Multi-band High-Impedance Surface Absorbers with a Single Resistive Sheet: Circuit Theory Model

Yashwanth R. Padooru Alexander B. Yakovlev* Chandra S. R. Kaipa Dept. of Electrical Engineering University of Mississippi University, Mississippi 38677-1848, USA Email: ypadooru@olemiss.edu Francisco Medina Dept. of Electronics and Electromagnetism University of Seville Seville 41012, Spain Email: medina@us.es Francisco Mesa Dept. of Applied Physics 1 University of Seville Seville 41012, Spain Email: mesa@us.es

Abstract—We present a simple and efficient circuit model for the analysis of multi-band absorption characteristics of a stack of sub-wavelength grids (fishnets or patch arrays) in a layered environment with a single resistive sheet placed on top. It is shown that at low frequencies the resonances of total absorption occur, which are explained in terms of Fabry-Pérot resonances associated with the individual reactively loaded dielectric slabs (that are strongly coupled through the sub-wavelength grids). It is observed that the number of absorption peaks in the absorption band corresponds to the number of reactively loaded dielectric slabs. The results of the proposed analytical model are validated with the full-wave simulations, showing good agreement.

I. INTRODUCTION

Electrically thin absorbers based on high-impedance surfaces (HIS) have been of interest over the last decade [1]-[3]. The absorption in these structures is achieved by combining the properties of the HIS with the absorption of a resistive sheet or surface-mounted resistors connected between the metallic parts of the frequency selective surfaces (FSS). The drawback of these absorbers is that they have an absorption peak (reflection null) at a single frequency. In [4], a design procedure for realizing dual-band absorbers was proposed, wherein the absorption resonances depend on the parameters of the unit-cell resonator. Hence, to achieve multiple absorption peaks, this procedure leads to complexity in the design of the unit-cell geometries. To overcome this problem, we use the concept described in [1] adapted to the multilayer structure with a resistive sheet (with the geometry shown in Fig. 1), and demonstrate that this structure exhibits multiple absorption peaks.

Recent advancements in the circuit theory models for the analysis of high-impedance surface absorbers [5]–[7] have attracted some attention because they provide simple and efficient modelling tools and help in understanding of physical phenomena in geometrically complicated structures. Also in [8], which is closely related to the work presented in this paper, the circuit theory approach has been applied for the analysis of transmission properties of a stack of subwavelength fishnet grids, wherein the analytical expressions for the grid impedances (admittances) are obtained in terms

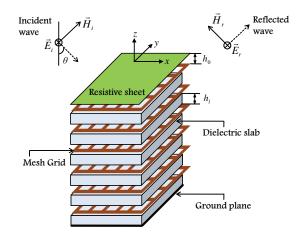


Fig. 1. Exploded schematic of the multilayer HIS absorber formed by mesh grids of sub-wavelength dimensions

of the effective circuit parameters from the full-wave scattering problem [9], [10].

In this paper, we first study a two-layer HIS formed by mesh grids without and with the resistive sheet, and relate the physical mechanisms of HIS and absorption resonances to the transmission resonances observed in a four-layer structure (without the ground plane) formed by the same mesh grids [8]. In addition, we study the electromagnetic field distribution associated with these resonances using the circuit model and full-wave commercial solver HFSS [11]. Moreover, for the case of multilayer mesh grid absorber, it is observed that the absorption spectra for each absorption band have a number of peaks (as Fabry-Pérot (FP) resonances) equal to the number of FP cavities (formed by reactively-loaded dielectric slabs) and all the peaks lie within a characteristic frequency band defined by the lower and upper band edges.

II. CIRCUIT MODEL ANALYSIS

Consider a *y*-polarized plane wave incident on the multilayer HIS absorber as shown in Fig. 1. A schematic representation of the transmission line network for the multilayer structure (loaded with sub-wavelength mesh grids separated by

2264

dielectric slabs) is shown in Fig. 2. The shunt reactances $Y_{g1}, Y_{g2}, \ldots, Y_{gm}$ (grid admittances) account for the belowcutoff higher-order modes scattered by the metal grids in each layer. The analytical expressions for the grid admittance Y_{gl}

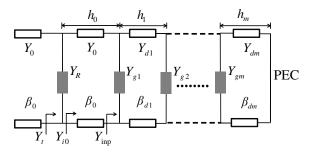


Fig. 2. Transmission-line network for the multilayer structure with subwavelength periodic elements; Y_0 , β_0 are the characteristic admittance and propagation constant of the air filled region; Y_{dl} and β_{dl} are the same parameters for the corresponding dielectric filled regions.

for TE and TM polarizations can be obtained as follows [10]:

$$Y_{gl}^{\text{TE}} = \frac{1}{j\omega L_{gl}^{\text{TE}}} \quad , \quad L_{gl}^{\text{TE}} = \frac{\eta_0 D}{2c\pi} \ln\left[\csc\left(\frac{\pi w}{2D}\right)\right] \tag{1}$$

$$Y_{gl}^{\rm TM} = \frac{1}{j\omega L_{gl}^{\rm TM}} \quad , \quad L_{gl}^{\rm TM} = \frac{\eta_0 D}{2c\pi} \ln\left[\csc\left(\frac{\pi w}{2D}\right)\right] \\ \times \left(1 - \frac{\sin^2 \theta}{2\varepsilon_l^{qs}}\right) \tag{2}$$

where csc is the co-secant function, $\eta_0 = \sqrt{\mu_0/\varepsilon_0} \approx 377 \,\Omega$ is the intrinsic impedance of free space, θ is the incidence angle, ω is the angular frequency, c is the speed of light in vacuum, $\varepsilon_l^{qs} = (\varepsilon_l + \varepsilon_{l+1})/2$ for interior grids (l = 2, 3,, m - 1)and $\varepsilon_l^{qs} = (\varepsilon_l + 1)/2$ for the grid located at the upper interface (l = 1). The geometrical parameters D and w of the inductive/capacitive grid are defined in Fig. 3(b). The admittance Y_R in parallel with the grid admittance Y_{g1} is the admittance of the lossy resistive sheet. The parameters of the transmission line in Fig. 2, propagation constants β_0 for the air filled section and β_{dl} for the dielectric filled sections, and the corresponding characteristic admittances Y_0 and Y_{dl} are known in closed form [9].

It is well known that for a HIS structure [12], [13] (singleband or multi-band) at resonance, the surface admittance or input admittance (Y_{inp} , shown in Fig. 2) is zero and the admittance of the open-circuit termination of the air-gap section (for small h_0) is given by

$$Y_{i0} = jY_0 \tan \beta_0 h_0 \approx 0 \tag{3}$$

where $Y_0 = \sqrt{\varepsilon_0}/\mu_0 \approx 1/377$ S is the intrinsic admittance of free space and $\beta_0 = \omega/c$. Thus, the total surface admittance Y_t of the HIS absorber is equal to admittance of the resistive sheet Y_R . If Y_R is chosen in such a way that, $Y_R = Y_0^{\text{TE,TM}}$ $(Y_0^{\text{TE}} = Y_0 \cos \theta$ and $Y_0^{\text{TM}} = Y_0/\cos \theta$, then the admittance of the structure is matched to the intrinsic admittance of free space. Under this condition, the incident field is absorbed by the structure and we have reflection nulls $(S_{11} = 0)$ or absorption peaks $(\mathcal{A} = 1 - S_{11}^2)$, since the perfect electric conductor (PEC) ensures $S_{12} = 0$. Since the system is assumed to be lossless, the total incident energy is absorbed by the resistive sheet and is dissipated as heat.

III. MESH GRID ABSORBER

A. Two-Layer Mesh Grid Absorber

The two-layer mesh grid absorber is shown in Fig. 3. The structure is formed with identical mesh grids separated by dielectric slabs on a PEC ground plane, with a resistive sheet placed on the top. The analysis is carried out using the circuit model formalism given in Section II.

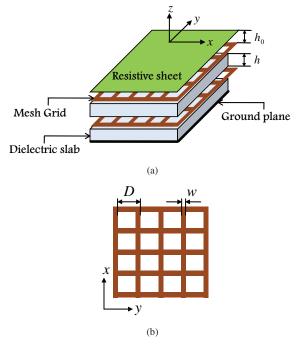


Fig. 3. (a) Exploded schematic of the two-layer mesh grid separated by dielectric slabs, with a resistive sheet placed on the top and a PEC ground plane at the bottom. (b) Top view of mesh grid.

Let us first consider the absorber shown in Fig. 3 without the resistive sheet. At resonance $(Y_{inp} = 0)$ this structure acts as a HIS, reflecting the incident field with the zero reflection phase. Since the HIS is impenetrable, in the absence of losses its reflectance is equal to unity. The reflection phase characteristics of the HIS structure for the case of normal incidence calculated using the circuit model (termed as "Analytical" in the figures) and HFSS are shown in Fig. 4. The first observation that can be drawn from Fig. 4 is that the reflection phase characteristics predicted by the circuit model agree well with HFSS. Moreover, the reflection phase behavior shows that the structure has two resonances labeled as A and B (corresponding to zero reflection phase) with the resonance frequencies 7.917 GHz and 12.16 GHz, respectively. It should be noted that the odd excitation (associated with placing a PEC wall in the middle) of the four layer structure

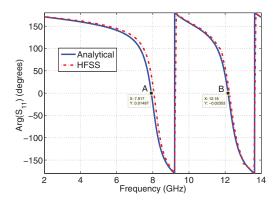


Fig. 4. Reflection phase behavior of the two-layer mesh structure in the absence of resistive sheet, calculated using the circuit model and HFSS for the following parameters [with notations used in Fig. 3]: D = 5.0 mm, w = 0.15 mm, h = 6.35 mm, and the permittivity of dielectric slab $\varepsilon_r = 3$.

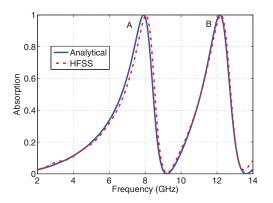


Fig. 5. Absorptivity of the two-layer HIS absorber as a function of frequency, calculated with the circuit model and HFSS for the following parameters [with notations used in Fig. 3]: D = 5.0 mm, w = 0.15 mm, h = 6.35 mm, permittivity of dielectric slab $\varepsilon_r = 3$, and the admittance of the resistive sheet $Y_R = 1/377$ S (for the normal plane-wave incidence).

formed by five mesh grids in [8] is equivalent to the HIS structure (two-layer mesh-grid absorber without the resistive sheet). Thus, following [8], we have two FP cavities that are strongly coupled through the square holes of each grid, i.e., two reactively loaded dielectric slabs. It is also observed that the resonances A and B have the same physical nature as the resonance B and D of the geometry analyzed in [8] with the resonance frequencies of 8.06 GHz and 12.2 GHz, respectively.

Next, consider a resistive sheet $(Y_R \approx 1/377 \text{ S})$ placed in a close proximity (the distance h_0 is assumed to be small) to the HIS structure. At resonance $(Y_{inp} = 0)$ the total admittance of the structure $(Y_t = Y_R)$ is matched to the intrinsic admittance of free space and hence there is no reflection from the structure, i.e., we have an absorption peak. Fig. 5 shows the absorptivity behavior of the two-layer HIS absorber for the case of normal incidence calculated using the circuit model and HFSS. It is observed that the

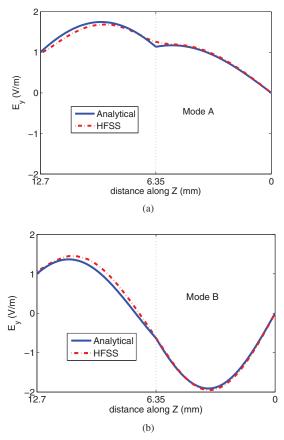


Fig. 6. Comparison of the analytical and HFSS results of the electric field distributions in the two-layer HIS mesh grid absorber for (a) mode A and (b) mode B. HFSS results are obtained by calculating the field along a line in the *z*-direction through the center of the holes.

structure still retains the same FP-like resonances A and B with the resonance frequencies of 7.917 GHz and 12.16 GHz, respectively. Also, it is worthy to point out that, in the two-layer HIS absorber and in the structure studied in [8], the corresponding resonances of absorption and transmission are associated with the perfect match of the surface admittance to the intrinsic admittance of free space. Hence, it is important to verify the field distributions for the two resonance modes A and B.

Fig. 6 shows the electric field distributions for the two resonance modes (A and B) calculated using the circuit model and HFSS. The field distributions corresponding to the modes A and B (shown in Fig. 6) have the same qualitative behavior as the fields in the first two layers (modes B and D) in [8]. Also, the circuit model results are in good agreement with the numerical results, which shows that our model captures the electromagnetic details of the structure.

B. Multilayer Mesh Grid Absorber

In the previous example, it is shown that the two-layer HIS absorber exhibits two FP-like resonances of complete absorption corresponding to the two FP cavities. To show that this effect is general, we apply the circuit theory model to

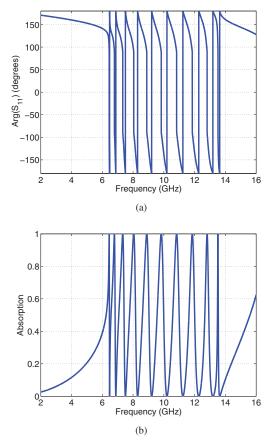


Fig. 7. Behavior of the ten-layer mesh grid HIS structure predicted by the analytical model: (a) Phase of the reflection coefficient (in the absence of the resistive sheet), (b) Absorptivity (in the presence of resistive sheet).

structures having a large number of identical layers (mesh grids and dielectric slabs with the last dielectric slab backed by a PEC ground plane). For instance, the frequency response of the ten-layer HIS structure (without and with the resistive sheet) with the same geometrical dimensions used in the previous example is shown in Fig. 7. It can be observed that both the structures have ten FP-like resonances corresponding to the ten coupled cavities. However, in one case the structure behaves as a HIS, reflecting the incident field with the zero reflection phase and in the other case it acts as an absorber (exhibiting absorption peaks). It should be noted that with an increase in the number of identical layers the number of absorption peaks is equal to the number of dielectric slabs and all the peaks are within the characteristic frequency band. The frequencies corresponding to the first (f_{LB}) and the last (f_{UB}) resonances for a different number of layers are given in Table I.

IV. CONCLUSION

In this paper, a simple and accurate circuit theory model has been applied for the analysis of the absorption properties of multilayer structures with sub-wavelength dimensions. It is observed that at low frequencies the resonances of complete

TABLE I FREQUENCIES OF THE LOWER AND UPPER BAND EDGES WITH RESPECT TO THE NUMBER OF LAYERS.

No. of Layers	$f_{\rm LB}$ (GHz)	$f_{\rm UB}$ (GHz)
2	7.917	12.16
5	6.713	13.25
10	6.447	13.52
20	6.363	13.60
30	6.347	13.62

absorption occur, and it is shown that these resonances are of FP type corresponding to the strongly coupled individual reactively-loaded dielectric slabs. The results obtained by the circuit theory model are in complete agreement with the numerical simulations. It has been noticed that the number of peaks in the absorption band is equal to the number of dielectric slabs, and all the peaks lie within a characteristic frequency band.

REFERENCES

- N. Engheta, "Thin absorbing screens using metamaterial surfaces," Proc. IEEE Int. Symp. Antennas Propag., vol. 2, pp. 392-395, 2002.
- [2] S. Simms and V. Fusco, "Thin radar absorber using artificial magnetic ground plane," Electron. Lett., vol. 41, pp. 1311-131, 2005.
- [3] H. Mosallaei and K. Sarabandi, "A one-layer ultra-thin meta-surface absorber," Proc. IEEE Int. Symp. Antennas Propag., vol. 1B, pp. 615-618, 2005.
- [4] Q. Y. Wen, H. W. Zhang, Y. S. Xie, Q. H. Yang, and Y. L. Liu, "Dual band terahertz metamaterial absorber: Design, fabrication, and characterization," Appl. Phys. Lett., vol. 95, pp. 241111-241111-3, Dec. 2009.
- [5] O. Luukkonen, F. Costa, C. R. Simovski, A. Monorchio, and S. A. Tretyakov, "A thin electromagnetic absorber for wide incidence angles and both polarizations," IEEE Trans. Antennas and Propag., vol. 57, no. 10, pp. 3119-3125, 2009.
- [6] Costa, F., A. Monorchio, and G. Manara, "Analysis and design of ultra thin electromagnetic absorbers comprising resistively loaded high impedance surfaces," IEEE Trans. Antennas and Propag., vol. 58, no. 5, pp. 1551-1558, May 2010.
- [7] S. A. Tretyakov and S. I. Maslovski, "Thin absorbing structure for all incident angles based on the use of a high-impedance surface," Microw. Opt. Technol. Lett., vol. 38, no. 3, pp. 175-178, 2003.
- [8] C. S. R. Kaipa, A. B. Yakovlev, F. Medina, F. Mesa, C. A. M. Butler, and A. P. Hibbins, "Circuit modeling of the transmissivity of stacked two-dimensional metallic meshes," Opt. Express, vol. 18, no. 13, pp. 13309-13320, 2010.
- [9] S. Tretyakov, Analytical Modeling in Applied Electromagnetics, Artech House: Norwood, MA, 2003.
- [10] O. Luukkonen, C. Simovski, G. Granet, G. Goussetis, D. Lioubtchenko, A. V. Raisanen, and S. A. Tretyakov, "Simple and accurate analytical model of planar grids and high-impedance surfaces comprising metal strips or patches," IEEE Trans. Antennas Propag., vol. 56, no. 6, pp. 1624-1632, 2008.
- [11] HFSS: High Frequency Structure Simulator based on the Finite Element Method, Ansoft Corporation, http://www.ansoft.com
- [12] D. F. Sievenpiper, L. Zhang, R. F. J. Broas, N. G. Alexopoulos, and E. Yablonovich, "High-impedance electromagnetic surfaces with a forbidden frequency band," IEEE Trans. Microwave Theory Tech., vol. 47, no. 11, pp. 2059-2074, 1999.
- [13] W. McKinzie and S. Rogers, "A multi-band artificial magnetic conductor comprised of multiple FSS layers," Proc. IEEE Int. Symp. Antennas Propag., vol. 2, pp. 423-426, June 2003.