

An Approach to Risk Quantification Based on Pseudo-Random Failure Rates

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Abstract: The risk quantification is one of the most critical areas in asset management (AM). The relevant information from the traditional models can be shown in risk matrices that represent a static picture of the risk levels and are according to its frequency and its impact (consequences). These models are used in a wide spectrum of knowledge domains. In this paper, we describe a quantitative model using the reliability and failure probability (as frequency in our risk model), and the preventive and corrective costs (as consequences in our risk model). The challenge here will be the treatment of reliability based on failure rate values with different random distributions (normal, triangular etc.) according to the available data. These possible values will enable the simulation of the behavior of the system in terms of reliability and, consequently, to use this information for making a risk based analysis. The traditional risk-cost-benefit models applied to maintenance usually provides an optimum for the time to apply a preventive task. But in this case, a time window is obtained showing minimum and maximum thresholds for the best time to apply the preventive maintenance task, together with other interesting statistics useful for the improvement of complex industrial asset management.

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1. INTRODUCTION

System reliability is usually modelled by using the mean time to repair (MTTR) registered in historical databases. This parameter is connected to the failure rate providing information related to the system probability to fail, $F(t)$, or not to fail, $R(t) = 1 - F(t)$, which are the reliability, both within a period of time. Failure databases generally provide information about the minimum, mean, maximum failure rates, as well as their standard deviation. These values depend mainly on the systems design and installation quality. Normally, the use of mean failure rates gives an insight into the physical asset behaviour, under controlled environments. The common definition of risk (associated with failure) is the probability that a failure will occur and the negative consequences of that failure. According to ISO 31000:2010, it is basically expressed as follows (i referred to event i):

$$Risk = \sum_{i=1} P_{f_i} * C_{f_i} \quad (1)$$

Where:

- R is the risk,
- P_{f_i} is the probability of failure
- C_{f_i} is the consequences of the unwanted event.

The objective of this study is to express risks in terms of maintenance costs (consequences) linked to parameter values given for the system reliability. In order to illustrate this goal,

an example is shown considering a Weibull distribution for modelling system reliability, and how considering different values for its failure rate (minimal, mean, maximal and pseudo-random), it is possible to analyse appropriately the subsequent risk, achieving a greater sensitivity of risk assessment in order to obtain relevant information about the potential costs to maintain the system at a specific time. In order to simplify the analysis, in this paper we consider an item from the Offshore Reliability Database (OREDA) with a specific failure mode. With the available data for failure rate and assuming specific costs for planned and unplanned maintenance, the result will aid in the decisions on preventive maintenance tasks. In other word, this methodology allows maintenance managers to better follow their risk appetite. With that purpose, this paper will start with a brief review of general risk indicators for maintenance and a proposed methodology for risk assessment. Then, with the support of a simple example, the study will approach the reliability uncertainty considering different alternatives for failure rate (with analytical and simulated values). The obtained results are shown and discussed in the following sections, providing different points of view for the analysis. Finally, the paper concludes with a summary of the main findings from the research.

2. RISK MANAGEMENT IN AM: RISK INDICATOR TO OPTIMIZE MAINTENANCE PERIODS

Risk management is one of the main aspects in the AM approach. ISO 55002:2013 introduces how the organizations

should determine the actions needed for addressing risks for its AM System. While addressing risks, the organization should determine the risk assessment criteria within the asset management decision making process. Given the contextual importance, of maintenance management in Asset management, it is interesting to present an example of risk-based maintenance decision making. According to Kaplan and Garrick, 1981, risk consists of three components; (1) the scenario, (2) the probability of the scenario and (3) the consequences of the scenario. They also suggest that one has to take all hazards into account and risk picture should be accomplished by summing up all possible scenarios with their consequences for a certain activity. Particularly for the calculation of probability, we refer to the failure occurrence and the reliability of the equipment, which depend directly on the parameters of life (MTTF) of its distribution function. The changes and evolution of life parameters impacts directly on the reliability and failure probability and, consequently, in the risk assumed for such a failure (Gonzalez-Prida et Al. 2014). The Risk Indicator (Ri) is applied in maintenance management processes with the objective of preserving the asset operation, maximizing operational performance and economic profitability. All this is achieved by applying the best maintenance strategies, inspections, and inventory control, in order to minimize the risks generated by different failure modes within the operational context (Woodhouse, 1993). Risk is a term which is probabilistic in nature and is commonly expressed in monetary units per time (e.g., EUR / year). In mathematical terms, the risk can be calculated from the following equation (Parra and Lopez, 2002):

$$Ri(xi) = F(xi) \cdot Co / xi \quad (2)$$

Where:

- xi: TTFi time to failure (hours, days, months, years, etc.)
- F(xi): probability of failure (%)
- Co: economic consequences of failure (in monetary units: Euros, etc.)

Therefore, this risk indicator integrates technical and economic factors, because, it combines failure probabilities (frequencies) with economic consequences (costs).

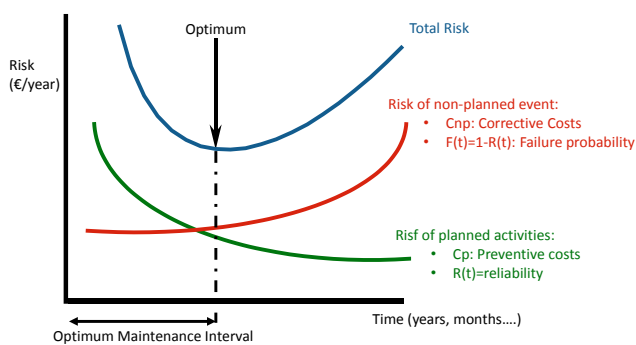


Fig. 1. Example of curves and the minimum expected cost per unit time.

The risk indicator quantifies the influence of both magnitudes (figure 1): failure probability and consequence of the failure, useful for maintenance optimization (Woodhouse, 1998). Risk indicator is useful to quantify the time for a preventive replacement at a lowest cost per unit of time (Campbell and Jardine, 2001) The mathematical expressions for calculating the time period that generates the minimum cost of a preventive maintenance replacement can be expressed as follows (Hastings, 2005):

$$Risk(t) = Cnp \cdot (F(t) / t) + Cp \cdot (R(t) / t) \quad (3)$$

Where:

- t: TTF time to failure (hours, days, months, years, etc.)
- Cnp: Corrective maintenance costs (or non-planned costs). It includes material, labour, lost profits, safety, environment, etc.
- F(t): probability of failure (%)
- Cp: Preventive maintenance costs (or planned costs). It includes materials, labour, lost prof-its, safety, environment, etc.
- R(t) = 1 – F(t): Reliability (%).

3. MODEL APPLICATION WITH ANALITICAL VALUES

3.1 Procedure

The value of failure rate (λ) is obtained in OREDA by an estimator, using data from multiple installations. Minimum and maximum values are also given with an uncertainty range of 90%. Considering this, assumptions are used in the calculations for different analysis in order to observe the system behaviour in reference to its reliability. In this case study, a Control and Safety Equipment, among the Fire & Gas Detectors has been selected with the following values from OREDA: (i) Lower Failure Rate: 1,32 (failures per million hours); (ii) Mean Failure Rate: 6,53 (failures per million hours); (iii) Upper Failure Rate: 15 (failures per million hours). The failure probability distribution for the example will be the Weibull distribution:

$$R(T) = \exp\left\{-\left(\frac{T}{MTTF}\right)^\beta\right\} \quad (4)$$

$$MTTF = \frac{1}{\lambda} \quad (5)$$

This case assumes Weibull distribution and equations refer to an exponential case (beta = 1 in Weibull). The scale parameter (MTTF) will be calculated applying the analytical values for failure rates given by OREDA. On the other hand, for the shape parameter (β) as well as for Corrective and Preventive maintenance costs (Cnp and Cp), specific values are given:

- Cp = 5000 EUR
- Cnp = 367200 EUR

- $\beta = 1,40$

The risk will be calculated bimonthly, till the 45th month. (Assumed end time). Following sub-sections provide charts together with the results.

3.2 Example with average failure rate

Applying the mean failure rates as provided by the OREDA database, the following results are obtained (Table 1):

Table 1: Results for risk with average failure rate

t (months)	R(t)	F(t)	Risk(t)
1	0,999449161	0,000551	5199,51
3	0,997438132	0,002562	1975,97
5	0,994769254	0,005231	1378,92
7	0,991635145	0,008365	1147,11
9	0,988128849	0,011871	1033,30
11	0,984308522	0,015691	971,22
13	0,980215159	0,019785	935,85
17	0,971327172	0,028673	905,02
19	0,966577552	0,033422	900,30
21	0,961648114	0,038352	899,57
23	0,956553651	0,043446	901,58
27	0,94592004	0,054080	910,66
29	0,940402724	0,059597	916,76
31	0,934764532	0,065235	923,49
33	0,929014001	0,070986	930,64
37	0,917206714	0,082793	945,61
39	0,911163944	0,088836	953,24
41	0,905036952	0,094963	960,87
43	0,898831619	0,101168	968,45
45	0,892553476	0,107447	975,94

From the table above, the minimal risk (899,57 EUR) with average failure rate occurs at the 21st month. That means that it is preferable to plan the replacement maintenance task at this specific moment (figure 2). The graph shows that the risk curve reaches a minimum, prior to increasing (very slowly in this case) indicating an increasing probability of failure. This behaviour may be considered by the decision-makers for scheduling the maintenance activities.

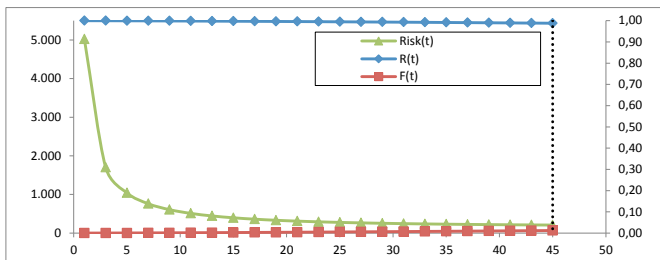


Fig. 2. Results for risk with average failure rate

3.3 Example with minimal and maximal failure rate

Applying the minimal and maximal failure rates as provided by the OREDA database, the following results are obtained (Table 2 and 3).

Table 2: Results for with minimal failure rate

t (months)	R(t)	F(t)	Risk(t)
1	0,999941243	0,000059	5021,28
3	0,999726486	0,000274	1699,69
5	0,999440879	0,000559	1040,50
7	0,999104611	0,000895	760,62
9	0,998727283	0,001273	606,78
11	0,998314796	0,001685	510,03
13	0,997871238	0,002129	443,93
17	0,99690241	0,003098	360,11
19	0,996381429	0,003619	332,14
21	0,9958383	0,004162	309,87
23	0,995274369	0,004726	291,81
27	0,994088585	0,005911	264,49
29	0,993468634	0,006531	253,99
31	0,992831734	0,007168	245,04
33	0,992178599	0,007821	237,36
37	0,990826149	0,009174	224,94
39	0,99012796	0,009872	219,89
41	0,989415804	0,010584	215,45
43	0,988690139	0,011310	211,54
45	0,987951389	0,012049	208,09

From the table 2, the minimal risk (208,09 EUR) with minimal failure rate occurs at the 45th month. That means that it is preferable to plan the replacement maintenance task in the 45th month (figure 3).

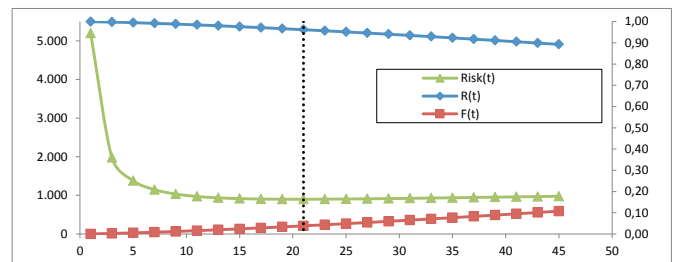


Fig. 3: Results for with minimal failure rate

Similarly for maximal failure rate (Table 3), the minimal risk (2066,18 EUR) with minimal failure rate occurs at the 9th month. That means that it is preferable to plan the replacement maintenance task in the 9th month (fig. 4).

Table 3: Results for risk with maximal failure rate

t (months)	R(t)	F(t)	Risk(t)
1	0,99823636	0,001764	5638,79
3	0,991815734	0,008184	2654,78
5	0,983338736	0,016661	2206,94
7	0,973447834	0,026552	2088,17
9	0,962463702	0,037536	2066,18
11	0,950593189	0,049407	2081,38
13	0,937986477	0,062014	2112,41
17	0,911010139	0,088990	2190,13
19	0,896815503	0,103184	2230,18
21	0,882245123	0,117755	2269,09
23	0,867358927	0,132641	2306,20
27	0,836845483	0,163155	2373,87
29	0,821308224	0,178692	2404,21
31	0,805636747	0,194363	2432,21
33	0,789866163	0,210134	2457,89
37	0,758153161	0,241847	2502,62
39	0,742266986	0,257733	2521,82

t (months)	R(t)	F(t)	Risk(t)
41	0,726394707	0,273605	2539,02
43	0,71055904	0,289441	2554,31
45	0,694780874	0,305219	2567,79

Basically, the time period associated with the value of minimum cost will be the period to implement the replacement activity (preventive maintenance at minimal cost).

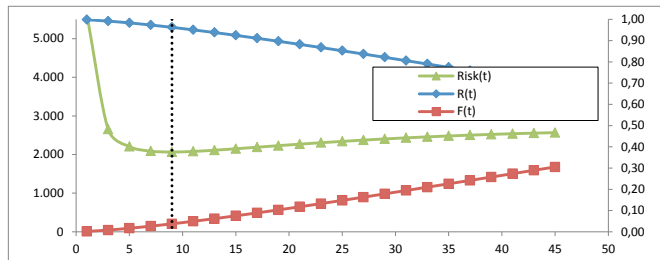


Fig. 4: Results for with maximal failure rate

A review of the three graphs provides a more accurate risk picture for the equipment maintenance. Now, more valuable considerations could be made than observing just one of the scenarios. For example, in principle, a time window for preventive maintenance can be set between 9 and 45 months. Refining this consideration, it could be argued that the period around month 20 would be an acceptable option, because: (i) On one hand, this is the minimum risk with mean λ ; (ii) on the other hand, the risk calculated for month 20 with a higher λ results a higher value for risk but the increment is not significant (just about 1% over the minimum risk at a higher λ). Additionally, it could be justified even a time-window between 20-25 months by a similar reasoning so that the maintenance could be schedules within these months.

4. MODEL APPLICATION WITH SIMULATED VALUES

4.1 Procedure

In the case of applying a pseudo-random failure rates, the analysis takes the data related to average failure rate and its standard deviation. With these two parameters, it is possible to calculate the inverse of a normal cumulative distribution. In this example, a probability associated to the normal distribution has been applied as a random number between 0.5 and 0.95. Applying just these formulas are not enough, as the possibility to obtain values under the minimal failure rates provided by OREDA. Therefore, the maximum value between the minimal failure rate provided by OREDA has been considered, and the result is obtained by the inverse of a normal accumulative distribution. For the average failure rates and the standard deviation, the values provided by OREDA are the following ones: (i) Mean Failure Rate: 6,53 (failures per million hours); (ii) Standard Deviation: 4,39 (failures per million hours)

4.2 Example with pseudo-random failure rates

Considering the mean failure rate and the standard deviation, also provided by OREDA, it is possible to obtain a pseudo-random value for λ from the inverse function of the cumulative normal distribution for the specified mean and standard deviation. In this function, the probability associated to the normal distribution is a random number between 5 and 95%, as commented before. With these values for λ and performing 50 simulations, results are obtained and shown in the value histograms and distribution functions for both minimal risk and time for minimal risk. Table 4 provides the results obtained for 50 simulations. In the Monte Carlo model, 50 simulations are performed where values of average failure rates (\pm standard deviation) are taken randomly, obtaining different values for minimal risk and for the time for minimal risk. These results are more clearly illustrated by histograms (figures 5 and 6).

Table 4: Results for risk with pseudo-random failure rate

#	Lambda	Minimal Risk:	t for minimal risk:
1	1,37	213,42	45,00
2	5,64	777,48	23,00
3	12,42	1710,36	11,00
4	4,10	564,85	33,00
5	3,57	491,61	37,00
6	9,77	1346,79	13,00
7	10,90	1501,71	13,00
8	7,64	1052,78	17,00
9	2,34	325,82	45,00
10	3,43	472,36	39,00
11	4,10	564,85	33,00
12	8,96	1234,09	15,00
13	8,22	1132,88	17,00
14	5,42	746,29	25,00
15	2,99	411,83	45,00
16	5,30	730,56	25,00
17	2,16	303,73	45,00
18	9,35	1289,04	15,00
19	4,96	682,70	27,00
20	4,72	650,12	29,00
21	12,42	1710,36	11,00
22	8,96	1234,09	15,00
23	5,30	730,56	25,00
24	3,71	510,50	37,00
25	1,32	208,09	45,00
26	4,35	599,64	31,00
27	4,72	650,12	29,00
28	3,14	432,53	43,00
29	3,71	510,50	37,00
30	8,96	1234,09	15,00
31	11,69	1610,34	11,00
32	4,84	666,51	27,00
33	11,47	1581,43	11,00
34	10,38	1430,34	13,00
35	4,35	599,64	31,00
36	8,83	1216,57	15,00
37	3,14	432,53	43,00
38	1,98	281,41	45,00
39	6,09	838,99	21,00
40	1,32	208,09	45,00
41	5,87	808,19	23,00
42	6,64	914,92	21,00
43	1,32	208,09	45,00
44	6,64	914,92	21,00
45	5,42	746,29	25,00
46	10,22	1408,38	13,00

#	Lambda	Minimal Risk:	t for minimal risk:
47	5,98	823,60	23,00
48	7,30	1006,38	19,00
49	4,96	682,70	27,00
50	7,08	975,45	19,00

In the graphic below, x-axis shows possible values for minimal risk (Min: 208,09 EUR/year; Max: 1710,36 EUR/year).

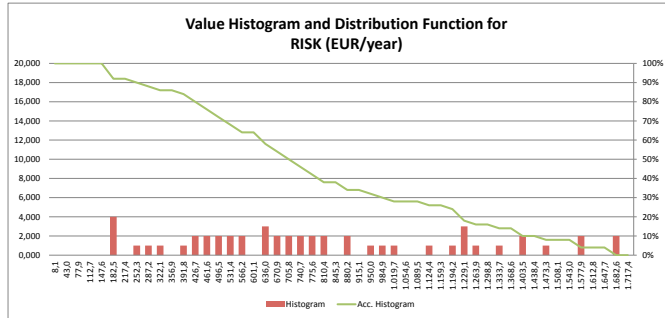


Fig. 5: Histogram and Distribution Function for min. risk

Y-axis at the left shows number of simulations. Thus, each red bar corresponds to the amount of simulations (left-y-axis) that resulted to a specific value for minimal risk (x-axis). At the right, y-axis shows the percentage of simulation results, which are over a specific value of minimal risk (x-axis). This is represented by the green line. Thus, for instance: (i) The 100 % of simulations provides results upper than 208,09 EUR/year for preventive maintenance at minimal cost; (ii) The 0 % of simulations provides results upper than 1710,36 EUR/year for preventive maintenance at minimal cost. Similarly, the results obtained after 50 Monte Carlo simulations for time period to implement the replacement activity at a minimal cost (time for minimal risk), is clearly illustrated in the next histogram (figure 6). In the graph above, x-axis shows the possible values for maintenance periods at a minimal cost (Min: 11 months; Max: 45 months). On the other hand, y-axis at the left shows number of simulations. Thus, each red bar corresponds to the amount of simulations (left-y-axis) that results to a specific value for maintenance periods at a minimal cost (x-axis). At the right, y-axis shows the percentage of simulation results, which are over a specific value of minimal risk (x-axis). This is represented by the green line.

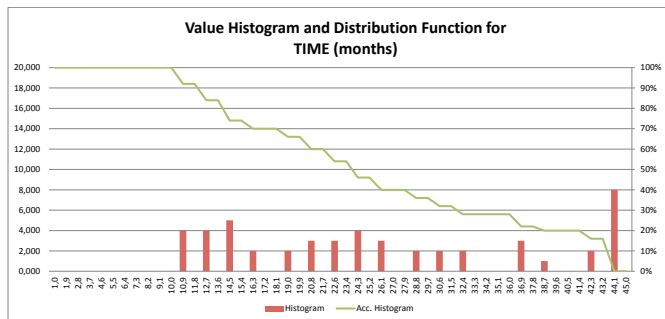


Fig. 6: Histogram and Distribution Function for time for minimal risk (months)

Thus, for instance: (i) The 100 % of simulations provides results higher than 11 months for periods of preventive maintenance at minimal cost; (ii) the 0 % of simulations provides results higher than 45 months for periods of preventive maintenance at minimal cost.

5. RESULTS AND DISCUSSIONS

The process of decision-making within the maintenance management must preferably integrate technical indicators of reliability together with economic information. In addition to this, the appropriate use of available data together with statistics and simulation tools may provide more valid forecasts. For instance, in the presented example, the following results are obtained using just analytical values (Table 5):

Table 5: Results using analytical values

	Minimal	Average	Maximal
Lambda (Failures/10⁶ hours)	1,32	6,53	15
Minimal Risk (EUR/year)	208,09	899,57	2066,18
t for minimal risk (months)	45	21	9

Nevertheless, the aim of this paper is to propose a procedure which helps to increase relevant information which is useful to take decisions according to the risk appetite of the company or the maintenance manager. Applying pseudo-random failure rates, relevant information about the preventive maintenance at a minimal cost can be obtained as shown in table 6:

Table 6: Results using pseudo-random values

	Minimal Risk: (EUR/year)	t for minimal risk: (months)
Average:	827,570465	26,76
Standard Deviation:	426,742419	11,680369
Most Likely	1234,09306	45

The use of histograms provides an interesting array of information such as the occurrence probability of assuming a specific value for minimal risk and time for that minimal risk (y-axis at the right in figures 5 and 6). In addition to this, the decision of when to apply a replacement activity can be taken being aware of the risk (in terms of cost). Comparing it with the results from the analytical values, a risk underestimation can be observed by using random λ . It occurs in both cases: in the months of minimal risk (26.76 on average, compared to month 20 for mean analytical λ), as well as lower risk value (1234 €/year vs. 2066 €/year). According to data from random λ , a maintenance policy could be justified considering preventive tasks between months 27-30, which would be an advantage of 6-10 months against the schedule estimated by analytical values. This paper does not consider a model free simulation to quantify the risk. The problem may be addressed, not just by sensitivity analyses and Monte Carlo simulations based on reliability databases but also when condition monitoring data can inform the estimation of reliability and remaining useful life. Although the methodology here is focused on single component/asset

maintenance, a key challenge is planning maintenance at system level and addressing groups of components. As commented, the process of decision-making within the maintenance management must integrate technical indicators of reliability together with maintenance cost which include the consequences of failure events. This consideration will enable organizations to maximize the profitability of its assets at a level of optimal reliability and safety. Other interesting aspects to be taken into account by organizations when designing technical and economic indicators are depicted by Nachlas, 1995.

6. CONCLUSIONS

This paper suggests a methodology to better decide the scheduling of a replacement activity, considering a minimization in maintenance costs for an assumed system. The proposed method provides to the reader an easy view about the effect over the system maintenance costs. The exercise considers values for failure rate. Frequently, failure rates are considered as a constant during the life cycle of the system in order to simplify calculations. Nevertheless, pseudo-random values may provide relevant information of the system without increasing the complexity of the analysis. The study presented the use of well-known simulation tools, whose results may substantially help the decision-making on aspects of the maintenance policies. Moreover, achieving a good level of maintenance, especially in groups of critical assets, requires an appropriate analysis for the prioritization in the allocation of resources. Therefore, a method for convenient and practical risk comparison becomes an important tool for the success of the maintenance function and, in some cases, its complement methodologies for auditing the resources allocation of critical maintenance activities. To conclude, this type of tool is also needed when the organization may change conditions modifying the values for $R(t)$ and, consequently the need for a replacement activity.

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