Analytical Circuit Model for Stacked Slit Gratings

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***Abstract*—This work presents a rigorous circuit model to compute the transmission/reflection properties of a finite number of stacked slit gratings printed on dielectric slabs of arbitrary thickness. A key aspect of the present approach is that the circuit model itself leads us to find fully analytical expressions for the finite stacked-grating structure. An analytical model to obtain the Brillouin diagram for the fully periodic structure (infinite number of identical unit cells) is also provided.**

***Index Terms*—Bloch waves, equivalent circuit modeling, metal gratings, periodic structures.**

I. Introduction

Periodic structures consisting of planar periodic distributions of metallic scatterers printed on one or more dielectric substrates have been studied for decades in the microwaves and optics literature [1], [2]. The simplest case corresponds to an infinite 1D periodic array of slits made in a thin metal plate printed on a dielectric slab [3]–[5]. Nowadays, the analysis of this kind of structures is mostly carried out using commercial software, although recently a lot of attention is being paid in the literature to the development of circuit-like models [6], [7]. These models provide closed-form analytical expressions for the transmission, reflection and absorption properties of that kind of geometries; see, for instance, the quasi-heuristic approaches for 2D arrays of slots in [8], for 2D arrays of patches in [9] or for strip/slit-like structures in [10], as well as the rigorous circuit-model derivation in [11] for 1D arrays of strips/slits. It should be noted that all the above mentioned papers only consider the case of a single structured periodic metallic surface (the periodicity is assumed to be in the transverse directions). The extension of the circuit-like models to the case of cascaded (stacked) structured surfaces is of great interest [see Fig. 1(a)]. Thus, for instance, controlled transmission and rejection bands appear associated with the quasi-periodic nature of the system along the propagation direction of the wave when $\varepsilon_1 = \varepsilon_2 = \ldots = \varepsilon_N$ and $d_1 = d_2 = \ldots = d_N$. This extension is relatively simple when the distance between the structured metal surfaces is large enough to preclude high-order mode interaction (i.e., when the interaction between metallic surfaces is due to the fundamental TEM propagating mode) [12]. However, interaction through the first few high-order modes drastically modifies the physics of the problem and makes the analysis much more complicated. The purpose of the present contribution is to present a simple and efficient procedure to extend the analysis reported in [11] in order to account for the electromagnetic properties of a densely packaged set of slit-like structures consisting of a finite (or infinite) number of screens [Fig. 1(a)]. To the authors' knowledge, a closed-form solution to this problem is here reported for the first time.

II. Derivation of the Circuit Model

Before addressing the case of the stacked gratings in Fig. 1(a), it will be first considered the problem of two coupled gratings shown in Fig. 1(b). An equivalent circuit solution for this problem was already reported in [11], where fully analytical circuit models are given for the even and odd excitations of the symmetric structure, corresponding respectively to the magnetic and electric wall half-problems shown in Fig. 1(c).

For the case of TM polarization of the incident wave (TE-polarized waves hardly interact with the gratings and thus they are not considered here), the equivalent admittance shown in Fig. 1(c) is found to be [11]

$$Y_{\text{eq}}^{(\text{e/o})} = \sum_{n=-\infty}^{\infty} A_n \left(Y_n^{(0)} + Y_n^{(\text{in})} \right) \qquad (1)$$

where

$$A_n = \left[\frac{J_0((k_n + k_t)w/2)}{J_0(k_t w/2)} \right]^2 \qquad (2)$$

$$Y_n^{(\text{in})} = \mathrm{j} Y_n^{(1)} \begin{cases} \tan(\beta_n^{(1)} d_1/2) & \text{even excitation} \\ -\cot(\beta_n^{(1)} d_1/2) & \text{odd excitation} \end{cases} \qquad (3)$$

$$Y_n^{(i)} = \frac{\omega \varepsilon_i}{\beta_n^{(i)}} \qquad (4)$$

$$\beta_n^{(i)} = \sqrt{\varepsilon_{r,i} k_0^2 - (k_n + k_t)^2} \qquad (5)$$

$$k_t = k_0 \sin\theta \, , \qquad k_n = \frac{2\pi n}{p} \qquad (6)$$

with θ being the incidence angle of the impinging plane wave. At this point, following [11], it is important to realize that all the modes excited at the discontinuities (that is, the different harmonics) are explicitly considered. It means that the present analysis rigorously account for the high-order mode interactions between metallic screens.

Our purpose now is to find the equivalent π-network shown in Fig. 2(a) that describes the behavior of the coupled-screens

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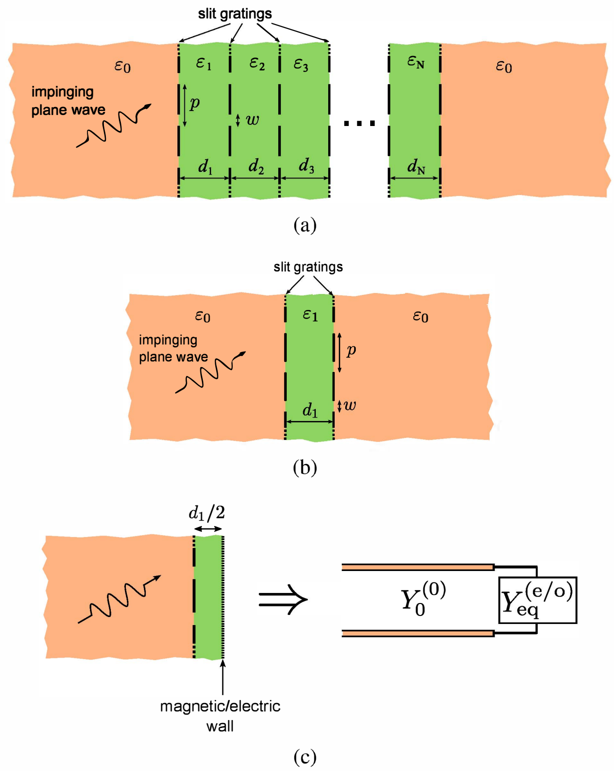

Fig. 1. (a) Stacked structure under study. (b) Transverse view of a symmetrical structure consisting of two coupled slit gratings with slit width w and period p printed on each side of a dielectric slab of permittivity $\varepsilon_1 = \varepsilon_{r1}\varepsilon_0$ and thickness d_1. (c) Equivalent even/odd excitation (magnetic/electric wall) half-problems and their equivalent circuits.

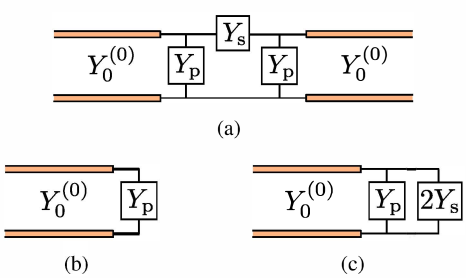

Fig. 2. (a) Equivalent circuit for the structure in Fig. 1(b), in which the coupled screens are represented by a π-network. (b) and (c) show the corresponding equivalent circuits for the even and odd excitation of the circuit in (a), respectively.

structure in Fig. 1(b). By placing open and short circuit terminations in the middle plane of this π circuit, the resulting equivalent circuits for the even and odd excitations are those shown in Figs. 2(b) and 2(c), respectively. By comparison with Fig. 1(c), it is clear that the parallel and series admittances in the π-circuit can readily be obtained from the even and odd excitation equivalent admittances as

$$Y_\mathrm{p} = Y_\mathrm{eq}^{(\mathrm{e})}\,, \qquad Y_\mathrm{s} = \frac{1}{2}\left[Y_\mathrm{eq}^{(\mathrm{o})} - Y_\mathrm{eq}^{(\mathrm{e})}\right] \tag{7}$$

and therefore

$$Y_\mathrm{p} = \sum_{n=-\infty}^{\infty}{}' A_n Y_n^{(0)} + \mathrm{j}\sum_{n=-\infty}^{\infty} A_n Y_n^{(1)} \tan(\beta_n^{(1)} d_1/2) \tag{8}$$

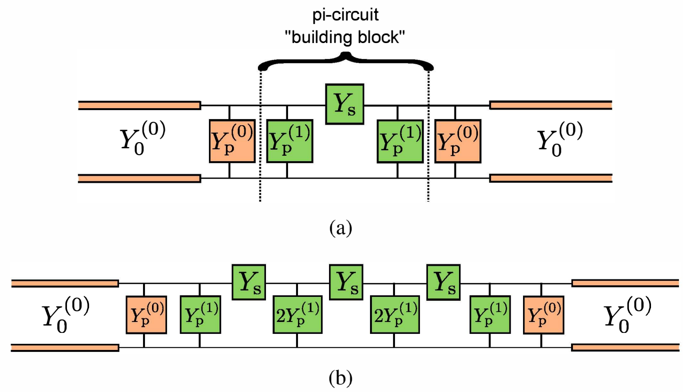

Fig. 3. (a) Equivalent circuit for the coupled gratings in Fig. 1(b), showing the decomposition of the parallel element into an external admittance $Y_\mathrm{p}^{(0)}$ and an internal admittance $Y_\mathrm{p}^{(1)}$. (b) Equivalent circuit for four stacked screens.

$$Y_\mathrm{s} = -\mathrm{j}\sum_{n=-\infty}^{\infty} A_n Y_n^{(1)} \csc(\beta_n^{(1)} d_1) \tag{9}$$

where the prime means that the fundamental harmonic $n = 0$ (namely, the harmonic associated with the impinging, reflected, and transmitted plane waves) is excluded in the series.

In principle, the admittances in (8) and (9) involve an infinite number of terms (i.e., a parallel connection of an infinite number of elements). However, the same approximation used in [11] to separate the contribution of low- and high-order harmonics can be used here. Thus, each of the above series can be split into a sum of a few low-order term plus a so-called "high-order" admittance that incorporates the contribution of *all* the remaining high order harmonics. This high-order admittance is given in terms of an infinite series that is independent of both frequency and incidence angle, and thus it should be computed only once in an eventual and usual frequency/angle sweeping analysis.

Now it is key to note that the series admittance in (9), Y_s, only depends on the characteristics of the medium in region (1) between the gratings (*internal* region). In turn, the parallel admittance in (8) can be written as the sum of an *external* admittance, $Y_\mathrm{p}^{(0)}$, and an *internal* admittance, $Y_\mathrm{p}^{(1)}$, as depicted in Fig. 3(a). As shown in this figure, the equivalent circuit consists of an internal π-circuit (formed by Y_s and $Y_\mathrm{s}^{(1)}$) which is connected to the parallel admittances $Y_\mathrm{s}^{(0)}$ that account for the field associated with the high-order harmonics ($|n| > 0$) in the external region and also to the transmission lines associated with the propagation of the impinging, reflected, and transmitted waves (harmonics with $n = 0$). This representation clearly suggests that, by cascading the building blocks corresponding to the internal π-circuits, it is possible to obtain a generalized circuit model for a stack with an arbitrary number of gratings. This idea is illustrated in Fig. 3(b) for the case of four screens (three dielectric slabs). The more general case with different dielectric slabs (either in thickness, permittivity, or both) separating each consecutive metallic screens is considered in the analysis by simply introducing the corresponding values of the thickness and/or permittivity

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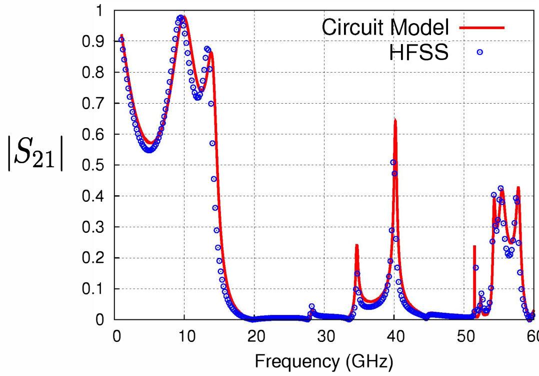

Fig. 4. Transmission coefficient of a TM-polarized plane wave obliquely ($\theta = 20°$) impinging on a structure consisting of three slit gratings separated by two different dielectric layers. Parameters: $w = 0.5$ mm, $p = 5$ mm, $d_1 = 2$ mm, $\varepsilon_1 = 2.2\varepsilon_0$, $d_2 = 1.5$ mm, and $\varepsilon_2 = 4\varepsilon_0$.

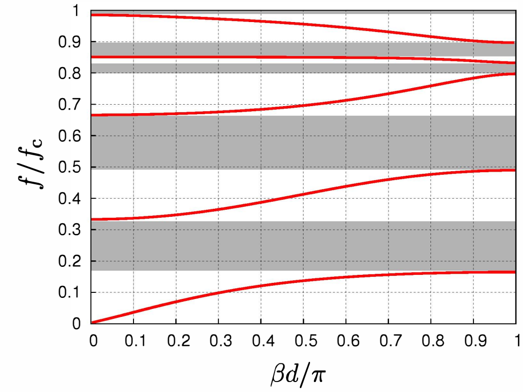

Fig. 5. Brillouin diagram of an infinite (z-periodic) slit grating structure. The parameters of the unit cell are: $w = 0.1p$, $d_1 = 0.2p$, $\varepsilon_1 = 9\varepsilon_0$. The upper frequency limit is $f_c = c/p$, with c being the speed of light in vacuum.

in the admittances of each π-circuit building block.

Finally, it is interesting to note that the equivalent circuit derived here also allows for an straightforward computation of the dispersion relation of the Bloch modes in the ideal infinite stack. Following [13], the dispersion relation is given by

$$\cos(\beta d) = 1 + \frac{Y_{\text{p}}^{(1)}}{Y_{\text{s}}} . \qquad (10)$$

III. Numerical Results

First, a finite stack with three slit gratings separated by two different dielectric layers is considered. Our analytical results for the transmission coefficient of a TM-polarized plane wave that impinges obliquely on this structure are shown in Fig. 4 together with the data provided by the commercial software HFSS [14]. The figure shows that the agreement between our results and HFSS data is very good over a very wide frequency band, even well within the grating-lobe regime (the onset of the grating-lobe regime takes place at 44.7 GHz). It is important to highlight that, due to the analytical nature of our model, our results are computed with an almost negligible computational effort while the results provided by HFSS require a considerable amount of CPU time.

Next, Fig. 5 shows the Brillouin diagram obtained using the circuit-model approach for an infinite longitudinal periodic stack of slit gratings separated by a dielectric substrate. The considered frequency range extends up to $f_c = c/p$ [c is the speed of light in free space]; namely, the frequency at which the free-space wavelength equals the value of the periodicity. This dispersion diagram shows five passbands and four stop bands (shaded regions in Fig. 5). The first three passbands are of forward nature whereas the last two passbands are backward. When this ideal infinitely periodic structure is simulated with a finite stack of six slit gratings (five dielectric layers), it can be observed in Fig. 6 that the transmission coefficient already follows the passbands and stopbands pattern described by the dispersion diagram in Fig. 5. Once again, our results are in very good agreement with those provided by HFSS.

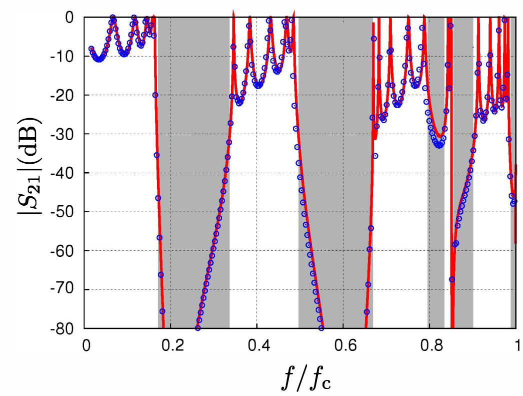

Fig. 6. Circuit-model (solid line) and HFSS (circles) results for the transmission coefficient (in dB) of a normally incident TM-polarized plane wave impinging on a finite set of 6 stacked slit gratings. The unit cell is the same as in Fig. 5. The shaded frequency regions correspond to the stopbands in Fig. 5.

IV. Conclusion

A fully analytical circuit-model approach to study the scattering and dispersion characteristics of periodic and quasi-periodic stacks of slit gratings embedded in a layered dielectric environment has been reported. The equivalent circuit accounts for the interaction of all the modes and is rigorously derived from the analysis of a pair of coupled slit gratings. The model has been found to be very accurate over a very wide frequency band. The procedure here presented can be extended to stacked 2-D periodic structures.

Acknowledgment

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