A Band-Pass/Stop Filter Made of SRRs and C-SRRs

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***Abstract*—Frequency Selective Surfaces (FSS) are usually classified into two big groups depending on wether they are made of unconnected elements or connected elements. Close to their resonance frequency, the first type behaves like a band-stop filter while the second type like a band-pass filter. In this paper we propose a new type of surface made of Split Ring Resonators (SRRs) and, at the same time, Complementary Split Ring Resonators (CSRRs), placed in such a way that the surface is self-complementary. The main result is that this FSS shows band-stop features for one linear polarization state and band-pass features for the orthogonal polarization. Therefore, it is in the middle between the two usual groups of FSS, what could drive us to new designs of band-pass/stop filters which are are easily switchable from band-pass to band-stop, and viceversa, by simply rotating the surface through 90 degrees.**

I. Introduction

Frequency Selective Surfaces (FSS) have been developed since many years ago (50's). Classical books on this topic were written by T. K. Wu [1] and B. A. Munk [2]. Both books agree in classifying FSSs into two big groups: those made of unconnected elements (unconnected pieces of metal), which show band-stop filter features, and those formed by connected elements (or unconnected slots), which show band-pass filter features. In the last decade this topic has received new breaths comming from the new concepts of metamaterials. In 1999 John Pendry proposed the Split Ring Resonator (SRR)[3] to get resonant magnetic properties at high frequencies without using magnetic materials. Soon after, the SRR was used to design the first bulk left-handed medium by David Smith's group [4]. Instead of bulk metamaterials, here we will focuse our attention into the possible use of the SRR and similar particles in the design of metasurfaces. One of the first atempts was developed by Falcone et al. in Ref. [5] where also the Complementary Split Ring Resonator (CSRR) was proposed. They demonstrated that a periodic screen formed by SRRs acts like a band-stop filter for certain linearly polarized plane wave, while the screen of CSRRs acts like a band pass filter for the orthogonal polarization. One of the advanteges is that the electrical size of the unit cell is considerably small (without the need of a substrate of high dielectric constant) so that grating lobes are avoided. After, surface admitance models of these two screens were proposed in [6] and the off-normal incidence was studied in [7]. Recently, an interesting self-complementary sub-wavelenght hole arrays has been proposed by Beruete et al. [8] in order to design a polarizer. However, in that work the unit cell size was similar to the periodicity so that grating lobes could make the surface not suitable for many typical applications of FSS.

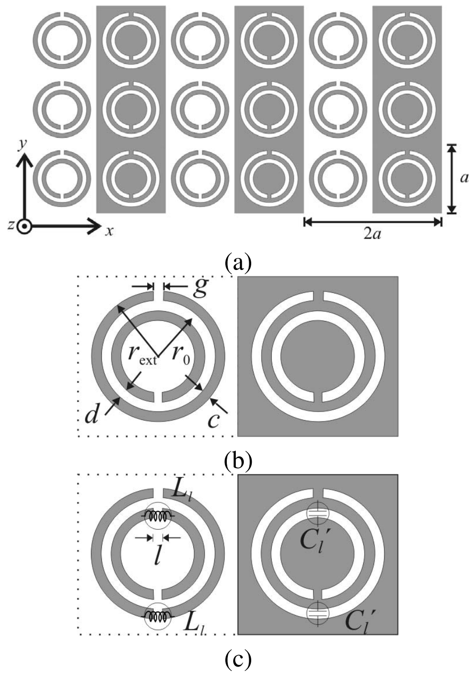

Fig. 1. The studied self-complementary metasurface (a), the unloaded unit cell (b), and the loaded unit cell (c). The geometrical parameters are: $a = 8$ mm, $r_{ext} = 3.5$ mm, $r_0 = 2.9$ mm, and $c = d = g = l = 0.4$ mm. L_l and C_l' of (c) are lump circuit elements used to push down the resonance frequency of the original particles (b).

In this paper we propose a new kind of FSS, based on a self-complementary metasurface made of SRRs and CSRRs (see Fig. 1(a)), which behaves like a band-pass filter for certain linear polarization and band-stop filter for the ortogonal polarization.

II. Theory

Fig. 1 shows the self-complementary metasurface under study, made of SRRs and CSRRs. In what follows we will always consider perfect conductors of infinitesimal thickness and no dielectric substrates (screen are hold in air), so that the duality principle and the Babinet's principle are strictly valid. For the lowest resonant frequency, and when the SRR size is much smaller than the resonant wavelenght, it can be modeled like an LC circuit [3]. Strong currents can be excited if a time varying magnetic field is applied orthogonally to the SRR. In some average sense, the currents over the two rings form a closed loop of currents which have an associated self-inductance L. The splits force some accumulation of charge, so that a capacitance C must be included into the circuit model. The small loop of current could be replaced by a magnetic dipole. A more accurate model was presented in [9], where analytical formulas of L and C were obtained and the magnetoelectric coupling effect was pointed out. This last effect means that the SRR is not purely a magnetic resonators, but that it has also an associated electric dipole and, reciprocally, can be excited by tangential electric field directed along the x-axis. This is a key point because it allows the resonant response of a surface of SRRs under normal incidence although the magnetic flux is zero. The bahavior of the CSRR can be infered by duality from the SRR (see the papers [5], [6], [7]).

However, Fig. 1(c) is showing a modification of the original resonators, which now appears simetrically loaded with lump inductors (L_l) for SRR and lump capacitors (C_l') for CSRR. They are just introduced in order to push down the resonant frequency of the original particles, which will be a key point for this paper as explained below. These lump elements are rounded by circles to stress the fact that they are blinded to any external field so that they do not affect the mechanism of excitation of the resonator. Introducing the lump elements implyies that the self-complementaryness is not perfect, but it will be clear below that this perturbation is not relevant, except for certain slight shift in frequency. The equivalent circuit models of single resonators are shown in Fig. 2. Since the magnetic resonator is the complementary counterpart of the electric resonator, their circuit models must be dual one each other. The rules for passing from Fig. 2(left) to Fig. 2(right) are very simple: change series connections to parallel connections, and interchange inductances and capacitances. In passing from an inductance to its dual capacitance we have to include a factor $4\epsilon_0/\mu_0$ [10], being the factor ϵ_0/μ_0 to correct the units and the factor 4 to take into account the different symmetry properties of the scattered fields (scattered electric field is even respect to the surface while scattered magnetic field is odd). Althought the duality is only applied to the circuit elements corresponding to the printed strips, we arbitrarily force the lump elements L_l and C_l' to follow the same rules. Then, by duality, both types of resonators will resonate at the same frequency.

Let us now imagine a linearly polarized plane wave normally impinging on the surface of Fig. 1(a). If its frequency is far from the resonant frequency of a single resonator, then the wave will mainly see a parallel strip grating without the effects of the resonators. For low frequencies, it should reject the wave when it is polarized with the **E** field parallel to the strips (the baseline for band-pass filters) while it allows the wave to go

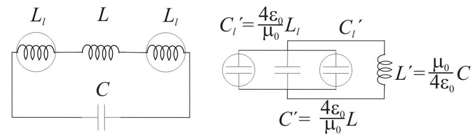

Fig. 2. Circuit models for a single SRR (left side) and a single CSRR (right side). The SRR circuit parameters are: $L = 13.0$ nH, $C = 6.31 \times 10^{-2}$ pF, and $L_l = 35.5$ nH. Duality relations gives the following CSRR circuit parameters: $C' = 0.365$ pF, $L' = 2.24$ nH, and $C_l' = 1$pF.

through when **E** is orthogonal to the strips (the baseline for band-stop filters). Resonators play an important role just when the frequency approach their resonant frequency. Then, for **E** along the y-axis (or y-polarized wave) the SRR is not excited, while the CSRR is excited by B_x. Based in our previous experience of Refs. [5], [6], [7], we expect that close to the resonant frequency the transmission coefficient should be 1. For the case of an x-polarized incident wave, the electric resonators will be excited by E_x while the magnetic resonators will not be excited, so that the scattered field will be important and the wave will be completelly rejected at some frequency close to the same resonant frequency. Therefore, the structure would behave as a band-pass filter for y-polarized waves and band-stop filter for x-polarized waves. Thus, it makes sense to use a new terminology *band-pass/stop* filter, because it can filter the wave in a double way: as band-pass or band-stop depending on the polarization state.

III. Numerical simulations

A. The metasurface with unloaded resonators

In order to demostrate the properties of the self-complementary metasurface shown in Fig. 1(a), we have numerically simulated the normal incidence of plane waves. The corresponding results are shown in Fig. 3(top). There exist a dip of total rejection for x-polarized waves at 6 GHz (solid line), while at the same frequency a peak of total transmission appears for the y-polarized waves (dashed line). However, the bands are very unsymmetric due to the rapid variation of the baselines because the wavelength is not much smaller than the periodicity.

It is worth to note that both sub-arrays – the structure with only SRRs or CSRRs – are independent. In Fig. 3(middle) only the SRRs are presents and thus only the stopband for x-polarization can be observed (solid line), while for y-polarization the resonance dissapears (dashed line). On the other hand, it is shown in Fig. 3(bottom) that the CSRRs sub-array losses the stopband for x-polarization (solid line) while keeps the passband for y-polarization (dashed line). In fact, this independency is also demonstrated by Fig. 4, where it is shown that for the case of x-polarization relevant currents are only excited over the SRR, while for y-polarization only the CSRR are strongly excited. It is also worth to note that the diagram of currents corresponds to the LC circuit model of [9]. Following the formulas therein we obtained the values of $L = 13.0$ nH and $C = 6.31 \times 10^{-2}$ pF which carry to a resonant frequency of 5.57 GHz. This frequency, which is for

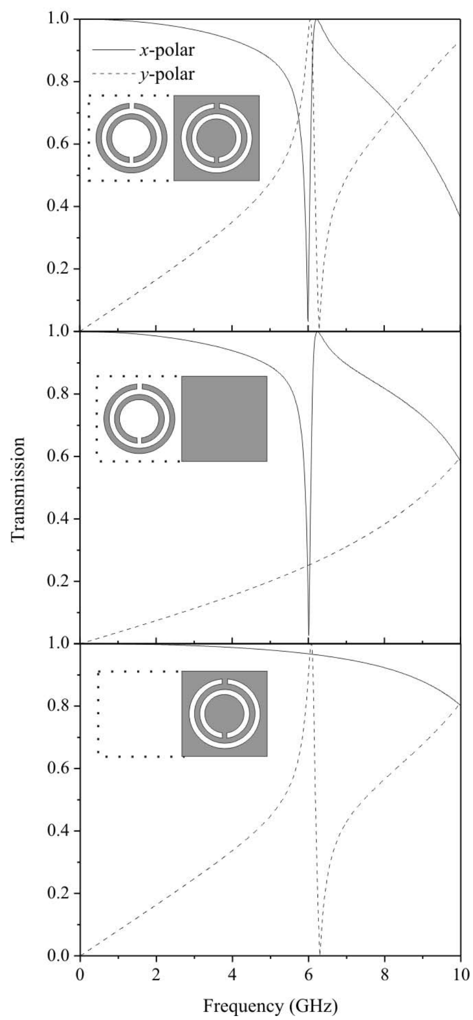

Fig. 3. Transmission coefficients for the case of unloaded resonators for different configurations: the full self-complementary metasurface (top), the sub-array of SRRs including the long metal strips without CSRRs (middle), and the sub-array of CSRRs (bottom). Solid lines represents the transmission for x-polarized waves and dashed lines for y-polarized waves.

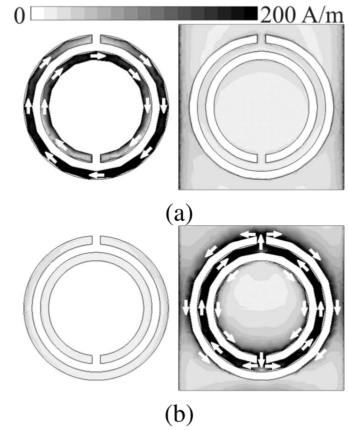

Fig. 4. Electric surface currents over the unloaded resonators for x-polarized waves at 6.00 GHz (a) and y-polarized at 6.06 GHz(b).

a single SRR, is not far from the simulated value of 6 GHz obtained for the whole coupled system of SRRs and CSRRs.

B. The metasurface with loaded resonators

With the aim of improving the shape of the stopband and passband we have loaded the resonators with lump circuit elements as shown in Fig. 1(c). In that way we expect to push down the resonant frequency to very low values where the baselines are more flat. Using the parameters of the caption of Fig. 2 it is easy to get a theoretical resonant frequency of 2.19 GHz. Of course, it can be much lower if we use higher values for L_l and C'_l. The numerical simulations for normal incidence drived us to the results shown in Fig. 5(top). Now, a double resonance appears being the lowest resonant frequency at 2.15 GHz. Actually, Fig. 3 for unloaded resonators should also show a second resonence if we would increase the frequency range of the simulation a few GHz more. This double resonance can be interpreted as the first antisymmetric (A1) and symmetric (S1) resonant modes of the SRR demonstrated in [11] (similarly for the CSRR). The bands related with the S1 mode are wider than the band of A1 because the effective distance between positive and negative charges for the S1 mode is higher than for the A1 mode, which makes the resonance to be stronger. Apart from this double resonance, it is clear that the bands of Fig. 5(top) look more symmetric than those of Fig. 3(top), which means an important improvement from a practical point of view when somebody wants to design a filter. Actually, the double resonance allows the design of a dual band filters. Apart from the double resonance, it is worth to stress the fact that we have got again the same filtering behavior: a stopband for x-polarization and a passband for y-polarization. And following the same reasoning at the end of Sec. III.A, the independency between the sub-arrays of loaded SRRs and loaded CSRRs is again demonstrated by results of Fig. 5(middle and bottom) and Fig. 6.

IV. CONCLUSION

In this paper we have demonstrated that a self-complementary metasurface made of SRRs and CSRRs can behaves like a band-pass filter for a certain linear polarization and band-stop filter for the orthogonal polarization. The idea was developed for SRR and its complementary version named CSRR, but it can be easily extended to other types of resonators. It is just important to satisfy two conditions. First the resonator should resonates under an applied electric or magnetic tangential field, in order to assure the response for normal incidence. Second, the particle must resonate at very low frequency in such a way that the resonant wavelenght is much bigger than the periodicity. This last condition assures that the baseline of the filter is flat enough. Since it is a new concept in the frame of FSS we have dealt with the ideal case of perfect conductor of infinitesimal thickness hold in air. Subseequent work about the effects of metal losses,

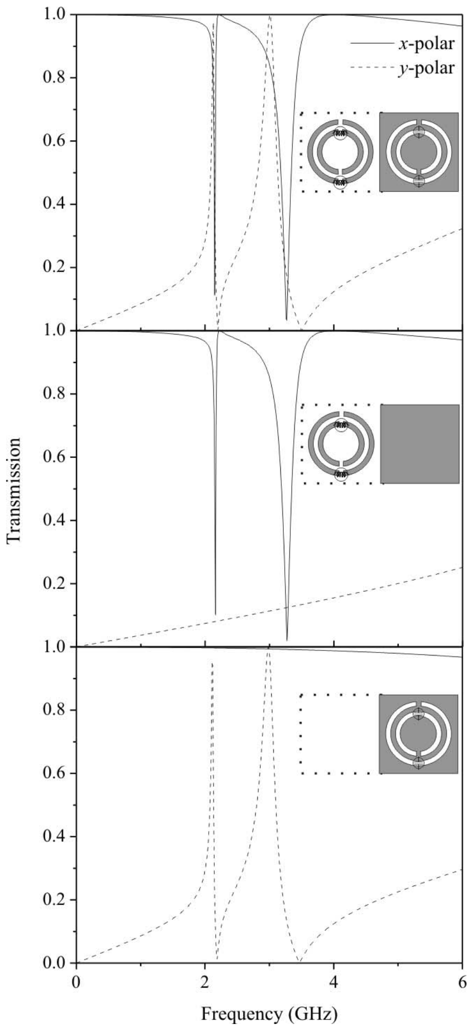

Fig. 5. Transmission coefficients for the case of loaded resonators for different configurations: the full self-complementary metasurface (top), the sub-array of SRRs including the long metal strips without CSRRs (middle), and the sub-array of CSRRs(bottom). Solid lines represents the transmission for x-polarized waves and dashed lines for y-polarized waves.

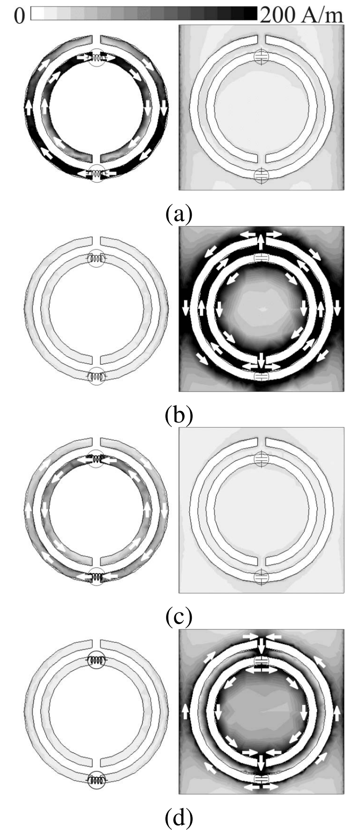

Fig. 6. Electric surface currents over the loaded resonators for x-polarization at 2.15 GHz (a), y-polarization at 2.13 GHz(b), x-polarization at 3.27 GHz (c), and y-polarization at 3.01 GHz(d). (a) and (b) correspond to the antisymmetric mode A1 at the lowest resonant frequency, while (c) and (d) correspond with the symmetric mode S1 at the second resonant frequency. This terminology (A1 and S1) was introduced in [11].

thickness and dielectric substrate is currently being done. We hope this idea could open the way to a new kind of FSS that can be switched from band-stop filter to band-pass filter by only rotating it through 90 degrees.

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