IV. Fabrication and Experimental Results

Fig. 7 shows the layer configuration of the fabricated chip-type balun. The designed chip-type baluns were fabricated with a multilayer configuration using ceramic sheets, which has a dielectric constant of 6.0 and a thickness of 60 μm, and Ag metal pattern. The thickness of the metal layer was kept at 30 μm. The physical overall size are 3.2 mm $\times$ 1.6 mm $\times$ 1.1 mm and 2.0 mm $\times$ 1.2 mm $\times$ 1.1 mm, which are sizes 3216 and 2012, respectively. There must be parasitic couplings between the fabricated inductor and capacitor. In order to resolve this parasitic coupling effects, each inductor and capacitor are separated using the ceramic isolation layers which are 100-μm thick.

Figs. 8 and 9 show the experimental results of the fabricated 3216 chip balun with 50 Ω : 50 Ω unbalanced ports termination and the fabricated 2012 chip balun with 50 Ω : 200 Ω unbalanced ports termination. The insertion and return losses are less than -1.3 and -10 dB over the operating frequency band, respectively. The measured phase characteristics between balance ports of the 3216 chip balun show the difference of 180° with $\pm$3% tolerance over the operating frequency range.

There must be some deviations of characteristics, such as the operation frequency, in fabricated baluns due to the multilayer process tolerance. In order to show the variation of the frequency characteristics for the designed chip balun dependent on the lumped-element value, three kinds of chip baluns, which have different capacitance, were designed and fabricated. Table I and Fig. 10 show the experimental results for the three kinds of chip baluns with different capacitance. Decreasing the capacitance increases the resonance frequency of the parallel *LC* circuit and the impedance. Thus, it increases the operating frequency.

V. Conclusions

The design method and equivalent circuit of the chip-type balun have been proposed in this paper to provide a simple design procedure, compact size, and excellent performance. Two kinds of chip-type baluns were designed based on the proposed equivalent model with the equivalent-circuit representation of the quarter-wavelength transformer and fabricated with a multilayer ceramic process. The proposed design method for chip-type balun is entirely based on a lumped-element equivalent circuit. Thus, the derived equivalent circuit could provide a quite simple and accurate design equation. Furthermore, the variation of the operating frequency with the changing of the lumped-element value has been discussed in order to investigate the deviation of the frequency characteristic in a fabricated chip-type balun. The presented equivalent-circuit model and design method of the multilayer chip balun can offer smaller efforts in designing multilayer chip baluns.

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A Suitable Integral Equation for the Quasi-TEM Analysis of Hybrid Strip/Slot-Like Structures

Jesús Martel and Francisco Medina

***Abstract*—This paper reports on a suitable formulation of the spectral-domain/integral-equation method for the quasi-TEM analysis of hybrid strip/slot-like planar lines. The free surface charge distribution is used as an unknown on the strip-like interface, whereas the electric field is used on the slot-like region. This formulation allows us to reduce the number of basis functions and makes possible a unified treatment of the problem. A single type of basis functions is used, leading to a quasi-analytical evaluation of the Galerkin matrix entries. The performance of the method is illustrated with a practical example structure useful for coupler design.**

***Index Terms*—Galerkin method, quasi-TEM analysis, strip/slot-like structures.**

I. Introduction

Planar multiconductor lines are widely used in microwave and millimeter-wave circuits. Although full-wave methods are available for the analysis of this type of transmission structures, quasi-static approaches are still useful for computer-aided design (CAD) purposes because they yield very fast codes still accurate enough for preliminary designs. Most of these transmission systems can be classified either as strip- or slot-like. The former can be described as a number of coupled conducting strips embedded in a layered medium, whereas the latter are best described as a number of slots practiced in a grounded metallization. The quasi-static analysis of structures of both categories has been carried out by means of a variety of techniques. In particular, integral-equation formulations (both in the spectral and spatial domain) making use of suitable basis functions and quasi-analytical techniques to evaluate numerical series or integrals have proven to be both fast and versatile in the analysis of strip-like [1]–[3] and slot-like [4], [5] structures. For strip-like structures, the integral equation should be formulated for the free surface charge distribution on the strips, while for slot-like ones, the problem is better posed in terms of the slots electric field. The per unit length (p.u.l.) capacitance matrix $[C]$ or its inverse $[P]$ are directly obtained from the solution of the above-mentioned integral equations. However, more complicated geometries in-

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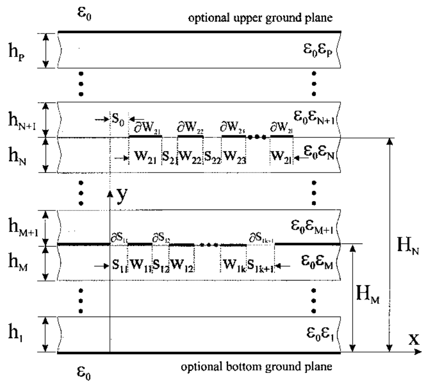


Fig. 1. Hybrid strip/slot-like structure in a P-layers iso/anisotropic dielectric medium.

volving slotted ground planes, floating conductors, etc. have been proposed to improve the performance of circuit components such as couplers, T-junctions, and baluns [6]–[8]. These structures can be considered neither strip-like, nor slot-like, but rather hybrid configurations. They are particular cases of the structure shown in Fig. 1. A slot-like geometry at the plane $y = H_M$ can be seen, whereas a strip-like configuration is found at $y = H_N$. The quasi-TEM analysis of the structure in Fig. 1 can be carried out in terms of the free surface charge or the electric-field integral equations, but numerical efficiency degrades when a surface charge integral equation is formulated for the slotted region or an electric-field integral equation is used for the strips region. This paper shows how to formulate the problem in such a way that the unknown quantities for the integral equations are defined on the bounded domains (strips for the strip-like interface and slots for the slot-like interface). Proceeding in this way, the strip- and slot-like interfaces are treated in a unified manner. In particular, the basis functions for the slot electric field and the strip surface charge density are the same, and the methods employed so as to accelerate spectral integral computations are common (we use techniques close to the ones reported in [4] and [5]). The extraction of the capacitance coefficients from the solution of the proposed integral equation is a simple task. The final product is a very fast and accurate computer code for the quasi-static analysis of the structure in Fig. 1, which is suitable for CAD applications. Some interesting examples are included in Section III.

II. Statement of the Problem

The goal of this paper is to obtain the p.u.l. capacitance matrix $[C]$ of the planar transmission system in Fig. 1. This implies to solve for an electrostatic two-dimensional (2-D) problem in the cross-sectional domain depicted in that figure. This is a Laplace's mixed boundary conditions problem. From previous experience with strip- and slot-like problems, it seems to be clear that our mixed problem would be more conveniently posed (from a numerical/analytical point-of-view) in terms of an integral equation involving as unknown functions the slots electric field at $y = H_M$, $E_x(x', H_M)$, and the free surface charge distribution on $y = H_N$, $\sigma(x', H_N)$. Defining $Q(x, H_M)$ as the amount of charge allocated in the interval $[0, x]$ at $y = H_M$ and $\phi(x, H_N)$ as the electrostatic potential along the plane $y = H_N$ (see Fig. 1), a pair of coupled integral equations for $E_x(x', H_M)$ and $\sigma(x', H_N)$ can be written as follows:

$$Q(x,H_M) = \int_{y=H_M} M_{11}(x, x')E_x(x',H_M)\,dx' + \int_{y=H_N} M_{12}(x, x')\sigma(x', H_N)\,dx' \tag{1}$$

$$\phi(x, H_N) = \int_{y=H_M} M_{21}(x, x')E_x(x', H_M)\,dx' + \int_{y=H_N} M_{22}(x, x')\sigma(x', H_N)\,dx' \tag{2}$$

with $Q(x \in \partial S_{1,j+1}, y = H_M) - Q(x \in \partial S_{1,j}, y = H_M) = Q_{1j}$ $(j = 1, \cdots, k$, k is the number of conducting strips at $y = H_M)$ and $\phi(x \in \partial W_{2,j}, y = H_N) = V_{2j}$ $(j = 1, \ldots, l$, l is the number of strips at $y = H_N)$. Q_{1j} is the charge supported by the jth strip at $y = H_M$ and V_{2j} is the voltage imposed on the jth strip at $y = H_N$.

Equations (1) and (2) can be written in the spectral domain where the kernel $M_{ij}(x, x')$ is known in closed form as follows:

$$\tilde{Q}(\alpha, H_M) = \tilde{M}_{11}(\alpha)\tilde{E}_x(\alpha, H_M) + \tilde{M}_{12}(\alpha)\tilde{\sigma}(\alpha, H_N) \tag{3}$$

$$\tilde{\phi}(\alpha, H_N) = \tilde{M}_{21}(\alpha)\tilde{E}_x(\alpha, H_M) + \tilde{M}_{22}(\alpha)\tilde{\sigma}(\alpha, H_N) \tag{4}$$

where $(\tilde{\ })$ stands for Fourier transform and α is the spectral variable.

The elements $\tilde{M}_{ij}(\alpha)$ $(i, j = 1, 2)$ in (3) and (4) can be easily computed in terms of the elements of the well-known spectral Green's function for the electrostatic potential $\tilde{G}_{ij}(\alpha)$ $(i, j = 1, 2)$. By using, for instance, the method and notation in [9]

$$\tilde{M}_{11}(\alpha) = \frac{1}{\alpha^2 \tilde{G}_{11}(\alpha)} \tag{5}$$

$$\tilde{M}_{12}(\alpha) = \frac{j\,\tilde{G}_{12}(\alpha)}{\alpha\,\tilde{G}_{11}(\alpha)} \tag{6}$$

$$\tilde{M}_{21}(\alpha) = \frac{j\,\tilde{G}_{21}(\alpha)}{\alpha\,\tilde{G}_{11}(\alpha)} \tag{7}$$

$$\tilde{M}_{22}(\alpha) = \tilde{G}_{22}(\alpha) - \frac{\tilde{G}_{12}\,\tilde{G}_{21}(\alpha)}{\tilde{G}_{11}(\alpha)}. \tag{8}$$

Thus, $\tilde{G}_{ij}(\alpha)$ are first calculated using [9] and then (5)–(8) are applied so as to get $\tilde{M}_{ij}(\alpha)$ $(i, j = 1, 2)$. Since the formulation in [9] includes anisotropic dielectric materials, the method here accounts for such possibility.

Integral equations (1) and (2) have to be solved for a set of N_c independent excitations (N_c is the number of conducting strips) so as to compute all the elements of $[C]$. This task has been carried out by using the Galerkin method with a proper set of basis functions to approximate the slots electric field at $y = H_M$ and the strips free charge density at $y = H_N$. These functions are Chebyshev polynomials weighed by the Maxwell edge condition. The application of this method yields a system of linear equations whose coefficients are quasi-analytically computed by using techniques similar to the ones in [4] and [5]. These techniques exploit the behavior of the functions $\tilde{M}_{ij}(\alpha)$ in (5)–(8) and the properties of the basis functions. The independent terms of the equations system depend on the particular excitation. Note that, in the case of strip-like systems, the computed charge on the strips directly provides a row of $[C]$ if one of the strips is set to voltage unity, while the rest of the strips are grounded. The elements of the matrix $[P] = [C]^{-1}$ are directly obtained for a slot-like system as the strip voltages provided one of the strips is charged to unity, the remaining being uncharged. However, in our hybrid situation, we have to combine both charge and voltage canonical excitations depending on the location of the excited strip (i.e., we distinguish strips located at the slot-like interface from those located at the strip-like interface). In order to obtain $[C]$ from the solution of (1) and (2) under these canonical excitations, let us define the following column vectors: $\overline{V}_1 = (V_{11} \quad V_{12} \quad \cdots \quad V_{1k})$ is a column vector whose elements are the voltages of the k strips located at the slots interface $(y = H_M)$; the elements of $\overline{V}_2 = (V_{21} \quad V_{22} \quad \cdots \quad V_{2l})$ are the voltages of the l strips at $y = H_N$ (note that $N_c = l + k$). Analogously, we can define the charge vectors $\overline{Q}_1 = (\, Q_{11} \quad Q_{12} \quad \cdots \quad Q_{1k} \,)$ and $\overline{Q}_2 = (\, Q_{21} \quad Q_{22} \quad \cdots \quad Q_{2l} \,)$. The matrix $[C]$ can be split into four sub-matrices $[c]_{ij}$ $(i,\, j = 1,\, 2)$ in such a way that

$$\begin{pmatrix} \overline{Q}_1 \\ \overline{Q}_2 \end{pmatrix} = \begin{pmatrix} [c]_{11} & [c]_{12} \\ [c]_{21} & [c]_{22} \end{pmatrix} \begin{pmatrix} \overline{V}_1 \\ \overline{V}_2 \end{pmatrix}. \tag{9}$$

In the context of our method, what we directly compute from the solution of the Galerkin system of equations for the N_c canonical excitations is a hybrid matrix $[T]$ relating charges and voltages as follows:

$$\begin{pmatrix} \overline{V}_1 \\ \overline{Q}_2 \end{pmatrix} = [T] \begin{pmatrix} \overline{Q}_1 \\ \overline{V}_2 \end{pmatrix} = \begin{pmatrix} [t]_{11} & [t]_{12} \\ [t]_{21} & [t]_{22} \end{pmatrix} \begin{pmatrix} \overline{Q}_1 \\ \overline{V}_2 \end{pmatrix}. \tag{10}$$

Simple algebraic manipulations of (9) and (10) allow us to get the matrix $[C]$ from matrix $[T]$ as follows:

$$[c]_{11} = [t]_{11}^{-1} \tag{11}$$

$$[c]_{12} = [t]_{11}^{-1}[t]_{12} \tag{12}$$

$$[c]_{21} = [t]_{21}[t]_{11}^{-1} \tag{13}$$

$$[c]_{22} = [t]_{21}[t]_{11}^{-1}[t]_{12} + [t]_{22}. \tag{14}$$

In this way, a numerically efficient and accurate code is obtained for the quasi-TEM analysis of the structure in Fig. 1.

III. Results

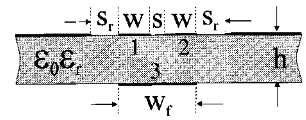
The numerical code developed on the basis of the theory in this paper has been exhaustively checked by means of convergence tests. Table I summarizes the results of one such test for a practical hybrid structure useful in directional-coupler design [8]. The cross section of the structure is shown on the top of the table: a coplanar-waveguide-coupled section with a floating conductor intended for increasing coupling. This is a good example of the type of structures considered in this paper. The four distinct elements of the computed p.u.l. capacitance matrix are shown as a function of the number of basis functions on the wide slot (N_{s_r}), the narrow slot (N_s) and the floating strip (N_{w_f}). The number of functions is increased systematically until five figures of C_{ij} remain unchanged. The table shows a very fast convergence pattern. Numerical errors and unstabilities are avoided thanks to the use of special asymptotic techniques to compute the Galerkin matrix entries. In this way, if the number of basis functions is increased, the numerical results reported in Table I do not change at all. This gives us high confidence in the numerical code. As was expected, the number of functions required to approximate the charge density or the electric field is larger for larger domains. In case a surface charge integral equation were used for both metallized interfaces, a large number of functions would have to be employed on the ground plane. In fact, the results appearing in the last row in Table I have been computed using that formulation such as implemented in [10]. Note that the agreement with the results in this paper is very good. However, much more CPU time was required by the code based on [10], which was designed for strip-like structures. Of course, a large number of functions would have been required to approximate the electric field along the floating conductor plane. Therefore, our hybrid integral equation is responsible for an important enhancement of the numerical efficiency when compared with strip-like or slot-like oriented codes. Since CPU time involved in the analysis of this structure is very small, our computer program could be used for the CAD of this device.

It can be seen in Table I that, if the number of basis functions is too low, nonphysical results are obtained: the computed C_{12} parameter is positive until at least two basis functions are used on the floating conductor. This is because a single basis function does not account for the asymmetric distribution of the charge on this conductor when one of the coplanar strips is excited and the other are grounded.

It was stated that the addition of the floating conductor to the coupled CPW structure increases coupling factor. This fact is illustrated

TABLE I

Convergence of the Elements of $[C]/\epsilon_0$ with the Number of Basis Functions for a CPW Directional Coupler with a Floating Conductor. Geometry Data: $h = 0.61$ mm, $s = 0.2$ mm, $s_r = 1.0$ mm, $w = 1.62$ mm, $w_f = 2w + s = 3.44$ mm, ϵ_r=10



N_s	N_{s_r}	N_{w_f}	C_{11}/ϵ_0	C_{12}/ϵ_0	C_{13}/ϵ_0	C_{33}/ϵ_0
1	1	1	28.037	2.7200	-28.462	64.265
2	1	1	27.985	2.6683	-28.353	64.035
3	1	1	27.985	2.6683	-28.353	64.035
3	2	1	27.392	2.0900	-27.066	61.205
3	3	1	27.354	2.0052	-26.962	60.909
3	4	1	27.353	2.0541	-26.959	60.903
3	5	1	27.353	2.0541	-26.959	60.903
3	5	2	33.888	-4.4806	-26.959	60.903
3	5	3	35.351	-3.0181	-29.289	64.612
3	5	4	37.516	-5.1828	-29.289	64.612
3	5	5	37.526	-5.1724	-29.296	64.617
3	5	6	37.665	-5.3113	-29.296	64.617
3	5	7	37.665	-5.3113	-29.296	64.617
method in [10]			37.662	-5.3137	-29.291	64.605

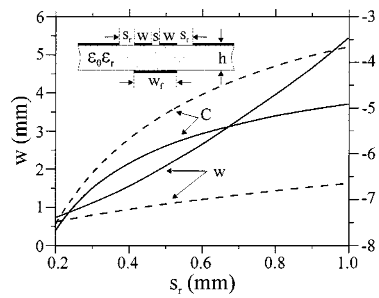


Fig. 2. Strip width (w) for a 50-Ω CPW directional coupler with (dashed line: $w_f = 2w + s$) and without (solid line: $w_f = 0$) a floating strip, and the corresponding coupling factor (C) versus the lateral slot width (s_r). The substrate and strips separation (s) are as in Table I.

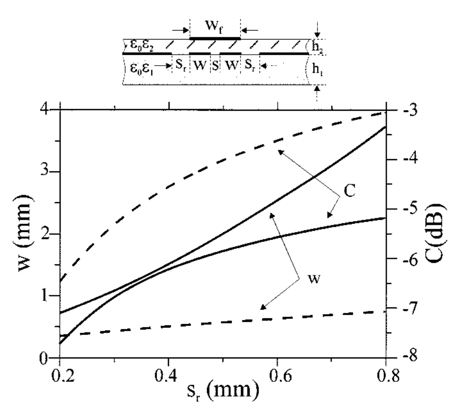


Fig. 3. Same curves as in Fig. 2, but for a structure with a cover substrate (see top figure). Dimensions: $h_1 = 0.61$ mm, $h_2 = 0.135$ mm, $s = 0.2$ mm, $w_f = 0$ (solid lines) or $w_f = 2w + s$ (dashed lines). Dielectric constants: $\epsilon_1 = 10$, $\epsilon_2 = 2.53$.

in Fig. 2. This figure compares the results obtained for a CPW directional coupler section with and without the floating strip. The separation distance between the signal strips has been set to a physically realizable value for hybrid microwave integrated circuit (MIC) technology ($s = 0.2$ mm). For practical design purposes, we have calculated and plotted the values of w yielding $Z_0 = \sqrt{Z_e Z_o} = 50\ \Omega$ (Z_e and Z_o are, respectively, the even- and odd-mode characteristic impedances of the coupled system). The coupling factor C is also plotted: solid lines stand for the case in which no floating conductor exists and dashed lines stand for the case in which the width of the floating conductor is set to $w_f = 2w + s$ (larger values of w_f do not lead to significant increasing of C [8]). It is clearly seen from Fig. 2 that the use of the floating conductor presents two advantages: first, as expected, the coupling is meaningfully enhanced; second, the values of w for each s_r are much smaller when the floating strip is present, thus reducing the lateral size of the circuit. With the substrate and geometry used in this example, it is not feasible to design a 3-dB coupler with practical dimensions. To overcome this problem, instead of printing the floating conductor on the opposite side of the substrate, we can add a thin dielectric layer and print the floating strip on it (see structure in Fig. 3). As is shown in this figure, coupling factors from 6 to 3 dB can be easily achieved for acceptable values of s_r and w.

IV. Conclusion

This paper has shown how to deal with the quasi-static spectral-domain analysis of hybrid strip/slot-like structures in an efficient way. The use of a hybrid surface charge/electric-field integral equation makes possible an unified treatment of both interfaces of the mixed boundary conditions problem. Previous experience in the efficient solution of strip- and slot-like structures (basis functions, asymptotic acceleration techniques, and etc.) can be directly applied to our problem. The performance of the method has been demonstrated using, as an example, a coplanar-waveguide coupled section with a floating strip intended for increasing coupling and reducing size.

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