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Title: Prospective mathematics teachers' development of noticing in an online teacher education program.

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In this study we aimed to characterize the development of prospective mathematics teachers' noticing in an online teacher education program. Prospective teachers wrote narratives about their own teaching during their traineeship period at schools, and shared them with their colleagues and with the university tutor in an online forum. A narrative is a description of a teaching-learning situation that incorporates reasons about what has happened during the teaching, and justifications of plausible future teaching actions. We analyzed changes in the prospective teachers' narratives by scrutinizing how prospective teachers identify Mathematically Significant Pedagogical Opportunities to Build on Student Thinking (MOSTs) and in which ways they take advantage of them. Additionally, we analyzed whether these changes could be related to the feedback provided in the online forum. The findings indicate changes in how prospective teachers notice their practice, evidenced by the manner in which the MOSTs are taken advantage of, and by the reasons given for their actions. These findings suggest that sharing narratives in online forums and receiving feedback from their colleagues and from the university tutor help prospective teachers to enhance the skill of noticing, leading to changes in their practice. This research provides new knowledge about how online contexts in teacher education programs promote opportunities for collaborative learning, noticing development, and changes in teachers' practices.

Key words: Distance Mathematics Teacher Education; Mathematically Significant Pedagogical Opportunities; Narrative; Online feedback; Professional noticing.

1. INTRODUCTION

The interest in online distance education, especially in mathematics teacher education, has grown in recent years, generating topics for further research (Borba et al. 2013; Borba and Llinares 2012). Some of these topics are knowledge-building practices in technology-mediated work group interactions, and the role of online interactions among teachers in generating new knowledge about mathematics teaching (Borba et al. 2018; Clay et al. 2012). Borba et al. (2018) highlighted that "it is necessary to create conditions to allow for the transmitting of information as well as for the construction of knowledge" (p. 284) in online distance courses. It is only with the interaction that occurs among students, tutors and teachers that this construction of knowledge may be realized. The research presented in this paper focused on the possibilities of online environments to enhance prospective teachers' skill in noticing teaching-learning situations (Mason 2002). Particular focus was placed on the role played by the feedback of the tutor and colleagues in an online forum where prospective teachers shared narratives about their own teaching during the period of teaching practice in secondary schools.

During the last decade a line of research focused on the skill of noticing has emerged, with research investigating different tools and contexts for its development (Amador 2019; Fernández et al. 2018; Schack et al. 2017; Sherin et al. 2011). Noticing is a teaching competence that allows teachers to recognize what is relevant in a teaching-learning situation, attending to some details and ignoring others in order to act accordingly for the purpose of supporting students' learning.

Noticing has been conceptualized from different perspectives (Mason 2002; Sherin et al. 2011). Mason (2002) claimed that noticing is a movement or shift of attention that is articulated by different ways of paying attention: (i) discerning details; (ii) recognizing relationships, that is, becoming aware of sameness and differences; (iii) perceiving properties, considered as becoming aware of particular relationships as instances of properties that could hold in other situations, and (iv) reasoning on the basis of agreed properties going beyond the assembling of things one thought one knew. The traineeship period in secondary schools in the teacher education programs is a suitable context in which to develop prospective teachers' noticing. However, there has not been enough information about how to support these shifts of attention during the traineeship period at schools in an online distance-learning teacher education program. In this

study, we considered the intersection of two aspects for the purpose of analyzing the development of noticing in an online distance-learning teacher training program, namely, the role of feedback in an online forum, and the task of writing and sharing narratives about one's own teaching during the traineeship period.

1.1. Feedback in online forums

Research has centered on the power of feedback in education, with particular emphasis on the fact that it can assist students in understanding, engaging, or developing effective strategies to process the information intended to be learned (Hattie and Timperley 2007). In order to be effective, "feedback needs to be clear, purposeful, meaningful, and compatible with students' prior knowledge and provide logical connections" (p. 104). Previous research on the influence of feedback in students' learning has pointed out two main functions (Coll et al. 2014; Van der Pol et al. 2008): a directive function occurring when the tutor provides specific information about which aspects students need to review, and a facilitative function taking place when feedback is used to give pointers, pose questions, make suggestions, and offer indirect guidance.

The online and blended approaches in teacher education have generated a growing interest in the study of feedback in online learning environments (Borba et al. 2016; Clay et al. 2012; Huisman et al. 2019; Ponte et al. 2009; Silverman 2012). Feedback in online learning environments includes prompts from the tutor to promote prospective teachers' learning, providing information that guides them towards the learning objectives (Wang et al. 2019). In this context, the role of feedback is relevant since it can help prospective teachers to change their ways of thinking (Narciss 2013).

It is of interest in our study to identify how feedback produced by university tutors and other colleagues as they participate in an online forum can help prospective teachers enhance their skill of noticing.

1.2. Writing narratives about one's own teaching, as a tool in mathematics teacher education

Writing narratives is a relevant tool in teacher education (Doyle and Carter 2003) and, particularly, in mathematics teacher training programs (Chapman, 2008). A narrative is a way of representing experience for oneself and for others and could be considered as the "primary form by which human experience is made meaningful" (Polkinghorne 1988, p. 1). A narrative is a story that tells a sequence of events that are significant for

the author and that have an internal logic that makes sense to him/her (Chapman 2008). In the field of mathematics education, Chapman (2008) argues that narratives allow teachers to express their practical understanding of mathematics teaching. From this perspective, prospective teachers are seen as storytellers of their own stories during their traineeship periods at schools, since narratives are seen as "a key form through which individuals come to know themselves, construct their lives, and make sense of their experiences" (Chapman 2008, p. 17). In this sense, a narrative involves episodes that are both personally significant for the prospective teachers, and relevant explanations.

During their training in schools, writing narratives can help prospective teachers to structure their attention towards noticing, particularly noticing students' mathematical thinking (Cavanagh and McMaster 2015; Ivars and Fernández 2018). Narratives can describe, on the one hand, the interactions between prospective teachers and students when the latter are solving a mathematical task, and on the other, the explanations about how prospective teachers consider these interactions able to influence students' opportunities to learn mathematics. The process of writing a narrative is a learning process in which prospective teachers describe students' answers and provide their interpretations of what happens in the classroom (Ponte et al. 2009). In other words, narratives make the teaching practice visible, and can become a shared object with the objective of improving the relation between theory and practice (Pulvermacher and Lefstein 2016). However, few studies have focused on how prospective teachers can develop the skill of noticing students' mathematical thinking during their teaching practice period in schools, when they write narratives of teaching contingencies and share them in an online forum.

1.3.Narratives, online feedback and the development of noticing

Narratives and online feedback provide the contexts in which prospective teachers can learn to pay attention to relevant aspects of students' mathematical thinking, interpret them and share their understanding of practice as a way to convey their pedagogical intentions. In fact, the online forum in which prospective teachers can share their narratives can be seen as an environment where "they can share experiences, meanings, knowledge, lessons and stories about the school's practice" (Ponte et al. 2009; p. 1). In this environment, the written feedback from the university tutors may play a significant role in improving prospective teachers' writing, in the same way as teachers' feedback helps to improve students' writing (Buhagiar 2013; Yang 2016; Zhu and Carless 2018).

Therefore, feedback from the tutor or other colleagues can help prospective teacher to revise their narratives, asking for more detailed descriptions of students' mathematical thinking, in order to establish relations between students' thinking and teaching moves (what the teacher does or should do).

To summarize, there is a lack of research concerning the development of the noticing skill in online distance-learning teacher training programs, notwithstanding the fact that these environments are at present common in mathematics teacher education in many countries. In fact, questions such as whether the strategies used in face to face environments for the developing of noticing skill can be applicable to online or blended learning environments, or whether completely new strategies are needed for this development, remain still unanswered.

From previous research, the benefits of feedback have been acknowledged (Buhagiar 2013; Clay et al. 2012; Rhoads et al. 2011), but there are few studies on online distance-learning mathematics teacher education. Our study contributes to this line of research by investigating how prospective teachers learn to notice students' mathematical thinking during their traineeship period in secondary schools as they write narratives about their own practices and share them, receiving feedback in an online forum. Therefore, the setting for this study provides an opportunity to investigate online feedback in order to develop prospective teachers' noticing, since it takes place in a fully online distance education university, where all interactions amongst tutors and students, and among students themselves, are performed online.

2. THEORETICAL FRAMEWORK

2.1. A sociocultural perspective on learning

We adopt a sociocultural perspective on learning (Wells 1999), which points out that learning is a process of cognitive and social transformation that occurs in a collaborative context in which prospective teachers learn by observing and participating with other individuals, through cultural artifacts in goal-directed activities. In an online distance mathematics teacher education program, online feedback on prospective teachers' narratives (considered as representations of practice) is a process that mediates how prospective teachers make sense of their experiences. In this process, a narrative can be considered as a cultural artifact, and writing as a goal-directed activity that creates a form of argumentation integrating *Thinking* and *Doing* (Wells 1999). *Thinking* is

conceptualized in the sense of recognizing and interpreting mathematics teaching events that may be turned into opportunities for the students' learning, and *Doing* is the other side of the coin, since a space is created to justify what has been done in the classroom or what could be done. From this point of view, writing narratives during the traineeship period and receiving online feedback from other colleagues and the tutor, can be understood as a process of socialization in socioculturally situated shared practices, and a process of sense-making (Philpott 2014) that may support the enhancement of prospective mathematics teachers' noticing.

When a prospective teacher shares a narrative in an online forum, the narrative is a tool for mediation between prospective teachers, their practices, and the feedback from the university tutors, through which the enhancement of noticing is possible. In this sense, the narratives are a form of situated rationality of the prospective mathematics teachers that allows to connect their representations of the practice with the online feedback, in order to learn from the experience. The changes in how prospective teachers describe the teaching-learning situations attending to specific teaching aspects, and how they interpret them to provide and justify their next teaching moves, can be considered evidence of the development of noticing. Therefore, changes in the substance and how prospective teachers write about it in their narratives can show features of prospective teachers' noticing enhancement.

2.2. MOST analytical framework as a tool to analyze changes in prospective teachers noticing

Taking into account students' mathematical thinking in mathematics teaching involves teachers recognizing contingent situations (Rowland and Zazskis 2013) that can turn into new learning possibilities. Learning to take advantage of these opportunities is a key aspect in practice-based mathematics teacher education programs. Mathematically Significant Pedagogical Opportunities to Build on Student Thinking (MOSTs) are "instances of student thinking that have considerable potential at a given moment to become the object of rich discussion about important mathematical ideas" (Leatham et al. 2015; p. 90). Recognizing these opportunities, and reasoning about them to decide how to act, integrates thinking and doing, and helps one to understand the practice of productively using student mathematical thinking during whole-group instruction (Teuscher et al. 2017). The MOST analytical framework provides a mechanism for

analyzing the skill of noticing by (i) providing a means for identifying instances of student thinking that can be mathematically important to notice in a given lesson (discerning); (ii) making a connection between a particular instance of student mathematical thinking and the broader education principle of building on student thinking (recognizing and perceiving) and, (iii) taking into account the classroom context when determining whether an instance is one that might provide leverage for moving the student forward in his or her mathematical understanding (providing next teaching moves).

MOSTs are in the intersection of three characteristics: student's mathematical thinking, mathematically significant events, and pedagogical opportunity (Leatham et al. 2015). For an event to be a MOST, it must be grounded in student's thinking. The occurrence must meet two criteria to be characterized as embodying student's mathematical thinking: (i) student's mathematics—if an observer can infer what the student is expressing mathematically and (ii) mathematical point—if there is a mathematical idea that is closely related to the student's mathematics of the event. Furthermore, in order for an occurrence to be a MOST, it must be mathematically significant. An event is characterized as being mathematically significant when it meets two criteria: (i) appropriate mathematics—the mathematical point must be accessible to the students according to their prior mathematical experiences, and should not be one that most students at this mathematical level would already understand and, (ii) central mathematics—if the mathematical point of the instance is closely related to a learning goal for the students in the classroom. Finally, a pedagogical opportunity is a student's event that provides an opportunity to make a specific type of pedagogical move—a move to build on the student's thinking. An occurrence embodies a pedagogical opportunity when it meets two criteria: (i) opening—an instance in which the expression of a student's mathematical thinking seems to create an intellectual need for students to make sense of the student's mathematics, and (ii) timing—the timing must be right to catch a pedagogical move. Timing takes into account the overall plan for the day's lesson, the preparation of other students of the class at that moment to engage with the idea that has emerged, and the context in which the opening emerges.

Although the MOST framework can be used as a lens for prospective teachers to analyze mathematics classroom lessons (Teuscher et al. 2017), in our study, this framework was used as an analytical tool for the researcher (Amador 2019). The MOST

analytical framework is used to identify mathematically productive instances of student's thinking described in the prospective teachers' narratives, in order to study how prospective teachers take advantage of them. We examined the development of prospective teachers' noticing through the changes in the written narratives generated by the feedback received from the others. Changes in the written narratives are seen through changes in the MOST that has been identified, and in prospective teachers' teaching moves in order to take advantage of the MOST.

2.3. Research questions

We formulated two research questions:

- How do prospective teachers learn to notice students' mathematical thinking during the trainee period through the writing of narratives?
- What is the role played by the feedback provided by other colleagues and the university tutor in online forums in supporting the development of noticing?

3. METHOD

3.1. Participants and instrument

The participants were five prospective secondary mathematics teachers (PTs) involved in a Mathematics Teaching Degree at the Distance State University of Costa Rica (UNED). The UNED implements an educational model that does not require face-to-face learning, using virtual environments supported by the institutional Moodle virtual platform.

The Mathematics Teaching Degree consists of four courses (12 four-month periods) and offers training in mathematics, pedagogy, psychology, legislation, and teaching resources for the teaching of mathematics, as well as a traineeship period in secondary education institutions. The data for this research were collected during the traineeship period.

This period is divided into eight stages of two weeks: three stages correspond to a period of classroom observation; and five stages correspond to a period of practice in the classroom where PTs have to implement a didactic unit. In the last five stages, PTs are involved in writing narratives about mathematics classroom situations that can be considered relevant for students' mathematical learning. PTs had guided questions to

write the narratives. These prompts focused on describing a situation that could help to explain how students were learning, interpreting it, and providing alternative directions to teaching. The narratives had to be shared with their colleagues and the university tutor in an online forum of the Moodle platform. In this online forum, PTs received feedback from the university tutor and their colleagues to clarify some ideas. Sharing their narratives in an online forum, receiving feedback from the tutor, and giving feedback to their peers, encourages PTs to develop responsibilities and a sense of participating in their peers' process of becoming mathematics teachers. The tutors' feedback focused on guiding PTs towards the identification of Mathematically Significant Pedagogical Opportunities to build on Student Thinking—MOSTs (Leatham et al. 2015; Rowland and Zazkis 2013), and towards the way to take advantage of them during the lesson.

PTs wrote five narratives during the period of teaching practice and participated in four online forums providing feedback to their classmates. During the classroom observation period, we provided PTs with three examples of narratives and they participated in three online forums sharing comments about the narratives (Figure 1). During these three stages (6 weeks), PTs became familiar with the structure of narratives and how to participate in a forum to discuss the narratives. PTs had theoretical documents related to mathematics teaching and learning that could help them to support their arguments in the narratives and in the online forums.

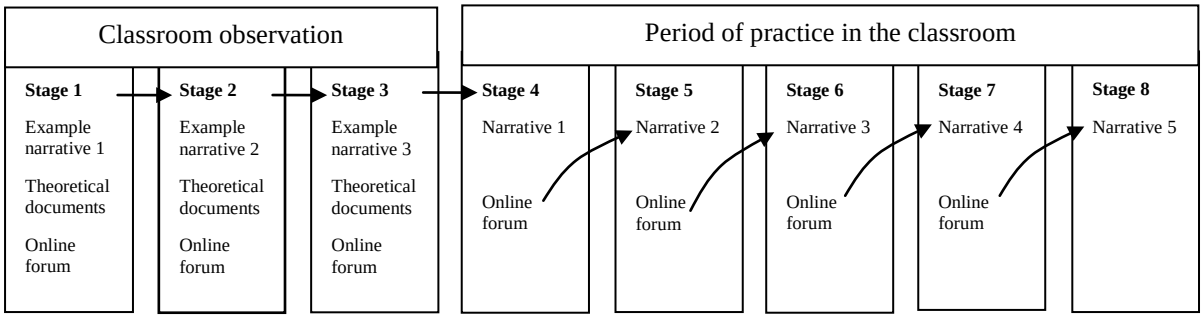

Figure 1. The different stages and activities during the traineeship period

3.2. Analysis

The data are comprised of the 25 narratives written by the five PTs, and the interactions between the four online forums. We analyzed these data in two phases.

In phase 1, we identified students' instances (observable students' actions or small collections of connected actions) in the PTs' narratives that could have the

characteristics of being a MOST (Leatham et al. 2015): students' mathematical thinking, mathematically significant event, and pedagogical opportunity. Next, we identified the moment that had generated each MOST and to which extent the PT took advantage of this moment. To identify the moment that had generated each MOST, we looked at the following aspects (Stockero and Van Zoest 2013) in the narratives:

- When students make a comment or ask a question that is grounded in, but goes beyond, the mathematics that the teacher had planned (extending).
- When students make public, incorrect mathematical thinking or an incorrect solution (incorrect mathematics).
- When the students try to make sense of the mathematics of the lesson (sense making).
- When students make public, two different answers to a problem that clearly should have only one answer (mathematical contradiction).
- When students articulate mathematically what they are confused about (mathematical confusion).

To identify if PTs took advantage of the MOSTs during the lessons, we looked at PTs teaching decisions taken (Stockero and Van Zoest 2013). The categories used were as follows:

- Ignores or dismisses.
- Acknowledges, but continues as planned.
- Emphasizes the mathematical meanings behind the issues, for instance, highlighting a definition or the mathematics underlying a procedure (emphasizes meaning).
- Finds out more about students' thinking by asking the students to provide more information about their thinking (pursues students' thinking).
- Goes beyond the topic in the lesson to revisit and make connections to past learning or to foresee or lay a foundation for future learning (extends/makes connections).

In phase 2, we compared the five narratives written by a PT to identify changes in relation to the MOSTs identified, the type of moment that generated the MOSTs, and

whether PTs took advantage of the MOSTs during the lessons. The nature of these changes provided us with information about the development of noticing. Finally, we analyzed whether these changes had some relation to the feedback provided in the online forum. In particular, we considered the facilitative function of feedback, when it was used to give pointers, pose questions, make suggestions, and offer indirect guidance. In order to carry out this process, we investigated whether changes in the narrative were connected to the content of the feedback generated in the prior narrative. Next, we compared the changes among the narratives of the five PTs to identify the role played by the online feedback in the changes identified. In the following section, we report these changes through the analysis of the narratives of two PTs.

4. RESULTS

Results show that writing and sharing narratives about their own practice in online forums can support the enhancement of PTs' noticing. The enhancement of noticing was observed in the changes in the MOSTs identified by PTs, taking into account whether they took advantage of them. Taking advantage of the MOST during the lesson depended on the type of action that the PT provided. To explain these changes, we describe characteristics from two narratives (narrative 1 and 2) written by a PT (JB) intermingled with the feedback received from the online forum, and from two narratives (narrative 2 and 3) written by another PT (GV) intermingled with the feedback received from the online forum. Both PTs' narratives illustrated changes in the MOST in the way in which they took advantage of the MOST during the lesson, allowing us to display how sharing the narrative in the online forum and receiving feedback supported these changes.

4.1. Characteristics of the first narrative written by JB

In his first narrative, JB described a classroom situation (26 third-year secondary school students, 15-16 years-old), in which students were solving tasks involving factoring polynomials using the methods seen in previous lessons. In his narrative, JB reproduced a dialogue with some of his students who were trying individually to factor the expression $9a^2-4b^2-4b-1$:

1 **Student 1**: Could you tell me if I am doing the exercise correctly?

2 **JB**: Tell me why you think that grouping in this way $(-4b^2-4b-1) + 9a^2$ is the best option.

Student 1: Because a perfect square trinomial is grouped, which is identified because it has the same letter, one square but not the other. On the other hand, a monomial is grouped because it has a different letter.

JB: Great! What you say is fine. But can you apply the difference of squares?

Student 1: No, because there is not a minus sign between the two expressions.

JB: Very well. Then, what can we do in this case?

Student 1: Multiplying by -1, or changing the sign that is the same.

JB: Well, you have to remember that before applying the difference of squares, it is necessary to have the trinomial factored. So, work on this and I'll return later to check it.

I went to answer the questions of another group of students, and as I notice that the majority of them had the same question I decided to make a general clarification on the board.

Student 2: I cannot solve this trinomial, because when I write the square root of -4 in the calculator, there is an error.

JB: That's because it's not a real number (it's a complex number), so check how you have grouped them because you have the extreme terms of the trinomial negative.

Student 2: I think the error occurred because I changed the sign of the monomial instead of the trinomial. If I had changed the sign of the trinomial, the extreme terms would have remained positive.

JB: Exactly, but what would have happened if the trinomial were all positive?

Student 2: Then, the way I did it before was fine.

JB: It is correct, remember the following: if the trinomial has negative extreme terms, it is written secondly for a change of signs to be applied. If the trinomial has positive extreme terms, it is written firstly and if it is necessary the sign of the monomial is changed.

In this part of the narrative, we can evidence that JB identified three MOSTs during the lesson, generated by two types of moments (Table 1): when the students try to make sense of the mathematics of the lesson (sense-making, lines 1-5 and 19-27) and when the students articulate mathematically what they are confused about (confusion, lines 15-18).

Table 1. Types of moments that generated the MOSTs in the narrative 1 of JB.

Student instance	**Description**	**Type of moment**
MOST 1 **Student 1:** Because a perfect square trinomial is grouped, which is identified because it has the same letter, one square but not the other. On the other hand, a monomial is grouped because it has a different letter.	Student 1 tries to make sense of the grouping obtained.	Sense-Making

MOST 2 **Student 2:** I cannot solve this trinomial, because when I write the square root of -4 in the calculator, there is an error.	Student 2 manifests confusion because of the result obtained by the calculator when he/she solved the square root.	Confusion
MOST 3 **Student 2:** I think the error occurred because I changed the sign of the monomial instead of the trinomial. If I had changed the sign of the trinomial, the extreme terms would have remained positive.	Student 2 tries to make sense of the error made in the polynomial factorization.	Sense-Making

The three situations fulfill the characteristics required for a MOST. Firstly, there is a focus on students' mathematical thinking, in this case, on students' difficulties in identifying the factoring method to use, taking into account the characteristics of the polynomials given. The difficulties showed in these three situations happened because students were not able to factor a *-1* to generate a perfect square trinomial and then a difference involving the remaining expression. Furthermore, these three situations showed aspects that were mathematically significant, since identifying the criteria by which monomials are grouped, as well as when a factor of *-1* in an expression can be used applying other factoring methods, are central and appropriate ideas for the achievement of the learning objectives of the lesson. Finally, the students' approaches to the problems created the need to continue building on students' thinking, particularly, towards the identification of the limits of using a factoring method; and the moment of the lesson in which they emerged was adequate, according to the objectives and lesson plan. Therefore, these situations could be considered pedagogical opportunities.

4.1.1. Taking advantage of the MOSTs during the lesson

JB took advantage of MOST1 by pursuing student thinking, for example, saying "Tell me why you think that grouping in this way $(-4b^2-4b-1) + 9a^2$ is the best option" and by emphasizing the mathematics underlined in the procedures "Well, you have to remember that before applying the difference of squares, it is necessary to have the trinomial factored." However, he did not take advantage of MOST2 during the lesson—he acknowledged it, but continued as planned. This is evidenced when JB mentions the term Complex Number, but quickly directed the attention of the student to the exercise that he was trying to factor. JB could have used the student's instance to make a connection with the topic of Complex Numbers, but he did not do it. Finally, JB took advantage of MOST3 by pursuing student thinking when he said "Exactly, but what

would have happened if the trinomial were all positive?" and, again emphasizing the mathematics underlined in the procedures "It is correct, remember …".

Therefore, JB took advantage of two out of the three MOSTs by focusing on pursuing the students' thinking, and by emphasizing the mathematics underlined in the procedures indicating when and how the procedure works. The characteristic of JB's actions is that he encouraged the students to think about when they can apply a factoring method, given a polynomial that had different specific characteristics compared to what students had previously seen. However, at the end of the interaction, JB emphasized the procedure that students have to use, thus providing them with the solution, thus not allowing them to continue building on their thinking (Figure 2).

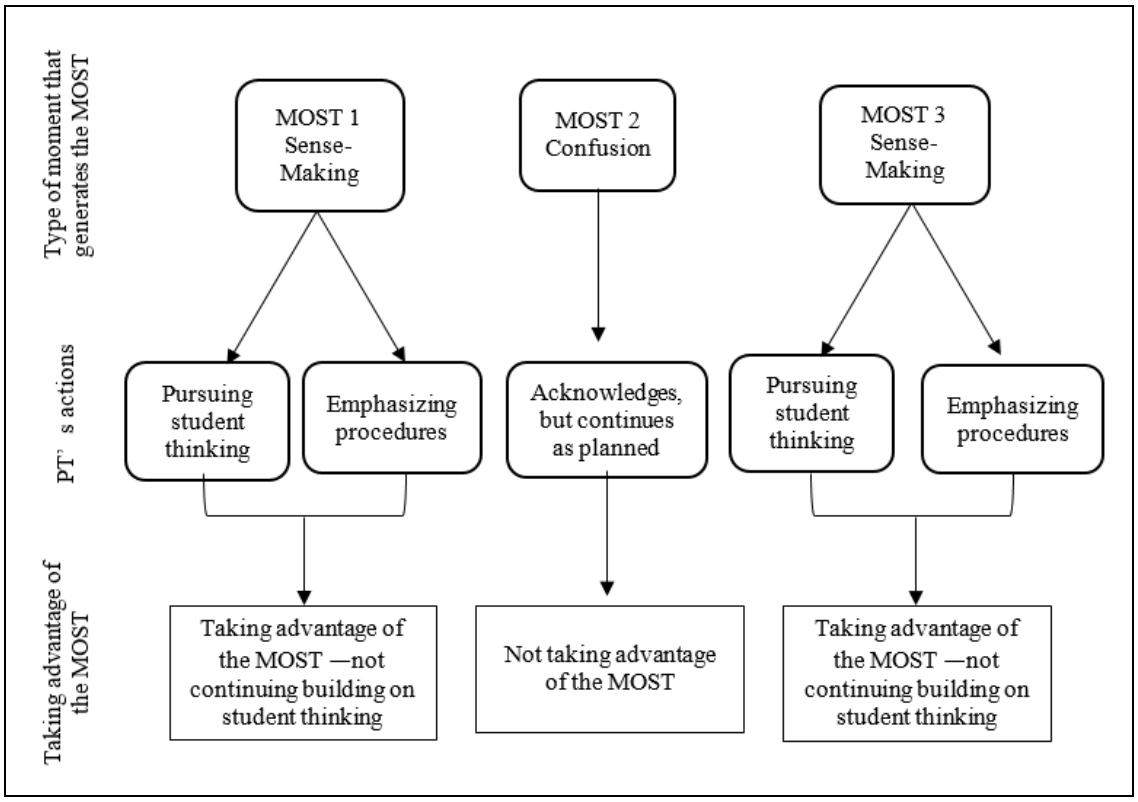

Figure 2. Narrative 1: MOSTs and JB's actions during the lesson.

4.1.2. Sharing narrative 1 in the online forum

The feedback provided by the other PTs and the tutor during the online forum focused on how JB had acted during the lesson. The feedback suggested that JB should promote students' reasoning about the factoring methods. Although JB had taken advantage of two MOST during the lesson, by pursuing students' thinking on when it is possible to apply the factoring methods, finally he emphasized the procedures (remembering the factoring method to be used). For instance, a PT pointed out:

> In many cases, students solve the exercises mechanically, as if all exercises could be solved in the same way. Teachers must promote students' reasoning on their methods.

In this feedback, JB's colleague insisted on the necessity of supporting students to think about whether a procedure (factoring method) can be applied. In this case, we can understand this feedback as trying to make students aware of why the procedures work.

In the same direction the tutor pointed out:

> ... apart from looking at students' mathematical thinking when interacting with them, you must understand this and use it to help the students continue building their thinking, making concrete decisions that help them progress in their learning.
>
> You have to note that students do not have a deep understanding of the use of factoring methods, so we should avoid giving them recipes and try to give them the opportunity to think about the difficulties they have, and why (emphasis added).

This feedback focused on the fact that JB should be more aware of students' thinking, considering the characteristics of the activities. Particularly, feedback aimed at helping JB to focus more on students' thinking, allowing students to reason about when they can use a factoring method (avoiding giving recipes or the correct answer). In this sense, the online feedback focused on being more aware of discerned details about students' mathematical thinking, in order to provide other actions that allow students to continue building their mathematical thinking.

4.2. Characteristics of the second narrative written by JB

After sharing the first narrative in the online forum, JB wrote the second narrative. In this narrative JB described an interaction with two students who are participating in two activities in which they must identify the appropriate factoring method and factor the expressions (Student 1, $5x(3x-2)-3x+2$ and Student 2, $w^2-z^2+4+4w-1-2z$). The two activities have different mathematical characteristics. In Student 1's activity, it is necessary to have a structural view of the polynomial to see the possibility of taking out $(3x-2)$ as common factor. In Student 2's activity, a regrouping of the terms is necessary in order to recognize the two square binomials. However, they have in common the need to factor *-1* first. The choice of JB of these two situations can be understood as being more aware of the role played by the characteristics of polynomials, when he paid attention to the students' thinking during the lesson, as was suggested in the feedback received in the first narrative. JB wrote about the following interaction:

1 **Student 1:** The exercise does not specify by which method it can be solved.

2 **JB**: Yes. The idea is that you identify the factoring method that can be used.

Student 1: In $5x(3x-2)-3x+2$, you must first solve the multiplication of monomials and then, group the expression.

JB: Tell me why you think that is the best option.

Student 1: Because when you have a monomial in front of a parenthesis you have to multiply, in this way, you get $15x^2-10x-3x+2$, that is, $15x^2-13x+2$.

JB: That's right, but once you have done it, how would you solve $15x^2-13x+2$?

Student 1: We can identify that it is a trinomial, so the factorization method of the perfect square trinomial is used.

JB: I'm going to see how Student 2 is solving the exercise and then, I will come back to see how you have done it.

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Student 2: I have a doubt with this exercise $w^2-z^2+4+4w-1-2z$. We can group it in two trinomials, but I do not know to which one I have to put the correct constant.

JB: Let's analyze it for a moment. What would happen if we use -1 as the term of c in w^2+4w, that is, w^2+4w-1.

Student 2: We cannot do it with the perfect square trinomial factorization method since the square root of -1 is not a real number.

JB: That's right. Now if we have w^2+4w+4, what would happen?

Student 2: It can be factored with no problem. If ax^2 is negative, it is grouped with the negative constant and afterwards we can get a -1 as common factor. If it is positive, it is grouped with the positive constant.

JB ends with Student 2 and returns to Student 1.

**

Student 1: I couldn't solve it with a perfect square trinomial because 2 and 15 do not have an exact root.

JB: Then, how can you solve this algebraic expression?

Student 1: Looking this expression $5x(3x-2)-3x+2$, we can take out $(3x-2)$ as common factor.

JB described two situations that can be considered MOSTs (Table 2): when the student 1 makes public incorrect mathematical thinking and an incorrect solution (incorrect mathematics, lines 1-12) and when the student 2 articulates mathematically what he is confused about (confusion, lines 14-23).

Table 2. Types of moments that generated the MOSTs in narrative 2 of JB.

Student instance	**Description**	**Type of moment**
MOST 1 **Student 1**: In 5x(3x-2)-3x+2, you must first solve the multiplication of monomials and then, group the expression.	Student 1 uses an incorrect method and she cannot factor the polynomial.	Incorrect mathematics
MOST 2 **Student 2:** I have a doubt with this exercise w2-z2+4+4w-1-2z. We can group it in two trinomials, but I do not know to which one I have to put the correct constant.	Student 2 knows how to continue but he manifests his confusion *I do not know to which one I have to put the correct constant.*	Confusion

The two situations have the characteristics required for them to be MOSTs. Firstly, students' mathematical thinking (in this case related to difficulties) can be inferred in the two situations. Student 1 has difficulties with the identification of the factoring method to be used since she approaches the task procedurally without reflecting on the structure of the polynomial (using *3x-2* as common factor). Student 2 manifests also confusion when he is grouping, since he does not recognize the two square binomials. Developing a structural view of polynomials in order to choose the right factorization method is central and appropriate for the achievement of the learning objectives of the lesson (mathematically significant event). Finally, the situations can be considered as pedagogical opportunities since they create the possibility of continuing building on students' thinking, in this case, moving from a procedural to a structural view of polynomials to choose the right factorization method. Furthermore, the moment of the lesson in which they emerge is adequate according to the objectives and lesson plan.

4.2.1. Taking advantage of the MOSTs during the lesson

JB took advantage of the MOST1 by pursuing Student 1's thinking, when he said "Tell me why you think that it is the best option," or "That's right, but once you have done it, how would you solve $15x^2$-13x+2?" JB invited Student 1 to investigate, and to realize

that the chosen method was not appropriate. Furthermore, JB took advantage of the MOST2 also by pursuing Student 2's thinking, through questions such as "Let's analyse it for a moment. What would happen if we use -1 as the term of c in w^2+4w, that is, w^2+4w-1" or "That's right. Now if we have w^2+4w+4, what would happen?" Again, JB invited Student 2 to inquire about his confusion and error.

JB took advantage of the two MOSTs during the lesson, pursuing students' thinking by asking questions that in both cases invited investigation about why the chosen factorization methods could not be applied. JB's actions help both students to continue building on their thinking (without reminding them or giving them the procedure to use). We can say that JB took advantage of the MOST and focused on continuing building on students' thinking (Figure 3).

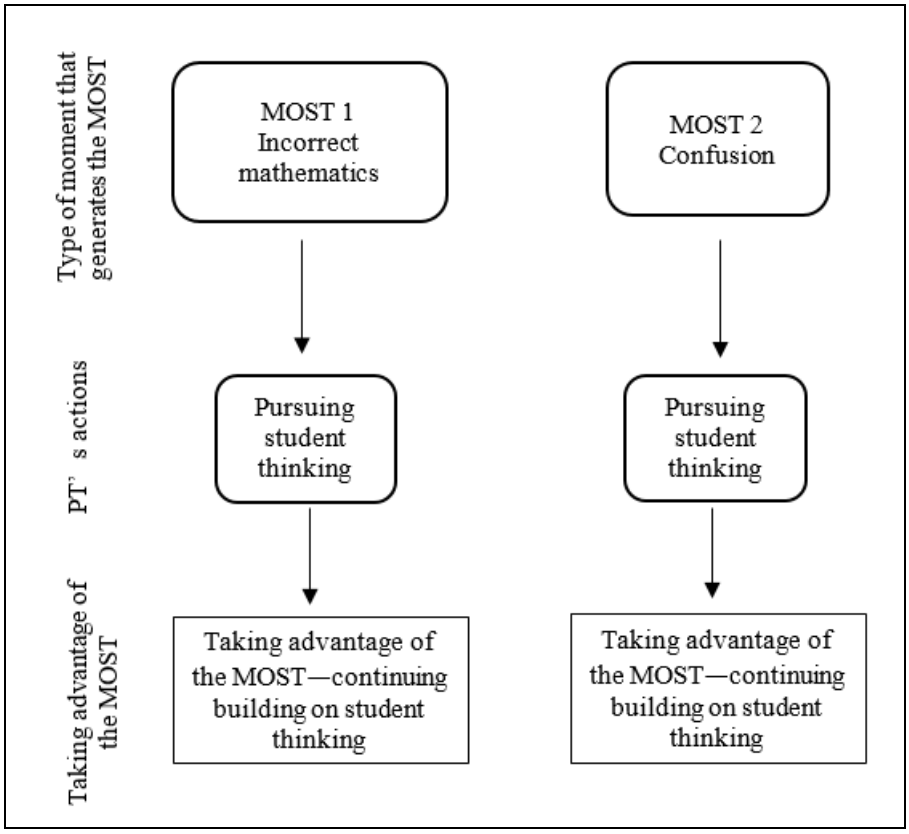
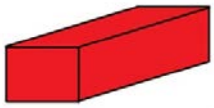


Figure 3. Narrative 2: MOSTs and JB's actions during the lesson.

4.3. Characteristics of the second narrative written by GV

In the second narrative, GV described an interaction with two students during a lesson whose objective was "to recognize the different elements in three-dimensional figures such as faces, edges and vertices, and to establish relationships between the elements".

Students, working in pairs, had to answer the following questions: (i) Identify a vertex, an edge and a plane; (ii) Which edges share the same vertex? (iii) Identify two parallel edges and two perpendicular edges; (iv) Identify two parallel and two perpendicular planes; (v) Explain the differences between the toothpaste box (red box) and the empty roll of toilet paper.




GV: What is the parallel face to ABCD?

Student 1: The face EFGH.

GV: Perfect. Could you explain it to me, please?

Student 1: Because, this face is behind.

GV: Fantastic. Remember that faces are part of planes and, they are parallel if they never touch each other.

…

Student 2: The roll has neither faces nor vertices, unlike the box.

GV: Right! You understood the idea. Later we will reinforce these differences.

GV identified two MOSTs during the lesson generated by the same type of moment (Table 3): when the students try to make sense of the mathematics of the lesson (sense-making; line 4 and line 8, respectively).

Table 3. Types of moments that generated the MOSTs in the narrative 2 of GV.

Student instance	**Description**	**Type of moment**
MOST 1 **Student 1:** Because, this face is behind.	Student 1 tries to make sense of the idea of parallel faces.	Sense-making
MOST 2 **Student 2:** The roll has neither faces nor vertices, unlike the box.	Student 2 tries to make sense of the differences between the box and the roll.	Sense-making

The two situations described by GV in his second narrative have the characteristics required for being a MOST. Firstly, students' mathematical thinking can be inferred. In the two situations, students were giving sense to some relationships between elements and properties of three-dimensional figures. Furthermore, recognizing the elements and their relationships is central and appropriate for the achievement of the learning

objectives of the lesson (mathematically significant event). Finally, students' mathematical thinking in each situation created a need to continue building on their mathematical thinking: the relationship between faces—parallel faces— (in the first case), and the different between curved geometric bodies and polyhedra (in the second case) The moment of the lesson in which they emerged is adequate according to the objectives and the lesson plan (pedagogical opportunity).

4.3.1. Taking advantage of the MOSTs during the lesson

GV took advantage of MOST1 by pursuing Student 1's thinking when he said "Perfect. Could you explain it to me, please?" and by emphasizing the mathematics underlined in the task (definition of parallelism): "Fantastic. Remember that faces are part of planes and they are parallel if they never touch each other". However, he did not take advantage of the MOST2 during the lesson —he acknowledged it, but continued as planned. This aspect was evidenced when GV identified that student 2 had understood the idea (differences between the box and the roll) but said that he will continue with these differences later. GV could have made use of that instance to make connections/differences between curve geometric bodies and polyhedra, but he did not do so.

Therefore, GV took advantage of one out of the two MOST by focusing on pursuing the students' thinking, and by emphasizing the mathematics underlined in the task. The characteristic of GV's actions was that he encouraged the students to think about the relationships between elements in a three-dimensional figure and about the definitions. However, at the end of the interaction with student 1, GV provided him with the definition (the student had delivered a vague idea of parallel faces), not allowing him to continue building on his thinking (Figure 4).

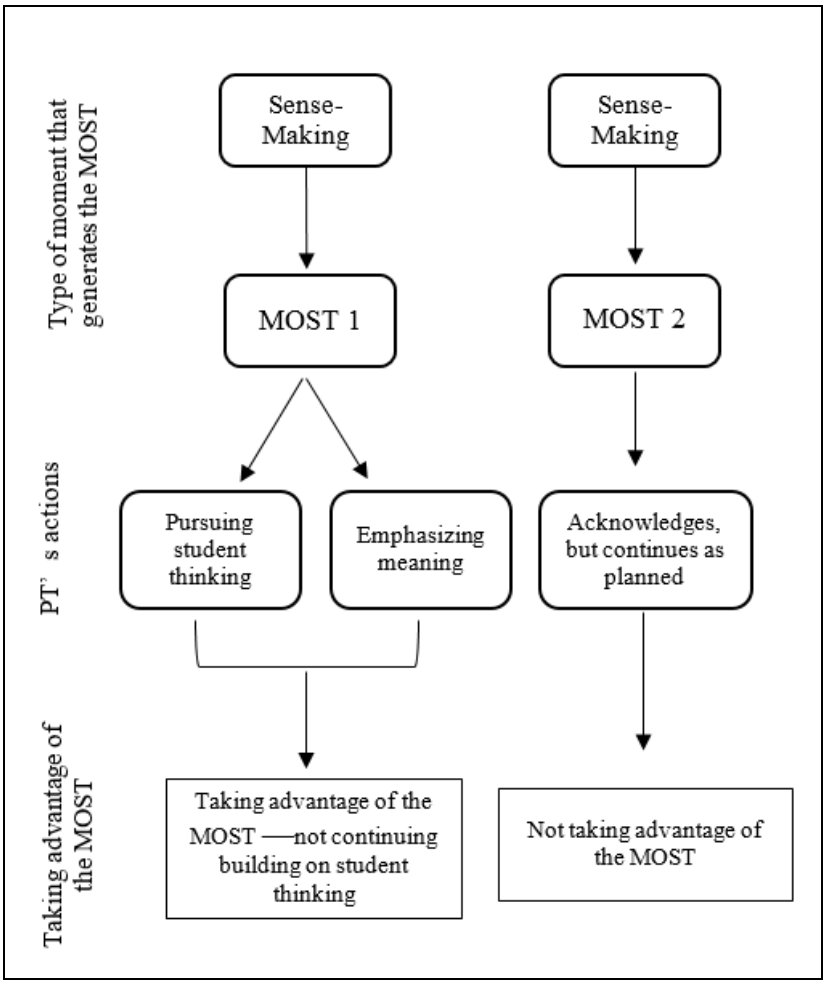


Figure 4. Narrative 2: MOSTs and GV's actions during the lesson.

4.3.2. Sharing narrative 2 in the online forum

The feedback provided to GV's narrative by the other PTs and the tutor during the online forum suggested that GV should promote students' reasoning about the definitions of the elements of three-dimensional figures and their relationships. Although GV had taken advantage of a MOST during the lesson by pursuing the student's thinking, inviting him to give explanations, finally he emphasized the mathematics (remembering the definition). For instance, a PT pointed out:

> Although GV not only explains the concepts but also tries to know what students think, it is important for students to be able to build their knowledge without too many indications.

In this feedback, GV's colleague insisted on the necessity of supporting students to think about the definitions of the elements of three-dimensional figures they used and the relationships between them. In this case, we can understand this feedback as an attempt to make students aware of the definitions they are providing.

In the same direction the tutor pointed out:

> ... You need to be aware of students' mathematical thinking when interacting with them, and must interpret it to help students continue building their thinking (emphasis added)
>
> The characteristics of an activity can generate difficulties in students and we need to identify them during the lesson and try to give students the opportunity to think about these difficulties. It is advisable to provide students with time to think about the difficulties before guiding them to the correct answer.

This feedback was aimed at helping GV to focus more on students' thinking, allowing students to think about the definitions they provided (avoiding facilitating them with a correct definition). In this sense (as in the case of JB), the online feedback focused on being more aware of discerned details about students' mathematical thinking, in order to provide alternative actions that would allow students to continue building on their mathematical thinking.

4.4. Characteristics of the third narrative written by GV

After sharing the second narrative in the online forum, GV wrote the third narrative. In this narrative GV described an interaction with three students during a lesson whose objective was to "recognize different angles: adjacent, complementary and supplementary". Students worked in groups of 5-6 students with a round frame (a hula hoop) and ropes. They needed to tie two ropes to the hula hoop with the instruction that both ropes should be intersected. Then, they had to study the measures of the different angles formed by the intersected ropes. GV's choice of this activity for writing the narrative can be understood as a higher awareness of the role played by the manipulatives in giving students time to think about the definitions and their relationships (in this case about the definitions of the different angles).

GV: What happened when you changed the position of a rope?

Student 1: The angles changed.

GV: How did they change?

Student 1: These angles became larger and these ones became smaller (pointing out the pairs of opposite angles by the vertex).

GV: Now, what is the measure of them?

Student 1: Before, this angle measured 80º and now it measures 95º.

GV: And the others?

Student 2: 120º before, and now it measures 105º.

GV: Well-done. What relationship do they have? You have noticed that something changes, but also that there are things that do not change. Could you find them?

Student 1: These (pointing out a pair of opposite angles in the intersected ropes) measure the same.

Student 3: And the same with these two angles (pointing out the other pair of opposite angles).

GV: Excellent. You have identified that the opposite angles by a vertex measure the same. But, what will happen with this and this (pointing out the adjacent angles)?

Student 3: They add to 180º.

GV describes a situation that can be considered a MOST (Table 4), namely, when student 1 tries to make sense of the mathematics of the lesson (sense-making; lines 2 and 4).

Table 4. Types of moments that generated the MOST in the narrative 3 of GV.

Student instance	Description	Type of moment
MOST 1 **Student 1:** The angles changed. **Student 1:** These angles became larger and these ones became smaller (pointing out the pairs of opposite angles by the vertex).	Student 1 tries to make sense of the angles formed by the two ropes.	Sense-making

This situation fulfills the characteristics required for being a MOST. Firstly, students' mathematical thinking (in this case related to giving meaning to the angles formed by the two ropes) can be inferred. Furthermore, to recognize the different angles formed is central and appropriate for the achievement of the learning objectives of the lesson (mathematically significant event). Finally, the situation can be considered as a pedagogical opportunity since it creates a need to continue building on students' thinking, in this case, identifying the relationships between the angles formed. Besides, the moment of the lesson in which they emerge is adequate according to the objectives and lesson plan.

4.4.1. Taking advantage of the MOST during the lesson

GV took advantage of the MOST during the lesson, pursuing students' thinking by asking questions that students could investigate about the different angles formed and the relationships between them. GV's action helps students to continue building on their thinking (providing them neither with the solution nor with the definitions of the

angles). We can say that GV took advantage of the MOST and focused on continuing building on students' thinking (Figure 5).

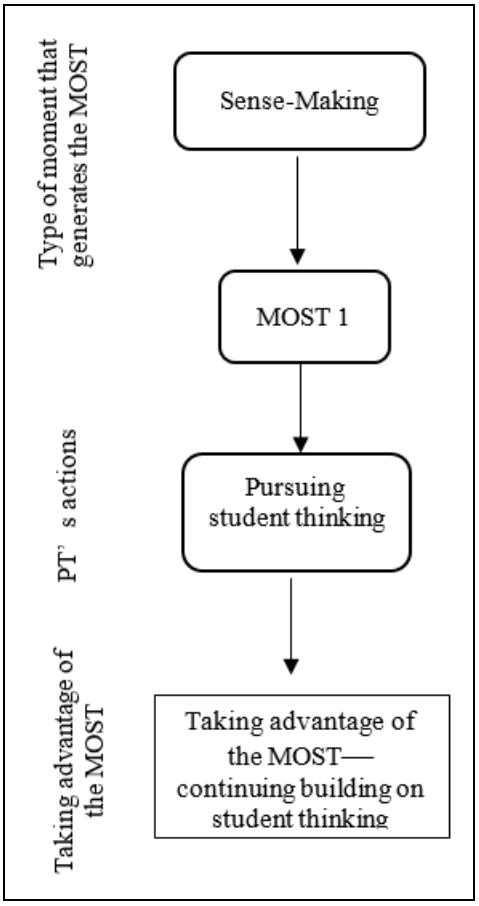


Figure 5. Narrative 3: MOST and GV's actions during the lesson.

5. DISCUSSION AND CONCLUSIONS

Our study contributes to the field of research based on how PTs learn to notice teaching-learning situations when they share narratives about their own teaching during the period of teaching traineeship in an online forum. We present our conclusions in two domains. First, we show the characteristics of how prospective teachers learn to notice students' mathematical thinking by writing and sharing their narratives in an online forum as a part of an online distance-learning teacher training program. Second, we discuss the role of the online feedback in enhancing prospective teachers' noticing.

5.1. The way PTs learn to notice: the role of writing and sharing narratives in an online forum

The results show how the PTs learned to notice in an online distance-learning teacher training program, writing and sharing narratives in an online forum. This learning was evidenced through changes in how the PTs identified MOSTs and took advantage of them. These changes were evidenced when the PTs described in more detail the students' instances as MOSTs, taking advantage of them through actions that allowed the students to continue building their thinking. These changes suggest that writing and sharing narratives in an online forum seem to favor the enhancement of PTs' noticing, since the PTs progressively focused more on students' mathematical thinking and provided actions to continue building on students' thinking. Writing narratives and sharing them in a forum, with the possibility of writing and reading feedback from others, is different to the situation of participating only in debriefing conversations between an observer-tutor and the prospective teacher. In this context, writing is the activity that mediates the process of noticing and how it is communicated to others. So, writing (narratives and feedback to other narratives) and reading (narratives and feedback from others) is what makes this learning context different to other traditional face-to-face contexts.

A characteristic of this learning is the fact that taking advantage of the MOSTs during the lesson depended on the type of action that the PTs generated. Actions such as ignoring or acknowledging, but continuing as planned (Stockero and Van Zoest 2013), did not allow students to take advantage of the MOSTs. Furthermore, the PTs who took advantage of the MOSTs by pursuing students' thinking, did not always continue building on students' mathematical thinking.

Prompts provided to PTs to write the narratives, and the online context in which they shared them, focused the PTs attention on students' thinking, and on what the role of the teacher should be in order to promote students' conceptual progression. This is a relevant aspect since there is a shift from a focus on themselves to a consideration of how their actions can influence students' learning outcomes (Cavahagh and McMaster 2015).

In the context of an online distance education program, narratives can be seen as cultural artifacts, and writing in the online forum as a goal-directed activity that creates a form of argumentation (Wells 1999). In fact, online forums provide a collaborative interface where meanings can be negotiated and knowledge-building can be supported (Fernández et al. 2012; Llinares and Valls 2009; 2010). The changes observed between

narratives suggest that the process of dialogic argumentation, in the sense of developing argumentative reasoning as a process of giving reasons, generated in the online forum, fostered that PTs were more aware of important details in teaching situations, thus helping them to identify students' instances as MOSTs and to take advantage of them. In this context, narratives can be considered as a mediation tool between PTs, their own practice and the feedback from colleagues and university tutors. From the results, writing and sharing narratives let the PTs make sense of their own practice, that is, to frame practical situations, supporting their noticing enhancement. However, perhaps other factors might also have favored the PTs' development of noticing.

5.2. The role of feedback in online forums for enhancing PTs' noticing

Our analysis shows that the facilitative function of feedback, in which the university tutor and other colleagues gave pointers, posed questions, made suggestions and offered indirect guidance, was a context to focus PT attention on students' thinking. Since noticing is a knowledge-based reasoning process, its development implies that PTs reason about students' thinking and about their actions. Feedback in online forums provided PTs with the opportunity to think more deeply about students' mathematical thinking. In this sense, the role of feedback in this online distance teacher education program has a role similar to the online interaction reported in other studies, indicating that providing and receiving feedback allowed the improvement of PTs' learning (Clay et al. 2012; Huisman et al. 2019; Yang, 2016; van der Pol et al. 2008). In the words of Borba and colleges (2018), a forum in an online distance course "is part of the collective of humans-with-media that produces knowledge at that moment" (p. 278). The notion of human-with-media refers to the idea that media shape the way humans think and, conversely, the way humans think shapes media. In these online distance courses, interaction (regarded in this research as the feedback provided in the online forum) plays a key role. In fact, interaction becomes necessary as the exchange of experiences and the exposure to reasoning are actions that favor the construction of knowledge.

Our findings show that the feedback was addressed to giving more details of students' thinking and to making explicit the reasons behind the teachers' actions. In fact, the feedback seemed to have made the PTs become more aware of the relation between students' mathematical thinking and their teaching actions. Therefore, the tutor's feedback pushed PTs to generate practical arguments (Fenstermacher and Richardson 1993) in the sense of providing reasons for their actions that could be seen as premises

connected to an action. Through this feedback, we obtained evidence of PTs' noticing enhancement and how PTs actions (practice) began to change. As has been previously mentioned, other factors of the online program may have influenced the PTs' changes; nonetheless the feedback in an online forum can be regarded as a tool that helped PTs' enhancement of noticing. In further research, it would be interesting to examine the PTs' views about the feedback provided by the tutor and the other colleagues in the online forum.

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