ under phase damping. When $p=0$, for the boundary values of γ and θ, QFI of $|W_3(\gamma,\theta)\rangle$ converges to $3-2/N$ but is decreased for the region $0<\gamma<2\pi$ and $0<\theta<2\pi$. On the other hand, independent from the values of γ and θ, QFI of decohered $W_3(\gamma,\theta)$ state is zero for all $p>0$. GHZ states have zero mean spin, i.e. $\langle J_x\rangle = \langle J_y\rangle = \langle J_z\rangle = 0$. As a pure GHZ state starts to decohere, the dominant Fisher information is in z direction and with the shrinking of the Bloch sphere in x-y plane, F_z smoothly decreases. On the other hand, for W states $\langle J_x\rangle = \langle J_y\rangle = 0$, $\langle J_z\rangle = 1/2$, i.e. the spin direction of the state is along the z axis but $F_z = 0$ and with the shrinking of the Bloch sphere in x-y plane, F_x and F_y drops to zero.

IV. CONLUSION

In conclusion, we have studied quantum Fisher information (QFI) of W states with respect to SU(2) rotations in three decoherence channels. We have shown that the QFI of W states when subjected to i) depolarization, as decoherence starts and increases, QFI starts at the level of pure W state, decreases smoothly and finally vanishes with full depolarization; ii) amplitude damping, as the decoherence starts, QFI drops to the shot noise limit, and with the increasing decoherence, QFI first vanishes and then starts to increase, reaching the shot-noise level at full decoherence; iii) phase damping: at any rate of decoherence, QFI vanishes. We also found that W states do not provide phase sensitivity in z direction and the phase sensitivities in x and y directions are equal to each other. Therefore on the contrary to GHZ states, QFI of W states do not exhibit sudden change points due to the competition between directions. Besides the decoherence effects, quantum Fisher information has also been studied considering photon losses [26–29]. On the other hand, an intense effort has been devoted to preparing large-scale photonic W states [30–38] and Dicke states [39] where environmental noise is generally not taken into account. Therefore we believe that our work may be useful for the efforts in preparing large scale W states, as well as the quantum critical phenomena and percolation in quantum networks [40, 41] when the unavoidable natural decoherence effects are taken into account.

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