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INNOVATIVE BUSINESS MODELS IN ONLINE RETAILING

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Declaration of Authorship

I hereby declare that this PhD dissertation is my original work
and it has been written by me in its entirety.
I have duly acknowledged all the sources of information
which have been used in this dissertation.

This PhD dissertation has also not been submitted for any degree
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Innovative Business Models in Online Retailing

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Abstract

Internet has opened the door for e-commerce and created a business avenue, online retailing. E-commerce presently shapes the manner in which consumers shop for products. The online retailing markets have grown by 56% during the past five years, while traditional retailing markets are only grown by 2% during the same time. The noticeable growth of online retailing creates numerous opportunities as well as challenges for the context of operations management.

Extensive literature in this domain focus on the conventional inventory management and pricing problems as in traditional retailing. However, the rapid development of information technology threatens the established business models and creates opportunities for new business models. Companies may find it increasingly difficult to make strategic decisions, such as how to deal with the challenge associated with online retailing and how to adapt to the new retailing environment. This thesis aims to investigate innovative business models involved in online retailing, to capture trendy phenomena that are under-studied, and provide managerial insights.

The first chapter focuses on dealing with the logistics challenge caused by the booming e-commerce activities. An urban consolidation center (UCC) or a peer-to-peer platform may alleviate the economic, social and environmental pressure on well-being. We compare the performance of these two business models to guide a consolidator to make efficient operational decisions. The second chapter focuses on the channel management decisions of a retailer who operates an offline (brick-and-mortar) channel and

an online channel. The two channels are either operated separately or integrated. We explore how the retailer can profitably integrate her offline and online channels, from a perspective of product descriptions and consumer reviews. The last chapter focuses on a seller's decisions in the process of entering the online market through online marketplaces. In addition to pure-play marketplaces, some marketplaces also sell their own products directly competing with sellers, which creates a new form of channel conflict. We analyze the optimal decisions for both the seller and the marketplaces to characterize the system equilibrium.

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Chapter 1

Introduction

The internet has opened the door for electronic commerce (e-commerce) and created a business avenue, online retailing. E-commerce presently shapes the manner in which consumers shop for products. The online retailing markets have grown by 56% during the past five years, while traditional retailing markets are only grown by 2% during the same time. The noticeable growth of online market creates numerous opportunities as well as challenges for the context of operations management. With such a large potential, online retailing has attracted tremendous attention from researchers in the past few years.

Extensive literature in online retailing focus on the conventional inventory management and pricing problems as in traditional retailing. However, the rapid development of information technology threatens the established business models and creates opportunities for new business models. Companies engaged in online retailing may find it increasingly difficult to make strategic decisions, such as how to deal with the challenge associated with online retailing and how to adapt to the new retailing environment. This thesis aims to investigate innovative business models involved in online retailing, to capture trendy phenomena that are under-studied, and provide managerial insights for practitioners.

Chapter 2 focuses on dealing with the logistics challenge caused by the booming e-commerce activities and the growing population. These phenomena create huge demand for urban last-mile delivery, exerting intense pressure on the cities' well-being. To build a city with congestion and pollution under control, a consolidator can operate an urban consolidation center (UCC) to bundle shipments from multiple carriers before the last-mile delivery. Alternatively, the consolidator can operate a peer-to-peer platform for the carriers to share their delivery capacity.

Our objective is to compare the performance of these two business models to guid the consolidator to make efficient operational decisions. Under each business model, we study the interactions between a consolidator and multiple carriers using a two-period game-theoretical model. In each period, the consolidator first chooses a delivery fee to maximize her expected profit. Each carrier then observes his task volume, and decides whether to deliver on his own or use the consolidator's service to minimize his expected cost.

Under the UCC model, the carriers become more dependent on the UCC to deliver their tasks as their variable delivery cost increases or their logistics reestablishment cost decreases. Under the platform model, the carriers generally keep their logistics capability (even if they purchase capacity from the platform) in equilibrium to ensure their flexibility of selling capacity on the platform. Between the two business models, it is generally more profitable for the consolidator to operate the UCC than the platform if the carriers' fixed delivery cost is large. Furthermore, the UCC becomes more dominant as there are more carriers. If the number of carriers is large, it is also more efficient for the consolidator to operate the UCC than the platform to reduce the expected social-environmental cost. Otherwise, the platform is more efficient.

Chapter 3 focuses on the channel management decisions of a retailer who sells a single product to consumers through an offline (brick-and-mortar) channel and an online channel. The consumers in each channel are heterogeneous such that the product fits the tastes of only a fraction of these consumers. The retailer provides a product description for each channel to help the consumers assess whether the product fits their tastes. The two channels are either operated separately with different product description levels or integrated with a common product description level.

To explore how the retailer can profitably integrate her offline and online channels, we construct a two-period game-theoretical model in which the retailer optimizes the product description levels to maximize her expected profit. We find that integrating the offline and online channels yields more profit for the retailer if and only if the offline channel's product description limit and the consumers' base product valuation are small.

We further consider a review system where the consumers who purchase the product in period 1 may post their reviews. The fraction of positive reviews in period 1 will influence the purchase intention of the upcoming consumers in period 2. In the presence of the consumer reviews, even if the offline product description limit is large, it can still be more profitable for the retailer to integrate the offline and online channels. Furthermore, the consumer reviews may reduce the retailer's profit if the consumers' base product valuation is sufficiently large.

Chapter 4 focuses on a seller's decisions in the process of entering the online market through online marketplaces. In addition to many pure-play marketplaces, some marketplaces also sell their own products directly to consumers. As a result, if the seller sells her products through those marketplaces, she may find herself in direct competition with the marketplaces. This creates a new form of channel conflict, which is one of the focus of this chapter. We are also interested in the competition between different marketplaces, since consumers may have their own preference over different marketplaces when they shop for products.

To analyze the optimal decisions of through which marketplace(s) to sell products for the seller, we construct a game-theoretical model to capture the main trade-off in this process. We consider a setting in which one pure-play marketplace only provides marketplace service to sellers, and one marketplace also sells its own product directly to consumers. We consider a seller, who would like to sell a single product to consumers and she will decide through which marketplace(s) to sell her products, and at what price(s). By hosting the seller, each marketplace decides a revenue sharing commission. We analyze the optimal decisions for both the retailer and the marketplaces, and characterize the system equilibrium.

In general, as the commission charged by one marketplace becomes higher, the seller tends to sell its product through the other marketplace. If the price of the competing product in the marketplace becomes higher, the seller tends to charge a lower price to further attract more consumers. However, if the price of the competing product becomes lower, the seller should charge a higher price to focus on a small group of consumers to maximize its profit. In equilibrium, the seller will only sell through the pure-play marketplace and set a lower price than the price of the competing product in the other marketplace.

Chapter 2

Urban Consolidation Center or Peer-to-Peer Platform? The Solution to Urban Last-Mile Delivery

2.1 Introduction

Last-mile delivery is the last leg of a supply chain that transfers freight or products from a distribution center to a receiver. It comprises up to 28% of the total delivery cost of a supply chain (Lopez, 2017, Wang et al., 2016). Managing last-mile delivery becomes especially challenging if it is performed in an urban area, where congestion increases fuel consumption, causes delay of delivery, and lowers delivery efficiency (Ranieri et al., 2018). In addition, last-mile delivery is the most expensive and critical operation for companies engaged in e-commerce (Lee and Whang, 2001). Due to the continuous growth of urban population and e-commerce activities, last-mile delivery to a city center exerts intense pressure on the city's economic, social, and environmental well-being (Quak and Tavasszy, 2011).

The economic impact of urban last-mile delivery includes the waste of resources due to extra waiting in traffic congestion and low utilization of uncoordinated vehicles transporting freight to the city center. The large number of small, individual customer orders in e-commerce further complicates urban last-mile delivery and incurs significant costs. The social-environmental impact includes the vicious effect of the increasing traffic incidents and pollution due to transport vehicles, which degrades the quality of life in the city. For example, based on the Beijing Municipal Environmental Monitoring Center's statistics, emissions of transport vehicles are the main source of PM2.5 that causes hazardous haze in Beijing (`http://www.bjmemc.com.cn/`).

To build a smart city with congestion and pollution under control, an *urban consolidation center* (UCC) is a potential solution to mitigate the repercussion of urban last-mile delivery. Also known as a *city distribution center* (van Duin et al., 2008) or an *urban distribution center* (Boudoin et al., 2014), a UCC consolidates shipments from multiple carriers and then delivers them to the city center using the UCC's own fleet of trucks. A consolidator operating a UCC usually requires a facility to sort the shipments according to their destinations before they are delivered. As a result of the consolidation with fewer trucks, higher truck utilization can be achieved, leading to a lower delivery cost. This shipment consolidation not only economically benefits stakeholders, including the consolidator, the carriers, and the public authorities (Ambrosini and Routhier, 2004), but also mitigates the social-environmental impact because of reduced traffic. Ideally, the resultant cost savings can be shared among the carriers, motivating them to use the UCC's service.

Despite the potential benefits, many UCC projects in practice are not successful. The UCCs of the Port Authority of New York and New Jersey were closed after five years of operations (Doig, 2001). Dablanc (2011) reports that 150 UCC projects were started in Europe during the last 25 years, but only five projects survive. Even if they survive, they usually have difficulty to break even and require significant subsidies from the government. For example, it costs a UCC in La Rochelle 3.8€ to deliver a parcel to a customer who is charged only 1.7–3€. A UCC in Monaco charges her customers 2.30€/100Kg, and receives 2.59€/100Kg as a subsidy from the local government (Dablanc, 2005). Many UCC projects failed because the carriers were reluctant to use their service. This is supported by a survey in the NYC metro, which reveals that less than 20% of the carriers would like to participate in a UCC project (Holguin-Veras et al., 2008). Their reluctance to participate is mainly due to a common concern that they may over rely on the UCCs. Many carriers reduce their own logistics capacity after using a consolidation service (Snapp, 2012, Vivaldini et al., 2012, Choe et al., 2017). For example, the logistics department of GOME, a Chinese retailer for electrical appliances, reduces its investment in delivery trucks and drivers after engaging a consolidation service (National Express, 2010). The substantial cost of reestablishing the logistics capability, which includes the costs to purchase trucks, recruit drivers, obtain licenses, and gain knowledge about local clients (Browne et al., 2005), makes the carriers reluctant to rely on a UCC's service.

More recently, some peer-to-peer platforms have been established for carriers to share their delivery capacity. Notable examples include Saloodo! by DHL, Freightos and Convoy in Europe, Loadsmart in U.S., and Cainiao and Truck Alliance in China. On such a platform, a carrier can sell his unused capacity to another carrier to fulfill the latter's delivery needs. It is attractive for a consolidator to operate a platform because it requires neither a sorting facility nor a fleet of delivery trucks. The peer-to-peer platform business model typically follows a sharing-economy approach: The platform takes a revenue share from each transaction of capacity for providing market access to the carriers and for processing the transaction (Gesing, 2017). In contrast to the UCCs' low success rate, the emergence of the capacity sharing platforms motivates us to investigate whether the latter can be a better alternative for a smart city to address the challenges of urban last-mile delivery.

Although bearing the delivery costs, a UCC can achieve a larger economy of scale as each truck of the UCC may consolidate the tasks of many carriers. In contrast, a capacity sharing platform does not incur any delivery cost, but each individual carrier on the platform has only very limited delivery capacity compared to the UCC's fleet. In this chapter, we investigate the performance of the two business models above (the UCC versus the capacity sharing platform) in terms of the consolidator's profit and the social-environmental impact. Specifically, for each business model we develop a two-period game-theoretical model to capture the interactions between a consolidator and multiple carriers. In each period, knowing that each carrier has a delivery task with a random volume to fulfill, the consolidator first determines the delivery fee to maximize her expected profit. Then, after knowing his task volume, each carrier decides whether to deliver on his own or use the consolidator's service to deliver his task to the city center such that his expected cost is minimized. We identify subgame perfect Nash Equilibrium with rational expectations for each model. We have obtained the following insights.

(i) Our results of the UCC model manifest the trade-off faced by the carriers in practice: The carriers can potentially save their delivery costs by using the UCC's service, while they face the risk of losing their logistics capability. As their variable delivery cost increases, the carriers become more dependent on the UCC to save their delivery costs. On the other hand, as the cost to reestablish their logistics capability increases, the carriers become less dependent on the UCC to avoid the risk of losing their logistics capability.

(ii) Our results of the capacity sharing platform model explain the increasing popularity of the capacity sharing platforms in practice. In equilibrium, we find that the carriers generally have their logistics capability on hand (even if they purchase capacity from the platform). This ensures sufficient capacity available on the platform to facilitate successful transactions. Since the platform can always earn a positive profit from each transaction, it can be more financially sustainable in the long run.

(iii) Comparing the UCC and the platform in terms of their expected profit shows that the UCC is generally more profitable than the platform if the carriers' fixed delivery cost is large. Moreover, it is easier for the UCC to dominate as the number of carriers becomes larger. We also compare the UCC and the platform in terms of their social-environmental impact. The analysis shows that if the number of carriers is large, then the UCC is more efficient in reducing the expected social-environmental cost than the platform. Furthermore, the condition for the UCC to outperform the platform varies with the distribution of the carriers' task volumes.

After reviewing the related literature in §2.2, we formulate and analyze the UCC model in §2.3 and the platform model in §2.4. We compare the two business models in terms of their expected profits and their social-environmental impact in §2.5. We study two extensions of our models in §2.6, before we provide concluding remarks in §2.7. All proofs are provided in Appendix A.

2.2 Literature review

This research is mainly related to two streams of literature. The first stream consists of papers on UCCs and the second stream is about peer-to-peer platforms. The majority of studies on UCCs is conceptual and descriptive. McDermott (1975) shows in a survey conducted in Columbus, Ohio that operating a UCC could bring substantial benefits to the shippers, carriers, consumers, society, and government. Based on a program in the European network, Dablanc (2007) concludes that the provision of urban logistics services emerges slowly despite their growing demand. Allen et al. (2012) review the feasibility studies, trials, and fully operational schemes of UCCs in 17 countries in the last 40 years.

Some analytical papers on UCCs focus on planning and allocation of delivery jobs among the carriers. For example, Crainic et al. (2009) consider a two-tier distribution structure and propose an optimization model to deal with job scheduling, resource management, and route selection. Handoko et al. (2016) propose an auction mechanism for last-mile delivery to match a UCC's truck capacity to the shipments such that the UCC's profit is maximized. Wang et al. (2015) study a rolling-horizon auction mechanism with virtual pricing of shipping capacity. Wang et al. (2018) consider cost uncertainty in last-mile delivery through a UCC, and propose approaches to solve the winner determination problem of an auction. Özener and Ergun (2008) study a logistics network in which shippers collaborate and bundle their shipment requests to negotiate better rates with a common carrier. They determine an optimal route covering all the demands such that the total cost is minimized. To the best of our knowledge, no papers have formally analyzed the stakeholders' incentives for a UCC project. Our research fills the gaps in the literature by providing a game-theoretical analysis of the carriers' incentive to participate in a UCC project.

The ideas of the capacity sharing platform relate our research to the literature on two-sided markets (Rochet and Tirole, 2006, Weyl, 2010, Hagiu and Wright, 2015). A typical setting of a two-sided market involves two types of players. On a platform, independent providers (such as drivers) offer service to consumers (such as riders). See, for example, Cachon et al. (2017), Bai et al. (2018), Taylor (2018), Bimpikis et al. (2016), Cohen and Zhang (2017), and Hu and Zhou (2017). In contrast, a carrier on the platform in our research is flexible to choose either to sell his remaining capacity like a service provider or to buy capacity like a consumer.

Several papers in operations management deal with peer-to-peer rental platforms, which are similar to our capacity sharing platform in spirit. For example, Fraiberger and Sundararajan (2015) analyze a peer-to-peer rental market where each consumer is either a supplier or a buyer. Benjaafar et al. (2018) analyze a model where players with different usage levels make decisions on whether to own a product. Non-owners can access the product through renting from owners on a needed basis. Jiang and Tian (2016) consider a setting in which consumers who purchased a product can derive different usage values and generate income by renting out their purchased product through a third-party sharing platform. Tian and Jiang (2018) further study how this consumer-to-consumer product sharing affects a distribution channel. Abhishek et al. (2016) consider a setting in which a consumer decides whether to purchase a durable good and whether to rent it when the rental market is available. In the stream of literature above, if an owner decides to rent out his product, he cannot use the product during the rental period. In contrast, a carrier on our capacity sharing platform does not rent out his entire truck. Instead, he uses his remaining truck capacity to deliver goods for another carrier to earn extra revenue. Benjaafar et al. (2017) consider a ride sharing platform on which individuals may rent out empty seats from their cars or find a ride. However, different from ride sharing, the carriers' random task volumes play a significant role in matching supply with demand of capacity on our capacity sharing platform. Furthermore, the carriers' task volumes in our research can change over time, which also affect their incentive to use the platform.

The collaboration among the carriers considered in our research shares some similarity with the paper by Agarwal and Ergun (2010), which considers the alliance formation among carriers. They study the design of large-scale networks and the allocation of limited capacity on a transportation network among the carriers in the alliance. Our research is also related to the literature of inventory transshipment, which typically considers a wholesaler distributes inventory to multiple retailers and the inventory can be transshipped among the retailers to fulfill demand. Papers most relevant to our work include Rudi et al. (2001) and Dong and Rudi (2004), where both the wholesaler's and the retailers' profits are considered. However, in this stream of literature, a player with demand must work with another player with supply to generate profits. In contrast, the carriers on our platform have the option to deliver by themselves and sell their remaining capacity to the platform, allowing them to be a seller or a buyer. Our platform model is also related to the literature of secondary markets, where resellers can buy and sell excess inventory (see, for example, Lee and Whang (2002), Mendelson and Tunca (2007), Milner and Kouvelis (2007), Broner et al. (2010), and Chen et al. (2013)). This stream of research focuses on the impact of secondary markets on supply chains' or firms' performance. In contrast, our objective is to compare the UCC with the capacity sharing platform. We do not see such a comparison in this stream of literature.

2.3 An urban consolidation center

2.3.1 Model

In this section, we consider a consolidator that operates a UCC to serve carriers $i = 1, 2, \ldots, n$ for their last-mile deliveries to the city center. We assume the carriers interact with the UCC in a two-period model, where period $t = 1$ captures the short-term impact of the UCC in practice, and period $t = 2$ captures its long-term impact. In period $t = 1, 2$, carrier i has a delivery task with volume v_{it}. We assume v_{it} equals v_L with a probability λ, or equals v_H $(> v_L)$ with a probability $1 - \lambda$, where $\lambda \in [0, 1]$. All the delivery tasks in each period must be fulfilled within the period. We assume each carrier is initially equipped with logistics capability that has a limited delivery capacity sufficient for his own task in each period. In contrast, the UCC owns a fleet of vehicles with a total capacity that is sufficiently large to accommodate all the carriers' tasks in each period.

In each period, the UCC first decides the pricing of her delivery service and each carrier then decides whether to deliver on his own or use the UCC's service to deliver his task to the city center. In period 1, if a carrier decides to use the UCC's service, then he can also choose to eliminate or keep his logistics capability for the future. In period 2, if a carrier decides to deliver on his own, then he needs to reestablish his logistics capability if it is eliminated in period 1.

Specifically, at the start of period $t = 1, 2$, the UCC first decides the price per unit volume $\bar{p}_t$ of her delivery service. After observing $\bar{p}_t$, each carrier i waits until his delivery task volume is realized. We assume each carrier i only knows his own realized task volume and decides independently on how to deliver his task to the city center. Let $\bar{d}_{it}$ denote the decision of carrier i for period $t = 1, 2$. In period 1, each carrier i has three possible options defined as follows. (i) $\bar{d}_{i1} = -1$: Carrier i delivers on his own. (ii) $\bar{d}_{i1} = 0$: Carrier i uses the UCC's service and eliminates his logistics capability. (iii) $\bar{d}_{i1} = 1$: Carrier i uses the UCC's service and keeps his logistics capability. We assume that each carrier's delivery capacity has no value after period

2. Thus, each carrier i has only two possible options in period 2 defined as follows. (i) $\bar{d}_{i2} = -1$: Carrier i delivers on his own. (ii) $\bar{d}_{i2} = 0$: Carrier i uses the UCC's service. As a result, we have $\bar{d}_{i1} \in \{-1, 0, 1\}$ and $\bar{d}_{i2} \in \{-1, 0\}$, for $i = 1, \ldots, n$. Figure 2.1 shows the sequence of decisions in the two periods.

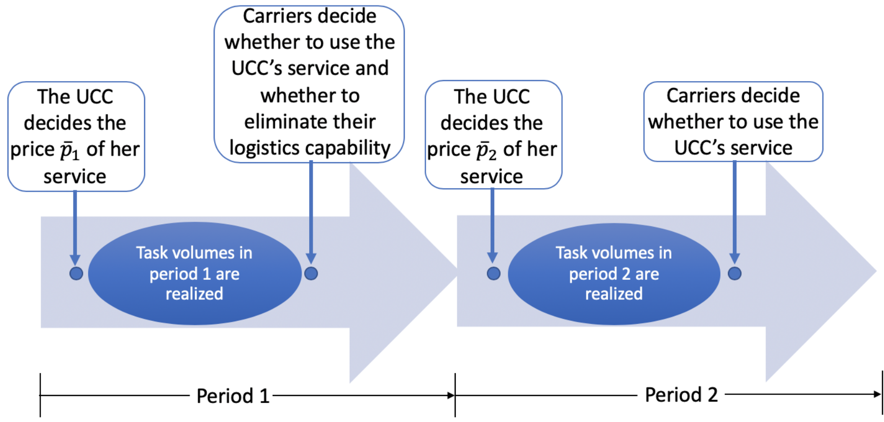


FIGURE 2.1: The sequence of decisions in the two periods under the UCC business model

If carrier i delivers on his own in period t ($\bar{d}_{it} = -1$), the carrier incurs a fixed cost $c > 0$ and a variable cost per unit volume $m > 0$. The fixed cost c includes the maintenance cost for the trucks, the license and permit fees for the trucks, and the salary of drivers. The variable cost includes the fuel cost and the loading-unloading cost.

If carrier i uses the UCC's service in period t, he pays $\bar{p}_t v_{it}$ to the UCC. If carrier i uses the UCC's service and eliminates his logistics capability in period 1 ($\bar{d}_{i1} = 0$), but decides to deliver on his own in period 2 ($\bar{d}_{i2} = -1$), he incurs an additional setup cost $f > 0$ to reestablish his logistics capability in period 2. The *reestablishment cost* f includes the costs to purchase trucks, to recruit drivers, and to learn about and reconnect with local clients.

If carrier i uses the UCC's service and keeps his logistics capability in period 1 ($\bar{d}_{i1} = 1$), then it incurs a fixed holding cost $h \in (0, c)$ to the carrier. The holding cost h includes the costs to maintain the unused trucks and to keep some relevant staff. Let $\delta \in (0, 1)$ denote a discount factor across the two periods. To rule out uninteresting cases, such as the carriers never keep their logistics capability, we assume $h < \delta f$ and $f > c(v_H - v_L)/v_L$.

Let n_t denote the expected number of carriers who use the UCC's delivery service in period t. To serve these carriers, the UCC incurs a fixed delivery cost that depends on n_t. Taking economies of scale into consideration, we assume that the fixed delivery cost equals $\sqrt{n_t}C > 0$ (Steinerberger, 2015). Furthermore, the UCC also incurs a variable cost per unit volume $M > 0$. To be consistent with reality, we assume the UCC may receive a subsidy $S \geq 0$ per unit volume of shipments from the local government or authority. Note that the UCC may not be subsidized ($S = 0$), which is considered as a special case of our model. Our main insights will remain the same.

In each period t in Figure 2.1, the UCC first sets the price per unit volume $\bar{p}_t$ for her service to maximize her expected profit. Given the price $\bar{p}_t$ and the realized task volume v_{it}, each carrier i determines his decision $\bar{d}_{it}$ to minimize his cost. We solve the problem in Figure 2.1 backward by first determining the optimal decisions of the carriers and the UCC in period 2, before we find their optimal decisions in period 1 in the following sections.

2.3.2 Analysis

We first find the optimal decision of each carrier i in period 2. Given the decision $\bar{d}_{i1}$ in period 1 and the price $\bar{p}_2$ in period 2, carrier i determines his optimal decision $\bar{d}^*_{i2}$ to minimize his cost in period 2. After that we substitute the optimal responses of all the carriers into the UCC's problem to find her optimal price $\bar{p}^*_2$.

Define $\bar{\phi}_{i2}\left(\bar{d}_{i2}; \bar{d}_{i1}, \bar{p}_2\right)$ as the cost of carrier i in period 2, which is a function of $\bar{d}_{i2}$ given $\bar{d}_{i1}$ and $\bar{p}_2$. Each carrier i minimizes his cost $\bar{\phi}_{i2}\left(\bar{d}_{i2}; \bar{d}_{i1}, \bar{p}_2\right)$ by comparing the following two options: (i) $\bar{d}_{i2} = -1$: Carrier i delivers on his own in period 2, which incurs a cost $\bar{\phi}_{i2}\left(-1; \bar{d}_{i1}, \bar{p}_2\right) = c + mv_{i2} - \left(\left|\bar{d}_{i1}\right| - 1\right) f$. (ii) $\bar{d}_{i2} = 0$: Carrier i uses the UCC's service in period 2, which incurs a cost $\bar{\phi}_{i2}\left(0; \bar{d}_{i1}, \bar{p}_2\right) = \bar{p}_2 v_{i2}$. The following lemma shows the optimal decision of each carrier i in period 2.

Lemma 2.1. (Optimal decision of carrier i in period 2)

1. If carrier i delivers on his own or uses the UCC's service and keeps his logistics capability in period 1 ($\bar{d}_{i1} = -1$ or 1), then in period 2, carrier i uses the UCC's service and eliminates his logistics capability ($\bar{d}^*_{i2} = 0$) if $\bar{p}_2 \leq m + c/v_{i2}$, or delivers on his own ($\bar{d}^*_{i2} = -1$) otherwise.

2. If carrier i uses the UCC's service and eliminates his logistics capability in period 1 ($\bar{d}_{i1} = 0$), then in period 2, carrier i uses the UCC's service ($\bar{d}^*_{i2} = 0$) if $\bar{p}_2 \leq m + (c + f)/v_{i2}$, or delivers on his own ($\bar{d}^*_{i2} = -1$) otherwise.

Part 1 of Lemma 2.1 shows that the carriers in period 1 who deliver on their own ($\bar{d}_{i1} = -1$), or who use the UCC's service and keep their logistics capability ($\bar{d}_{i1} = 1$) will make the same decision in period 2. This is because in both cases, the carriers own their logistics capability in period 2, leading to the same delivery cost. Furthermore, Lemma 2.1 also shows that carrier i is more likely to use the UCC's service in period 2 if his task volume in the period is smaller (because $\bar{p}_2 \leq m + c/v_{i2}$ and $\bar{p}_2 \leq m + (c + f)/v_{i2}$ are more likely to hold if v_{i2} is smaller). In this case, it is not worthwhile to pay the fixed cost c to deliver on his own. It is also worth noting that if carrier i uses the UCC's service and eliminates his logistics capability in period 1 ($\bar{d}_{i1} = 0$), then he is more likely to engage the UCC in period 2 because of the additional reestablishment cost f.

Let V_2 denote the expected total task volume of the carriers who use the UCC's service in period 2. Given the carriers' optimal responses in Lemma 2.1, the UCC chooses the price $\bar{p}_2$ to maximize her expected profit in period 2:

$$\bar{\pi}_2 (\bar{p}_2) \quad = \quad (\bar{p}_2 + S - M) V_2 - \sqrt{n_2} C. \tag{2.1}$$

Define n_e as the number of carriers who use the UCC's service and eliminate their logistics capability in period 1 (that is, the carriers with $\bar{d}_{i1} = 0$). Note that n_e is known in period 2. The following lemma shows the UCC's optimal pricing decision in period 2.

Lemma 2.2. (Optimal decision of the UCC in period 2)

1. If $n_e > 0$, the optimal price of the UCC's service in period 2 is

$$\bar{p}_2^* = \begin{cases} m + (c+f)/v_L, & \text{if } m < \min\{b_1, b_2, b_3\}; \\ m + (c+f)/v_H, & \text{if } b_1 \leq m < \min\{b_4, b_5\}; \\ m + c/v_L, & \text{if } \max\{b_2, b_4\} \leq m < b_6; \\ m + c/v_H, & \text{if } m \geq \max\{b_3, b_5, b_6\}. \end{cases}$$

2. If $n_e = 0$, the optimal price of the UCC's service in period 2 is

$$\bar{p}_2^* = \begin{cases} m + c/v_L, & \text{if } m < b_7; \\ m + c/v_H, & \text{if } m \geq b_7. \end{cases}$$

The terms $b_j, j = 1, \ldots, 7$, are defined in the proof of Lemma 2.2 in the online supplement. Lemma 2.2 shows that if no carriers eliminate their logistics capability ($n_e = 0$), then the UCC is forced to charge lower prices to attract the carriers. Note that the proof of Lemma 2.2 shows that $b_j, j = 1, \ldots, 7$, decrease as the subsidy S increases. Thus, Lemma 2.2 implies that if the government provides a higher subsidy to the UCC, the latter can afford to charge a lower price $\bar{p}_2^*$ for her service.

After obtaining the optimal decisions d_{i2}^* and $\bar{p}_2^*$, we use them to find the carriers' and the UCC's optimal decisions in period 1 as follows. We first determine the optimal decision of each carrier i in period 1. Given a price $\bar{p}_1$, carrier i determines his optimal decision $\bar{d}_{i1}^*$ to minimize his expected total cost. After that we substitute all the carriers' optimal responses into the UCC's problem to find her optimal price $\bar{p}_1^*$.

Given $\bar{p}_1$, each carrier i chooses $\bar{d}_{i1}$ to minimize his expected total discounted cost $\bar{\Phi}_i\left(\bar{d}_{i1}; \bar{p}_1\right)$ over the two periods by comparing the three options: $\bar{d}_{i1} = -1, 0$, or 1. Note that, to evaluate $\bar{\Phi}_i\left(\bar{d}_{i1}; \bar{p}_1\right)$, one needs to form some belief about the number of carriers who use the UCC's service and eliminate their logistics capability in period 1 (that is, the value of n_e). Following Su and Zhang (2008) and Cachon and Swinney (2009), we seek to identify a subgame perfect Nash Equilibrium with rational expectations. This means that each player (including the carriers and the UCC) chooses their optimal action given their belief about how the others will play. Furthermore, these beliefs are correct, which are identical to the corresponding actions in equilibrium. In our context, all the carriers and the UCC form the same rational belief $\tilde{n}_e$ about n_e when they optimize their decisions in period 1, and in equilibrium, $\tilde{n}_e = n_e\left(\bar{p}_1^*; \bar{d}_{i1}^*, i = 1, 2, \ldots, n\right)$.

For notational convenience, given $\bar{d}_{i1}$, define $\bar{\phi}_{i2}^*\left(\bar{d}_{i1}\right) = \bar{\phi}_{i2}\left(\bar{d}_{i2}^*\left(\bar{d}_{i1}\right); \bar{d}_{i1}, \bar{p}_2^*\left(\bar{d}_{i1}\right)\right)$ as the optimal cost of carrier i in period 2. Given $\bar{p}_1$, each carrier i minimizes $\bar{\Phi}_i\left(\bar{d}_{i1}; \bar{p}_1\right)$ by choosing one of the following options: (i) $\bar{d}_{i1} = -1$: Carrier i delivers on his own, which incurs an expected total discounted cost $\bar{\Phi}_i(-1; \bar{p}_1) = c + m v_{i1} + \delta\bar{\phi}_{i2}^*(-1)$. (ii) $\bar{d}_{i1} = 0$: Carrier i uses the UCC's service and eliminates his logistics capability, which incurs an expected total discounted cost $\bar{\Phi}_i(0; \bar{p}_1) = \bar{p}_1 v_{i1} + \delta\bar{\phi}_{i2}^*(0)$. (iii) $\bar{d}_{i1} = 1$: Carrier i uses the UCC's service and keeps his logistics capability, which incurs an expected total discounted cost $\bar{\Phi}_i(1; \bar{p}_1) = \bar{p}_1 v_{i1} + h + \delta\bar{\phi}_{i2}^*(1)$. The following lemma shows the optimal decision of carrier i in period 1.

Lemma 2.3. (Optimal decision of carrier i in period 1)

1. If $\tilde{n}_e > 0$, the optimal decision of carrier i is determined as follows.

(a) If $m < \min\left\{\tilde{b}_1, \tilde{b}_2, \tilde{b}_3\right\}$, then

$$\bar{d}_{i1}^* = \begin{cases} 1, & \text{if } \bar{p}_1 \le m + (c-h)/v_{i1}; \\ -1, & \text{otherwise.} \end{cases}$$

(b) If $\tilde{b}_1 \le m < \min\left\{\tilde{b}_4, \tilde{b}_5\right\}$, then

$$\bar{d}_{i1}^* = \begin{cases} 1, & \text{if } \bar{p}_1 \le m + (c-h)/v_{i1} \text{ and } h \le \delta(c+f)(\lambda v_L/v_H + 1 - \lambda) - \delta c; \\ 0, & \text{if } \bar{p}_1 \le m + (1+\delta)c/v_{i1} - \delta(c+f)(\lambda v_L/v_H + 1 - \lambda)/v_{i1} \\ & \quad \text{and } h > \delta(c+f)(\lambda v_L/v_H + 1 - \lambda) - \delta c; \\ -1, & \text{otherwise.} \end{cases}$$

(c) If $\max\left\{\tilde{b}_2, \tilde{b}_4\right\} \le m < \tilde{b}_6$, then

$$\bar{d}_{i1}^* = \begin{cases} 1, & \text{if } \bar{p}_1 \le m + (c-h)/v_{i1} \text{ and } h \le \delta c(\lambda + (1-\lambda)v_H/v_L) - \delta c; \\ 0, & \text{if } \bar{p}_1 \le m + (1+\delta)c/v_{i1} - \delta c(\lambda + (1-\lambda)v_H/v_L)/v_{i1} \\ & \quad \text{and } h > \delta c(\lambda + (1-\lambda)v_H/v_L) - \delta c; \\ -1, & \text{otherwise.} \end{cases}$$

(d) If $m \geq \max\left\{\tilde{b}_3, \tilde{b}_5, \tilde{b}_6\right\}$, *then*

$$\bar{d}_{i1}^* = \begin{cases} 0, & \text{if } \bar{p}_1 \leq m + c/v_{i1}; \\ -1, & \text{otherwise.} \end{cases}$$

2. If $\tilde{n}_e = 0$, *the optimal decision of carrier* i *is determined as follows.*

(a) If $m < b_7$, *then*

$$\bar{d}_{i1}^* = \begin{cases} 1, & \text{if } \bar{p}_1 \leq m + (c-h)/v_{i1} \text{ and } h \leq \delta c(\lambda + (1-\lambda)v_H/v_L) - \delta c; \\ 0, & \text{if } \bar{p}_1 \leq m + (1+\delta)c/v_{i1} - \delta c(\lambda + (1-\lambda)v_H/v_L)/v_{i1} \\ & \quad \text{and } h > \delta c(\lambda + (1-\lambda)v_H/v_L) - \delta c; \\ -1, & \text{otherwise.} \end{cases}$$

(b) If $m \geq b_7$, *then*

$$\bar{d}_{i1}^* = \begin{cases} 0, & \text{if } \bar{p}_1 \leq m + c/v_{i1}; \\ -1, & \text{otherwise.} \end{cases}$$

The terms $\tilde{b}_j$, $j = 1, \ldots, 6$, are defined in the proof of Lemma 2.3 in the online supplement. Lemma 2.3 shows that if the task volume v_{i1} of carrier i becomes smaller in period 1, then the carrier is more likely to use the UCC's service to avoid the fixed cost c. In case carrier i chooses to use the UCC's service in period 1, he will eliminate his logistics capability ($\bar{d}_{i1}^* = 0$) if m is sufficiently large (that is, if $m \geq \max\left\{\tilde{b}_3, \tilde{b}_5, \tilde{b}_6\right\}$ or $m \geq b_7$); otherwise, he will keep his logistics capability ($\bar{d}_{i1}^* = 1$) if the holding cost h is sufficiently small.

Let V_1 denote the expected total task volume of the carriers who use the UCC's service in period 1. Recall that $\bar{\pi}_2(\bar{p}_2^*)$ is the UCC's expected profit in period 2 given by Equation (2.1). Assuming all the carriers respond optimally according to Lemma 2.3, the UCC optimizes her price $\bar{p}_1$ to maximize her expected total discounted profit over the two periods:

$$\bar{\Pi}\left(\bar{p}_1\right) = \left(\bar{p}_1 + S - M\right) V_1 - \sqrt{n_1} C + \delta \bar{\pi}_2\left(\bar{p}_2^*\left(\bar{p}_1\right)\right). \tag{2.2}$$

2.3.3 Equilibrium decisions

The following theorem determines the rational expectation equilibrium. To rule out uninteresting cases in which the carriers never keep their logistics capability, we assume $h \leq \min\{\delta(c+f)(\lambda v_L/v_H + 1 - \lambda) - \delta c, \delta c(\lambda + (1-\lambda)v_H/v_L) - \delta c\}$.

Theorem 2.4. (Equilibrium decisions of the UCC model) *Assume $h \leq \min\{\delta(c+f)(\lambda v_L/v_H + 1 - \lambda) - \delta c, \delta c(\lambda + (1-\lambda)v_H/v_L) - \delta c\}$. There are three candidates of the equilibrium characterized as follows.*

1. *If $m < \min\{b_7, m_1\}$, then we have the following candidate of the equilibrium.*

   Period 1: The UCC's equilibrium price is $\bar{p}_1^* = m + (c-h)/v_L$. Under this price, each carrier i uses the UCC's service and keeps his logistics capability if $v_{i1} = v_L$, and delivers on his own otherwise.

   Period 2: The UCC's equilibrium price is $\bar{p}_2^* = m + c/v_L$. Under this price, each carrier i uses the UCC's service if $v_{i1} = v_L$, and delivers on his own otherwise.

2. *If $\min\{b_7, m_1\} \leq m < b_7$, then we have the following candidate of the equilibrium.*

   Period 1: The UCC's equilibrium price is $\bar{p}_1^* = m + (c-h)/v_H$. Under this price, all the carriers use the UCC's service and keep their logistics capability.

   Period 2: The UCC's equilibrium price is $\bar{p}_2^* = m + c/v_L$. Under this price, each carrier i uses the UCC's service if $v_{i1} = v_L$, and delivers on his own otherwise.

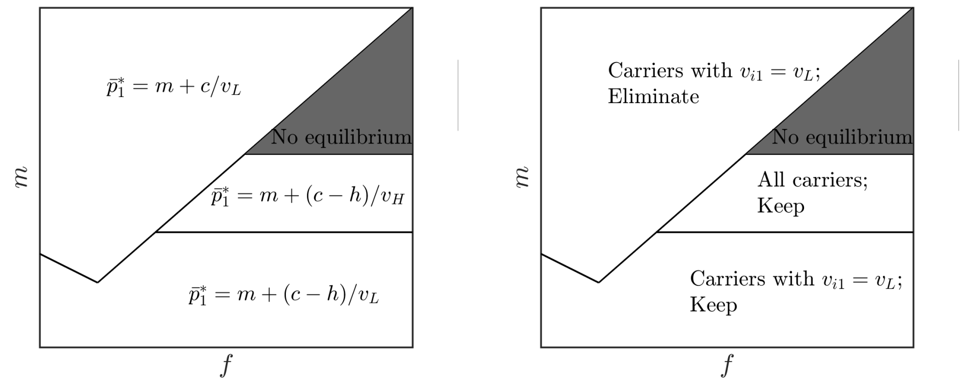
3. *If $m \geq \max\{m_2, m_3, m_4\}$, then we have the following candidate of the equilibrium.*

   Period 1: The UCC's equilibrium price is $\bar{p}_1^* = m + c/v_L$. Under this price, each carrier i uses the UCC's service and eliminates his logistics capability if $v_{i1} = v_L$, and delivers on his own otherwise.

   Period 2: The UCC's equilibrium price is $\bar{p}_2^* = m + c/v_H$. Under this price, all the carriers use the UCC's service.

The terms $m_j, j = 1, \ldots, 4$, are defined in the proof of Theorem 2.4 in the online supplement. Note that the three intervals of m in Theorem 2.4 may overlap. Given a set of parameters (including m), the equilibrium is the candidate with the highest expected total discounted profit for the UCC. According to the proof of Theorem 2.4, $m_j, j = 1, \ldots, 4$, decrease as the subsidy S increases. Thus, if the government provides a higher subsidy S to the UCC, then the third equilibrium in Theorem 2.4 becomes more likely to exist (that is, $m \geq \max\{m_2, m_3, m_4\}$ becomes easier to hold). Since all the carriers will use the UCC's service in period 2 in this equilibrium, the UCC is more likely to sustain in the long run. This result is aligned with the observation that many UCC projects require government subsidies in practice.

The equilibrium of the UCC model can be characterized by the reestablishment cost f and the variable delivery cost m. Figure 2.2(a) shows the UCC's equilibrium price in period 1. If f is sufficiently small (corresponding to the left end of Figure 2.2(a)), then the UCC's price $\bar{p}_1^*$ increases as m increases. This is because if m is getting larger, the carriers are more likely to use the UCC's service. Anticipating this, the UCC charges a higher price in period 1.



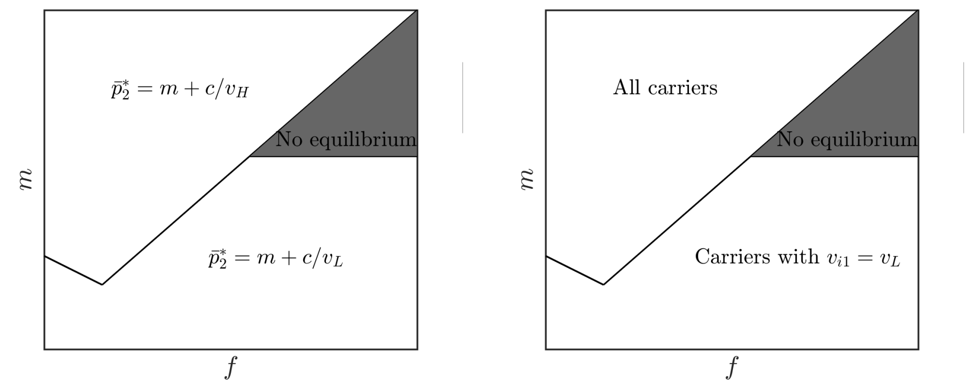
(a) The UCC's equilibrium pricing strategy

(b) The equilibrium decisions of the carriers using the UCC's service

FIGURE 2.2: The equilibrium decisions in period 1 under the UCC model

Figure 2.2(b) illustrates the equilibrium decisions of the carriers who use the UCC's service in period 1. If f is sufficiently small and m is sufficiently large (corresponding to the top-left corner of Figure 2.2(b)), then the carriers who use the UCC's service will eliminate their logistics capability. This is because the carriers anticipate that they are likely to continue to use the

UCC's service in period 2. Even if they need to deliver on their own in period 2, it is affordable to reestablish their logistics capability. In contrast, if f is sufficiently large and m is sufficiently small (corresponding to the bottom-right corner of Figure 2.2(b)), the carriers who use the UCC's service will keep their logistics capability. Furthermore, as m increases all the carriers will use the UCC's service and keep their logistics capability.



(a) The UCC's equilibrium pricing strategy (b) Carriers using the UCC's service in equilibrium

FIGURE 2.3: The equilibrium decisions in period 2 under the UCC model

Figures 2.3(a) and (b) show the UCC's equilibrium price and the carriers who use the UCC's service, respectively, in period 2. If f is sufficiently small and m is sufficiently large, all the carriers will use the UCC's service (see the top-left corner of Figure 2.3(b)). However, as f increases and m decreases, the carriers will keep their logistics capability in period 1 (see the bottom-right corner of Figure 2.2(b)), thus fewer carriers will use the UCC's service in period 2 (see the bottom-right corner of Figure 2.3(b)).

In general, as m increases, the carriers are more dependent on the UCC to deliver their tasks. That is, in period 1 the carriers who use the UCC's service will eliminate their logistics capability, and in period 2 more carriers will use the UCC's service. However, as f increases, the carriers become less dependent on the UCC. That is, in period 1 the carriers who use the UCC's service will keep their logistics capability, and in period 2 fewer carriers will use the UCC's service.

2.4 A capacity sharing platform

Instead of having a physical UCC, the consolidator can operate a platform for the carriers to share their delivery capacity. On the platform, a carrier delivering by himself to the city center can sell his remaining truck capacity to another carrier, so that the latter can outsource his delivery task by paying a fee. If the transaction is successful, then the platform retains a portion of this fee as her revenue.

2.4.1 Model

Similar to §2.3.1, we consider a two-period model. At the start of each period t, the platform first decides the price per unit volume $\hat{p}_t$ of the delivery service. After observing the price $\hat{p}_t$, each carrier i waits until his delivery task volume v_{it} is realized. We make the same assumption as in §2.3.1 that $v_{it} = v_L$ with a probability λ, and $v_{it} = v_H$ with a probability $1 - \lambda$, where $\lambda \in [0, 1]$. For convenience, define $\mathcal{N} = \{1, 2, \ldots, n\}$, $\mathcal{N}_{L,t} = \{i | v_{it} = v_L, i \in \mathcal{N}\}$, and $\mathcal{N}_{H,t} = \{i | v_{it} = v_H, i \in \mathcal{N}\}$, for $t = 1, 2$.

Motivated by the fact that the delivery capacity of each individual carrier is usually very limited compared to the UCC's fleet, we assume in contrast to the UCC model, if $v_{it} = v_H$, then carrier i has to deliver by himself to the city center in period t (the other carriers cannot help him) and his remaining capacity is insufficient to help any other carrier to deliver. Thus, for each carrier $i \in \mathcal{N}_{H,t}$, his action is $\hat{d}_{it} = -1$, for $t = 1, 2$.

In each period t, only carrier i with $v_{it} = v_L$ will participate (purchase or sell capacity) in the capacity sharing platform. Among these carriers in $\mathcal{N}_{L,t}$, we assume that each carrier can serve (or can be served by) at most one other carrier on the platform. In period 1, each carrier $i \in \mathcal{N}_{L,1}$ has three possible options. (i) $\hat{d}_{i1} = -1$: Carrier i delivers on his own and sells his remaining capacity to the platform. (ii) $\hat{d}_{i1} = 0$: Carrier i purchases capacity from the platform and eliminates his logistics capability. (iii) $\hat{d}_{i1} = 1$: Carrier i purchases capacity from the platform and keeps his logistics capability. In consistent with the UCC model, we assume that all the delivery capacity has no value after period 2. Thus, in period 2 each carrier $i \in \mathcal{N}_{L,2}$ has only two possible options defined as follows. (i) $\hat{d}_{i2} = -1$: Carrier i delivers on his own and sells his remaining capacity to the platform. (ii) $\hat{d}_{i2} = 0$: Carrier i purchases capacity from the platform. As a result, for $i \in \mathcal{N}_{L,1}$, we have $\hat{d}_{i1} \in \{-1, 0, 1\}$, and for $i \in \mathcal{N}_{L,2}$, we have $\hat{d}_{i2} \in \{-1, 0\}$. Figure 2.4 shows the sequence of decisions in the two periods.

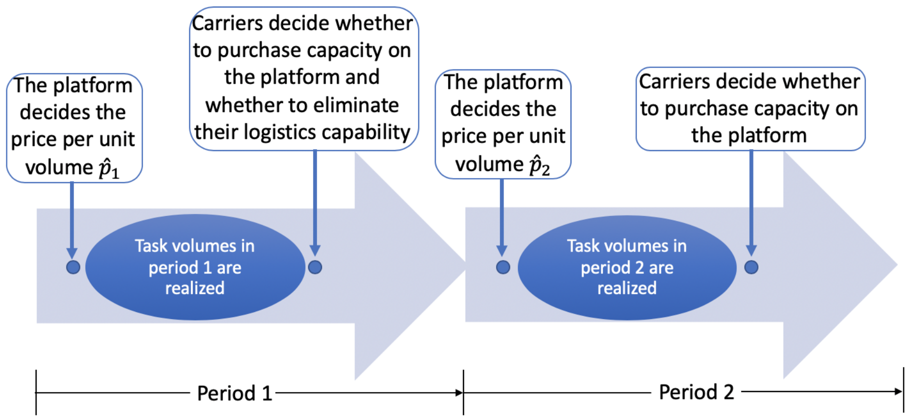


FIGURE 2.4: The sequence of decisions in the two periods under the platform business model

Given that all the delivery tasks must be fulfilled in each period t, if carrier $i \in \mathcal{N}_{L,t}$ wants to purchase capacity from the platform, we assume the carrier can always obtain the required capacity v_L. The platform can guarantee this by outsourcing the delivery task of carrier i to an external party, if necessary. We assume that the platform does not make any profit in this outsourcing process. On the other hand, if carrier $i \in \mathcal{N}_{L,t}$ wants to sell his remaining capacity to the platform, whether his capacity can be successfully sold depends on the demand and the supply of capacity on the platform. If the demand is no less than the supply, then all the carriers who wish to sell their remaining capacity can successfully sell it. However, if the demand is less than the supply, then only a subset of these carriers can sell their remaining capacity. In this situation, the platform will randomly distribute the tasks with an equal probability to the carriers willing to sell their remaining capacity.

For notational convenience, define $n_{s,t}$ as the expected number of carriers who deliver on their own and sell their remaining capacity to the platform in period t (that is, the carriers who choose $\hat{d}_{it} = -1$). Define $n_{p,t}$ as the expected number of carriers who purchase capacity from the platform in period t (that is, the carriers who choose $\hat{d}_{i1} = 0$ or 1 in period 1, and the carriers who choose $\hat{d}_{i2} = 0$ in period 2). Therefore, the supply and the demand of capacity on the platform in period t are propotional to $n_{s,t}$ and $n_{p,t}$ respectively.

If carrier i delivers on his own in period t ($\hat{d}_{it} = -1$), then the carrier incurs a fixed cost $c > 0$ and a variable cost per unit volume $m > 0$. If carrier $i \in \mathcal{N}_{L,t}$ purchases capacity from the platform in period t ($\hat{d}_{it} = 0$ or 1), then he pays $\hat{p}_t v_L$. The platform receives a portion $\alpha \hat{p}_t v_L$, where $\alpha \in (0, 1)$ represents the platform's *revenue share*. The remaining portion $(1 - \alpha)\hat{p}_t v_L$ goes to the other carrier who serves carrier i. To ensure that selling capacity on the platform is profitable, we assume $(1 - \alpha)\hat{p}_t > m$. If carrier $i \in \mathcal{N}_{L,1}$ eliminates his logistics capability in period 1 ($\hat{d}_{i1} = 0$), but decides to deliver on his own in period 2 ($\hat{d}_{i2} = -1$), he incurs a setup cost $f > 0$ to reestablish his logistics capability. If carrier $i \in \mathcal{N}_{L,1}$ purchases capacity from the platform and keeps his logistics capability in period 1 ($\hat{d}_{i1} = 1$), then it incurs a fixed holding cost $h \in (0, c)$ to the carrier. Similar to §2.3.1, we assume $h < \delta f$.

For each period t in Figure 2.4, the platform first sets the price per unit volume $\hat{p}_t$ to maximize her expected profit. Given the price $\hat{p}_t$ and the realized task volume v_{it}, each carrier $i \in \mathcal{N}_{L,t}$ determines his decision $\hat{d}_{it}$ to minimize his expected cost. We solve the problem in Figure 2.4 backward by first identifying the optimal decisions of each carrier $i \in \mathcal{N}_{L,2}$ and the platform in period 2, before we find their optimal decisions in period 1 in the following sections.

2.4.2 Analysis

Given the decision $\hat{d}_{i1}$ in period 1 and the price $\hat{p}_2$ in period 2, we first determine the optimal decision $\hat{d}^*_{i2}$ of each carrier $i \in \mathcal{N}_{L,2}$ to minimize his expected cost. After that we substitute the carriers' optimal responses into the platform's problem to find her optimal price $\hat{p}^*_2$.

Each carrier $i \in \mathcal{N}_{L,2}$ minimizes his expected cost $\hat{\phi}_{i2}\left(\hat{d}_{i2}; \hat{d}_{i1}, \hat{p}_2\right)$ in period 2 by comparing the two options: $\hat{d}_{i2} = -1$ or 0. If carrier i delivers by himself and sells his remaining capacity to the platform ($\hat{d}_{i2} = -1$), then the expected revenue generated from selling his remaining capacity depends on the supply (proportional to $n_{s,2}$) and the demand (proportional to $n_{p,2}$) of capacity on the platform in period 2. Following Su and Zhang (2008) and Cachon and Swinney (2009), we aim to identify a subgame perfect Nash Equilibrium with rational expectations. We assume all the carriers in $\mathcal{N}_{L,2}$ form the same rational beliefs $\tilde{n}_{s,2}$ and $\tilde{n}_{p,2}$ about $n_{s,2}$ and $n_{p,2}$, respectively, when they optimize their decisions in period 2. Furthermore, $\tilde{n}_{s,2} = n_{s,2}\left(\hat{d}^*_{i2}, i \in \mathcal{N}_{L,2}\right)$ and $\tilde{n}_{p,2} = n_{p,2}\left(\hat{d}^*_{i2}, i \in \mathcal{N}_{L,2}\right)$ in equilibrium. Define $\theta_t = \min\{\tilde{n}_{p,t}/\tilde{n}_{s,t}, 1\}$, for $t = 1, 2$.

Specifically, each carrier $i \in \mathcal{N}_{L,2}$ minimizes $\hat{\phi}_{i2}\left(\hat{d}_{i2}; \hat{d}_{i1}, \hat{p}_2\right)$ by comparing the following options. (i) $\hat{d}_{i2} = -1$: Carrier i delivers on his own and sells his remaining capacity to the platform, which incurs an expected cost $\hat{\phi}_{i2}\left(-1; \hat{d}_{i1}, \hat{p}_2\right) = c + mv_L - \left(\left|\hat{d}_{i1}\right| - 1\right) f - \theta_2 \left[(1-\alpha)\hat{p}_2 - m\right] v_L$. (ii) $\hat{d}_{i2} = 0$: Carrier i purchases capacity from the platform, incurring a cost $\hat{\phi}_{i2}\left(0; \hat{d}_{i1}, \hat{p}_2\right) = \hat{p}_2 v_L$. Note that for both periods 1 and 2, if the cost of delivering by himself is identical to the cost of purchasing capacity from the platform, we assume that carrier i will choose either option with an equal probability. This random tie-breaking rule is to avoid the extreme situation where the carriers with identical costs choose the same option on the platform. In this extreme situation, all the carriers will either deliver on their own or purchase capacity from the platform, and the platform will never earn a positive profit. To facilitate an interesting comparison between the UCC and the platform, we rule out this extreme case.

After we determine the optimal decision $\hat{d}_{i2}^*$ of carrier $i \in \mathcal{N}_{L,2}$, we can substitute it into the platform's problem to find her optimal price in period 2. The platform chooses $\hat{p}_2$ to maximize her expected profit in period 2:

$$\hat{\pi}_2(\hat{p}_2) \quad = \quad \alpha \hat{p}_2 v_L \min\{n_{s,2}, n_{p,2}\}. \tag{2.3}$$

After obtaining the optimal decisions $\hat{d}_{i2}^*$ and $\hat{p}_2^*$ in period 2, we use them to find the carriers' and the platform's optimal decisions in period 1.

Each carrier $i \in \mathcal{N}_{L,1}$ in period 1 minimizes his expected total discounted cost $\hat{\Phi}_i\left(\hat{d}_{i1}; \hat{p}_1\right)$ over the two periods by comparing the three options: $\hat{d}_{i1} = -1, 0$, or 1. If $\hat{d}_{i1} = -1$, then the expected cost of carrier i in period 1 depends on $n_{s,1}$ and $n_{p,1}$. To identify a subgame perfect Nash Equilibrium with rational expectations, we assume all the carriers in $\mathcal{N}_{L,1}$ form the same rational beliefs $\tilde{n}_{s,1}$ and $\tilde{n}_{p,1}$ about $n_{s,1}$ and $n_{p,1}$, respectively, when optimizing their decisions in period 1. Furthermore, $\tilde{n}_{s,1} = n_{s,1}\left(\hat{d}_{i1}^*, i \in \mathcal{N}_{L,1}\right)$ and $\tilde{n}_{p,1} = n_{p,1}\left(\hat{d}_{i1}^*, i \in \mathcal{N}_{L,1}\right)$ in equilibrium.

For notational convenience, given $\hat{d}_{i1}$, define $\hat{\phi}_{i2}^*\left(\hat{d}_{i1}\right) = \hat{\phi}_{i2}\left(\hat{d}_{i2}^*\left(\hat{d}_{i1}\right); \hat{d}_{i1}, \hat{p}_2^*\left(\hat{d}_{i1}\right)\right)$ as the optimal expected cost of carrier i in period 2. Given $\hat{p}_1$, carrier $i \in \mathcal{N}_{L,1}$ minimizes $\hat{\Phi}_i\left(\hat{d}_{i1}; \hat{p}_1\right)$ by choosing one of the following options:

(i) $\hat{d}_{i1} = -1$: Carrier i delivers on his own and sells his remaining capacity to the platform, which incurs an expected total discounted cost $\hat{\Phi}_i(-1; \hat{p}_1) = c + m v_L - \theta_1\left[(1-\alpha)\hat{p}_1 - m\right] v_L + \delta \hat{\phi}_{i2}^*(-1)$.

(ii) $\hat{d}_{i1} = 0$: Carrier i purchases capacity from the platform and eliminates his logistics capability, which incurs an expected total discounted cost $\hat{\Phi}_i(0; \hat{p}_1) = \hat{p}_1 v_L + \delta \hat{\phi}_{i2}^*(0)$.

(iii) $\hat{d}_{i1} = 1$: Carrier i purchases capacity from the platform and keeps his logistics capability, which incurs an expected total discounted cost $\hat{\Phi}_i(1; \hat{p}_1) = \hat{p}_1 v_L + h + \delta \hat{\phi}_{i2}^*(1)$.

We then substitute all the carriers' optimal responses $\hat{d}_{i1}^*$ into the platform's problem to find her optimal price $\hat{p}_1^*$ that maximizes her expected total discounted profit:

$$\hat{\Pi}\left(\hat{p}_1\right) = \alpha \hat{p}_1 v_L \min\left\{n_{s,1}, n_{p,1}\right\} + \delta \hat{\pi}_2\left(\hat{p}_2^*\left(\hat{p}_1\right)\right), \tag{2.4}$$

where $\hat{\pi}_2\left(\hat{p}_2^*\left(\hat{p}_1\right)\right)$ represents the platform's optimal expected profit in period 2 given $\hat{p}_1$ (see Equation (2.3)).

2.4.3 Equilibrium decisions

The following theorem summarizes the platform's and the carriers' decisions for each period in the equilibrium with rational expectations. Define $\underline{f} = \frac{(2-2\lambda+\frac{\alpha\lambda^2}{4})mv_L+(1-\frac{3\lambda}{2}+\frac{\lambda(\lambda+\alpha)}{4})c}{(2-\alpha)\frac{\lambda}{2}(1-\frac{\lambda}{4})}$ and $\underline{f}' = \frac{(2-3\lambda+\frac{\alpha\lambda^2}{2})mv_L+(1-\frac{5\lambda}{2}+\lambda^2+\frac{\alpha\lambda(2-\lambda)}{4})c}{(2-\alpha)\frac{\lambda}{2}(2-\lambda)}$.

Theorem 2.5. (Equilibrium decisions of the platform model)

1. If $f \geq \frac{h}{\delta(1-\lambda)}$, then we have the following results.

Period 1: The platform's equilibrium price is $\hat{p}_1^* = (c + 2mv_L - h)/[(2-\alpha)v_L]$. Under this price, each carrier $i \in \mathcal{N}_{L,1}$ chooses $\hat{d}_{i1}^* = -1$ or $\hat{d}_{i1}^* = 1$ with an equal probability.

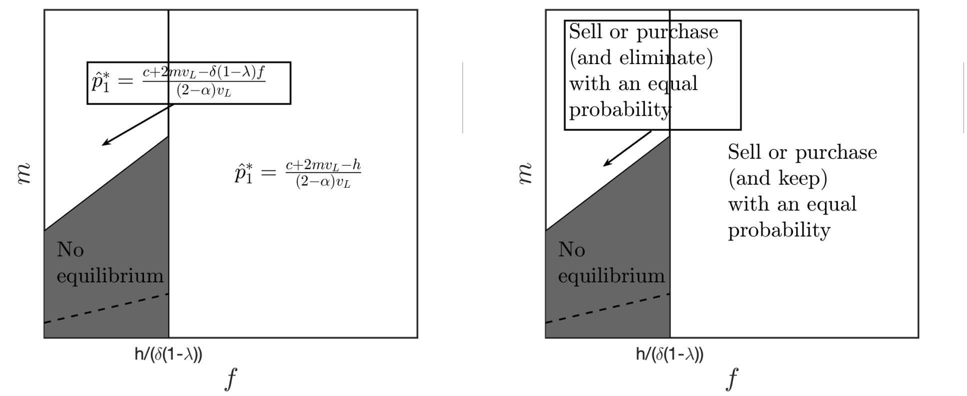
Period 2: The platform's equilibrium price is $\hat{p}_2^* = (c+2mv_L)/[(2-\alpha)v_L]$. Under this price, each carrier $i \in \mathcal{N}_{L,2}$ chooses $\hat{d}_{i2}^* = -1$ or $\hat{d}_{i2}^* = 0$ with an equal probability.

2. If $f < \min\left\{\frac{h}{\delta(1-\lambda)}, \underline{f}, \underline{f}'\right\}$, then we have the following results.

Period 1: The platform's equilibrium price is $\hat{p}_1^* = [c + 2mv_L - \delta(1-\lambda)f]/[(2-\alpha)v_L]$. Under this price, each carrier $i \in \mathcal{N}_{L,1}$ chooses $\hat{d}_{i1}^* = -1$ or $\hat{d}_{i1}^* = 0$ with an equal probability.

Period 2: The platform's equilibrium price is $\hat{p}_2^* = (c+2mv_L)/[(2-\alpha)v_L]$. Under this price, carrier $i \in \mathcal{N}_{L,2}$ chooses $\hat{d}_{i2}^* = -1$ or $\hat{d}_{i2}^* = 0$ with an equal probability, if $\hat{d}_{i1}^* = -1$; or chooses $\hat{d}_{i2}^* = 0$, if $\hat{d}_{i1}^* = 0$.

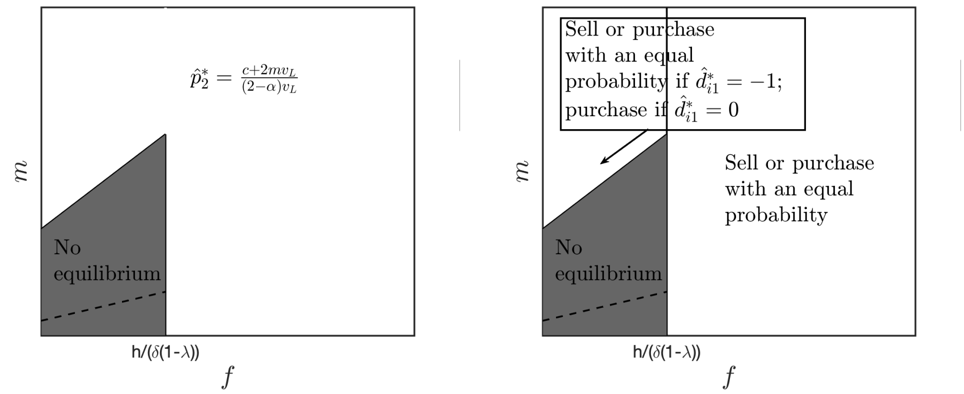
Theorem 2.5 is illustrated by Figures 2.5 and 2.6. Figure 2.5(a) shows the platform's equilibrium price in period 1. Figure 2.5(b) shows that each carrier $i \in \mathcal{N}_{L,1}$ sells or purchases capacity on the platform in period 1 with an equal probability. If the reestablishment cost f is sufficiently large ($f \geq h/[\delta(1-\lambda)]$), then the carriers who purchase capacity from the platform should keep their logistics capability. Otherwise, these carriers should eliminate their logistics capability.



(a) The platform's equilibrium pricing strategy (b) The equilibrium decision of each carrier

FIGURE 2.5: The equilibrium decisions of the platform and each carrier i in period 1

Figures 2.6(a) and (b) show the equilibrium decisions of the platform and each carrier $i \in \mathcal{N}_{L,2}$, respectively, in period 2. Figure 2.6(b) shows that if $f \geq h/[\delta(1-\lambda)]$, then each carrier i sells or purchases capacity on the platform in period 2 with an equal probability. Otherwise, the carrier's decision depends on his decision in period 1. If he delivers on his own in period 1 (that is, $\hat{d}_{i1}^* = -1$), then he sells or purchases capacity on the platform in period 2 with an equal probability. On the other hand, the carriers who purchase capacity and eliminate their logistics capability in period 1 (that is, $\hat{d}_{i1}^* = 0$) will continue to purchase capacity from the platform in period 2. In this situation, although the reestablishment cost f is affordable, but with a large variable delivery cost m, it is expensive to make their own delivery.



(a) The platform's equilibrium pricing strategy (b) The equilibrium decision of each carrier

FIGURE 2.6: The equilibrium decisions of the platform and each carrier i in period 2

2.5 Comparing the UCC and the capacity sharing platform

We compare the UCC and the capacity sharing platform in terms of the expected profit and the expected social-environmental cost. We focus on three regions where the equilibria exist in both models: (i) When f is sufficiently small and m is sufficiently large: $f < \min\left\{\frac{h}{\delta(1-\lambda)}, f_1, f_2\right\}$ and $m > \max\{b_7, m_4, m_5, m_6\}$. (ii) When f is sufficiently large and m is intermediate: $f > \max\left\{\frac{h}{\delta(1-\lambda)}, f_1, f_2, f_3, f_4\right\}$ and $\min\{b_7, m_1\} \leq m < b_7$. (iii) When f is sufficiently large and m is sufficiently small: $f > \max\left\{\frac{h}{\delta(1-\lambda)}, f_1, f_2, f_3, f_4\right\}$ and $m < \min\{b_7, m_1\}$. The terms m_5, m_6, and $f_j, j = 1, \ldots, 4$ are defined in the proof of Theorem 2.6 in the online supplement.

2.5.1 Expected profit

Between the UCC and the platform, which business model is more profitable for the consolidator? We determine the consolidator's preference by comparing the equilibrium profits $\bar{\Pi}\left(\bar{p}_1^*\right)$ of the UCC in §2.3 and $\hat{\Pi}\left(\hat{p}_1^*\right)$ of the capacity sharing platform in §2.4. The following theorem identifies the conditions under which the UCC (or the platform) is more profitable for the consolidator.

Theorem 2.6. (Comparing the UCC's and the platform's profits) *In each region, the UCC is more profitable than the platform ($\bar{\Pi}\left(\bar{p}_1^*\right) > \hat{\Pi}\left(\hat{p}_1^*\right)$) if and only if*

Region (i): $c > c_1$;

Region (ii): one of the following conditions holds: (a) $c > c_2$ and $\delta > \delta_1$, (b) $c < c_2$ and $\delta < \delta_1$;

Region (iii): one of the following conditions holds: (a) $c > c_3$, (b) $h < h_1$.

In Region (i), the UCC is more profitable than the platform if the carriers' fixed delivery cost $c > c_1$. This is because when c is large, the carriers are more likely to outsource their delivery tasks to avoid the fixed cost. This will benefit the consolidator if she operates a UCC because there will be many carriers using her service. On the other hand, if the consolidator operates a platform, there will not be many successful transactions because the supply of capacity is low. This reduces her profit. Furthermore, the proof of Theorem 2.6 shows that c_1 decreases with n and S. As n increases, the carriers enjoy more savings by using the UCC because of the economies of scale in shipment consolidation, making the UCC more likely to outperform the platform. The UCC also becomes more dominant as the government subsidy S increases. If $c < c_1$, then the carriers are more likely to deliver on their own. Thus, more capacity will be available on the platform, making the platform more profitable than the UCC.

In Region (ii), the UCC is more profitable than the platform if both the fixed delivery cost c and the discount factor δ are large ($c > c_2$ and $\delta > \delta_1$). A large c pushes more carriers to outsource their delivery tasks. Furthermore, a large reestablishment cost f persuades the carriers who eliminate their logistics capability to continue outsourcing the delivery in the long run. A large δ magnifies this effect. Under the platform model, these carriers are less likely to supply capacity in period 2. This creates excessive demand for capacity on the platform, leading to a severe imbalance of supply and demand, which yields a lower profit for the platform. On the other hand, if δ is small ($c > c_2$ and $\delta < \delta_1$), the carriers are less sensitive to their costs in period 2 and become more likely to do their own delivery. This mitigates the supply-demand imbalance on the platform, making the platform more profitable than the UCC.

In contrast, if both c and δ are small ($c < c_2$ and $\delta < \delta_1$), the affordable delivery costs (small c and intermediate m) attract more carriers to deliver on their own. This is especially so for a small δ, which encourages the carriers, who eliminate their logistics capability in period 1, to deliver on their own in period 2. This creates excessive supply of capacity on the platform, which reduces the number of successful transactions, making the platform less profitable than the UCC. However, if δ is large ($c < c_2$ and $\delta > \delta_1$), the large f makes the carriers, who eliminate their logistics capability in period 1, to outsource their delivery tasks in period 2. This increases the demand for capacity on the platform, which mitigates the imbalance of supply and demand, leading to a higher profit for the platform than the UCC.

Lastly, in Region (iii), the UCC is more profitable than the platform if $c > c_3$ because of the same reason mentioned in Region (i). The second condition ($h < h_1$) for the UCC to outperform the platform needs more explanations. We first consider the opposite case with $h > h_1$. If the holding cost h is large, the carriers are less likely to hold their logistics capability in period 1. Meanwhile, the large f deters the carriers from eliminating their logistics capability. Therefore, more carriers will deliver on their own to avoid these large costs, reducing the UCC's profit. However, if h is small, then the carriers can always use the UCC's service and hold their logistics capability in period 1, avoiding a costly reestablishment in the next period. This makes the UCC more profitable than the platform. Furthermore, the proof of Theorem 2.6 shows that c_3 decreases and h_1 increases with n, making it easier for the UCC to dominate as n increases.

2.5.2 Expected social-environmental cost

Between the UCC and the platform, which business model leads to a lower expected social-environmental cost? As a result of the consolidation, both the UCC and the platform yield higher truck utilization with fewer trucks used. This not only economically benefits the consolidator and the carriers, but also mitigates the social-environmental impact (in terms of reduced congestion and pollution) because of reduced traffic to the city center. In this section, we compare the UCC and the platform with respect to their impact to the society and the environment.

To quantify the impact, define ψ as the social-environmental cost associated with a carrier's delivery to the city center. This includes, for example, the cost to the society due to congestion and the cost to the environment due to pollution. Define $\bar{\Delta}_\psi$ and $\hat{\Delta}_\psi$ as the *expected total social-environmental cost reduction* achieved by the UCC and the platform respectively. Under the UCC model, although additional trucks are required, each UCC's truck can potentially consolidate multiple tasks. In contrast, under the platform model, although no additional trucks are required, each carrier can at most serve one other carrier's task. It is unclear that which business model is more effective in reducing the social-environmental cost.

We first analyze the expected total social-environmental cost reduction achieved by the UCC. Recall that n_t represents the expected number of carriers served by the UCC in period $t = 1, 2$. Using the same setup cost's formula due to the consolidation by the UCC in §2.3.1, the expected total social-environmental cost in each period t is reduced from $n\psi$ to $\sqrt{n_t}\psi + (n - n_t)\psi$. This leads to $\bar{\Delta}_\psi = n\psi - [\sqrt{n_1}\psi + (n - n_1)\psi] + n\psi - [\sqrt{n_2}\psi + (n - n_2)\psi]$.

In contrast, the task of a carrier who purchases capacity from the platform is fulfilled by another carrier, leading to a social-environmental cost reduction ψ. In case the platform does not have sufficient supply of capacity, we assume that the unmatched delivery tasks are outsourced to a third party without incurring any additional social-environmental cost. Recall that $n_{p,t}$ represents the expected number of carriers who purchase capacity from the platform in period t. The expected total social-environmental cost reduction in each period t is $n_{p,t}\psi$. Thus, we have $\hat{\Delta}_\psi = n_{p,1}\psi + n_{p,2}\psi$.

The following theorem compares $\bar{\Delta}_\psi$ and $\hat{\Delta}_\psi$. We focus on the same three regions in Theorem 2.6 where the equilibria exist in both models.

Theorem 2.7. (Comparing the UCC's and the platform's social-environmental cost reductions) *In each region, the UCC is more efficient than the platform in reducing the expected total social-environmental cost ($\bar{\Delta}_\psi > \hat{\Delta}_\psi$) if and only if*

Region (i): $n > \left(\frac{1+\sqrt{\lambda}}{1-\lambda/4}\right)^2$;

Region (ii): $n > \left(1+\sqrt{\lambda}\right)^2$;

Region (iii): $n > 4/\lambda$.

Theorem 2.7 shows that if the number of carriers n is large, then the UCC is more efficient in reducing the social-environmental cost than the platform. This is because if n is large, the UCC's trucks (each can serve multiple tasks) can achieve a larger economy of scale in shipment consolidation. This significantly reduces the traffic congestion and pollution caused by the last-mile delivery. On the other hand, if n is small, the UCC may not be efficient in reducing the social-environmental cost. In contrast, the platform, which matches a carrier's task with another carrier without employing any additional trucks, becomes more efficient.

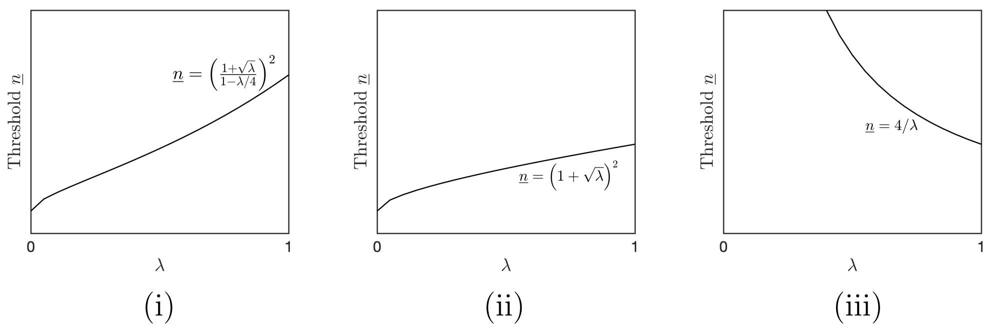


FIGURE 2.7: Thresholds of n in Regions (i), (ii), and (iii)

Figure 2.7 shows how the threshold of n in each region varies with λ. In Regions (i) and (ii), as λ increases, the thresholds $\left(\frac{1+\sqrt{\lambda}}{1-\lambda/4}\right)^2$ and $\left(1+\sqrt{\lambda}\right)^2$ also increase, making the platform more likely to outperform the UCC in reducing the social-environmental cost. As the probability of low task volume (λ) increases, more carriers will engage the platform. Many of these carriers want to purchase capacity from the platform because of the large and intermediate variable delivery cost m in Regions (i) and (ii). This significantly reduces the social-environmental cost, making the platform more efficient than the UCC.

In Region (iii), as λ increases, the threshold $4/\lambda$ decreases, making the UCC more likely to outperform the platform in reducing the social-environmental cost. This is because as λ increases, more carriers will engage the platform. However, the large f and small m in Region (iii) make the carriers more likely to deliver on their own. This is especially so under the platform model because the carriers can earn extra revenue by selling their remaining capacity. In contrast, the UCC can achieve a larger scale of shipment consolidation, which reduces the social-environmental cost more efficiently than the platform.

Table 2.1 shows the preferred business model with respect to the profit and the social-environmental impact. To maximize the expected profit, the consolidator should choose the UCC if the carriers' fixed delivery cost c is large in general. Otherwise, the capacity sharing platform is preferred. To minimize the expected social-environmental cost, the UCC is preferred if the number of carriers n is large. Otherwise, the consolidator should choose the platform.

Table 2.1: The preferred business model

	small c **small** n	**small** c **large** n	**large** c **small** n	**large** c **large** n
To maximize expected profit	platform	platform	UCC	UCC
To minimize expected social-environmental cost	platform	UCC	platform	UCC

2.6 Extensions

2.6.1 A hybrid model

We consider a hybrid business model that combines the ideas of both the UCC and the capacity sharing platform. In this hybrid model, the consolidator simultaneously operates a UCC, which fulfills the carriers' delivery tasks, and a platform, which matches supply and demand for capacity among the carriers. This hybrid model is inspired by Amazon that sells products to consumers by itself, and also allows peer-to-peer selling on its platform.

For analytical tractability, we consider a one-period model in which the consolidator operates both the UCC and the platform. Through the UCC, the consolidator charges the carriers for her delivery service. Through the platform, the consolidator receives a revenue share $\alpha \in (0, 1)$ from each successful transaction of capacity. The consolidator first chooses the prices $\bar{p}$ and $\hat{p}$ per unit volume of delivery service for the UCC and the platform, respectively, to maximize her expected profit.

After observing the prices $\bar{p}$ and $\hat{p}$, each carrier i waits until his delivery task volume v_i is realized. Depending on v_i, each carrier i has different options to fulfill his task. If $v_i = v_L$ (which occurs with a probability λ), then carrier i has three possible options: (i) He delivers on his own and sells his remaining capacity to the platform. (ii) He uses the UCC's service. (iii) He purchases capacity from the platform. If $v_i = v_H$ (which occurs with a probability $1-\lambda$), then carrier i has two possible options: (i) He delivers on his own. (ii) He uses the UCC's service. Each carrier independently decides how to fulfill his task to minimize his expected cost. To ensure that selling capacity on the platform is profitable and the options do not always dominate each other, we assume $m < (1-\alpha)\hat{p} < (1/v_L - 1/v_H)\, c$. The following theorem summarizes the consolidator's and the carriers' equilibrium decisions.

Theorem 2.8. (Equilibrium decisions of the hybrid model)

1. If $m < \min\{m_7, m_8\}$, then it is optimal for the consolidator to charge any $\bar{p}^* > (c+2mv_L)/((2-\alpha)v_L)$ and $\hat{p}^* = (c+2mv_L)/((2-\alpha)v_L)$. Under these prices, each carrier i with $v_i = v_H$ delivers on his own, and each carrier i with $v_i = v_L$ delivers on his own (and sells his remaining capacity to the platform) or purchases capacity from the platform with an equal probability.

2. If $m_7 \leq m < m_9$, then it is optimal for the consolidator to charge $\bar{p}^* = m + c/v_L$ and any $\hat{p}^* \geq m + c/v_L$. Under these prices, each carrier i with $v_i = v_H$ delivers on his own, and each carrier i with $v_i = v_L$ uses the UCC's service.

3. If $m \geq \max\{m_8, m_9\}$, then it is optimal for the consolidator to charge $\bar{p}^* = m + c/v_H$ and any $\hat{p}^* \geq m + c/v_L$. Under these prices, all the carriers use the UCC's service.

The terms $m_j, j = 7, \ldots, 9$, are defined in the proof of Theorem 2.8 in the online supplement.

The conditions of the above equilibrium result determine the source from which the consolidator generates her profit. If the carriers' variable delivery cost m is small ($m < \min\{m_7, m_8\}$), then the consolidator will generate profit from the platform. This is because the affordable delivery cost m makes it difficult to attract the carriers to use the UCC's service. However, as m becomes moderate or large ($m_7 \leq m < m_9$ or $m \geq \max\{m_8, m_9\}$), more carriers would like to outsource their delivery tasks. Specifically, if $m_7 \leq m < m_9$, then only the carriers with a high task volume will deliver on their own. If $m \geq \max\{m_8, m_9\}$, then no carriers will make their own delivery. Both cases eliminate the supply of capacity on the platform. Thus, the consolidator will optimize her prices to induce the carriers to engage the UCC (rather than the platform), such that her expected profit is maximized. In both cases, the consolidator generates profit from the UCC.

Note that some equilibrium in Theorem 2.8 leads to a lower social-environmental cost than the others. For example, it is straightforward to show that if $n > 1/(1-\lambda/2)^2$, then the third equilibrium (when $m \geq \max\{m_8, m_9\}$) results in the lowest expected total social-environmental cost. In this equilibrium, all the carriers use the UCC's service. The government can promote the third equilibrium by increasing the variable delivery cost m, such as imposing variable tax to the carriers who deliver on their own. Furthermore, the proof of Theorem 2.8 shows that m_7, m_8, and m_9 decrease with the government subsidy S for the UCC's service. Thus, to make the third equilibrium more achievable, the government can provide a higher subsidy to the consolidator for the UCC's service. Conversely, if $n \leq 1/(1-\lambda/2)^2$, then the first equilibrium (when $m < \min\{m_7, m_8\}$) yields the lowest social-environmental cost. In this equilibrium, all the carriers with a low task volume sell or purchase capacity on the platform. In this situation, the government can act in a reverse manner to make the first equilibrium more attainable.

2.6.2 Demand correlation

In the UCC and the platform models, some carriers are reluctant to eliminate their logistics capability in period 1 because of the reestablishment cost f. This decision depends on the carrier's delivery task volume in the next period. In practice, each carrier's demands across the periods are sometimes correlated such that the carriers can roughly predict their task volumes in the near future. This helps them plan ahead with their logistics requirement.

In this section, we analyze the UCC model with correlated demands for each carrier between the two periods. Specifically, we assume the demands for each carrier in the two periods are positively correlated. That is, if the carrier's task volume is low (high) in period 1, then his task volume is also low (high) in period 2. The rest of the model is identical to that of §2.3.1. The following theorem summarizes the equilibrium results.

Theorem 2.9. (Equilibrium decisions of the UCC model with correlated demands)

Assume $h \leq \min\{\delta(c+f)v_L/v_H - \delta c, \delta c(v_H/v_L - 1)\}$*. There are three cases:*

1. *If* $\max\{m_{11}, m_{12}\} \leq m < \min\{m_4, m_{10}\}$*, then we have the following results.*

   Period 1: The UCC's equilibrium price is $\bar{p}_1^* = m + c/v_L$*. Under this price, each carrier* i *uses the UCC's service and eliminates his logistics capability if* $v_{i1} = v_L$*, and delivers on his own otherwise.*

   Period 2: The UCC's equilibrium price is $\bar{p}_2^* = m + c/v_L$*. Under this price, each carrier* i *uses the UCC's service if* $v_{i1} = v_L$*, and delivers on his own otherwise.*

2. *If* $\max\{m_{10}, m_{11}, m_{12}\} \leq m < m_4$*, then we have the following results.*

   Period 1: The UCC's equilibrium price is $\bar{p}_1^* = m + (c-h)/v_H$*. Under this price, each carrier* i *uses the UCC's service and eliminates his logistics capability if* $v_{i1} = v_L$*, and uses the UCC's service and keeps his logistics capability otherwise.*

   Period 2: The UCC's equilibrium price is $\bar{p}_2^* = m + c/v_L$*. Under this price, each carrier* i *uses the UCC's service if* $v_{i1} = v_L$*, and delivers on his own otherwise.*

3. *If* $m \geq \max\{m_2, m_3, m_4\}$*, then we have the following results.*

   Period 1: The UCC's equilibrium price is $\bar{p}_1^* = m + c/v_L$*. Under this price, each carrier* i *uses the UCC's service and eliminates his logistics capability if* $v_{i1} = v_L$*, and delivers on his own otherwise.*

   Period 2: The UCC's equilibrium price is $\bar{p}_2^* = m + c/v_H$*. Under this price, all the carriers use the UCC's service.*

The terms $m_j, j = 10, \ldots, 12$, are defined in the proof of Theorem 2.9. Note that the carriers eliminate their logistics capability in period 1 if they will continue to use the UCC's service in period 2. On the other hand, the carriers keep their logistics capability in period 1 if they will deliver on their own in period 2. This is because in period 1 the carriers already know their task volumes in the future, so they can plan ahead with their logistics capability.

We also analyze the platform model with positively correlated demands across the two periods for each carrier. We find that there is no Nash Equilibrium with rational expectations in that model. This is because if the *expected* number of carriers who eliminate their logistics capability in period 1 is small, then the carriers anticipate that the platform will charge a low price in period 2. This in turn encourages the carriers to eliminate their logistics capability in period 1, leading to deviations. Similar deviations exist if the expected number of carriers eliminating their logistics capability in period 1 is large. Therefore, there is no equilibrium.

We have also obtained the equilibrium results for the UCC and the platform models for a case where each carrier's task volumes across the two periods are negatively correlated. Compared to Theorem 2.9, the negative demand correlation induces more carriers to use the UCC's service. We omit the details here.

2.7 Summary

We study two different business models to make urban last-mile delivery in a smart city more economically sustainable and operationally efficient. Under the first business model, a consolidator operates a UCC, which requires a sorting facility and a fleet of trucks to deliver the tasks of carriers. The consolidator bears the delivery costs, but charges the carriers a service fee for the last-mile delivery. Under the second business model, the consolidator operates a platform for the carriers to share their delivery capacity. The consolidator does not need a facility and trucks. There is no delivery cost incurred to the consolidator, who receives a revenue share from each transaction of capacity on the platform. The UCC can achieve a larger economy of scale as each truck of the UCC may consolidate the tasks from many carriers. In contrast, each carrier on the platform has only very limited delivery capacity compared to the UCC's fleet.

For each business model, we develop a two-period game-theoretical model capturing the interactions between the consolidator and the multiple carriers. In each period, the consolidator first determines the delivery fee per unit volume to maximize her expected profit. Then, after knowing his task volume, each carrier minimizes his expected cost by choosing to (i) deliver on his own, (ii) use the consolidator's service and eliminate his own logistics capability, or (iii) use the consolidator's service but keep his own logistics capability.

In practice, the carriers under the UCC business model face the following trade-off: They can potentially save their delivery costs by using the UCC's service, but they are subject to the risk of losing their logistics capability. Our game-theoretical model delicately demonstrates this trade-off through its equilibrium results (see Figures 2.2 and 2.3). As the carriers' variable delivery cost m increases, they become more dependent on the UCC to save their delivery costs: In period 1 the carriers who use the UCC's service will eliminate their logistics capability, and in period 2 more carriers will use the UCC's service. On the other hand, as the carriers' logistics reestablishment cost f increases, they become less dependent on the UCC to avoid the risk of losing their logistics capability: In period 1 the carriers who use the UCC's service will keep their logistics capability, and in period 2 fewer carriers will use the UCC's service. We also find that if the UCC receives a sufficient government subsidy, then all the carriers will use the UCC's service in period 2, making the UCC more sustainable in the long run. This echoes the phenomenon in practice that many UCC projects rely on government subsidies.

Under our capacity sharing platform model, the carriers generally have their logistics capability on hand in equilibrium (even if they purchase capacity from the platform). This ensures sufficient capacity available on the platform to facilitate successful transactions. Since the platform can always earn a positive profit (revenue share) from each transaction, our equilibrium results partially explain the increasing popularity of the capacity sharing platforms in practice. Only if f is sufficiently small and m is sufficiently large, the carriers who purchase capacity from the platform in period 1 will eliminate their logistics capability, and will purchase capacity again from the platform in period 2.

We compare the UCC and the capacity sharing platform in terms of their expected profits. In general, the UCC is more profitable than the platform if the carriers' fixed delivery cost c is large. If c is large, the carriers are more likely to outsource their delivery service, leading to a low supply of capacity on the platform. Thus, there will not be sufficiently many successful transactions on the platform, causing it to be less profitable than the UCC. Moreover, it is easier for the UCC to dominate as the number of carriers n becomes larger because of her economy of scale in shipment consolidation. However, there is an exception if f is sufficiently large and m is intermediate (Region (ii) of §2.5.1). In this situation, the platform outperforms the UCC if the discount factor δ is small. Since the carriers are less sensitive to their costs in period 2, they become more likely to do their own delivery (and sell their remaining capacity to the platform). This mitigates the imbalance of supply and demand on the platform.

We also compare the UCC and the platform in terms of their social-environmental impact. Although additional trucks are required by the UCC model, each truck of the UCC can potentially consolidate multiple carriers' tasks. In contrast, no additional trucks are required by the platform model, but each carrier on the platform can only serve at most one other carrier because of his limited capacity. We find that if n is large, then the UCC is more efficient in reducing the expected social-environmental cost than the platform. This is because the UCC's trucks (each can serve multiple tasks) can achieve a larger economy of scale in shipment consolidation when n is large. This significantly reduces the traffic congestion and pollution of the last-mile delivery. Note that this is nontrivial because the threshold of n for the UCC to outperform the platform varies with the probability of a low task volume λ in different manners under different situations (see Figure 2.7).

We study two extensions of our models. The first extension considers a hybrid model in which the consolidator concurrently operates a UCC and a platform. We also analyze an extension with correlated demands between two periods for each carrier. Other future research directions include endogenizing the government subsidy S and considering the construction costs of the UCC and the platform.

Chapter 3

Should Retailers Integrate Their Offline and Online Channels? A Perspective of Product Descriptions and Consumer Reviews

3.1 Introduction

During the past decades, many retailers have supplemented their brick-and-mortar (offline) stores with an online channel (Gao and Su, 2017). Conventionally, these retailers operate their offline and online channels separately (for example, in terms of human resource, inventory, product information, etc.). To provide consumers a seamless shopping experience across the offline and online channels, some retailers start to adopt an *omni-channel strategy* that employs new technology to integrate the two channels (Cisco Study, 2013).

For some product categories such as apparel, consumers' purchase decisions are affected by their trials of the products in brick-and-mortar stores.

However, for many other product categories such as electronics, cosmetics, drugs, food, and books, consumers may not be able to exhaustively try the products before purchases. Thus, for these product categories, the consumers' purchase decisions are largely determined by *product descriptions*. For example, it is difficult to judge a digital camera's versatility in handling different lighting conditions without extensive use of the product. Such information is usually provided in product descriptions.

According to the literature (see, for example, Gu and Xie (2013) and Sun and Tyagi (2017)), a consumer's purchase decision depends on how well a product's attributes fit the consumer's personal taste. For many product categories, it is difficult for consumers to assess a product's attributes even if the product can be inspected physically. In contrast, a more comprehensive product description can reduce the uncertainty of product fit, which affects the consumers' purchase decisions. We focus on such product categories in this work.

While the online channel can offer virtually unlimited space to describe a product (Berman and Thelen, 2004), a detailed product description is usually lacking in the offline channel. According to a survey by Digimarc Corporation (2015), 85% of U.S. adult shoppers indicated that their in-store purchase decisions were influenced by the descriptive information on product packages. However, 78% of them mentioned that they did not find sufficient information that they required. This reflects the limitation of the product descriptions in the offline channel. If the offline and online channels are operated separately, then it is hard to resolve the offline consumers' product-fit uncertainty because of the product description limitation (Mayzlin and Shin, 2011). Many consumers expect offline stores to provide the same product descriptions as their online counterparts, and 71% of them prefer in-store access to digital contents (Cisco Study, 2013).

Nowadays, with the advance in information technology, some retailers are able to disseminate a product description to both the offline and online consumers seamlessly, overcoming the product description limitation in the offline channel. For example, the Chinese e-commerce giant, Alibaba has proposed a "smart store" concept recently. In a smart store, offline consumers can access additional product descriptions about sizes, colors, and functions from an online channel through a "cloud shelf", which is a digital interactive wall screen (Chen, 2017, Dudarenok, 2018). Maserati has transformed two of its dealers in China into smart stores, where customers can scan different parts of a car to learn about its features (Chou, 2018). Similarly, the beauty and body-care retailer, Sephora encourages in-store customers to scan products using Sephora's mobile app to access additional product descriptions (Lawson, 2016). The Indian e-commerce platform, Paytm Mall collaborates with a fashion brand, Redtape to enable QR codes for their entire offline catalogue. When offline consumers scan a QR code, they receive a product description from an online platform (Nair, 2018, Rai, 2018).

By integrating the offline and online channels with such technologies, an omni-channel retailer is able to provide a more detailed product description to her offline consumers, reducing their product-fit uncertainty. However, the retailer delivers the same product description through both the offline and online channels, albeit the consumers' decisions may be different across the two channels. For example, if a consumer decides not to purchase a product from the retailer, then he can visit another retailer. Switching to another retailer usually incurs a larger hassle cost to the offline consumers (Balasubramanian, 1998, Forman et al., 2009, Mehra et al., 2017). This is because the cost of searching and transportation for an offline consumer to visit another brick-and-mortar store is higher than the cost of switching to another website for an online consumer. In this case, integrating the offline and online channels causes inflexibility that prevents the retailer to take advantage of the difference between her offline and online consumers' decisions. Thus, it is not clear whether adopting new technologies to integrate the offline and online channels can always benefit the retailer.

The Internet and information technology also provide an opportunity for consumers who purchase a product to share their reviews online. *Consumer reviews* can be influential on the upcoming consumers' purchase decisions

(Chen and Xie, 2008). Among all the ratings and reviews that may influence the consumers' purchase decisions, the reviews that are endogenously generated by the consumers on retailers' websites are most influential (Cisco Study, 2013). It is known that positive reviews can increase the consumers' intention to purchase a product (Bickart and Schindler, 2001, Shaffer and Zettelmeyer, 2002, Huang and Chen, 2006, Park et al., 2007). However, whether a consumer will post a positive review depends on his experience of the product (Li and Hitt, 2008, Moe and Trusov, 2011). Generally, the review will be positive if the product fits the consumer's taste. The consumer can find out whether the product fits his taste from the product description. Thus, the product description will affect the consumer reviews, which in turn will affect the retailer's sales.

Given that most consumer review systems are available only online, a retailer who operates the offline and online channels separately is typically unable to deliver consumer reviews to the offline consumers. However, the technology integrating the two channels allows both the offline and online consumers to access the same consumer reviews (in addition to a consistent product description). For example, in the Alibaba smart stores, consumers can read reviews through the cloud shelf (Chen, 2017, Dudarenok, 2018). In Sephora offline stores, consumers can also read reviews by scanning a product using a Sephora's mobile app. Since the presence of consumer reviews will complicate a retailer's product-description decision, it in turn makes the retailer's decision on whether or not to integrate the offline and online channels challenging.

The above observations motivate the following research questions: (i) Under what conditions should a retailer integrate her offline and online channels? (ii) What is the impact of consumer reviews on this decision process? Will the consumer reviews make integrating the two channels more favorable? (iii) Lastly, will the consumer reviews improve or hurt the retailer's expected profit? To the best of our knowledge, there is no previous research that optimizes the product descriptions with endogenously generated consumer reviews in a multi-channel retail setting.

To address the above questions, we consider a retailer selling a single product to consumers through an offline channel and an online channel. The consumers in each channel are heterogeneous such that the product fits the tastes of only a fraction of these consumers. We also consider the difference of the outside options between the offline consumers and the online consumers. We construct a two-period game-theoretical model to study the retailer's decision-making process. If the retailer operates the two channels separately, she optimizes a product description level for each individual channel. In contrast, if the retailer integrates the two channels, she optimizes a common product description level for both channels. In each period, the retailer first determines the product description level(s). Upon observing the product description, each consumer updates his belief about the product fit and makes a purchase decision to maximize his expected utility. After the consumers make their purchase decisions, the profit of the retailer is realized.

We further consider that if a consumer review system is available, the consumers who purchase the product in period 1 may post their reviews on the system, which will affect the period 2 consumers' purchase intention. Although the consumer reviews in practice may provide some information mitigating the product fit uncertainty (Kwark et al., 2014), we focus on the reviews which only affects consumers' valuation about the product, to make our model tractable. It's common in practice that consumers only provide a review stating that they "liked" the product or that they "disliked" it (Ifrach et al., 2015). This type of reviews and star ratings do not contain any fit information, which cannot help consumers further resolve their fit uncertainty. In our model, the consumers only rely on the product descriptions provided by the retailer to update their belief about the product fit. In contrast, the fraction of positive consumer reviews generated in period 1 only affects period 2 consumers' product valuation. If the offline and online channels are operated separately, only the online consumers can post and read the reviews, whereas the consumers in both channels can post and read the reviews if the two channels are integrated.

We have obtained the following insights:

(i) Without consumer reviews, we find that it is more profitable for the retailer to integrate the offline and online channels than to operate them separately if the limit of the offline product description is low and the consumers' base product valuation is small. This is because if the two channels are operated separately, the offline consumers are unlikely to purchase the product given the high product-fit uncertainty caused by the limited product description. In contrast, integrating the offline and online product descriptions makes more offline consumers willing to purchase the product with a more detailed product description, and making the retailer more profitable. However, if the base product valuation is sufficiently large, it is more profitable for the retailer to operate the two channels separately. In this situation, it is easier to attract the offline consumers who have a low utility from the outside option. Differentiating the product description levels between the two channels yields a higher profit for the retailer.

(ii) In the presence of consumer reviews, even if the offline product description limit is high, it can still be more profitable for the retailer to integrate the offline and online channels. This is because it allows the offline consumers to post and read the reviews. This retailer provides a sufficiently rich product description in the first period to generate a large fraction of positive reviews, which in turn raise the offline consumers' product valuation in the second period. This increases the retailer's expected profit. However, since the retailer sets the same product description level for both channels, the higher product description level in the first period can deter some offline consumers whom the product does not fit from purchasing the product. If the profit gain from the upcoming consumers in the second period can compensate this profit loss in the first period, it is more profitable to integrate the offline and online channels.

(iii) Consumer reviews may reduce the retailer's profit if the consumers' base product valuation is sufficiently large. This is because when there are no reviews, the retailer can already attract the consumers with large base product valuation easily even with a low product description level. With the addition of the consumer reviews, however, the retailer has to set a high product description level in the first period to prevent the negative impact of the reviews on the upcoming consumers. The increase in the product description level drives down the retailer's profit in the first period as it prevents some consumers whom the product does not fit from purchasing the product, which is detrimental to the retailer.

This chapter is organized as follows. After reviewing the relevant literature in §3.2, we analyze the models without and with consumer reviews in §3.3 and §3.4 respectively. We consider an extended model in §3.5. §3.6 concludes this chapter. All proofs are provided in Appendix B.

3.2 Literature review

We consider a retailer that operates an offline channel and an online channel separately with different product description levels or integrates the two channels with a common product description level. In contrast, a stream of literature on traditional channel management studies different channels operated by different companies. See, for example, Chiang et al. (2003), Cattani et al. (2006), Chen et al. (2008), and Netessine and Rudi (2006). Our work is also related to Bernstein et al. (2008), who consider bricks-and-mortar retailers opening their online channels in an oligopoly setting.

There is growing literature on omni-channel retail management. Most operations management papers in this area focus on fulfillment. Gallino and Moreno (2014) empirically test the impact of the practice of buy-online, pick-up-in-store (BOPS) on a retailer's sales in both online and offline channels. Gao and Su (2017) study the implications of BOPS on channel coordination based on an analytical model. Gallino et al. (2017) investigate another omni-channel fulfillment strategy – ship to store. They empirically demonstrate that within a group of stock-keeping units, the strategy increases the sales of bottom-selling items. Lim et al. (2016) describe last-mile supply network configurations in omni-channel retailing. Harsha et al. (2019) propose two pricing policies for an omni-channel retailer in the presence of cross-channel fulfillment.

A few papers study how to effectively provide information to consumers in an omni-channel environment. Bell et al. (2017) investigate the impact of physical showrooms on consumers' channel choice. Gao and Su (2017) study the individual as well as the joint impacts of physical showrooms, virtual showrooms, and inventory-availability information on consumer behavior and retail operational efficiency. Gao et al. (2018) investigate how an online channel influences a retailer's decisions regarding physical stores, where customers can inspect products. In contrast, our work considers product descriptions and consumer reviews. The former helps consumers assess whether a product fits their tastes, whereas the latter can influence the future consumers' purchase decisions. We study how a retailer strategically provide the product descriptions in different channels with and without consumer reviews. We identify conditions in which the retailer is better off by integrating the offline and online channels.

Our research is also related to the literature on how a firm can induce purchases from consumers facing product value and fit uncertainty. Gu and Xie (2013) examine firms' equilibrium fit-revelation decisions in a competitive market. Ofek et al. (2011) focus on the impact of adding an online channel on a retailer's pricing and fit-revelation decisions. Kwark et al. (2014) and Sun and Tyagi (2017) examine the disclosure of product-fit information in the context of a distribution channel. Liu et al. (2019) investigates the optimal information provision strategy to resolve uncertainty in the presence of consumer search costs. Different from these papers, our study focuses on the retailer's decisions on the product descriptions under two different channel-management strategies: operating the offline and online channels separately or integrating them.

This research also adds to the literature that studies consumer reviews. Chen and Xie (2008) study how to adjust a retailer's marketing communication strategy in response to consumer reviews. They identify when the retailer benefits from facilitating the consumer reviews. Yu et al. (2016) examine the impact of consumer reviews on a firm's dynamic pricing strategy in the presence of strategic consumers. Liu et al. (2017) study how online reviews and past-sales-volume information jointly affect consumer purchase decisions and firms' pricing strategies. In contrast, we investigate how consumer reviews affect a retailer's decisions on the product descriptions and her incentive to integrate the offline and online channels.

3.3 Model without consumer reviews

3.3.1 Model description

We consider a retailer that sells a single product through an offline (brick-and-mortar) channel and an online channel to consumers in two periods $t = 1, 2$. Let $i = b, o$ denote the offline and online channels respectively. For each channel, we assume there are n distinct consumers in each period. For simplicity, we assume the retail price of the product is fixed at p for both channels and for both periods.

The product is characterized by several attributes related to its design and functionality. For example, the attributes that characterize a digital camera are its size, weight, sensor, image processor, AF points, ISO range, etc. The consumers are heterogeneous in their tastes (preferences) for each product attribute. We assume that each consumer knows his own taste for each attribute, but the actual details of the attribute of the product are unknown to him. Therefore, before purchasing the product, the consumer is unsure about whether each product attribute matches his expectation. We say the product *fits* a consumer if *all* its attributes fit the consumer's taste. Otherwise, the product *does not fit* the consumer. Let $m = 0$ denote a scenario where the product fits the consumer and assume this occurs with a probability $P\{m = 0\} = \theta \in (0, 1)$ in each period. Let $m = 1$ denote a scenario where the product does not fit a consumer and assume this occurs with a probability $P\{m = 1\} = 1 - \theta$ in each period. Given the definition of m, each consumer's valuation of the product is $V - cm$, where V represents base product valuation and $c > 0$ captures a valuation reduction due to *product misfit*. Note that for the sake of tractability, we neglect the difference between a misfit caused by a single attribute and a misfit caused by multiple attributes.

In each period $t = 1, 2$, the retailer decides on the *product description level* $d_{i,t} \in [0, 1]$ for channel $i = b, o$. The product description level $d_{i,t}$ represents the amount of information of the product's attributes disclosed to the consumers. This information helps the consumers resolve the uncertainty of m. A larger $d_{i,t}$ corresponds to a more informative product description for channel i in period t. Similar to a common approach in the literature (see, for example, Lewis and Sappington (1994), Chen and Xie (2008), and Kwark et al. (2014)), we assume that after knowing $d_{i,t}$, a *private signal* s is generated for each consumer. Specifically, after the consumer reads the product description in channel i, if this description suggests that the product fits the consumer, then $s = 0$. Otherwise, we have $s = 1$. If the product *actually does not fit* the consumer ($m = 1$), the signal s is more likely to reveal the misfit (s is more likely to appear as 1) as more information about the product's attributes is provided. Thus, we have $P\{s = 1|m = 1\} = d_{i,t}$ and $P\{s = 0|m = 1\} = 1-d_{i,t}$, for $i = b, o$ and $t = 1, 2$. On the other hand, if the product *actually fits* the consumer ($m = 0$), then all its attributes fit the consumer's taste. In this case, the signal s will not reveal any misfit (s will appear as 0) regardless of the product description level $d_{i,t}$. That is, $P\{s = 0|m = 0\} = 1$.

Let U denote each consumer's utility of purchasing the product from the retailer. Given the signal s and using the above probabilities, we can derive each consumer's conditional expected utility of purchasing the product as $E[U|s] = V - cE[m|s] - p$, where $E[m|s]$ represents the *expected degree of misfit given* s. According to Bayes' Theorem (Stuart and Ord, 1994), we have $E[m|s = 0] = \frac{(1-\theta)(1-d_{i,t})}{1-(1-\theta)d_{i,t}}$ and $E[m|s = 1] = 1$. Based on the above results, we can derive each consumer's conditional expected utility in channel i for each period t as

$$\begin{array}{l} E[U|s = 0] = V - cE[m|s = 0] - p = V - \frac{c(1-\theta)(1-d_{i,t})}{1-(1-\theta)d_{i,t}} - p, \\ E[U|s = 1] = V - cE[m|s = 1] - p = V - c - p. \end{array} \tag{3.1}$$

Each consumer in channel i decides whether to purchase the product from the retailer by comparing his conditional expected utility in Equations (3.1) with an outside option. Note that we assume that if a consumer decides not to purchase the product from the retailer through channel i, then his outside option is to visit and purchase it from another retailer. Let u_i denote the consumer's utility from the outside option of channel i. We assume $u_o > u_b$ because switching to an outside option usually incurs a larger hassle cost to the offline consumers (Balasubramanian, 1998, Forman et al., 2009, Mehra et al., 2017) as mentioned in §3.1. Furthermore, to exclude uninteresting cases in which the consumers make purchases for any product description level or always purchase from only one channel, we assume $p+u_o \leq V < p+u_b+(1-\theta)c$ and $(1-\theta)c/2 < u_o-u_b < \min\{\theta c, (1-\theta)c\}$.

Let $\phi_{i,t}(d)$ denote the probability of each consumer in channel i to purchase the product in period t given a product description level d. The retailer's expected profit from channel i in period t is denoted as $\pi_{i,t}(d_{i,t}) = p \times n \times \phi_{i,t}(d_{i,t})$, for $i = b, o$ and $t = 1, 2$. We assume that providing a product description incurs a fixed cost that is independent of the product description level $d_{i,t}$, and we normalize it to zero in our analysis. This is reasonable in practice because the extra cost of adding a few more lines of text about the product is often negligible. To refine the equilibrium, we assume that if $\pi_{i,t}(d'_{i,t}) = \pi_{i,t}(d''_{i,t})$ and $d'_{i,t} < d''_{i,t}$, then the retailer always chooses $d'_{i,t}$.

The retailer can operate the offline and online channels separately, or integrate the two channels. Depending on the retailer's channel-management strategy, the retailer makes different decisions to maximize her total expected profit. Specifically, a retailer who operates two channels separately chooses $d_{b,t}$ to maximize $\pi_{b,t}$ and chooses $d_{o,t}$ to maximize $\pi_{o,t}$, resulting in a total expected profit $\pi_{dual,t} = \pi_{b,t} + \pi_{o,t}$. To capture the offline channel's limitation in the product description, we assume that $d_{b,t} \leq \bar{d}$, where $\bar{d} \in (0, 1)$. In contrast, with the technology mentioned in §3.1, a retailer who integrates the offline and online channels does not face such a limitation in her offline channel. Instead, the retailer chooses an identical product description level $d_{omni,t}$ for the two channels to maximize her total expected profit $\pi_{omni,t} = p \times (n \times \phi_{b,t}(d_{omni,t}) + n \times \phi_{o,t}(d_{omni,t}))$.

In each period t, the retailer first decides the product description level for each channel to maximize her total expected profit. After reading the product description, a private signal s is generated for each consumer in channel i according to the probability $P\{s|m\}$. The consumer then decides whether to purchase the product by comparing his conditional expected utility $E[U|s]$ with the outside option of channel i. Figure 3.1 shows the sequence of the decisions in the two periods.

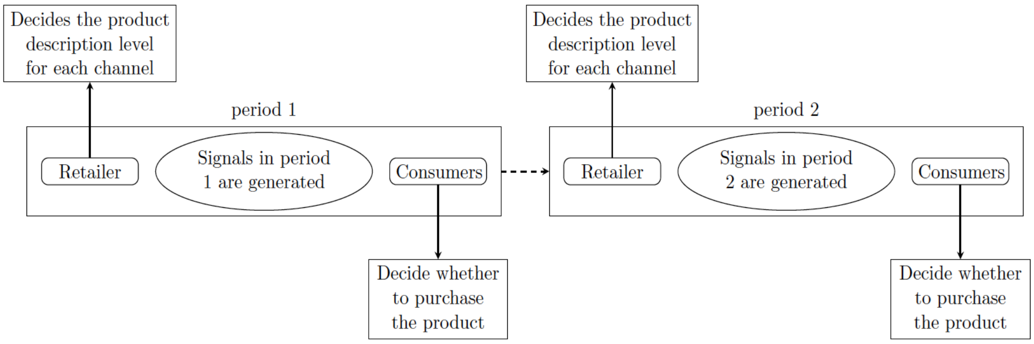


FIGURE 3.1: The sequence of decisions in the two periods without consumer reviews

We first determine the optimal decisions of the retailer when she operates the offline and online channels separately in §3.3.2, before we find the optimal decisions of the retailer when she integrates the offline and online channels in §3.3.3. Note that in the absence of consumer reviews, the optimal decisions are the same across the two periods and we use a superscript $(^*)$ to denote all the optimal decisions and the equilibrium outcomes. We identify conditions under which integrating the offline and online channels benefits the retailer. Although the decisions are the same across the periods, it is worth considering the two-period model here so that we can compare it with a model with consumer reviews, in which the decisions are different across the periods.

3.3.2 Operating offline and online channels separately

We first analyze the retailer's optimal decisions for the offline channel and then for the online channel. In the offline channel, the retailer chooses

$d_{b,t} \leq \bar{d}$ to maximize her expected profit $\pi_{b,t}$ in each period t. Lemma 3.1 shows the optimal decisions of the retailer for the offline channel. Let $\hat{d} = \frac{(1-\theta)c-(u_o-u_b)}{(1-\theta)[c-(u_o-u_b)]}$ and $\hat{V} = p + u_b + \frac{(1-\theta)(1-\bar{d})c}{1-(1-\theta)\bar{d}}$.

Lemma 3.1. *The retailer sets a product description level $d^*_{b,t} = 0$ for the offline channel with an expected profit $\pi^*_{b,t} = 0$ if $\bar{d} \leq \hat{d}$ and $V < \hat{V}$, and sets a product description level $d^*_{b,t} = \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_b-c)}$ with an expected profit $\pi^*_{b,t} = \frac{\theta npc}{p-V+u_b+c}$ otherwise, for $t = 1, 2$.*

For the online channel, the retailer can choose any $d_{o,t} \in [0, 1]$ to maximize her expected profit $\pi_{o,t}$ in each period t. Lemma 3.2 shows the retailer's optimal decisions.

Lemma 3.2. *The retailer sets a product description level $d^*_{o,t} = \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}$ for the online channel with an expected profit $\pi^*_{o,t} = \frac{\theta npc}{p-V+u_o+c}$, for $t = 1, 2$.*

Lemma 3.1 shows that if the product description limit for the offline channel is low and the consumers' base product valuation is small, then the retailer provides a minimum product description for the offline channel. Since the base product valuation is small, the consumers will not purchase the product if their fit uncertainty is high. The retailer cannot resolve the consumers' fit uncertainty even with the highest level of product description. Therefore, the retailer would rather not sell through the offline channel. In contrast, Lemma 3.2 shows that the retailer can always induce her online channel's consumers to purchase the product, and make a profit by optimizing the product description level.

3.3.3 Integrating offline and online channels

The retailer sets a common product description level $d_{omni,t}$ for the offline and online channels to maximize her total expected profit $\pi_{omni,t}$ in each period t. Lemma 3.3 shows the optimal decisions of the retailer.

Lemma 3.3. *The retailer sets a product description level* $d^*_{omni,t} = \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}$ *for both the offline and online channels with a total expected profit* $\pi^*_{omni,t} = \frac{2\theta npc}{p-V+u_o+c}$, *for* $t = 1, 2$.

By comparing Lemmas 3.3 and 3.1, we observe that for the offline channel, the retailer provides a more detailed product description when the offline and online channels are integrated than that when the two channels are operated separately (that is, $d^*_{omni,t} > d^*_{b,t}$). This can be achieved by enabling the offline consumers to access the online product description with the technology mentioned in §3.1.

3.3.4 Comparing the two strategies

By comparing the retailer's total expected profits under the two strategies in §3.3.2 and §3.3.3, we identify conditions under which integrating the offline and online channels yields a higher expected profit for the retailer.

Theorem 3.4. *The retailer is more profitable by integrating the offline and online channels if and only if* $\bar{d} \leq \hat{d}$ *and* $V < \hat{V}$.

Figure 3.2 illustrates whether integrating the offline and online channels is beneficial for the retailer under different values of $\bar{d}$ and V. It is intuitive that integrating the two channels is more profitable if $\bar{d}$ is small. This is because the integration removes the product description limit from the offline channel, and induces the consumers who are deterred by an insufficient product description to make purchases.

Figure 3.2 shows that, contrary to a common belief that integrating the offline and online channels is more beneficial, operating the two channels separately turns out to be more profitable if V is sufficiently large. This is because when the base product valuation is large, it is easier to attract the offline consumers whose utility from the outside option is low.

To take advantage of this, the retailer should differentiate the product description levels for the two channels to maximize her profit. Specifically, the

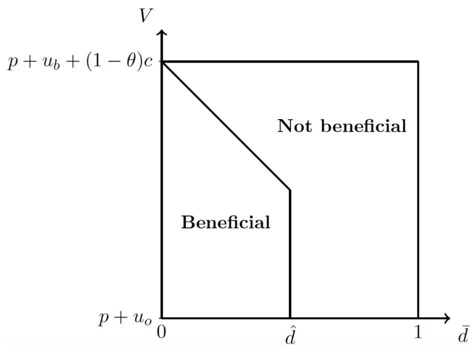


FIGURE 3.2: Region in which integrating the offline and online channels benefits the retailer

retailer should set a lower product description level for the offline channel (than the online channel) because the outside option there is less attractive. In contrast, the retailer does not have this flexibility if the two channels are integrated. Furthermore, a higher product description level for the offline channel (recall that $d^*_{omni,t} > d^*_{b,t}$) will increase the number of offline consumers who think the product does not fit them and will not purchase the product. When V is large, this effect dominates the benefit of integrating the two channels, hurting the retailer's profitability.

3.4 Model with consumer reviews

3.4.1 Model description

In this section, we assume that the consumers who purchase the product in period 1 can post their reviews about the product through a review system. The consumers in period 2 can read these reviews before they decide whether to purchase the product. To align with the practice that consumer reviews are typically posted online, we assume that if the retailer operates the offline and online channels separately, only the online consumers can post and read the reviews, whereas both the offline and online consumers can post and read the reviews if the retailer integrates the two channels.

We study the equilibrium outcomes and the implications of adding such a consumer review system.

Given the retailer's decision on the product descriptions and the generated private signals, the conditional expected utility and the purchase decision of each consumer in period 1 are the same as in the model without consumer reviews in §3.3. After a consumer purchases the product in period 1, he learns that whether the product fits him or not (that is, m is realized). The consumer will post a review with a probability $\eta \in [0, 1]$. Based on a common assumption in the literature of product reviews (Li and Hitt, 2008, Moe and Trusov, 2011), the consumer's review can be either positive or negative, depending on whether the product fits him. Specifically, if his *realized* utility from purchasing the product is no less than that from the outside option (that is, if $U = V - cm - p \geq u_i$), then the consumer will write a positive review. Otherwise, he will write a negative review. Note that this approach is commonly used in modeling consumer reviews (Ifrach et al., 2015). Furthermore, this approach ensures that a consumer, in our model, will write a positive review if and only if the product truly fits his taste, or write a negative review otherwise.

As mentioned in §3.1, we only focus on the consumer reviews that do not contain product fit information, to make our model tractable. For the consumers in period 2, they still rely on the product descriptions provided by the retailer to update their belief about the product fit $E[m|s]$. The consumer reviews generated in period 1 only affect their perceived value of the product. Extensive literature shows that only when the quantity of positive reviews is sufficiently large to overcome the negative attitudes from negative reviews, will those positive reviews improve consumers' purchase intentions (Huang and Chen, 2006, Bickart and Schindler, 2001, Shaffer and Zettelmeyer, 2002, Park et al., 2007). Thus, we assume that the fraction of positive reviews will affect period 2 consumers' product valuation ultimately. Specifically, let λ denote the fraction of positive reviews among all the consumer reviews generated in period 1. Let $\underline{\lambda}$ denote a threshold value, above which the consumer reviews will make a positive impact on period 2 consumer's product valuation. The period 2 consumers' ultimate valuation about the product is updated as $V - cE[m|s] + \psi(\lambda)$, where $\psi(\lambda) \equiv a(\lambda - \underline{\lambda})$ captures the impact of the consumer reviews.

Therefore, in period 2, the conditional expected utility of each consumer in channel i from purchasing the product can be derived as follows:

$$\begin{aligned}E[U|s=0] &= V - cE[m|s=0] + \psi(\lambda) - p = V - \tfrac{c(1-\theta)(1-d_{i,2})}{1-(1-\theta)d_{i,2}} + \psi(\lambda) - p,\\ E[U|s=1] &= V - cE[m|s=1] + \psi(\lambda) - p = V - c + \psi(\lambda) - p.\end{aligned} \tag{3.2}$$

The outside option of each channel i stays unchanged. Each consumer in channel i decides whether to purchase the product by comparing his conditional expected utility in Equations (3.2) with the outside option of the channel.

Note that in Equations (3.2), the fraction of positive reviews (λ) only affects the consumers' valuation of the product but cannot help them to determine whether the product fits them. This is suitable when the consumer reviews are expressed as thumbs up or down, or star ratings, which do not carry the information to resolve the uncertainty of product fit (Hu et al., 2006). A model that incorporates a more comprehensive review system providing the information about product fit is, unfortunately, intractable in our setting. Furthermore, we assume that $\theta + \frac{(1-\theta)c}{a} < \underline{\lambda} < 1 - \frac{(1-\theta)c}{a}$ and $a > 2c/\theta^2$ to exclude trivial cases in which the consumer reviews have no impact or a single-sided (only positive or negative) impact on the equilibrium outcomes.

Similar to the model without consumer reviews, in each period t, if the offline and online channels are operated separately, the retailer determines the product description level $d_{i,t}$ for each channel i, but if the two channels are integrated, the retailer determines a common product description level $d_{omni,t}$ for both channels. The sequence of the decisions is similar to that of the model without consumer reviews in Figure 3.1, except that in period 1, the retailer chooses the product description level for each channel to maximize her total expected profit *over the two periods*. Figure 3.3 shows the sequence of the decisions in the two periods. We solve the optimal decisions of the retailer and the consumers through backward induction to obtain the perfect Bayesian equilibrium.

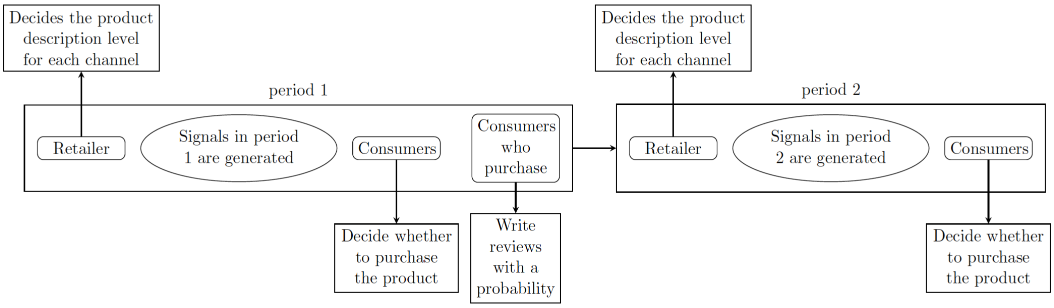


FIGURE 3.3: The sequence of decisions in the two periods with consumer reviews

We determine the optimal decisions and the equilibrium outcomes if the offline and online channels are operated separately in §3.4.2 and if the two channels are integrated in §3.4.3. We use a superscript (†) to denote all the optimal decisions and the equilibrium outcomes in the presence of consumer reviews. We identify conditions under which the consumer review system and the integration of the offline and online channels benefit the retailer.

3.4.2 Operating offline and online channels separately

If the retailer operates the offline and online channels separately, only the online consumers can post and read the reviews. Thus, the offline consumers have the same conditional expected utilities and optimal decisions as in the model without consumer reviews in §3.3.2. Thus, the retailer's optimal decision $d^\dagger_{b,t}$ for the offline channel is identical to $d^*_{b,t}$ in Lemma 3.1.

For the online channel, we determine the retailer's optimal decisions backward by first finding $d^\dagger_{o,2}$ in period 2, before we find $d^\dagger_{o,1}$ in period 1. The retailer takes the consumer reviews in period 1 into account, and chooses a product description level $d_{o,2}$ for the online channel in period 2 to maximize her expected profit $\pi_{o,2}(d_{o,2}) = p \times n \times \phi_{o,2}(d_{o,2})$. Recall that $\psi(\lambda) = a(\lambda - \underline{\lambda})$ represents the impact of consumer reviews.

Lemma 3.5. *In the presence of consumer reviews, the retailer sets a product description level $d_{o,2}^{\dagger}$ with an expected profit $\pi_{o,2}^{\dagger}$ in period 2 for the online channel as follows.*

1. $d_{o,2}^{\dagger} = 0$ *and* $\pi_{o,2}^{\dagger} = 0$ *if*
   (a) $\psi(\lambda) \leq u_o - u_b - (1-\theta)c$, *or*
   (b) $u_o - u_b - (1-\theta)c < \psi(\lambda) < 0$ *and* $V < p + u_o - \psi(\lambda)$.
2. $d_{o,2}^{\dagger} = \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V+\psi(\lambda)-p-u_o-c)}$ *and* $\pi_{o,2}^{\dagger} = \frac{\theta npc}{p-V+u_o+c-\psi(\lambda)}$ *if*
   (a) $u_o - u_b - (1-\theta)c < \psi(\lambda) < 0$ *and* $V \geq p + u_o - \psi(\lambda)$, *or*
   (b) $0 \leq \psi(\lambda) \leq u_o - u_b$, *or*
   (c) $u_o - u_b < \psi(\lambda) < (1-\theta)c$ *and* $V < p + u_o + (1-\theta)c - \psi(\lambda)$.
3. $d_{o,2}^{\dagger} = 0$ *and* $\pi_{o,2}^{\dagger} = pn$ *if*
   (a) $u_o - u_b < \psi(\lambda) < (1-\theta)c$ *and* $V \geq p + u_o + (1-\theta)c - \psi(\lambda)$, *or*
   (b) $\psi(\lambda) \geq (1-\theta)c$.

Figure 3.4 shows the retailer's optimal decision $d_{o,2}^{\dagger}$ for the online channel in period 2 under different values of $\psi(\lambda)$ and V. For Case 1 where $\psi(\lambda) < 0$ and V is small, the reviews have a negative impact on the consumers' product valuation in period 2, it is optimal for the retailer to set a minimum product description level ($d_{o,2}^{\dagger} = 0$). In this case, the consumers in period 2 will not purchase the product even if the retailer provides the maximum product description. Thus, the retailer sets $d_{o,2}^{\dagger} = 0$.

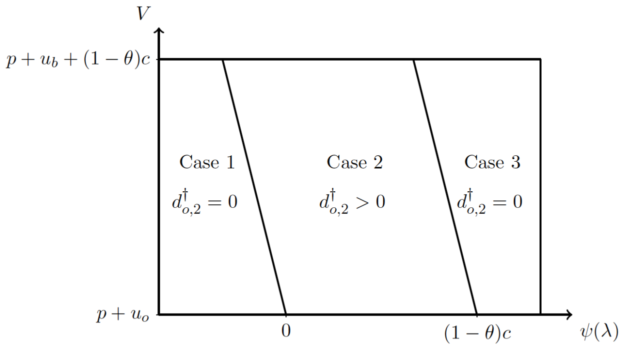


FIGURE 3.4: The retailer's optimal decision $d_{o,2}^{\dagger}$ for the online channel in period 2

For Case 2 where the reviews have a negative impact on the consumers' product valuation in period 2 but V is large, or the reviews have a moderate positive impact, the retailer can induce the consumers to purchase the product by choosing a positive product description level. For Case 3 where the reviews have a strong positive impact on the consumers' product valuation, the retailer can induce all the consumers in period 2 to purchase the product even if she provides a minimum product description.

By comparing Lemma 3.5 and Lemma 3.2 (without consumer reviews), we have the following findings. (i) While the retailer always makes a profit in the absence of reviews, she may not make any profit from the online channel in the presence of reviews. For example, in Case 1 of Lemma 3.5 where the reviews have a negative impact on the consumers' product valuation and V is small, the retailer earns no profit and she is hurt by the consumer review system. (ii) While the retailer always sets a positive product description level in the absence of reviews, she may choose to provide no product description for the online channel in the presence of reviews. For example, in Case 3 of Lemma 3.5 where the reviews have a strong positive impact on the consumers' product valuation, the retailer can make a higher profit. In this case, the consumer review system benefits the retailer.

In period 1, the retailer chooses a product description level $d_{o,1}$ for the online channel to maximize her total expected profit $\Pi_o = \pi_{o,1} + \pi_{o,2}$ over the two periods. Let $\bar{\lambda} = \theta + \frac{(u_o - u_b)(a-c)}{ac}$ and $\bar{V}_1 = p + u_o + (1-\theta)c - \frac{ca(\underline{\lambda}-\theta)}{a-c}$.

Lemma 3.6. *In the presence of consumer reviews, the retailer sets a product description level $d_{o,1}^{\dagger}$ with a total expected profit $\Pi_o^{\dagger}$ over the two periods for the online channel as follows.*

1. $d_{o,1}^{\dagger} = \frac{1}{1-\theta} + \frac{\theta a}{(1-\theta)(V-p-a\underline{\lambda}-u_o-(1-\theta)c)}$ *and* $\Pi_o^{\dagger} = \frac{\theta npa}{p-V+a\underline{\lambda}+u_o+(1-\theta)c} + pn$ *if* $\underline{\lambda} \geq \bar{\lambda}$ *and* $V \geq \bar{V}_1$.
2. $d_{o,1}^{\dagger} = \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}$ *and* $\Pi_o^{\dagger} = \frac{\theta npc}{p-V+u_o+c} + pn$ *otherwise.*

The proof of Lemma 3.6 shows that the retailer sets a high product description level in period 1 to reduce the number of negative reviews from the consumers whom the product does not fit. This is because, after reading a more detailed product description, the consumers whom the product does not fit (with $m = 1$) are more likely to find out the misfit ($s = 1$) and they will neither purchase the product nor write a review. Therefore, the retailer is able to generate a large fraction of positive reviews λ such that $\psi(\lambda)$ has a positive impact on the consumers' product valuation in period 2.

Comparing Lemma 3.6 with Lemma 3.2, if the consumers are very sensitive to the negative reviews and have large base product valuation (that is, $\underline{\lambda} \geq \bar{\lambda}$ and $V \geq \bar{V}_1$), then we have $d^{\dagger}_{o,1} > d^{*}_{o,1}$. This means the retailer needs to provide a more detailed product description in the presence of reviews. This is because, without a sufficiently large λ, the reviews will hurt the consumers' product valuation in period 2 ($\psi(\lambda)$ is negative), making it difficult to sell the product even if V is large. Therefore, to make λ sufficiently large, the retailer needs a higher product description level $d^{\dagger}_{o,1}$ to reduce the number of consumers, whom the product does not fit, to purchase the product and write negative reviews.

To find out the impact of the consumer reviews on retailer, we compare the retailer's total expected profits Π^{*}_{o} and $\Pi^{\dagger}_{o}$ and the corresponding optimal decisions $d^{*}_{o,1}$ and $d^{\dagger}_{o,1}$ without and with the consumer reviews respectively. Let $\bar{V}_2 = p + u_o + (1-\theta)c - \frac{\theta c - a(\theta + \underline{\lambda}) + \sqrt{(a(\theta+\underline{\lambda}) - \theta c)^2 + 4\theta c a(\underline{\lambda} - \theta)}}{2}$.

Theorem 3.7. *The consumer review system has the following impact on the retailer.*

1. *If $\underline{\lambda} < \bar{\lambda}$, or $\underline{\lambda} \geq \bar{\lambda}$ and $V < \bar{V}_1$, then the consumer review system benefits the retailer ($\Pi^{\dagger}_{o} > \Pi^{*}_{o}$) and it does not affect the product description level in period 1 ($d^{\dagger}_{o,1} = d^{*}_{o,1}$).*

2. *If $\underline{\lambda} \geq \bar{\lambda}$ and $\bar{V}_1 \leq V < \bar{V}_2$, then the consumer review system benefits the retailer ($\Pi^{\dagger}_{o} > \Pi^{*}_{o}$) and it increases the product description level in period 1 ($d^{\dagger}_{o,1} > d^{*}_{o,1}$).*

3. *If $\underline{\lambda} \geq \bar{\lambda}$ and $V \geq \bar{V}_2$, then the consumer review system hurts the retailer ($\Pi_o^\dagger < \Pi_o^*$) and it increases the product description level in period 1 ($d_{o,1}^\dagger > d_{o,1}^*$).*

Adding the consumer review system benefits the retailer (that is, $\Pi_o^\dagger > \Pi_o^*$) except when both $\underline{\lambda}$ and V are large. There are three cases in Theorem 3.7. First, if the consumers are not very sensitive to negative reviews (that is, $\underline{\lambda} < \bar{\lambda}$), then the reviews are likely to have a positive impact on the consumers' product valuation in period 2. In this case, adding the consumer review system increases the retailer's profit in period 2 without affecting the product description level ($d_{o,1}^\dagger = d_{o,1}^*$) and the expected profit in period 1. As a result, the retailer's total expected profit over the two periods increases after adding the consumer review system (that is, $\Pi_o^\dagger > \Pi_o^*$).

If $\underline{\lambda}$ is large but V is sufficiently small ($\underline{\lambda} \geq \bar{\lambda}$ and $V < \bar{V}_1$), then, even without reviews, the retailer already needs to set a high product description level to help the consumers resolve product-fit uncertainty and to induce them to purchase the product (because the conditional expected utility in Equation (3.1) increases with $d_{o,1}$). Adding the consumer review system does not affect the product description level in period 1 (that is, $d_{o,1}^\dagger = d_{o,1}^*$). This is because the high product description level can already deter the consumers, whom the product does not fit, to purchase the product in period 1. This leads to a large fraction of positive reviews λ and generates a larger expected profit for the retailer in period 2. Thus, we have $\Pi_o^\dagger > \Pi_o^*$.

Second, if $\underline{\lambda}$ is large and V is moderate (Case 2 of Theorem 3.7) then adding the consumer review system increases the retailer's total expected profit (that is, $\Pi_o^\dagger > \Pi_o^*$) and increases the product description level in period 1 (that is, $d_{o,1}^\dagger > d_{o,1}^*$). This is because a moderate V means that there is a considerable number of consumers purchasing the product even without the consumer reviews. Thus, adding the consumer review system will benefit the retailer ($\Pi_o^\dagger > \Pi_o^*$) only when λ is sufficiently large. To achieve that, the retailer needs to increase the product description level in period 1 ($d_{o,1}^\dagger > d_{o,1}^*$) to reduce the number of consumers whom the product does not fit to make purchases and write negative reviews.

Lastly, adding the consumer review system hurts the retailer's expected profit (that is, $\Pi_o^\dagger < \Pi_o^*$) and increases the product description level in period 1 (that is, $d_{o,1}^\dagger > d_{o,1}^*$), if $\underline{\lambda}$ and V are both large (Case 3 of Theorem 3.7). In this case, V is sufficiently large that even without the consumer reviews, the retailer can easily attract many consumers to purchase in both periods with a low product description level. However, in the presence of reviews, the retailer has to set a higher product description level in period 1 ($d_{o,1}^\dagger > d_{o,1}^*$) to ensure a larger λ such that the reviews will not negatively affect her profit in period 2. This increase in the product description level drives down the retailer's profit in period 1 because some consumers will find out the product misfit and will not purchase the product. The profit gain in period 2 due to the positive reviews is insufficient to compensate the profit loss in period 1. Thus, the retailer is worse off by having the consumer review system ($\Pi_o^\dagger < \Pi_o^*$).

Overall, Theorem 3.7 reveals that whether the consumer review system is beneficial to the retailer depends on two important factors: the consumers' sensitivity to negative reviews ($\underline{\lambda}$) and the consumers' base product valuation (V). Figure 3.5 illustrates the region in which adding the consumer review system benefits the retailer.

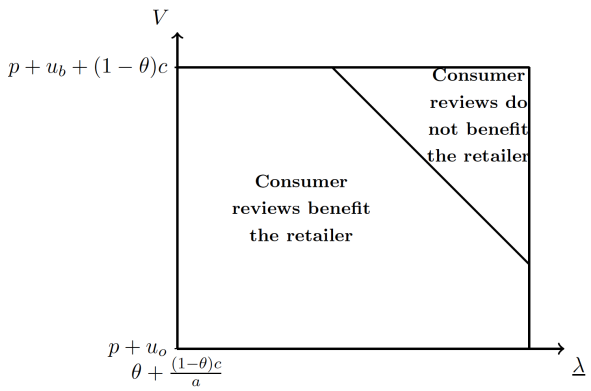


FIGURE 3.5: Adding the consumer review system may benefit the retailer

3.4.3 Integrating offline and online channels

If the offline and online channels are integrated, the retailer sets the same product description level $d_{omni,t}$ for the two channels and enables the consumers in both channels to post and read the reviews. To facilitate the offline consumers to provide their feedback, the retailer can enable them to create online accounts so that they can post their reviews (Chen, 2017, Yang, 2018). To allow the consumers to read the reviews conveniently in the offline stores, the retailer can install the "cloud shelf."

Taking into account of the consumer reviews, the retailer optimizes her decision $d_{omni,1}$ in period 1 to maximize her total expected profit $\Pi_{omni} = \pi_{omni,1} + \pi_{omni,2}$ over the two periods, and chooses $d_{omni,2}$ to maximize her expected profit $\pi_{omni,2}$ in period 2. We determine the retailer's optimal decisions backward by first finding $d^{\dagger}_{omni,2}$ in period 2, before we find $d^{\dagger}_{omni,1}$ in period 1.

Lemma 3.8. *In the presence of consumer reviews, the retailer sets a product description level $d^{\dagger}_{omni,2}$ with an expected profit $\pi^{\dagger}_{omni,2}$ in period 2 as follows.*

1. $d^{\dagger}_{omni,2} = 0$ *and* $\pi^{\dagger}_{omni,2} = 0$ *if*
   (a) $\psi(\lambda) \leq -(1-\theta)c$, *or*
   (b) $-(1-\theta)c < \psi(\lambda) < -(u_o - u_b)$ *and* $V < p + u_b - \psi(\lambda)$.
2. $d^{\dagger}_{omni,2} = \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V+\psi(\lambda)-p-u_b-c)}$ *and* $\pi^{\dagger}_{omni,2} = \frac{\theta npc}{p-V+u_b+c-\psi(\lambda)}$ *if*
   (a) $-(1-\theta)c < \psi(\lambda) < -(u_o - u_b)$ *and* $V \geq p + u_b - \psi(\lambda)$, *or*
   (b) $-(u_o - u_b) \leq \psi(\lambda) \leq u_o - u_b - (1-\theta)c$, *or*
   (c) $u_o - u_b - (1-\theta)c < \psi(\lambda) < 0$ *and* $V < p + u_o - \psi(\lambda)$.
3. $d^{\dagger}_{omni,2} = \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V+\psi(\lambda)-p-u_o-c)}$ *and* $\pi^{\dagger}_{omni,2} = \frac{2\theta npc}{p-V+u_o+c-\psi(\lambda)}$ *if*
   (a) $u_o - u_b - (1-\theta)c < \psi(\lambda) < 0$ *and* $V \geq p + u_o - \psi(\lambda)$, *or*
   (b) $0 \leq \psi(\lambda) \leq u_o - u_b$, *or*
   (c) $u_o - u_b < \psi(\lambda) < (1-\theta)c$ *and* $V < p + u_o + (1-\theta)c - \psi(\lambda)$.
4. $d^{\dagger}_{omni,2} = 0$ *and* $\pi^{\dagger}_{omni,2} = 2pn$ *if*

(a) $u_o - u_b < \psi(\lambda) < (1-\theta)c$ *and* $V \geq p + u_o + (1-\theta)c - \psi(\lambda)$*, or*
(b) $\psi(\lambda) \geq (1-\theta)c$*.*

According to Lemma 3.8, the retailer sets a minimum product description level $d^{\dagger}_{omni,2} = 0$ in period 2, if the reviews have a strong negative (Case 1) or strong positive (Case 4) impact on the product valuation. In Case 1, the reviews will significantly reduce the consumers' product valuation in period 2 so that no consumers will purchase the product even if the retailer provides a maximum product description. Conversely, in Case 4, the reviews can significantly increase the product valuation in period 2 such that all the consumers will purchase the product even if the retailer provides a minimum product description.

The consumer reviews have a moderate impact in Cases 2 and 3. The retailer sets different product description levels depending on $\psi(\lambda)$ and V. Figure 3.6 shows the retailer's optimal decision $d^{\dagger}_{omni,2}$ in period 2 for different values of $\psi(\lambda)$ and V.

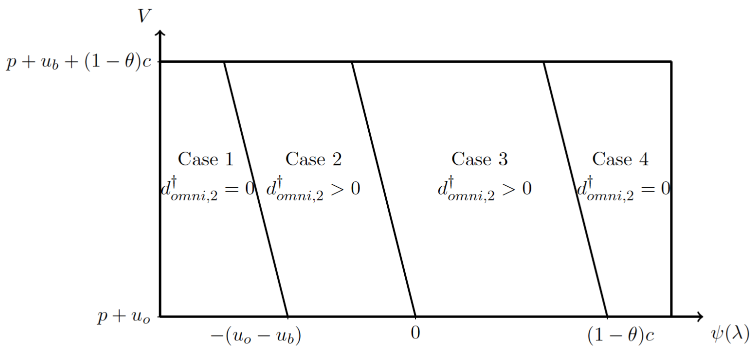


FIGURE 3.6: The retailer's optimal decision $d^{\dagger}_{omni,2}$ in period 2

Comparing Figure 3.4 (the retailer's optimal decision $d^{\dagger}_{o,2}$ if the offline and online channels are operated separately) with Figure 3.6, we observe that if $\psi(\lambda) < 0$, $d^{\dagger}_{o,2} = 0$ in Figure 3.4 but $d^{\dagger}_{omni,2} > 0$ in Case 2 of Figure 3.6 . Due to the negative impact of the reviews ($\psi(\lambda) < 0$), if the two channels are operated separately, the retailer cannot induce the online consumers to purchase the product even if she provides a maximum product description. Thus, she chooses a minimum product description for the online channel. In contrast, if the offline and online channels are integrated, the retailer sets a positive product description level even if the reviews have a negative impact ($\psi(\lambda) < 0$). Although she cannot generate sales from the online channel, the retailer still can generate sales from the offline consumers whose outside option is less attractive.

Comparing Lemma 3.8 to Lemma 3.3 (the retailer's optimal decision without consumer reviews), we observe that $d^{\dagger}_{omni,2} < d^{*}_{omni,2}$ in many cases. This suggests that the consumer reviews may serve as an alternative device to induce purchases, while reducing the retailer's dependence on the product description.

The following lemma determines the retailer's optimal decision in period 1. Recall that $\bar{\lambda} = \theta + \frac{(u_o-u_b)(a-c)}{ac}$ and $\bar{V}_1 = p + u_o + (1-\theta)c - \frac{ca(\underline{\lambda}-\theta)}{a-c}$.

Lemma 3.9. *In the presence of consumer reviews, the retailer sets a product description level $d^{\dagger}_{omni,1}$ with a total expected profit $\Pi^{\dagger}_{omni}$ over the two periods as follows.*

1. $d^{\dagger}_{omni,1} = \frac{1}{1-\theta} + \frac{\theta a}{(1-\theta)(V-p-a\underline{\lambda}-u_o-(1-\theta)c)}$ *and* $\Pi^{\dagger}_{omni} = \frac{2\theta npa}{p-V+a\underline{\lambda}+u_o+(1-\theta)c} + 2np$
   if $\underline{\lambda} > \bar{\lambda}$ *and* $V \geq \bar{V}_1$.
2. $d^{\dagger}_{omni,1} = \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}$ *and* $\Pi^{\dagger}_{omni} = \frac{2\theta npc}{p-V+u_o+c} + 2np$ *otherwise.*

Similar to Lemma 3.6, the proof of Lemma 3.9 shows that, in response to the addition of the consumer review system, the retailer will choose a high product description level in period 1 to generate a large fraction of positive reviews λ. Similar to Theorem 3.7 where the offline and online channels are operated separately, Corollary 3.10 shows the condition under which adding the consumer review system benefits the retailer if the offline and online channels are integrated.

Corollary 3.10. *The consumer review system benefits the retailer ($\Pi^{\dagger}_{omni} > \Pi^{*}_{omni}$) except for both $\underline{\lambda}$ and V are large.*

3.4.4 Comparing the two strategies

In the presence of consumer reviews, if the two channels are operated separately, we can obtain the retailer's total expected profit over the two periods $\Pi^{\dagger}_{dual} = \pi^*_{b,1} + \pi^*_{b,2} + \Pi^{\dagger}_o$ from Lemmas 3.1 and 3.6. Comparing $\Pi^{\dagger}_{dual}$ with $\Pi^{\dagger}_{omni}$ in Lemma 3.9, Theorem 3.11 identifies the conditions under which integrating the offline and online channels yields a higher profit for the retailer. Let $\hat{\lambda} = \theta + \frac{(u_o-u_b)(a-c)}{ac} + \frac{\theta(1-\theta)(a-c)c\bar{d}}{1-(1-\theta)\bar{d}}$ and $\tilde{V} = p + u_o + (1-\theta)c + \frac{\theta c-(u_o-u_b)-\sqrt{(\theta c-(u_o-u_b))^2+8\theta c(u_o-u_b)}}{2}$. Recall that $\hat{d} = \frac{(1-\theta)c-(u_o-u_b)}{(1-\theta)[c-(u_o-u_b)]}$ and $\hat{V} = p + u_b + \frac{(1-\theta)(1-\bar{d})c}{1-(1-\theta)\bar{d}}$.

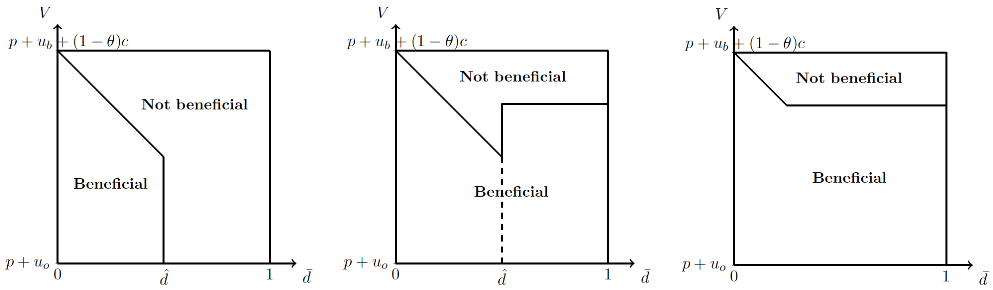
Theorem 3.11. *In the presence of consumer reviews, we have the following results.*

1. *If $u_o - u_b > \frac{(1-\theta)c}{1+\theta}$, it is more profitable for the retailer to integrate the offline and online channels if and only if $\bar{d} \le \hat{d}$ and $V < \hat{V}$.*

2. *If $u_o - u_b \le \frac{(1-\theta)c}{1+\theta}$ and $\underline{\lambda} > \hat{\lambda}$, it is more profitable for the retailer to integrate the offline and online channels if and only if $\bar{d} \le \hat{d}$ and $V < \hat{V}$, or $\bar{d} > \hat{d}$ and $V < \tilde{V}$.*

3. *If $u_o - u_b \le \frac{(1-\theta)c}{1+\theta}$ and $\underline{\lambda} \le \hat{\lambda}$, it is more profitable for the retailer to integrate the offline and online channels if and only if $\bar{d} \le \hat{d}$ and $V < \max\{\hat{V}, \tilde{V}\}$, or $\bar{d} > \hat{d}$ and $V < \tilde{V}$.*

Theorem 3.11 shows that integrating the offline and online channels yields a higher total expected profit for the retailer if and only if $u_o - u_b$ is large and both $\bar{d}$ and V are small (Case 1), or both $u_o - u_b$ and V are small (Cases 2 and 3). Figures 3.7(a), (b), and (c) illustrate under what conditions integrating the offline and online channels is beneficial for the retailer in the presence of consumer reviews for Cases 1, 2, and 3 respectively. Similar to Figure 3.2 for the model without consumer reviews, Figure 3.7(a) shows that if $\bar{d}$ and V are small, it is more profitable to integrate the offline and online channels. This is because if the two channels are operated separately, the retailer can hardly attract the offline consumers with small base product valuation V to purchase the product given a very limited product description. In contrast, integrating the offline and online channels removes this limit.



(a) $u_o - u_b > \frac{(1-\theta)c}{1+\theta}$ (b) $u_o - u_b \le \frac{(1-\theta)c}{1+\theta}$ and $\underline{\lambda} > \hat{\lambda}$ (c) $u_o - u_b \le \frac{(1-\theta)c}{1+\theta}$ and $\underline{\lambda} \le \hat{\lambda}$

FIGURE 3.7: Region in which integrating the offline and online channels benefits the retailer

In the presence of consumer reviews, even if $\bar{d}$ is large ($\bar{d} > \hat{d}$), it is still more profitable for the retailer to integrate the offline and online channels as long as $u_o - u_b$ is small (see Figures 3.7(b) and 3.7(c)). Note that this is different from Figure 3.2, where it is more profitable to integrate the two channels only if $\bar{d}$ is small ($\bar{d} \le \hat{d}$).

If $\bar{d} > \hat{d}$, integrating the offline and online channels is a double-edged sword that can benefit or hurt the retailer with consumer reviews. On the one hand, this strategy allows the offline consumers to post and read the reviews. Since the retailer sets a high product description level in period 1 to generate a large fraction of positive reviews λ (see Lemma 3.9), which increases the offline consumers' product valuation in period 2 and thus the retailer's expected profit. In contrast, integrating the offline and online channels cannot create this effect in the absence of the consumer reviews.

On the other hand, if the offline and online channels are integrated, the retailer cannot differentiate the product descriptions of the two channels. Since the outside option of the offline consumers is less attractive than that of the online consumers ($u_b < u_o$), the retailer could set a lower product description level for the offline channel. However, if the offline and online channels are integrated, the product description level of the offline channel is increased ($d^{\dagger}_{omni,1} > d^{\dagger}_{b,1}$), which will deter some offline consumers whom the product does not fit from purchasing the product in period 1.

Therefore, when $\bar{d}$ is large ($\bar{d} > \hat{d}$), whether integrating the offline and online channels is more profitable for the retailer depends on the trade-off between the retailer's profit gain in period 2 from the positive impact of consumer reviews and the profit loss in period 1 due to the inflexibility of setting a common product description level for both channels. If $u_o - u_b$ is small (the difference between the outside options of the offline and online channels is small), the retailer incurs a limited profit loss in period 1 due to the inflexibility. Consequently, it is more profitable to integrate the offline and online channels (see Figures 3.7(b) and 3.7(c)). However, if $u_o - u_b$ is large ($u_o - u_b > \frac{(1-\theta)c}{1+\theta}$), the profit gain in period 2 from the positive impact of consumer reviews is insufficient to compensate the profit loss in period 1 due to the inflexibility. Thus, the retailer can be worse off by integrating the offline and online channels (see Figure 3.7(a)).

Further comparing Figures 3.7(b) and 3.7(c), we observe that when $\bar{d}$ and $u_o - u_b$ are small, integrating the offline and online channels is more dominating if $\underline{\lambda}$ is small (see Figure 3.7(c)). This is intuitive because with a small $\underline{\lambda}$, it is more likely for the reviews to have a positive impact ($\psi(\lambda) = a(\lambda - \underline{\lambda})$ is more likely to be large), magnifying the benefit of integrating the offline and online channels.

In summary, without consumer reviews, it is beneficial for the retailer to integrate the offline and online channels only if $\bar{d}$ and V are small (see Figure 3.2). In the presence of consumer reviews, even if $\bar{d}$ is large ($\bar{d} > \hat{d}$), it is still beneficial to integrate the offline and online channels when the difference between the outside options of the two channels ($u_o - u_b$) is small (see Figures3.7(b) and 3.7(c)).

3.5 Extended model with consumers searching for the product description

If the retailer operates the offline and online channels separately, we assume that the offline consumers cannot access the online product description. In practice, some offline consumers may search for the online product description even if the retailer does not facilitate that (for example, they may search using their mobile devices). In this section, we consider a model without consumer reviews and assume that the offline consumers search for the online product description with a probability $\eta \in [0,1]$. The rest of the model is identical to that of §3.3.

In this extended model, the retailer chooses $d_{o,t}$ to maximize her expected profit from both the online consumers and the offline consumers who search for the online product description: $\pi_{o,t}(d_{o,t}) = p \times n \times \phi_{o,t}(d_{o,t}) + p \times n \times \eta \times \phi_{b,t}(d_{o,t})$. On the other hand, the retailer chooses $d_{b,t}$ to maximize her expected profit from the offline consumers who do not search for the online product description: $\pi_{b,t}(d_{b,t}) = p \times n \times (1-\eta) \times \phi_{b,t}(d_{b,t})$. The following corollary summarizes the optimal decisions of the retailer.

Corollary 3.12. *Suppose the offline consumers search for the online product description with a probability η. The retailer sets an offline product description level $d^*_{b,t} = 0$ with an expected profit $\pi^*_{b,t} = 0$ if $\bar{d} \leq \hat{d}$ and $V < \hat{V}$, and sets an offline product description level $d^*_{b,t} = \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_b-c)}$ with an expected profit $\pi^*_{b,t} = \frac{\theta(1-\eta)npc}{p-V+u_b+c}$ otherwise, for $t = 1, 2$. The retailer sets an online product description level $d^*_{o,t} = \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}$ with an expected profit $\pi^*_{o,t} = \frac{\theta(1+\eta)npc}{p-V+u_o+c}$, for $t = 1, 2$.*

By comparing Corollary 3.12 with Lemmas 3.1 and 3.2, we observe that the retailer makes identical optimal decisions as in §3.3, but the total expected profits are different. Note that the retailer's optimal decisions and expected profits are given by Lemma 3.3 if the offline and online channels are integrated. By comparing the retailer's total expected profit when the two channels are operated separately with that when the two channels are integrated, Corollary 3.13 shows the conditions under which it is more profitable for the retailer to integrate the two channels.

Corollary 3.13. *Suppose the offline consumers search for the online product description with a probability* η*. The retailer is more profitable by integrating the offline and online channels if and only if* $\bar{d} \leq \hat{d}$ *and* $V < \hat{V}$*.*

Corollary 3.13 shows that Theorem 3.4 continues to hold for this extended model.

3.6 Summary

We consider a retailer that sells a single product through an offline (brick-and-mortar) channel and an online channel to consumers in two periods. The product has several different attributes and the consumers are heterogeneous in their tastes (preferences) for these attributes. Each consumer knows his own taste for each attribute, but the actual details of the product for that attribute are unknown to him. Therefore, before purchasing the product, the consumer is unsure about whether the product fits his taste. The consumer, however, can learn about the product by reading the product description provided by the retailer. We study the retailer's decisions on the provision of the product descriptions in this multi-channel environment, and investigate the impact of different channel-management strategies on the retailer's expected profit.

We develop a game-theoretical model to study the decision-making process of a retailer who either operates an offline channel and an online channel separately or in an integrated manner. Specifically, the retailer may differentiate the product description level of the offline channel from that of the online channel if she operates the two channels separately. In contrast, if the retailer integrates the two channels, she sets the same product description level for both channels. In each period of our model, the retailer first decides the product description level(s) to maximize her expected profit. After reading the product description, a private signal is generated for each consumer in each channel about whether the product fits the consumer. The consumer then evaluates his expected utility of purchasing the product conditioned on the private signal, and decides whether to make the purchase by comparing this utility with that of an outside option. After the consumers make their purchase decisions, the retailer's profit is realized.

We find that it is more profitable for the retailer to integrate the offline and online channels if the limit on the offline product description $\bar{d}$ is low and the consumers' base product valuation V is small (see Theorem 3.4 and Figure 3.2). This is because if the retailer operates the offline and online channels separately, the offline consumers are unlikely to purchase the product given the high product-fit uncertainty caused by the limited product description. In contrast, by integrating the offline and online channels, the retailer can provide an identical product description for both channels using new technology such as "cloud shelf." A more detailed product description makes more offline consumers willing to purchase the product, making the retailer more profitable. This partially explains why facilitating online product descriptions in brick-and-mortar stores becomes popular in practice.

However, the retailer needs to be cautious about integrating the offline and online channels. Even if the offline product description limit is low, integrating the two channels does not always benefit the retailer. Specifically, if the base product valuation V is sufficiently large, it is more profitable to operate the two channels separately (see Figure 3.2). This is because when the base product valuation is large, it is easier to attract the offline consumers who have a low utility from the outside option. Differentiating the product description levels between the two channels yields a higher total expected profit for the retailer. However, the retailer loses this flexibility if she integrates the offline and online channels. Thus, it is important for the retailer to take both the offline product description limit and the base product valuation into account when deciding whether to integrate the offline and online channels.

We further study a model in which the consumers who purchase the product in the first period can post their reviews about the product online through a review system. The consumers in the second period can read these reviews before they decide whether to purchase the product. If the retailer operates the offline and online channels separately, only the online consumers can post and read the reviews, while both the offline and online consumers can post and read the reviews if the retailer integrates the two channels. We analyze the equilibrium outcomes and investigate the implications of adding such a review system.

In the presence of consumer reviews, even if the offline product description limit $\bar{d}$ is high, it may still be beneficial for the retailer to integrate the offline and online channels as long as the outside-option utility difference between the two channels $u_o - u_b$ is small (see Figures 3.7(b) and 3.7(c)). In this situation, integrating the offline and online channels is a double-edged sword that can benefit or hurt the retailer. On the one hand, it allows the offline consumers to post and read the reviews. Since the retailer provides a sufficiently rich product description in the first period to generate a large fraction of positive reviews (see Lemma 3.9), these reviews can increase the offline consumers' product valuation in the second period, and thus increase the retailer's expected profit. On the other hand, the retailer sets the same product description level for both channels. The higher product description level in the first period ($d^{\dagger}_{omni,1} > d^{\dagger}_{b,1}$) can deter some offline consumers whom the product does not fit from purchasing the product.

Therefore, if $\bar{d}$ is large, whether integrating the two channels is more profitable for the retailer depends on the trade-off between the profit gain in period 2 from the consumer reviews' positive impact and the profit loss in period 1 due to the inflexibility of setting the same product description level for both channels. If $u_o - u_b$ is small, the retailer incurs a limited profit loss in period 1 due to the product description inflexibility. Consequently, it is more profitable to integrate the offline and online channels (see Figures 3.7(b) and 3.7(c)). However, if $u_o - u_b$ is large, the profit gain in period 2 from the positive impact of the consumer reviews is insufficient to compensate the profit loss due to the inflexibility. Thus, the retailer can be worse off by integrating the two channels (see Figure 3.7(a)).

We also find that the consumer reviews may hurt the retailer's profit if the consumers' base product valuation V is sufficiently large (see Theorem 3.7 and Corollary 3.10). This is because when there are no reviews, the retailer can already attract the consumers with large base product valuation easily even with a low product description level. In the presence of consumer reviews, however, the retailer has to set a high product description level in period 1 to prevent the negative impact of the reviews on the upcoming consumers. The increase in the product description level drives down the retailer's profit in period 1 as it prevents some consumers whom the product does not fit from purchasing the product. This is detrimental to the retailer.

Finally, it is worth noting that understanding how consumer reviews affect future purchases is not straightforward in practice. For example, some retailers allow consumers to rate a product in multiple dimensions. These multi-dimensional ratings may reflect how the product fits the consumers' tastes, and may help future consumers figure out whether the product fits them. Investigating the effects of such reviews can be an interesting direction for future work.

Chapter 4

Should Sellers Sell through Pure-Play Marketplace or Marketplace with Competing Product?

4.1 Introduction

The increasing prevalence of online marketplaces, where sellers use the online platform to sell their products to customers, has sparked researchers' interest in studying the economics of online marketplaces (Mantin et al. , 2014). The best known examples of online marketplace include Amazon and eBay in U.S., Lazada and Qoo10 in Singapore. While Qoo10 and eBay operate as a pure-play marketplace, Amazon and Lazada also have the business of selling their own products. As a result, a seller selling products through a marketplace like Lazada may find itself in direct competition with the marketplace (Ryan et al. , 2012). Specifically, for some similar or substitutable products, the seller and Lazada are competing for the same customers. A customer has the option of buying it directly from Lazada, or through the seller. This potential conflict, and its impact on both the marketplace and the seller is one of the focus of this chapter.

We are also interested in the competition between different marketplaces, since consumers may have their own preference over different marketplaces when they shop for products. To decide through which marketplace to sell its product, a seller should take into account all of the above impacts. We consider a setting in which a pure-play marketplace is operating like Qoo10 and a marketplace is operating like Lazada who also sells its own product directly to consumers. We consider a seller who decides through which marketplace(s) to sell its product, and at what price(s). By hosting the seller, the marketplaces charge a revenue sharing commission. We are also interested in the marketplaces' decision regarding its product price which competes with the seller. We will determine the equilibrium outcomes for the seller and both marketplaces under this setting. Given this motivation and problem setting, our research questions include:

(i) Under what conditions should the seller sell its product through the pure-play marketplace, and under what conditions should the seller sell through the marketplace who sells a competing product? What will be the corresponding optimal pricing decisions?

(ii) What will be the optimal product price charged by the marketplace who also sells its own product?

(iii) What will be the equilibrium outcomes?

To address the above questions, we develop a game-theoretical model that captures the key trade-offs arising in this decision process. In our model, a retailer consider selling a single product to consumers through a pure-play marketplace m, or through a marketplace c who also sells its own product competing with the seller, or though both marketplaces. The consumers are heterogenous in their preferences over the marketplaces c and m, as well as in their preferences over the products sold by the marketplace c and sold by the seller. The marketplace c will first determine its product price, and then the seller will decide through which marketplace(s) to sell its product, and the corresponding product price(s). After the consumers make their purchase decisions which are to maximize their utility, the profit of the seller and the marketplaces are realized.

We have obtained the following insights:

(i) In general, as the commission charged by one marketplace becomes higher, the seller tends to sell its product through the other marketplace. If the price of the competing product in marketplace c becomes higher, the seller tends to charge a lower price to further attract more consumers. However, if the price of the competing product in marketplace c becomes lower, the seller should charge a higher price to focus on a small group of consumers to maximize its profit.

(ii) In equilibrium, if the commission of the marketplace c is lower than that of the marketplace m, the marketplace c will decide its product price such that the seller only sells through it. On the other hand, if the commission of the marketplace c is higher than that of the marketplace m, the marketplace c will decide its product price such that the seller sells through both marketplaces.

This chapter is organized as follows. After reviewing the relevant literature in §4.2, we describe the model in §4.3, before analyzing it in §4.4. We also analyze an extended model in §4.5. §4.6 concludes this chapter. All proofs are provided in the Appendix C.

4.2 Literature review

We consider a seller that decides through which online marketplaces to sell product. Some existing papers have developed on the analysis of online marketplaces. see Greiger (2003) and Wang et al. (2008) for reviews of this strand of literature. We focus on those papers that are most relevant to our research problems. Bernstein et al. (2008) study a traditional retailer who considers selling its product online. They consider two options for the retailer: developing its own website or aligning with pure etailers. Wang et al. (2004) analyze channel performance under a consignment contract, inspired by Amazon's marketplace. Ru and Wang (2010) consider a similar model to study the inventory control problem. While these papers are motivated by similar practical examples as our research, they differ in that they do not consider the competition, either between the similar products sold by retailer and the marketplace, or between different marketplaces.

Since our research considers that a seller may sell product through a marketplace which also sells its own competing product, a related question is whether a marketplace would want to offer its own product competing with the seller, or whether the marketplace would allow the seller with a competing product to sell through it at the first place. Hagiu and Spulber (2013) study a platform's incentive to introduce first-party content alongside third-party content in two-sided markets. Zhu and Liu (2018) empirically show that Amazon is more likely to compete with its third-party sellers in product categories that are more successful. Ryan et al. (2012) consider that a retailer selling its product through its own website, may also choose to sell through a marketplace, and the marketplace firm who has its own product may or may not offer the marketplace service to the seller. Hagiu et al. (2020) study conditions under which a firm can profitably turn itself into a marketplace by hosting a competing retailer. In contrast to these papers, our research does the reverse: it studies whether a seller should sell through a marketplace to compete with product of that marketplace.

4.3 Model description

We consider the following problem setting. A seller sells a single product to n consumers through online marketplace. There is a pure-play marketplace m, and a marketplace c who also sells its own product in addition to providing the marketplace service. The seller has the option to sell through either marketplace or both. To sell through each marketplace i, $i = m, c$, the seller pays a proportion commission α_i for each unit sold to marketplace i. We assume α_i is exogenous in our model.

The marketplace c and the marketplace m are *horizontally* differentiated, which are located at the ends of a unit Hotelling line $[0, 1]$. The consumers are heterogenous in their preferences for the two marketplaces. The consumer preferences are uniformly distributed along the Hotelling line with an individual's preference denoted by $x \in [0, 1]$. For instance, the two online marketplaces may vary in their user interface and order system, for which consumers have different preferences.

The consumers are also heterogenous in their preferences over the product sold by the seller and the product sold by the marketplace c. For example, some consumers may perceive higher value from the product directly sold by the marketplace, because they believe the marketplace is more reliable than the individual seller. We capture this heterogeneity by assuming there are two types of consumers in the market. A fraction $\theta \in [0, 1]$ of consumers perceive a base utility V from the seller's product, and perceive a higher base utility V_H from the product sold by the marketplace c. The remaining fraction $1 - \theta$ of consumers perceive same base utility V from the two products. We call the former consumers high-type consumers, the latter consumers low-type consumers.

The marketplace c first decides a price p_c of its own product, and the seller decides through which marketplace(s) to sell her product and decides price $p_{s,i}$ for her product sold through marketplace i. If the seller sell her product through both marketplaces, we assume that the seller charges a common price $p_s = p_{s,m} = p_{s,c}$. In §4.5, we relax this assumption and consider a setting in which the seller can price discriminately on the two marketplaces.

All the consumers have unit demands, and their utility of purchasing a product is given by the sum of a base utility, a disutility measuring the distance between an individual's ideal location and the location of the product, less the price. A constant marginal disutility t is incurred for a unit distance between a consumer's ideal preference and the position of the product. We assume that $t < V - p_i$, i.e., the incurred disutility is low enough that the market is fully covered. We also discuss what happens in the partially covered market at the end of §4.4. Let $u_{s,m}$ and $u_{s,c}$ denote the consumer's utility of purchasing the seller's product through marketplace m, and marketplace c respectively. Let u_c denote the consumer's utility of purchasing the product of the marketplace c. A high-type consumer's utility is as follows.

$$\begin{aligned}
&u_c = V_H - tx - p_c \\
&u_{s,c} = V - tx - p_{s,c} \\
&u_{s,m} = V - t(1-x) - p_{s,m}
\end{aligned} \tag{4.1}$$

A low-type consumer's utility is as follows.

$$\begin{aligned}
&u_c = V - tx - p_c \\
&u_{s,c} = V - tx - p_{s,c} \\
&u_{s,m} = V - t(1-x) - p_{s,m}
\end{aligned} \tag{4.2}$$

The sequence of events and decisions in our model is specified as follows, which is also depicted in Figure 4.1. At first, the marketplace c decides the price p_c to maximize its expected profit. Then, the seller decides through which marketplace(s) i to sell her product, after which she decides price(s) $p_{s,i}$ for her product, to maximize her expected profit. Finally, each consumer makes purchase decision to maximize his utility.

4.4 Model analysis

In this section, we analyze the model backward, first determining the optimal decisions of the seller for product price and marketplace choice in §4.4.1, and then determining the optimal decisions of the marketplace c for its product price, as well as the equilibrium outcomes in §4.4.2.

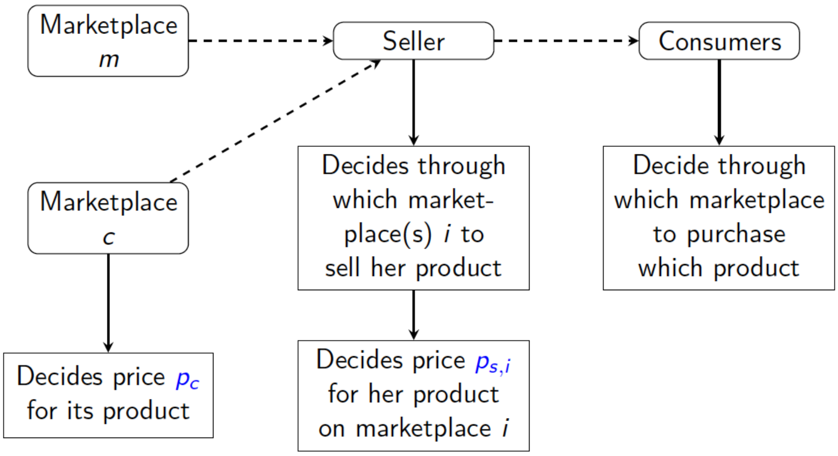


FIGURE 4.1: The sequence of decisions

4.4.1 Optimal decisions of the seller

We first find the optimal pricing decision of the seller in the cases selling through marketplace c, selling through marketplace m, and selling through both marketplaces, in §4.4.1.1, §4.4.1.2, §4.4.1.3 respectively.

4.4.1.1 Seller sells through marketplace c

Define $D_{s,c}$ as the seller's demand from selling through marketplace c. The seller's profit of selling through marketplace c will be $\pi_{s,c} = D_{s,c} \times p_{s,c} \times (1 - \alpha_c)$. To maximize her profit, the seller will set her product price $p^*_{s,c}$ according to the following Lemma 4.1.

Lemma 4.1. *If the seller only sells her product through marketplace c, then the optimal pricing decisions of the seller are as follows.*

1. *If $p_c > \frac{V_H - V}{\theta}$, the seller sets a price $p^*_{s,c} = p_c + V - V_H$.*

2. *If $p_c \leq \frac{V_H - V}{\theta}$, the seller sets a price $p^*_{s,c} = p_c$.*

4.4.1.2 Seller sells through marketplace m

Define $D_{s,m}$ as the seller's demand from selling through marketplace m. The seller's profit of selling through marketplace m will be $\pi_{s,m} = D_{s,m} \times p_{s,m} \times (1 - \alpha_m)$. To maximize her profit, the seller will set her product price $p^*_{s,m}$ according to the following Lemma 4.2.

Lemma 4.2. *If the seller only sells her product through marketplace m, then the optimal pricing decisions of the seller are as follows.*

1. *If $p_c > (1 + \frac{2}{\theta})t$, the seller sets a price $p^*_{s,m} = p_c - t + V - V_H$.*
2. *If $p_c \leq (1 + \frac{2}{\theta})t$, the seller sets a price $p^*_{s,m} = p_c - t$.*

4.4.1.3 Seller sells through both marketplaces

We assume that if the seller sells through both marketplaces c and m, then the seller set a same price $p_s = p_{s,c} = p_{s,m}$ for the product on both marketplaces. The seller's profit of selling through both marketplaces will be $\pi_s = D_{s,c} \times p_s \times (1 - \alpha_c) + D_{s,m} \times p_s \times (1 - \alpha_m)$. To maximize her profit, the seller will set her product price p^*_s according to the following Lemma 4.3.

Lemma 4.3. *If the seller sells her product through both marketplaces c and s, then the optimal pricing decisions of the seller are as follows.*

1. *$p_c > \frac{V_H - V}{\theta}$, the seller sets a price $p^*_s = p_c + V - V_H$.*
2. *If $p_c \leq \frac{V_H - V}{\theta}$, the seller sets a price $p^*_s = p_c$.*

From lemmas 4.1, 4.2, and 4.3, we can observe that, in general, no matter which marketplace the seller decides to sell through, her pricing decision depends on the price p_c charged by marketplace c. If p_c is high, the seller will set a lower price, and if p_c is low, the seller will set a higher price. The reason is as follows. Since some consumers have higher valuation for the product of marketplace c, only if p_c is sufficiently high, the seller has an incentive to lower her price to aggressively compete for those consumers. Otherwise, the seller would rather charge a higher price to focus on other consumers.

4.4.1.4 Optimal marketplace choice

By comparing the expected profits in lemmas 4.1, 4.2, and 4.3, the seller decides through which marketplace(s) to sell her product to maximize her profit. The following theorem shows the seller's optimal decisions on marketplace choice.

Theorem 4.4. *The seller's optimal decisions on marketplace choice are as follows.*

1. *If* $p_c > (1 + \frac{2}{\theta})t$, *and*
   (a) $\alpha_c > \alpha_m + \frac{2(1-\alpha_m)t}{p_c+V-V_H}$, *the seller only sells through marketplace* m, *and sets a price* $p^*_{s,m} = p_c - t + V - V_H$.
   (b) $\alpha_m < \alpha_c \leq \alpha_m + \frac{2(1-\alpha_m)t}{p_c+V-V_H}$, *the seller sells through both marketplaces* m *and* c, *and sets a price* $p^*_s = p_c + V - V_H$.
   (c) $\alpha_c \leq \alpha_m$, *the seller only sells through marketplace* c, *and sets a price* $p^*_{s,c} = p_c + V - V_H$.

2. *If* $\frac{V_H - V}{\theta} < p_c \leq (1 + \frac{2}{\theta})t$, *and*

   (a) $\alpha_c > \alpha_m + \frac{(1-\alpha_m)[\theta(V_H - V)(p_c - t) - 2t(V_H - V - t)]}{(p_c + V - V_H)t}$, *the seller only sells through marketplace* m, *and sets a price* $p^*_{s,m} = p_c - t$.

   (b) $\alpha_m < \alpha_c \leq \alpha_m + \frac{(1-\alpha_m)[\theta(V_H - V)(p_c - t) - 2t(V_H - V - t)]}{(p_c + V - V_H)t}$, *the seller sells through both marketplaces* m *and* c, *and sets a price* $p^*_s = p_c + V - V_H$.

   (c) $\alpha_c \leq \alpha_m$, *the seller only sells through marketplace* c, *and sets a price* $p^*_{s,c} = p_c + V - V_H$.

3. *If* $p_c \leq \frac{V_H - V}{\theta}$, *and*

   (a) $\alpha_c > \alpha_m + \frac{(1-\alpha_m)[\theta p_c(V_H - V - 2t) + t(2t - \theta(V_H - V))]}{(1-\theta)tp_c}$, *the seller only sells through marketplace* m, *and sets a price* $p^*_{s,m} = p_c - t$.

   (b) $\alpha_m < \alpha_c \leq \alpha_m + \frac{(1-\alpha_m)[\theta p_c(V_H - V - 2t) + t(2t - \theta(V_H - V))]}{(1-\theta)tp_c}$, *the seller sells through both marketplaces* m *and* c, *and sets a price* $p^*_s = p_c$.

   (c) $\alpha_c \leq \alpha_m$, *the seller only sells through marketplace* c, *and sets a price* $p^*_{s,c} = p_c$.

In general, we observe that on matter p_c is high, medium, or low, the seller has similar marketplace choice depending on α_c and α_m. If α_c is much larger than α_m, then the seller only sells through marketplace m, but if α_c is slightly larger than α_m, then the seller sells through both marketplaces. On the other hand, if α_c is smaller than α_m, then the seller only sells through marketplace c. It's intuitive that when one marketplace commission is quite lower than another's, the seller only sells through this marketplace.

The interesting part is when one commission is slightly lower or higher than another one. We see that when α_c is slightly higher than α_m, the seller will sell through both marketplaces, but when α_c is slightly lower than α_m, the seller will only sell through marketplace c. This is because if the seller also sells on marketplace m in this case, the marketplace c will always lower its own product price to compete for consumers, the seller will lose some consumers and her profit from marketplace m cannot compensate this loss. Thus, the seller would rather only sell through marketplace c.

4.4.2 Equilibrium outcomes

Define D_c as the demand from the product of marketplace c. The marketplace c decides a price p_c of its product to maximize its profit $\pi_c = D_{s,c} \times p_{s,c} \times \alpha_c + D_c \times p_c$. After obtaining the optimal decisions of the seller and the marketplaces c, we can determine their equilibrium decisions. The following theorem characterizes the equilibrium outcomes.

Theorem 4.5. *There are two equilibrium outcomes exist as follows.*

1. *If $\alpha_c \leq \alpha_m$, the marketplace c can set any price $p_c^* > \bar{p}$. The seller only sells through marketplace c, and sets a price $p_{s,c}^* = p_c^* + V - V_H$.*
2. *If $\alpha_c > \alpha_m$, the marketplace c sets price $p_c^* = V_H - V + \frac{2(1-\alpha_m)t}{\alpha_c - \alpha_m}$. The seller sells through both marketplaces, and sets a price $p_s^* = \frac{2(1-\alpha_m)t}{\alpha_c - \alpha_m}$.*

In equilibrium, if the commission α_c of the marketplace c is lower than the commission α_m of the marketplace m, the seller will always choose to only sell her product through marketplace c. If the seller also sells through marketplace m, then the marketplace c always has the incentive to reduce its product price to compete for the consumers from the seller and the marketplace m. This is because the marketplace c cannot generate sufficient profit from the commission, due to a low α_c. Since the seller cannot earn too much by selling through marketplace m due to a high α_m, so the seller would rather not sell on the marketplace m to avoid this situation.

On the other hand, if the commission α_m of the marketplace m is lower than the commission α_c of the marketplace c, the seller would like to sell through the marketplace m, but she will still sell through the marketplace c. This ensures the marketplace c to generate a sufficiently large profit from the commission because of a high α_c, so that the marketplace c will not compete with the seller using its own product. However, if the seller does not sell through the marketplace c, then the marketplace c will always reduce its product price to compete with the seller to generate a profit, which is detrimental to the seller. Therefore, the seller will sell through both marketplaces to avoid this situation.

Furthermore, we also analyze the model in a partially covered market. We find that the equilibrium result is a special case of Theorem 4.5. Specifically, the seller will always choose to sell through both marketplaces. This is because she will become a local monopoly on marketplace m, and she can optimize her price to compete with marketplace c on it as well. The following corollary shows the equilibrium result.

Corollary 4.6. *In a partially covered market, the seller, in equilibrium, sells through both marketplaces with a price* $p_s^* = V/2$. *The marketplace* c *sets a price* $p_{s,c}^* = V_H/2$.

4.5 Extension: Discriminate for two marketplaces

So far we have assumed that the seller will charge a common price if she chooses to sell through both marketplaces, while ignoring the possibility of pricing discriminately for the two marketplaces. Since the consumers in our model have heterogeneous preference for the two marketplaces m and c, the seller is likely to take advantage of the pricing flexibility to optimize her prices for the two marketplaces individually. This may allow the seller to compete with the marketplace c more effectively. In this section, we relax the single-price assumption and consider the setting in which the seller is able to price discriminately if she chooses to sell through both marketplaces. The rest of the model is identical to that of §4.3. The following theorem characterizes the equilibrium decisions of the seller in this setting.

Theorem 4.7. ***(Equilibrium decisions under the discriminate price model)***

1. *If $\frac{1-\alpha_c}{1-\alpha_m} > \max\{\bar{\alpha}_1, \bar{\alpha}_2\}$, the seller will only sell through the marketplace c, and set a price $p^*_{s,c} = 3t - \theta(V_H - V)$.*

2. *If $\bar{\alpha}_3 < \frac{1-\alpha_c}{1-\alpha_m} \leq \bar{\alpha}_2$, the seller will sell through both marketplaces, and set prices $p^*_{s,c} = 3t - \theta(V_H - V)$ and $p^*_{s,m} = 4t - (1+\theta)(V_H - V)$.*

3. *If $\frac{1-\alpha_c}{1-\alpha_m} \leq \bar{\alpha}_1$, the seller will only sell through the marketplace m, and set a price $p^*_{s,m} = 2t - \theta(V_H - V)$.*

Similar to Theorem 4.5, the first two cases of Theorem 4.7 show that if the commission α_m of the marketplace m is larger or slightly smaller than the commission α_c of the marketplace c, the seller will only sell through marketplace c and sell through both marketplaces, respectively. However, if α_m is sufficiently smaller than α_c, the seller would give up selling on marketplace c and choose to sell through marketplace m only. This is because, in contrast to the main model, the marketplace c cannot always leverage its own price to force the seller to sell through it. Since the seller has the flexibility to price discriminately for the two marketplaces, she can always charge a lower price to compete fiercely with marketplace c on it. Both of the seller and the marketplace c will be worse off in this situation. Thus, in equilibrium, the seller will only sell through marketplace m.

4.6 Summary

We consider a seller that sells a single product through online marketplace to consumers. We consider one marketplace m operate as a pure-play marketplace like eBay and Qoo10, and another marketplace c who also sells its own competing product to consumers like Amazon and Lazada. The consumers are heterogenous in their preferences for these two marketplaces as well as for the products sold by the seller and the marketplace. We study the seller's decisions on the choice of the marketplace and the price of the product, and the pricing decision of marketplace c. We develop a game-theoretical model which captures the competition between the seller and the marketplace as well as the competition between the two marketplaces.

Specifically, at the first stage, the marketplace c will determine its product price, and then the seller will decide through which marketplace(s) to sell its product, before determining the product price(s). Finally, the consumers make their purchase decisions to maximize their utility, and the profit of the seller and the marketplaces are realized. We analyze this decision process to determine the equilibrium outcomes.

We find that, in general, as the commission charged by one marketplace becomes higher, the seller tends to sell its product through the other marketplace. If the price of the competing product in marketplace c becomes higher, the seller tends to charge a lower price to further attract more consumers. However, if the price of the competing product in marketplace c becomes lower, the seller should charge a higher price to focus on a small group of consumers to maximize its profit. In equilibrium, if the commission of the marketplace c is lower than that of the marketplace m, the marketplace c will decide its product price such that the seller only sells through it. On the other hand, if the commission of the marketplace c is higher than that of the marketplace m, the marketplace c will decide its product price such that the seller sells through both marketplaces.

Furthermore, we study an extended model in which the seller is able to price discriminately for the two marketplaces. With the pricing flexibility, the seller may only sell through marketplace m as well in equilibrium. This chapter provides a formal analysis of how a seller can strategically choose the online marketplaces to sell through. Naturally, there are some other factors relevant to the decision that we did not capture in this chapter. There are some interesting extensions that future research can explore.

A key feature of the online marketplace is the commission rate, which is assumed to be exogenous in our model. In practice, some marketplaces may compete with each other in the commission rate to attract the seller. Thus, one future research direction is endogenizing the commission rates of the two marketplaces. Furthermore, some marketplaces or online platforms in practice offer exclusive sale contract with the sellers, through which the sellers can benefit from a lower commission rate. Another future research direction is to take into account this exclusive sale contract, in line with the endogenous commission rate.

Chapter 5

Conclusion

The growth of online retailing market creates numerous opportunities and challenges for the context of operations management. Extensive literature in online retailing focus on the conventional inventory management and pricing problems as in traditional retailing. However, the rapid development of information technology threatens the established business models and creates opportunities for new business models. Companies engaged in online retailing may find it increasingly difficult to make strategic decisions in the new retailing environment. This thesis investigates innovative business models which are under-studied in this domain to provide managerial insights for practitioners.

In Chapter 2, we study how to deal with the pressure on well-beings caused by the last-mile delivery of online retailing. To alleviate the congestion and pollution, a consolidator can operate an urban consolidation center (UCC) to bundle shipments from multiple carriers before the last-mile delivery. Alternatively, the consolidator can operate a peer-to-peer platform for the carriers to share their delivery capacity. For each business model, we construct a two-period game-theoretical model to study the interactions between a consolidator and multiple carriers. In each period, the consolidator first chooses a delivery fee to maximize her expected profit. Each carrier then observes his task volume, and decides whether to deliver on his own or use the consolidator's service to minimize his expected cost.

We compare the performance of these two models to guid the consolidator to make efficient operational decisions. Under the UCC model, the carriers become more dependent on the UCC to deliver their tasks as their variable delivery cost increases or their logistics reestablishment cost decreases. Under the platform model, the carriers generally keep their logistics capability (even if they purchase capacity from the platform) in equilibrium to ensure their flexibility of selling capacity on the platform. Between the two business models, it is generally more profitable for the consolidator to operate the UCC than the platform if the carriers' fixed delivery cost is large. Furthermore, the UCC becomes more dominant as there are more carriers. If the number of carriers is large, it is also more efficient for the consolidator to operate the UCC than the platform to reduce the expected social-environmental cost. Otherwise, the platform is more efficient.

In Chapter 3, we investigate the channel management decisions of a retailer who sells a single product to consumers through an offline (brick-and-mortar) channel and an online channel. The consumers in each channel are heterogenous in their tastes for the product. Each consumer is unsure about whether the product fits his taste before purchasing the product.

The consumer, however, can learn about the product by reading the product description provided by the retailer. We study the retailer's decisions on the provision of the product descriptions in this multi-channel environment, and investigate the impact of these decisions on the retailer's channel management strategy.

The retailer may operate the offline and online channels separately with different product description levels or collectively with a common product description level. For each strategy, we develop a two-period game-theoretical model in which the retailer optimizes the product description levels to maximize her expected profit. We find that integrating the offline and online channels yields more profit for the retailer if and only if the offline channel's product description limit and the consumers' base product valuation are small.

To explore how the retailer can profitably integrate her offline and online channels, we further consider a review system where the consumers who purchase the product in period 1 may post their reviews. The fraction of positive reviews in period 1 will influence the purchase intention of the upcoming consumers in period 2. In the presence of the consumer reviews, even if the offline product description limit is large, it can still be more profitable for the retailer to integrate the offline and online channels. Furthermore, the consumer reviews may reduce the retailer's profit if the consumers' base product valuation is sufficiently large.

In Chapter 4, we study a seller's decisions in the process of entering the online market through online marketplaces. While some pure-play marketplaces only provides an online platform for sellers to sell their products, some marketplaces not only provide this service but also sell their own products directly to consumers. If the seller sells her products through the latter marketplaces, she may find herself in direct competition with these marketplaces. This creates a new form of channel conflict, which is one of the focus of this chapter. In addition, the seller may engage in the competition between different marketplaces, since consumers have their own preference over different marketplaces when they shop for products.

To analyze the optimal decisions of through which marketplace(s) to sell products for the seller, we construct a game-theoretical model to capture the main trade-off in this decision process. We consider a setting in which a seller would like to sell a single product to consumers through marketplace(s). We consider one pure-play marketplace and one marketplace who also sells its own product which is competing with the seller's product. By hosting the seller, each marketplace decides a revenue sharing commission. We analyze the optimal decisions for both the retailer and the marketplaces to characterize the system equilibrium.

In general, as the commission charged by one marketplace becomes higher, the seller tends to sell its product through the other marketplace. If the price of the competing product in the marketplace becomes higher, the seller tends to charge a lower price to further attract more consumers. However, if the price of the competing product becomes lower, the seller should charge a higher price to focus on a small group of consumers to maximize its profit. In equilibrium, to alleviate the competition with the marketplace's product, if the commission of the pure-play marketplace higher, the seller will only sell on the other marketplace. On the other hand, if the commission of the pure-play marketplace is lower, the seller will sell on both marketplaces.

Appendix A

Proofs of Chapter 2

Proof of Lemma 2.1. By solving $\bar{\phi}_{i2}(0; \bar{d}_{i1}, \bar{p}_2) \leq \bar{\phi}_{i2}(-1; \bar{d}_{i1}, \bar{p}_2)$ for v_{i2}, we obtain that

1. $\bar{p}_2 \leq m + \frac{c}{v_{i2}}$ if $\bar{d}_{i1} = -1$ or 1. Thus, $\bar{d}^*_{i2} = 0$ if $\bar{p}_2 \leq m + \frac{c}{v_{i2}}$, and $\bar{d}^*_{i2} = -1$ otherwise.

2. $\bar{p}_2 \leq m + \frac{c+f}{v_{i2}}$ if $\bar{d}_{i1} = 0$. Thus, $\bar{d}^*_{i2} = 0$ if $\bar{p}_2 \leq m + \frac{c+f}{v_{i2}}$, and $\bar{d}^*_{i2} = -1$ otherwise. $\square$

Proof of Lemma 2.2. Define $b_1 = M - S + \frac{\left(\sqrt{n_e}-\sqrt{\lambda n_e}\right)C+\left(\lambda\left(1-\frac{v_L}{v_H}\right)-(1-\lambda)\right)(c+f)n_e}{(1-\lambda)n_e v_H}$, $b_2 = M - S + \frac{\left(\sqrt{\lambda(n-n_e)+n_e}-\sqrt{\lambda n_e}\right)C+\lambda n_e f-\left((1-\lambda)n_e\frac{v_H}{v_L}+\lambda(n-n_e)\right)c}{(1-\lambda)n_e v_H+\lambda(n-n_e)v_L}$, $b_3 = M - S + \frac{\left(\sqrt{n}-\sqrt{\lambda n_e}\right)C+\lambda(c+f)n_e-\left(\lambda\frac{v_L}{v_H}+(1-\lambda)\right)cn}{(1-\lambda)n v_H+\lambda(n-n_e)v_L}$, $b_4 = M - S + \frac{\left(\sqrt{\lambda(n-n_e)+n_e}-\sqrt{n_e}\right)C+\left(\lambda\frac{v_L}{v_H}+(1-\lambda)\right)(c+f)n_e-\left(\lambda n+(1-\lambda)n_e\frac{v_H}{v_L}\right)c}{\lambda(n-n_e)v_L}$, $b_5 = M - S + \frac{\left(\sqrt{n}-\sqrt{n_e}\right)C+\left(\lambda\frac{v_L}{v_H}+(1-\lambda)\right)(c+f)n_e-\left(\lambda\frac{v_L}{v_H}+(1-\lambda)\right)cn}{\lambda(n-n_e)v_L+(1-\lambda)(n-n_e)v_H}$, $b_6 = M - S + \frac{\left(\sqrt{n}-\sqrt{\lambda(n-n_e)+n_e}\right)C+\left(\lambda n\left(1-\frac{v_L}{v_H}\right)+(1-\lambda)n_e\frac{v_H}{v_L}-(1-\lambda)n\right)c}{(1-\lambda)(n-n_e)v_H}$, and $b_7 = M - S + \frac{\left(\sqrt{n}-\sqrt{\lambda n}\right)C+\left(\lambda\left(1-\frac{v_L}{v_H}\right)-(1-\lambda)\right)nc}{(1-\lambda)n v_H}$.

To derive V_2 and n_2 in the UCC's expected profit function in Equation (2.1), we need to distinguish the following four types of carriers:

Type 1 ($\bar{d}_{i1} = -1$ or 1, and $v_{i2} = v_L$): Each carrier i of this type uses the UCC's service and eliminates his logistics capability in period 2 ($\bar{d}^*_{i2} = 0$) if $\bar{p}_2 \leq m + \frac{c}{v_L}$. The expected number of carriers of this type is $\lambda(n - n_e)$, and if those carriers use the UCC's service in period 2, then the expected task volumes served by the UCC are $\lambda(n - n_e)v_L$.

Type 2 ($\bar{d}_{i1} = -1$ or 1, and $v_{i2} = v_H$): Each carrier i of this type uses the UCC's service and eliminates his logistics capability in period 2 ($\bar{d}^*_{i2} = 0$) if $\bar{p}_2 \leq m + \frac{c}{v_H}$. The expected number of carriers of this type is $(1-\lambda)(n-n_e)$, and if those carriers use the UCC's service in period 2, then the expected task volumes served by the UCC in period 2 are $(1-\lambda)(n-n_e)v_H$.

Type 3 ($\bar{d}_{i1} = 0$, and $v_{i2} = v_L$): Each carrier i of this type uses the UCC's service in period 2 ($\bar{d}^*_{i2} = 0$) if $\bar{p}_2 \leq m + \frac{c+f}{v_L}$. The expected number of carriers of this type is λn_e, and if those carriers use the UCC's service in period 2, then the expected task volumes served by the UCC in period 2 are $\lambda n_e v_L$.

Type 4 ($\bar{d}_{i1} = 0$, and $v_{i2} = v_H$): Each carrier i of this type uses the UCC's service in period 2 ($\bar{d}^*_{i2} = 0$) if $\bar{p}_2 \leq m + \frac{c+f}{v_H}$. The expected number of carriers of this type is $(1-\lambda)n_e$, and if those carriers use the UCC's service in period 2, then the expected task volumes served by the UCC in period 2 are $(1-\lambda)n_e v_H$.

Note that the expected number of Type 3 and Type 4 carriers will be 0 if $n_e = 0$. We first analyze the UCC's optimal decision in the case that $n_e > 0$, before we analyze the case that $n_e = 0$. According to the assumption $f > \frac{c(v_H - v_L)}{v_L}$, we can derive $m + \frac{c+f}{v_L} > m + \frac{c+f}{v_H} > m + \frac{c}{v_L} > m + \frac{c}{v_H}$, so the optimal choice of the UCC is among the following four:

1. Choose a price $\bar{p}_2 \in \left(m + \frac{c+f}{v_H}, m + \frac{c+f}{v_L}\right]$ to attract type 3 carriers only, then n_2 and V_2 equal to the expected number and expected task volumes of type 3 carriers, that is $n_2 = \lambda n_e$ and $V_2 = \lambda n_e v_L$. Substituting them into Equation (2.1), the UCC's expected profit is

$$\bar{\pi}_2(\bar{p}_2) = (\bar{p}_2 + S - M)\lambda n_e v_L - \sqrt{\lambda n_e}C, \tag{A.1}$$

which increases in $\bar{p}_2$, so it is optimal for the UCC to choose $\bar{p}^*_2 = m + \frac{c+f}{v_L}$ to maximize profit. Substituting $\bar{p}^*_2 = m + \frac{c+f}{v_L}$ into Equation (A.1), we obtain that $\bar{\pi}_2\left(m + \frac{c+f}{v_L}\right) = \left(m + \frac{c+f}{v_L} + S - M\right)\lambda n_e v_L - \sqrt{\lambda n_e}C$.

2. Choose a price $\bar{p}_2 \in \left(m + \frac{c}{v_L}, m + \frac{c+f}{v_H}\right]$ to attract type 3 and type 4 carriers, then n_2 and V_2 equal to the total expected number and expected task volumes of those carriers, that is $n_2 = \lambda n_e + (1-\lambda)n_e$ and $V_2 = \lambda n_e v_L + (1-\lambda)n_e v_H$. Substituting them into Equation (2.1), the UCC's expected profit is

$$\bar{\pi}_2(\bar{p}_2) = (\bar{p}_2 + S - M)(\lambda n_e v_L + (1-\lambda)n_e v_H) - \sqrt{\lambda n_e + (1-\lambda)n_e}C, \tag{A.2}$$

which increases in $\bar{p}_2$, so it is optimal for the UCC to choose $\bar{p}^*_2 = m + \frac{c+f}{v_H}$ to maximize profit. Substituting $\bar{p}^*_2 = m + \frac{c+f}{v_H}$ into Equation (A.2), we obtain that $\bar{\pi}_2\left(m + \frac{c+f}{v_H}\right) = \left(m + \frac{c+f}{v_H} + S - M\right)(\lambda n_e v_L + (1-\lambda)n_e v_H) - \sqrt{n_e}C$.

3. Choose a price $\bar{p}_2 \in \left(m + \frac{c}{v_H}, m + \frac{c}{v_L}\right]$ to attract type 3, type 4, and type 1 carriers, then n_2 and V_2 equal to the total expected number and expected task volumes of those carriers, that is $n_2 = \lambda n_e + (1-\lambda)n_e + \lambda(n-n_e)$ and $V_2 = \lambda n_e v_L + (1-\lambda)n_e v_H + \lambda(n-n_e)v_L$. Substituting them into Equation (2.1), the UCC's expected profit is

$$\bar{\pi}_2(\bar{p}_2) = (\bar{p}_2+S-M)(\lambda n_e v_L+(1-\lambda)n_e v_H+\lambda(n-n_e)v_L) - \sqrt{\lambda n_e + (1-\lambda)n_e + \lambda(n-n_e)}C, \tag{A.3}$$

which increases in $\bar{p}_2$, so it is optimal for the UCC to choose $\bar{p}_2^* = m + \frac{c}{v_L}$ to maximize profit. Substituting $\bar{p}_2^* = m + \frac{c}{v_L}$ into Equation (A.3), we obtain that $\bar{\pi}_2\left(m + \frac{c}{v_L}\right) = \left(m + \frac{c}{v_L} + S - M\right)(\lambda n v_L + (1-\lambda)n_e v_H) - \sqrt{\lambda(n-n_e)+n_e}C$.

4. Choose a price $\bar{p}_2 \in \left(0, m + \frac{c}{v_H}\right]$ to attract all types of carriers, then $n_2 = n$ and V_2 equals to the total expected task volumes of all the carriers, that is $V_2 = \lambda n_e v_L + (1-\lambda)n_e v_H + \lambda(n-n_e)v_L + (1-\lambda)(n-n_e)v_H = \lambda n v_L + (1-\lambda)n v_H$. Substituting them into Equation (2.1), the UCC's expected profit is

$$\bar{\pi}_2(\bar{p}_2) = (\bar{p}_2 + S - M)(\lambda n v_L + (1-\lambda)n v_H) - \sqrt{n}C, \tag{A.4}$$

which increases in $\bar{p}_2$, so it is optimal for the UCC to choose $\bar{p}_2^* = m + \frac{c}{v_H}$ to maximize profit. Substituting $\bar{p}_2^* = m + \frac{c}{v_H}$ into Equation (A.4), we obtain that $\bar{\pi}_2\left(m + \frac{c}{v_H}\right) = \left(m + \frac{c}{v_H} + S - M\right)(\lambda n v_L + (1-\lambda)n v_H) - \sqrt{n}C$.

By comparing the profits of the UCC under choices 1, 2, 3, and 4, we can obtain that $\bar{\pi}_2\left(m + \frac{c+f}{v_L}\right)$ is the maximum if $m < \min\{b_1, b_2, b_3\}$; $\bar{\pi}_2\left(m + \frac{c+f}{v_H}\right)$ is the maximum if $b_1 \le m < \min\{b_4, b_5\}$; $\bar{\pi}_2\left(m + \frac{c}{v_L}\right)$ is the maximum if $\max\{b_2, b_4\} \le m < b_6$; and $\bar{\pi}_2\left(m + \frac{c}{v_H}\right)$ is the maximum if $m \ge \max\{b_3, b_5, b_6\}$. Therefore, the corresponding prices $\bar{p}_2^*$ under those choices are optimal for the UCC, and the results in Lemma 2.2 follow.

Similarly, we analyze the case that $n_e = 0$. Since there is only type 1 and type 2 carriers, the optimal decision of the UCC is among the following two:

1. Choose a price $\bar{p}_2 \in \left(m + \frac{c}{v_H}, m + \frac{c}{v_L}\right]$ to attract type 1 carriers only, then n_2 and V_2 equal to the expected number and expected task volumes of type 1 carriers, that is $n_2 = \lambda(n-n_e) = \lambda n$ and $V_2 = \lambda(n-n_e)v_L = \lambda n v_L$. Substituting them into Equation (2.1), the UCC's expected profit is

$$\bar{\pi}_2(\bar{p}_2) = (\bar{p}_2 + S - M)\lambda n v_L - \sqrt{\lambda n}C, \tag{A.5}$$

which increases in $\bar{p}_2$, so it is optimal for the UCC to choose $\bar{p}_2^* = m + \frac{c}{v_L}$ to maximize profit. Substituting $\bar{p}_2^* = m + \frac{c}{v_L}$ into Equation (A.5), we obtain that $\bar{\pi}_2\left(m + \frac{c}{v_L}\right) = \left(m + \frac{c}{v_L} + S - M\right)\lambda n v_L - \sqrt{\lambda n}C$.

2. Choose a price $\bar{p}_2 \in \left(0, m + \frac{c}{v_H}\right]$ to attract both types of carriers, then $n_2 = n$ and V_2 equals to the total expected task volumes of all the carriers, that is $V_2 = \lambda(n - n_e)v_L + (1 - \lambda)(n - n_e)v_H = \lambda n v_L + (1 - \lambda)n v_H$. Substituting them into Equation (2.1), the UCC's expected profit is

$$\bar{\pi}_2(\bar{p}_2) = (\bar{p}_2 + S - M)(\lambda n v_L + (1 - \lambda)n v_H) - \sqrt{n}C, \tag{A.6}$$

which increases in $\bar{p}_2$, so it is optimal for the UCC to choose $\bar{p}_2^* = m + \frac{c}{v_H}$ to maximize profit. Substituting $\bar{p}_2^* = m + \frac{c}{v_H}$ into Equation (A.6), we obtain that $\bar{\pi}_2\left(m + \frac{c}{v_H}\right) = \left(m + \frac{c}{v_H} + S - M\right)(\lambda n v_L + (1 - \lambda)n v_H) - \sqrt{n}C$.

By comparing the profits of the UCC under choices 1 and 2, we can obtain that $\bar{\pi}_2\left(m + \frac{c}{v_L}\right) > \bar{\pi}_2\left(m + \frac{c}{v_H}\right)$ if $m < b_7$. Therefore, it is optimal for the UCC to choose $\bar{p}_2^* = m + \frac{c}{v_L}$ if $m < b_7$, and $\bar{p}_2^* = m + \frac{c}{v_H}$ otherwise. The results in Lemma 2.2 thus follow.

□

Proof of Lemma 2.3. Define $\tilde{b}_1 = M - S + \frac{(\sqrt{\tilde{n}_e} - \sqrt{\lambda \tilde{n}_e})C + \left(\lambda\left(1 - \frac{v_L}{v_H}\right) - (1-\lambda)\right)(c+f)\tilde{n}_e}{(1-\lambda)\tilde{n}_e v_H}$,

$\tilde{b}_2 = M - S + \frac{\left(\sqrt{\lambda(n-\tilde{n}_e)+\tilde{n}_e} - \sqrt{\lambda \tilde{n}_e}\right)C + \lambda \tilde{n}_e f - \left((1-\lambda)\tilde{n}_e \frac{v_H}{v_L} + \lambda(n-\tilde{n}_e)\right)c}{(1-\lambda)\tilde{n}_e v_H + \lambda(n-\tilde{n}_e)v_L}$,

$\tilde{b}_3 = M - S + \frac{(\sqrt{n} - \sqrt{\lambda \tilde{n}_e})C + \lambda(c+f)\tilde{n}_e - \left(\lambda \frac{v_L}{v_H} + (1-\lambda)\right)cn}{(1-\lambda)n v_H + \lambda(n-\tilde{n}_e)v_L}$,

$\tilde{b}_4 = M - S + \frac{\left(\sqrt{\lambda(n-\tilde{n}_e)+\tilde{n}_e} - \sqrt{\tilde{n}_e}\right)C + \left(\lambda \frac{v_L}{v_H} + (1-\lambda)\right)(c+f)\tilde{n}_e}{\lambda(n-\tilde{n}_e)v_L} - \frac{\left(\lambda n + (1-\lambda)\tilde{n}_e \frac{v_H}{v_L}\right)c}{\lambda(n-\tilde{n}_e)v_L}$,

$\tilde{b}_5 = M - S + \frac{(\sqrt{n} - \sqrt{\tilde{n}_e})C + \left(\lambda \frac{v_L}{v_H} + (1-\lambda)\right)(c+f)\tilde{n}_e - \left(\lambda \frac{v_L}{v_H} + (1-\lambda)\right)cn}{\lambda(n-\tilde{n}_e)v_L + (1-\lambda)(n-\tilde{n}_e)v_H}$,

and $\tilde{b}_6 = M - S + \frac{\left(\sqrt{n} - \sqrt{\lambda(n-\tilde{n}_e)+\tilde{n}_e}\right)C - \left(\lambda n\left(1 - \frac{v_L}{v_H}\right) + (1-\lambda)\tilde{n}_e \frac{v_H}{v_L} - (1-\lambda)n\right)c}{(1-\lambda)(n-\tilde{n}_e)v_H}$.

We first determine a carrier's optimal decision when $\tilde{n}_e > 0$. Note that $\tilde{n}_e$ is rational and hence is equal, in equilibrium, to the corresponding actual value n_e. Thus, according to case 1(a) of Lemma 2.2, if $m < \min\left\{\tilde{b}_1, \tilde{b}_2, \tilde{b}_3\right\}$, then $\bar{p}_2^* = m + \frac{c+f}{v_L}$. Given $\bar{p}_2^*$, each carrier i minimizes his total discounted cost $\bar{\Phi}_i(\bar{d}_{i1}; \bar{p}_1)$ by comparing the following 3 options:

1. $\bar{d}_{i1} = -1$: In this case, according to Lemmas 2.1 and 2.2, carrier i will deliver on his own in period 2. This incurs an expected cost $\bar{\Phi}_i(-1; \bar{p}_1) = c + m v_{i1} + \delta(\lambda(c + m v_L) + (1 - \lambda)(c + m v_H))$.

2. $\bar{d}_{i1} = 0$: In this case, according to Lemmas 2.1 and 2.2, carrier i will use the UCC's service in period 2 if $v_{i2} = v_L$ and deliver on his own otherwise. This incurs an expected cost $\bar{\Phi}_i(0; \bar{p}_1) = \bar{p}_1 v_{i1} + \delta(\lambda \bar{p}_2^* v_L + (1 - \lambda)(c + m v_H + f)) = \bar{p}_1 v_{i1} + \delta(\lambda(c + m v_L) + (1 - \lambda)(c + m v_H) + f)$.

3. $\bar{d}_{i1} = 1$: In this case, according to Lemmas 2.1 and 2.2, carrier i will deliver on his own in period 2. This incurs an expected cost $\bar{\Phi}_i(1; \bar{p}_1) = \bar{p}_1 v_{i1} + h + \delta(\lambda(c + m v_L) + (1 - \lambda)(c + m v_H))$.

By comparing the above three options, we obtain that $\bar{d}_{i1}^* = 1$ if $\bar{p}_1 \leq m + \frac{c-h}{v_{i1}}$, and $\bar{d}_{i1}^* = -1$ otherwise. This proves case 1(a) of Lemma 2.3. Next we determine the carrier's optimal decision in case 1(b) of Lemma 2.3. Similarly, according to case 1 of Lemma 2.2, if $\tilde{b}_1 \leq m < \min\left\{\tilde{b}_4, \tilde{b}_5\right\}$, then $\bar{p}_2^* = m + \frac{c+f}{v_H}$. Given $\bar{p}_2^*$, each carrier i minimizes his total discounted cost $\bar{\Phi}_i(\bar{d}_{i1}; \bar{p}_1)$ by comparing the following three options:

1. $\bar{d}_{i1} = -1$: In this case, according to Lemmas 2.1 and 2.2, carrier i will deliver on his own in period 2. This incurs an expected cost $\bar{\Phi}_i(-1; \bar{p}_1) = c + mv_{i1} + \delta(\lambda(c + mv_L) + (1-\lambda)(c + mv_H))$.

2. $\bar{d}_{i1} = 0$: In this case, according to Lemmas 2.1 and 2.2, carrier i will use the UCC's service in period 2. This incurs an expected cost $\bar{\Phi}_i(0; \bar{p}_1) = \bar{p}_1 v_{i1} + \delta(\lambda \bar{p}_2^* v_L + (1-\lambda)\bar{p}_2^* v_H) = \bar{p}_1 v_{i1} + \delta\left(m + \frac{c+f}{v_H}\right)(\lambda v_L + (1-\lambda)v_H)$.

3. $\bar{d}_{i1} = 1$: In this case, according to Lemmas 2.1 and 2.2, carrier i will deliver on his own in period 2. This incurs an expected cost $\bar{\Phi}_i(1; \bar{p}_1) = \bar{p}_1 v_{i1} + h + \delta(\lambda(c + mv_L) + (1-\lambda)(c + mv_H))$.

By comparing the above three options, we obtain that $\bar{d}_{i1}^* = 1$ if $\bar{p}_1 \leq m + \frac{c-h}{v_{i1}}$ and $h \leq \delta(c+f)\left(\lambda\frac{v_L}{v_H} + 1 - \lambda\right) - \delta c$, $\bar{d}_{i1}^* = 0$ if $\bar{p}_1 \leq m + \frac{(1+\delta)c}{v_{i1}} - \frac{\delta(c+f)\left(\lambda\frac{v_L}{v_H}+1-\lambda\right)}{v_{i1}}$ and $h > \delta(c+f)\left(\lambda\frac{v_L}{v_H} + 1 - \lambda\right) - \delta c$, and $\bar{d}_{i1}^* = -1$ otherwise. This proves case 1(b) of Lemma 2.3. The proofs of cases 1(c) and 2(a) are similar to the proof of case 1(b), and the proofs of cases 1(d) and 2(b) are similar to the proof of case 1(a), and thus omitted. □

Proof of Theorem 2.4. Define $m_1 = M - S + \frac{(\sqrt{n}-\sqrt{\lambda n})C + \left(\lambda\left(1-\frac{v_L}{v_H}\right)-(1-\lambda)\right)n(c-h)}{(1-\lambda)nv_H}$,
$m_2 = M - S + \frac{(1-\lambda)\sqrt{n}C + \lambda^2 nf - \left(\lambda\frac{v_L}{v_H} + (1-\lambda)^2\right)nc}{(1-\lambda)nv_H + \lambda(1-2\lambda)nv_L}$,
$m_3 = M - S + \frac{(\sqrt{n}-\sqrt{\lambda n})C + \left(\lambda\frac{v_L}{v_H}+1-\lambda\right)(c+f)\lambda n - \left(\lambda\frac{v_L}{v_H}+1-\lambda\right)nc}{(1-\lambda)(\lambda nv_L + (1-\lambda)nv_H)}$,
and $m_4 = M - S + \frac{\left(\sqrt{n}-\sqrt{\lambda(1-\lambda)n+\lambda n}\right)C + \left(\lambda\left(1-\frac{v_L}{v_H}\right)-(1-\lambda)\left(1-\lambda\frac{v_H}{v_L}\right)\right)nc}{(1-\lambda)^2 nv_H}$.

The UCC's expected profit $\bar{\Pi}(\bar{p}_1)$ in Equation (2.2) depends on V_1 and n_1. The different cases in Lemma 2.3 corresponding to different decisions of each carrier will lead to different values of V_1 and n_1. In the following, we analyze each case of Lemma 2.3 to derive V_1 and n_1 and obtain the UCC's expected total discounted profit and then determine the equilibrium price. To derive V_1 and n_1, we need to distinguish the following two typs of carriers:

Type A $(v_{i1} = v_L)$**:** The expected number of carriers of this type is λn.

Type B $(v_{i1} = v_H)$**:** The expected number of carriers of this type is $(1-\lambda)n$.

We first analyze the cases that $\tilde{n}_e > 0$ of Lemma 2.3, that is cases 1(a), 1(b), 1(c), and 1(d). Note that $\tilde{n}_e$ is rational and hence equal to to the corresponding actual value in equilibrium, and thus $\tilde{n}_e = n_e(\bar{p}_1^*, \bar{p}_2^*) > 0$, $\tilde{b}_1 = b_1$, $\tilde{b}_2 = b_2$, $\tilde{b}_3 = b_3$, $\tilde{b}_4 = b_4$, $\tilde{b}_5 = b_5$, and $\tilde{b}_6 = b_6$.

In case 1(a) ($\tilde{n}_e > 0$ and $m < \min\left\{\tilde{b}_1, \tilde{b}_2, \tilde{b}_3\right\}$), $\bar{d}_{i1}^* = 1$ if $\bar{p}_1 \le m + \frac{c-h}{v_{i1}}$, or $\bar{d}_{i1}^* = -1$ otherwise. Thus, type A carriers use the UCC's service and keep their logistics capability if $\bar{p}_1 \le m + \frac{c-h}{v_L}$, and type B carriers use the UCC's service and keep their logistics capability if $\bar{p}_1 \le m + \frac{c-h}{v_H}$. In this case, no carrier will use the UCC's service and eliminate logistics capability, which means $n_e = 0$, and thus cannot happen in equilibrium.

In cases 1(b) ($\tilde{n}_e > 0$ and $\tilde{b}_1 \le m < \min\left\{\tilde{b}_4, \tilde{b}_5\right\}$) and 1(c) ($\tilde{n}_e > 0$ and $\max\left\{\tilde{b}_2, \tilde{b}_4\right\} \le m < \tilde{b}_6$), since we focus on the case that $h \le \min\Big\{\delta(c + f)\left(\lambda\frac{v_L}{v_H} + 1 - \lambda\right) - \delta c, \delta c\left(\lambda + (1-\lambda)\frac{v_H}{v_L}\right) - \delta c\Big\}$, thus $\bar{d}_{i1}^* = 1$ if $\bar{p}_1 \le m + \frac{c-h}{v_{i1}}$, or $\bar{d}_{i1}^* = -1$ otherwise. Similar to the above case 1(a), these cases will never happen in equilibrium.

In case 1(d) ($\tilde{n}_e > 0$ and $m \ge \max\left\{\tilde{b}_3, \tilde{b}_5, \tilde{b}_6\right\}$), $\bar{d}_{i1}^* = 0$ if $\bar{p}_1 \le m + \frac{c}{v_{i1}}$, or $\bar{d}_{i1}^* = -1$ otherwise. Thus, type A carriers use the UCC's service and eliminate their logistics capability if $\bar{p}_1 \le m + \frac{c}{v_L}$, and type B carriers use the UCC's service and eliminate their logistics capability if $\bar{p}_1 \le m + \frac{c}{v_H}$. In case 1(d), we have obtained that $\bar{p}_2^* = m + \frac{c}{v_H}$ according to Lemma 2.2. The optimal choice of the retailer in period 1 is among the following two:

1. Choose a price $\bar{p}_1 \in \left(m + \frac{c}{v_H}, m + \frac{c}{v_L}\right]$ to attract type A carriers only, then n_1 and V_1 equal to the expected number and task volumes of type A carriers, that is $n_1 = \lambda n$ and $V_1 = \lambda n v_L$. Since type A carriers use the UCC's service and eliminate their logistics capability, thus $n_e = n_1 = \lambda n$. Substituting them into Equation (2.2), the UCC's expeted total discounted profit is

$$
\begin{aligned}
\bar{\Pi}(\bar{p}_1) &= (\bar{p}_1 + S - M)\lambda n v_L - \sqrt{\lambda n}C + \delta\bar{\pi}_2\left(m + \frac{c}{v_H}\right) \\
&= (\bar{p}_1 + S - M)\lambda n v_L - \sqrt{\lambda n}C + \delta\left[\left(m + \frac{c}{v_H} + S - M\right)(\lambda n v_L + (1-\lambda)n v_H) - \sqrt{n}C\right],
\end{aligned} \tag{A.7}
$$

which increases in $\bar{p}_1$, so it is optimal for the UCC to choose $\bar{p}_1^* = m + \frac{c}{v_L}$ to maximize profit. This could be in equilirbium only if $n_e = \lambda n$ satisfies the conditions that $n_e > 0$ (which is satisfied) and $m \ge \max\{b_3, b_5, b_6\}$. Substituting $n_e = \lambda n$ into b_3, b_5, and b_6, we can rewrite the latter conition as $m \ge \max\{m_2, m_3, m_4\}$. This leads to the results in case 3 of Theorem 2.4.

2. Choose a price $\bar{p}_1 \in \left(0, m + \frac{c}{v_H}\right]$ to attract both types of carriers, then $n_1 = n$ and V_1 equals to the total expected task volumes of all the carriers, that is $V_1 = \lambda n v_L + (1-\lambda) n v_H$. Since all the carriers use the UCC's service and

eliminate their logistics capability, thus $n_e = n_1 = n$. Substituting them into Equation (2.2), the UCC's expeted total discounted profit is

$$\begin{aligned}\bar{\Pi}(\bar{p}_1) &= (\bar{p}_1 + S - M)(\lambda n v_L + (1-\lambda) n v_H) - \sqrt{n}C + \delta\bar{\pi}_2\left(m + \tfrac{c}{v_H}\right)\\ &= (\bar{p}_1 + S - M)(\lambda n v_L + (1-\lambda) n v_H) - \sqrt{n}C\\ &\quad + \delta\left[\left(m + \tfrac{c}{v_H} + S - M\right)(\lambda n v_L + (1-\lambda) n v_H) - \sqrt{n}C\right],\end{aligned} \tag{A.8}$$

which increases in $\bar{p}_1$, so it is optimal for the UCC to choose $\bar{p}_1^* = m + \frac{c}{v_H}$ to maximize profit. This could be in equilirbium only if $n_e = n$ satisfies the conditions that $n_e > 0$ (which is satisfied) and $m \geq \max\{b_3, b_5, b_6\}$. Substituting $n_e = n$ into b_3, b_5, and b_6, we find that the latter condition can never be satisfied as b_5 and b_6 go to infinity.

Next we analyze the cases that $\tilde{n}_e = 0$ of Lemma 2.3, that is cases 2(a) and 2(b). Similarly, since $\tilde{n}_e$ is rational and hence equal to the corresponding actual value in equilibrium, and thus $n_e(\bar{p}_1^*, \bar{p}_2^*) = \tilde{n}_e = 0$.

In case 2(a) ($\tilde{n}_e = 0$ and $m < b_7$, since we focus on the case that $h \leq \min\left\{\delta(c + f)\left(\lambda\frac{v_L}{v_H} + 1 - \lambda\right) - \delta c, \delta c\left(\lambda + (1-\lambda)\frac{v_H}{v_L}\right) - \delta c\right\}$, thus $\bar{d}_{i1}^* = 1$ if $\bar{p}_1 \leq m + \frac{c-h}{v_{i1}}$, or $\bar{d}_{i1}^* = -1$ otherwise. Thus, type A carriers use the UCC's service and keep their logistics capability if $\bar{p}_1 \leq m + \frac{c-h}{v_L}$, and type B carriers use the UCC's service and eliminate their logistics capability if $\bar{p}_1 \leq m + \frac{c-h}{v_H}$. In case 2(a), we have obtained that $\bar{p}_2^* = m + \frac{c}{v_L}$ according to Lemma 2.2. The optimal choice of the retailer in period 1 is among the following two:

1. Choose a price $\bar{p}_1 \in \left(m + \frac{c-h}{v_H}, m + \frac{c-h}{v_L}\right]$ to attract type A carriers only, then n_1 and V_1 equal to the expected number and task volumes of type A carriers, that is $n_1 = \lambda n$ and $V_1 = \lambda n v_L$. Since no carrier will use the UCC's service and keeplogistics capability, thus $n_e = 0$. Substituting them into Equation (2.2), the UCC's expeted total discounted profit is

$$\begin{aligned}\bar{\Pi}(\bar{p}_1) &= (\bar{p}_1 + S - M)\lambda n v_L - \sqrt{\lambda n}C + \delta\bar{\pi}_2\left(m + \tfrac{c}{v_L}\right)\\ &= (\bar{p}_1 + S - M)\lambda n v_L - \sqrt{\lambda n}C + \delta\left[\left(m + \tfrac{c}{v_L} + S - M\right)\lambda n v_L - \sqrt{\lambda n}C\right],\end{aligned} \tag{A.9}$$

which increases in $\bar{p}_1$, so it is optimal for the UCC to choose $\bar{p}_1^* = m + \frac{c-h}{v_L}$ to maximize profit. This could be in equilirbium only if $n_e = 0$ satisfies the conditions that $n_e = 0$ (which is satisfied) and $m - (M - S) < m_1$. Substituting $\bar{p}_1^* = m + \frac{c-h}{v_L}$ into Equation (A.9), we can obtain that $\bar{\Pi}\left(m + \frac{c-h}{v_L}\right) = (1 + \delta)(m + S - M)\lambda n v_L + ((1+\delta)c - h)\lambda n - (1+\delta)\sqrt{\lambda n}C$.

2. Choose a price $\bar{p}_1 \in \left(0, m + \frac{c-h}{v_H}\right]$ to attract both types of carriers, then $n_1 = n$ and V_1 equals to the total expected task volumes of all the carriers,

that is $V_1 = \lambda n v_L + (1-\lambda) n v_H$. Since no carrier will use the UCC's service and eliminate logistics capability, thus $n_e = 0$. Substituting them into Equation (2.2), the UCC's expeted total discounted profit is

$$\begin{aligned}\bar{\Pi}(\bar{p}_1) &= (\bar{p}_1 + S - M)(\lambda n v_L + (1-\lambda) n v_H) - \sqrt{n}C + \delta \bar{\pi}_2 \left(m + \frac{c}{v_L}\right) \\ &= (\bar{p}_1 + S - M)(\lambda n v_L + (1-\lambda) n v_H) - \sqrt{n}C + \delta \left[\left(m + \frac{c}{v_L} + S - M\right) \lambda n v_L - \sqrt{\lambda n}C\right],\end{aligned} \tag{A.10}$$

which increases in $\bar{p}_1$, so it is optimal for the UCC to choose $\bar{p}_1^* = m + \frac{c-h}{v_H}$ to maximize profit. This could be in equilirbium only if $n_e = 0$ satisfies the conditions that $n_e = 0$ (which is satisfied) and $m < b_7$. Substituting $\bar{p}_1^* = m + \frac{c-h}{v_H}$ into Equation (A.10), we can obtain that $\bar{\Pi}\left(m + \frac{c-h}{v_H}\right) = (m + S - M)((1+\delta)\lambda n v_L + (1-\lambda) n v_H) + \delta \lambda c n + (c-h) n \left(\lambda \frac{v_L}{v_H} + 1 - \lambda\right) - \left(\sqrt{n} + \delta\sqrt{\lambda n}\right) C$.

By comparing $\bar{\Pi}\left(m + \frac{c-h}{v_L}\right)$ and $\bar{\Pi}\left(m + \frac{c-h}{v_H}\right)$ with respect to m, we obtain that $\bar{\Pi}\left(m + \frac{c-h}{v_L}\right) > \bar{\Pi}\left(m + \frac{c-h}{v_H}\right)$ if $m < m_1$. Therefore, we have $\bar{p}_1^* = m + \frac{c-h}{v_L}$ if $m < \min\{b_7, m_1\}$, and $\bar{p}_1^* = m + \frac{c-h}{v_H}$ if $\min\{b_7, m_1\} \le m < b_7$. This leads to the results in cases 1 and 2 of Theorem 2.4.

In case 2(b) ($\tilde{n}_e = 0$ and $m - (M - S) \ge b_7$), $\bar{d}_{i1}^* = 0$ if $\bar{p}_1 \le m + \frac{c}{v_{i1}}$, or $\bar{d}_{i1}^* = -1$ otherwise. Thus, type A carriers use the UCC's service and eliminate their logistics capability if $\bar{p}_1 \le m + \frac{c}{v_L}$, and type B carriers use the UCC's service and eliminate their logistics capability if $\bar{p}_1 \le m + \frac{c}{v_H}$. In this case, $n_e = 0$ will never happen which indicates that it will never be in equilibrium.

□

Lemma A.1. (Optimal decision of carrier $i \in \mathcal{N}_{L,2}$ in period 2)

1. If $\hat{d}_{i1} = -1$ or 1, then in period 2 carrier i purchases capacity from the platform and eliminates his logistics capability ($\hat{d}_{i2}^* = 0$) if $\hat{p}_2 < \frac{c + (1+\theta_2) m v_L}{[1+\theta_2(1-\alpha)]v_L}$, or delivers on his own ($\hat{d}_{i2}^* = -1$) if $\hat{p}_2 > \frac{c + (1+\theta_2) m v_L}{[1+\theta_2(1-\alpha)]v_L}$.

2. If $\hat{d}_{i1} = 0$, then in period 2, carrier i purchases capacity from the platform ($\hat{d}_{i2}^* = 0$) if $\hat{p}_2 < \frac{c + f + (1+\theta_2) m v_L}{[1+\theta_2(1-\alpha)]v_L}$, or delivers on his own ($\hat{d}_{i2}^* = -1$) if $\hat{p}_2 > \frac{c + f + (1+\theta_2) m v_L}{[1+\theta_2(1-\alpha)]v_L}$.

Proof of Lemma A.1. By solving $\hat{\phi}_{i2}(0; \hat{d}_{i1}, \hat{p}_2) \le \hat{\phi}_{i2}(-1; \hat{d}|_{i1}, \hat{p}_2)$ for v_{i2}, we obtain that

1. $\hat{p}_2 < (c + 2m v_L)/((2-\alpha)v_L)$ if $\hat{d}_{i1} = -1$ or 1. Thus, $\hat{d}_{i2}^* = 0$ if $\hat{p}_2 < (c + 2m v_L)/((2-\alpha)v_L)$, and $\hat{d}_{i2}^* = -1$ if $\hat{p}_2 > (c + 2m v_L)/((2-\alpha)v_L)$.

2. $\hat{p}_2 < (c + 2m v_L + f)/((2-\alpha)v_L)$ if $\hat{d}_{i1} = 0$. Thus, $\hat{d}_{i2}^* = 0$ if $\hat{p}_2 < (c + 2m v_L + f)/((2-\alpha)v_L)$, and $\hat{d}_{i2}^* = -1$ if $\hat{p}_2 > (c + 2m v_L + f)/((2-\alpha)v_L)$.

□

Lemma A.2. (Optimal decision of the platform in period 2) *Define n_e as the number of carriers who purchase capacity on the platform and elimate their logistics capability in period 1.*

1. If $n_e > n/2$, the optimal price of the platform in period 2 is as follows. If $(c+f)[2(n-n_e)(2n-\alpha n_e)-(2-\alpha)(2n-n_e)n_e] \leq 2mv_L[(2-\alpha)nn_e-2(n-n_e)(2n-\alpha n_e)]$, then $\hat{p}_2^* = \frac{(2n-n_e)(c+f)+2nmv_L}{(2n-\alpha n_e)v_L}$; if $(c+f)[2(n-n_e)(2n-\alpha n_e)-(2-\alpha)(2n-n_e)n_e] > 2mv_L[(2-\alpha)nn_e-2(n-n_e)(2n-\alpha n_e)]$, then $\hat{p}_2^* = \frac{c+2mv_L+f}{(2-\alpha)v_L}-\epsilon$.

2. If $n_e \leq n/2$, the optimal price of the UCC's service in period 2 is as follows. If $(c+f)[2(n-n_e)(2n-\alpha n_e)-(n-\alpha n_e)(2n-n_e)] \leq 2mv_L[n(n-\alpha n_e)-n(2n-\alpha n_e)]$ and $(c+f)(2-\alpha)(2n-n_e)n_e-(2n-\alpha n_e)(n-n_e)c > 2mv_L[(2n-\alpha n_e)(n-n_e)-(2-\alpha)nn_e]$, then $\hat{p}_2^* = \frac{(2n-n_e)(c+f)+2nmv_L}{(2n-\alpha n_e)v_L}$; if $(c+f)[2(n-n_e)(2n-\alpha n_e)-(n-\alpha n_e)(2n-n_e)] > 2mv_L[n(n-\alpha n_e)-n(2n-\alpha n_e)]$ and $2(c+f)(2-\alpha)(n-n_e)n_e-(n-\alpha n_e)(n-n_e)c > 2mv_L[(n-\alpha n_e)(n-n_e)-(2-\alpha)nn_e]$, then $\hat{p}_2^* = \frac{(c+f)(n-n_e)+nmv_L}{(n-\alpha n_e)v_L}-\epsilon$; if $2(c+f)(2-\alpha)(n-n_e)n_e-(n-\alpha n_e)(n-n_e)c \leq 2mv_L[(n-\alpha n_e)(n-n_e)-(2-\alpha)nn_e]$ and $(c+f)(2-\alpha)(2n-n_e)n_e-(2n-\alpha n_e)(n-n_e)c \leq 2mv_L[(2n-\alpha n_e)(n-n_e)-(2-\alpha)nn_e]$, then $\hat{p}_2^* = \frac{c+2mv_L}{(2-\alpha)v_L}$.

Proof of Lemma A.2. To derive $n_{s,2}$ and $n_{p,2}$ in the platform's expected profit function in Equation (2.3), we need to distinguish the following two types of carriers:

Type 1 ($\hat{d}_{i1} = -1$ or 1): Each carrier i of this type purchases capacity from the platform and eliminates his logistics capability in period 2 ($\hat{d}_{i2}^* = 0$) if $\hat{p}_2 < \frac{c+2mv_L}{(2-\alpha)v_L}$, or delivers on his own and sell capacity on the platform ($\hat{d}_{i2}^* = 1$) if $\hat{p}_2 > \frac{c+2mv_L}{(2-\alpha)v_L}$, or chooses either option with same probability if $\hat{p}_2 = \frac{c+2mv_L}{(2-\alpha)v_L}$. The expected number of carriers of this type is $\lambda(\lambda n - n_e) + \lambda(1-\lambda)n$.

Type 2 ($\hat{d}_{i1} = 0$): Each carrier i of this type purchases capacity from the platform and eliminates his logistics capability in period 2 ($\hat{d}_{i2}^* = 0$) if $\hat{p}_2 < \frac{c+2mv_L+f}{(2-\alpha)v_L}$, or delivers on his own and sell capacity on the platform ($\hat{d}_{i2}^* = 1$) if $\hat{p}_2 > \frac{c+2mv_L+f}{(2-\alpha)v_L}$, or chooses either option with same probability if $\hat{p}_2 = \frac{c+2mv_L+f}{(2-\alpha)v_L}$. The expected number of carriers of this type is λn_e.

To maximize her profit, the optimal choice of the platform is among the following three:

1. Choose a price $\hat{p}_2 = \frac{c+2mv_L+f}{(2-\alpha)v_L}$ to incentivize type 1 carriers to sell capacity, and type 2 carriers to purchase or sell capacity with same probability. Then we can obtain that $n_{s,2} = \lambda(\lambda n - n_e) + \lambda(1-\lambda)n + \lambda n_e/2$, and $n_{p,2} = \lambda n_e/2$. Substituting them into Equation (2.3), the platform's expected profit is

$$
\begin{aligned}
\hat{\pi}_2\big(\tfrac{c+2mv_L+f}{(2-\alpha)v_L}\big) &= \alpha\tfrac{c+2mv_L+f}{(2-\alpha)v_L}\min\{[\lambda(\lambda n-n_e)+\lambda(1-\lambda)n+\lambda n_e/2]v_L, \lambda n_e v_L/2\} \\
&= \tfrac{\lambda\alpha(c+2mv_L+f)}{2-\alpha}\min\{n-n_e/2, n_e/2\} \\
&= \tfrac{\lambda\alpha(c+2mv_L+f)n_e}{2(2-\alpha)}.
\end{aligned}
$$

2. Choose a price $\hat{p}_2 = \frac{c+2mv_L+f}{(2-\alpha)v_L} - \epsilon$ to incentivize type 1 carriers to sell capacity, and type 2 carriers to purchase capacity from the platform. Then we can obtain that $n_{s,2} = \lambda(\lambda n - n_e) + \lambda(1-\lambda)n$, and $n_{p,2} = \lambda n_e$. Substituting them into Equation (2.3), the platform's expected profit is

$$
\begin{aligned}
\hat{\pi}_2\big(\tfrac{c+2mv_L+f}{(2-\alpha)v_L}-\epsilon\big) &= \alpha(\tfrac{c+2mv_L+f}{(2-\alpha)v_L}-\epsilon)\min\{[\lambda(\lambda n-n_e)+\lambda(1-\lambda)n]v_L, \lambda n_e v_L\} \\
&= \tfrac{\lambda\alpha(c+2mv_L+f)}{2-\alpha}\min\{n_e, n-n_e\}-\epsilon \\
&= \begin{cases} \frac{\lambda\alpha(c+2mv_L+f)n_e}{2-\alpha}-\epsilon, & \text{if } n_e < n/2; \\ \frac{\lambda\alpha(c+2mv_L+f)(n-n_e)}{2-\alpha}-\epsilon, & \text{if } n_e \geq n/2. \end{cases}
\end{aligned}
$$

3. Choose a price $\hat{p}_2 = \frac{c+2mv_L}{(2-\alpha)v_L}$ to incentivize type 1 carriers to sell or purchase capacity with same probability, and type 2 carriers to purchase capacity from the platform. Then we can obtain that $n_{s,2} = [\lambda(\lambda n - n_e) + \lambda(1-\lambda)n]/2$, and $n_{p,2} = [\lambda(\lambda n - n_e) + \lambda(1-\lambda)n]/2 + \lambda n_e$. Substituting them into Equation (2.3), the platform's expected profit is

$$
\begin{aligned}
\hat{\pi}_2\big(\tfrac{c+2mv_L}{(2-\alpha)v_L}\big) &= \alpha\tfrac{c+2mv_L}{(2-\alpha)v_L}\min\{[\lambda(\lambda n-n_e)+\lambda(1-\lambda)n]v_L/2, \lambda n_e v_L\} \\
&= \tfrac{\lambda\alpha(c+2mv_L)}{2-\alpha}\min\{(n-n_e)/2, (n+n_e)/2\} \\
&= \tfrac{\lambda\alpha(c+2mv_L)(n-n_e)}{2(2-\alpha)}.
\end{aligned}
$$

By comparing the profits of the platform under choices 1, 2 and 3, we can obtain that $\hat{\pi}_2\big(\frac{c+2mv_L+f}{(2-\alpha)v_L}\big)$ is the maximum if $n_e \geq \frac{2n}{3}$; $\hat{\pi}_2\big(\frac{c+2mv_L+f}{(2-\alpha)v_L}-\epsilon\big)$ is the maximum if $\frac{(c+2mv_L)n}{2(c+2mv_L)+f} \leq n_e < \frac{2n}{3}$; and $\hat{\pi}_2\big(\frac{c+2mv_L}{(2-\alpha)v_L}\big)$ is the maximum if $n_e < \frac{(c+2mv_L)n}{2(c+2mv_L)+f}$. Therefore, the results in Lemma A.2 follow.

□

Lemma A.3. (Optimal decision of carrier i in period 1) *Assume all carriers and the capacity sharing platform have a common rational belief $\tilde{n}_e$ about n_e.*

1. If $\tilde{n}_e > n/2$; or $\tilde{n}_e \leq n/2$, $(c+f)[2(n-n_e)(2n-\alpha n_e)-(2-\alpha)(2n-n_e)n_e] \leq 2mv_L[(2-\alpha)nn_e - 2(n-n_e)(2n-\alpha n_e)]$, and $(c+f)(2-\alpha)(2n-n_e)n_e - (2n-\alpha n_e)(n-n_e)c > 2mv_L[(2n-\alpha n_e)(n-n_e)-(2-\alpha)nn_e]$; or $\tilde{n}_e \leq n/2$, $(c+f)[2(n-n_e)(2n-\alpha n_e)-(2-\alpha)(2n-n_e)n_e] > 2mv_L[(2-\alpha)nn_e-2(n-n_e)(2n-\alpha n_e)]$, and $2(c+f)(2-\alpha)(n-n_e)n_e-(n-\alpha n_e)(n-n_e)c > 2mv_L[(n-\alpha n_e)(n-n_e)-(2-\alpha)nn_e]$, the optimal decisions of carrier i are as follows. If $\hat{p}_1 < \frac{c+2mv_L-h}{(2-\alpha)v_L}$, then $\hat{d}^*_{i1} = 1$; if $\hat{p}_1 > \frac{c+2mv_L-h}{(2-\alpha)v_L}$, then $\hat{d}^*_{i1} = -1$; if $\hat{p}_1 = \frac{c+2mv_L-h}{(2-\alpha)v_L}$, then $\hat{d}^*_{i1} = 1$ or -1 with an equal probability.

2. If $\tilde{n}_e \leq n/2$, $2(c+f)(2-\alpha)(n-n_e)n_e - (n-\alpha n_e)(n-n_e)c \leq 2mv_L[(n-\alpha n_e)(n-n_e)-(2-\alpha)nn_e]$, and $(c+f)(2-\alpha)(2n-n_e)n_e-(2n-\alpha n_e)(n-n_e)c \leq 2mv_L[(2n-\alpha n_e)(n-n_e)-(2-\alpha)nn_e]$, the optimal decisions of carrier i are as follows.

(a) If $h \leq \delta(1-\lambda)f$, then if $\hat{p}_1 < \frac{c+2mv_L-h}{(2-\alpha)v_L}$, $\hat{d}^*_{i1} = 1$; if $\hat{p}_1 > \frac{c+2mv_L-h}{(2-\alpha)v_L}$, $\hat{d}^*_{i1} = -1$; if $\hat{p}_1 = \frac{c+2mv_L-h}{(2-\alpha)v_L}$, $\hat{d}^*_{i1} = 1$ or -1 with an equal probability.

(b) If $h > \delta(1-\lambda)f$, then if $\hat{p}_1 < \frac{c+2mv_L-\delta(1-\lambda)f}{(2-\alpha)v_L}$, $\hat{d}^*_{i1} = 0$; if $\hat{p}_1 > \frac{c+2mv_L-\delta(1-\lambda)f}{(2-\alpha)v_L}$, $\hat{d}^*_{i1} = -1$; if $\hat{p}_1 = \frac{c+2mv_L-\delta(1-\lambda)f}{(2-\alpha)v_L}$, $\hat{d}^*_{i1} = 0$ or -1 with an equal probability.

Proof of Lemma A.3. We first determine a carrier's optimal decision when $\tilde{n}_e \geq \frac{2n}{3}$. Note that $\tilde{n}_e$ is rational and hence is equal, in equilibrium, to the corresponding actual value n_e. Thus, according to Lemma A.2, if $\tilde{n}_e = n_e \geq \frac{2n}{3}$, then $\hat{p}^*_2 = \frac{c+2mv_L+f}{(2-\alpha)v_L}$. Each carrier i minimizes his total discounted cost $\hat{\Phi}_{i1}(\hat{d}_{i1};\hat{p}_1,\hat{p}_2)$ by comparing the following three options:

1. $\hat{d}_{i1} = -1$: In this case, according to Lemma A.2, carrier i will deliver on his own and sell capacity in period 2 (if he has low task volume in period 2, i.e., $v_{i2} = v_L$). This incurs an expected total discounted cost $\hat{\Phi}_{i1}(-1;\hat{p}_1,\hat{p}_2) = c+mv_L-[(1-\alpha)\hat{p}_1-m]v_L+\delta(\lambda(c+mv_L-[(1-\alpha)\hat{p}^*_2-m]v_L)+(1-\lambda)(c+mv_H))$.

2. $\hat{d}_{i1} = 0$: In this case, according to Lemma A.2, carrier i will purchase capacity from the platform or deliver on his own and sell capacity in period 2 with same probability (if he has low task volume in period 2, i.e., $v_{i2} = v_L$). This incurs an expected total discounted cost $\hat{\Phi}_{i1}(0;\hat{p}_1,\hat{p}_2) = \hat{p}_1 v_L + \delta(\lambda(\hat{p}^*_2 v_L/2 + (c+mv_L - [(1-\alpha)\hat{p}^*_2 - m]v_L + f)/2) + (1-\lambda)(c+mv_H))$.

3. $\hat{d}_{i1} = 1$: In this case, according to Lemma A.2, carrier i will deliver on his own and sell capacity in period 2 (if he has low task volume in period 2, i.e., $v_{i2} = v_L$). This incurs an expected total discounted cost $\hat{\Phi}_{i1}(-1;\hat{p}_1,\hat{p}_2) = \hat{p}_1 v_L + h + \delta(\lambda(c+mv_L-[(1-\alpha)\hat{p}^*_2-m]v_L) + (1-\lambda)(c+mv_H))$.

By comparing the above three options, we obtain that, if $h \leq \delta\lambda f$, then $\hat{d}^*_{i1} = 1$ if $\hat{p}_1 < (c+2mv_L-h)/((2-\alpha)v_L)$, or $\hat{d}^*_{i1} = -1$ if $\hat{p}_1 > (c+2mv_L-h)/((2-\alpha)v_L)$. If $h > \delta\lambda f$, then $\hat{d}^*_{i1} = 0$ if $\hat{p}_1 < (c+2mv_L-\delta\lambda f)/((2-\alpha)v_L)$, or $\hat{d}^*_{i1} = -1$ if $\hat{p}_1 > (c+2mv_L-\delta\lambda f)/((2-\alpha)v_L)$.

Next we determine the carrier's optimal decision when $\frac{(c+2mv_L)n}{2(c+2mv_L)+f} \leq \tilde{n}_e < \frac{2n}{3}$. Similarly, according to Lemma A.2, $\hat{p}^*_2 = \frac{c+2mv_L+f}{(2-\alpha)v_L} - \epsilon$. Each carrier i minimizes his expected total discounted cost $\hat{\Phi}_{i1}(\hat{d}_{i1};\hat{p}_1,\hat{p}_2)$ by comparing the following three options:

1. $\hat{d}_{i1} = -1$: In this case, according to Lemma A.2, carrier i will deliver on his own and sell capacity in period 2 (if he has low task volume in period 2, i.e., $v_{i2} = v_L$). This incurs an expected total discounted cost $\hat{\Phi}_{i1}(-1;\hat{p}_1,\hat{p}_2) = c+mv_L-[(1-\alpha)\hat{p}_1-m]v_L+\delta(\lambda(c+mv_L-[(1-\alpha)\hat{p}^*_2-m]v_L)+(1-\lambda)(c+mv_H))$.

2. $\hat{d}_{i1} = 0$: In this case, according to Lemma A.2, carrier i will purchase capacity from the platform in period 2 (if he has low task volume in period 2, i.e., $v_{i2} = v_L$). This incurs an expected total discounted cost $\hat{\Phi}_{i1}(0; \hat{p}_1, \hat{p}_2) = \hat{p}_1 v_L + \delta(\lambda \hat{p}_2^* v_L + (1-\lambda)(c + m v_H))$.

3. $\hat{d}_{i1} = 1$: In this case, according to Lemma A.2, carrier i will deliver on his own and sell capacity in period 2 (if he has low task volume in period 2, i.e., $v_{i2} = v_L$). This incurs an expected total discounted cost $\hat{\Phi}_{i1}(-1; \hat{p}_1, \hat{p}_2) = \hat{p}_1 v_L + h + \delta(\lambda(c + m v_L - [(1-\alpha)\hat{p}_2^* - m]v_L) + (1-\lambda)(c + m v_H))$.

By comparing the above three options, we obtain the same results as in the case that $\tilde{n}_e \geq \frac{2n}{3}$. Thus, the carrier's optimal decision is same as in that case.

Finally, we determine the carrier's optimal decision when $\tilde{n}_e < \frac{(c+2mv_L)n}{2(c+2mv_L)+f}$. According to Lemma A.2, $\hat{p}_2^* = \frac{c+2mv_L}{(2-\alpha)v_L}$. Each carrier i minimizes his expected total discounted cost $\hat{\Phi}_{i1}(\hat{d}_{i1}; \hat{p}_1, \hat{p}_2)$ by comparing the following three options:

1. $\hat{d}_{i1} = -1$: In this case, according to Lemma A.2, carrier i will purchase capacity from the platform or deliver on his own and sell capacity in period 2 with same probability (if he has low task volume in period 2, i.e., $v_{i2} = v_L$). This incurs an expected total discounted cost $\hat{\Phi}_{i1}(-1; \hat{p}_1, \hat{p}_2) = c + m v_L - [(1-\alpha)\hat{p}_1 - m]v_L + \delta(\lambda(\hat{p}_2^* v_L/2 + (c + m v_L - [(1-\alpha)\hat{p}_2^* - m]v_L)/2) + (1-\lambda)(c + m v_H))$.

2. $\hat{d}_{i1} = 0$: In this case, according to Lemma A.2, carrier i will purchase capacity from the platform in period 2 (if he has low task volume in period 2, i.e., $v_{i2} = v_L$). This incurs an expected total discounted cost $\hat{\Phi}_{i1}(0; \hat{p}_1, \hat{p}_2) = \hat{p}_1 v_L + \delta(\lambda \hat{p}_2^* v_L + (1-\lambda)(c + m v_H))$.

3. $\hat{d}_{i1} = 1$: In this case, according to Lemma A.2, carrier i will purchase capacity from the platform or deliver on his own and sell capacity in period 2 with same probability (if he has low task volume in period 2, i.e., $v_{i2} = v_L$). This incurs an expected total discounted cost $\hat{\Phi}_{i1}(-1; \hat{p}_1, \hat{p}_2) = \hat{p}_1 v_L + h + \delta(\lambda(\hat{p}_2^* v_L/2 + (c + m v_L - [(1-\alpha)\hat{p}_2^* - m]v_L)/2) + (1-\lambda)(c + m v_H))$.

By comparing the above three options, we obtain that, $\hat{d}_{i1}^* = 0$ if $\hat{p}_1 < (c + 2mv_L)/((2-\alpha)v_L)$, or $\hat{d}_{i1}^* = -1$ if $\hat{p}_1 > (c + 2mv_L)/((2-\alpha)v_L)$. Combining the results in the above three cases together, Lemma A.3 follows.

□

Proof of Theorem 2.5. Similar to the proof of Theorem 2.4, we analyze each case of Lemma A.3 to derive the platform's expected total discounted profit and determine the equilibrium price.

We can obtain that case 1 of Lemma A.3 is not in equilibrium, because the conditions of this case cannot be satisfied under any $\hat{p}_1$. For case 2 of Lemma A.3, according to Lemma A.2, we have $\hat{p}_2^* = \frac{c+2mv_L}{(2-\alpha)v_L}$. If $h \leq \delta(1-\lambda)f$, that is $f \geq \frac{h}{\delta(1-\lambda)}$, then according to Lemma A.3, each carrier purchases capacity from the platform and keeps his logistics capability if $\hat{p}_1 < \frac{c+2mv_L-h}{(2-\alpha)v_L}$, or delivers on his own and sell remaining capacity if $\hat{p}_1 > \frac{c+2mv_L-h}{(2-\alpha)v_L}$, or with same probability to choose either option if $\hat{p}_1 = \frac{c+2mv_L-h}{(2-\alpha)v_L}$. It's optimal for the platform to choose $\hat{p}_1^* = \frac{c+2mv_L-h}{(2-\alpha)v_L}$ to maximize her profit. This leads to $n_e = 0 = \tilde{n}_e$, with which the conditions of case 2 are always satisified. This completes the proof of case 1 of Theorem 2.5.

If $h > \delta(1-\lambda)f$, that is $f < \frac{h}{\delta(1-\lambda)}$, according to Lemma A.3, each carrier purchases capacity from the platform and eliminates his logistics capability if $\hat{p}_1 < \frac{c+2mv_L-\delta(1-\lambda)f}{(2-\alpha)v_L}$ or delivers on his own and sell remaining capacity if $\hat{p}_1 > \frac{c+2mv_L-\delta(1-\lambda)f}{(2-\alpha)v_L}$, or with same probability to choose either option if $\hat{p}_1 = \frac{c+2mv_L-\delta(1-\lambda)f}{(2-\alpha)v_L}$. It is optimal for the platform to choose $\hat{p}_1 = \frac{c+2mv_L-\delta(1-\lambda)f}{(2-\alpha)v_L}$ to maximize her profit. This leads to $n_e = \frac{\lambda}{2}n = \tilde{n}_e$. Substituting them into the conditions of case 2, we obtain that $f < \frac{(2-2\lambda+\frac{\alpha\lambda^2}{4})mv_L+(1-\frac{3\lambda}{2}+\frac{\lambda(\lambda+\alpha)}{4})c}{(2-\alpha)\frac{\lambda}{2}(1-\frac{\lambda}{4})}$ and $f < \frac{(2-3\lambda+\frac{\alpha\lambda^2}{2})mv_L+(1-\frac{5\lambda}{2}+\lambda^2+\frac{\alpha\lambda(2-\lambda)}{4})c}{(2-\alpha)\frac{\lambda}{2}(2-\lambda)}$. Combining them with the condition $f < \frac{h}{\delta(1-\lambda)}$, the result in case 2 of Theorem 2.5 follows.

$\square$

Proof of Theorem 2.6. Define $m_5 = \frac{\frac{(2-\alpha)\left(1-\frac{\lambda}{4}\right)\lambda h}{2\delta(1-\lambda)}-\left(1-\frac{3\lambda}{2}+\frac{\lambda(\lambda+\alpha)}{4}\right)c}{\left(2-2\lambda+\frac{\alpha\lambda^2}{4}\right)v_L}$,

$$m_6 = \frac{\frac{(2-\alpha)(2-\lambda)\lambda h}{2\delta(1-\lambda)}-\left(1-\frac{5\lambda}{2}+\lambda^2+\frac{\alpha\lambda(2-\lambda)}{4}\right)c}{\left(2-3\lambda+\frac{\alpha\lambda^2}{2}\right)v_L},$$

$$f_1 = \frac{[(1-\lambda)v_H+\lambda(1-2\lambda)v_L]\left[\left(\sqrt{n}-\sqrt{\lambda(1-\lambda)n+\lambda n}\right)C+\left(\lambda n\left(1-\frac{v_L}{v_H}\right)-(1-\lambda)\left(1-\lambda\frac{v_H}{v_L}\right)n\right)c\right]}{\lambda^2(1-\lambda)^2nv_H}+\frac{\left(\lambda\frac{v_L}{v_H}+(1-\lambda)^2\right)nc-(1-\lambda)\sqrt{n}C}{\lambda^2 n},$$

$$f_2 = \frac{[(1-\lambda)v_H+\lambda v_L]\left[\left(\sqrt{n}-\sqrt{\lambda(1-\lambda)n+\lambda n}\right)C+\left(\lambda n\left(1-\frac{v_L}{v_H}\right)-(1-\lambda)\left(1-\lambda\frac{v_H}{v_L}\right)n\right)c\right]}{\lambda(1-\lambda)nv_H\left(\lambda\frac{v_L}{v_H}+1-\lambda\right)}+\frac{(1-\lambda)\left(\lambda\frac{v_L}{v_H}+1-\lambda\right)nc-\left(\sqrt{n}-\sqrt{\lambda n}\right)C}{\lambda n\left(\lambda\frac{v_L}{v_H}+1-\lambda\right)},$$

$$f_3 = \frac{[m-(M-S)][(1-\lambda)nv_H+\lambda(1-2\lambda)nv_L]-(1-\lambda)\sqrt{n}C+\left(\lambda\frac{v_L}{v_H}+(1-\lambda)^2\right)nc}{\lambda^2 n}, \text{ and}$$

$$f_4 = \frac{[m-(M-S)](1-\lambda)(\lambda nv_L+(1-\lambda)nv_H)-\left(\sqrt{n}-\sqrt{\lambda n}\right)C+(1-\lambda)\left(\lambda\frac{v_L}{v_H}+1-\lambda\right)nc}{\lambda n\left(\lambda\frac{v_L}{v_H}+1-\lambda\right)}.$$

In Region (i), the equilibrium expected total discounted profits of the UCC and the platform are $\bar{\Pi}(m+\frac{c}{v_L})$ and $\hat{\Pi}\left(\frac{c+2mv_L-\delta(1-\lambda)f}{(2-\alpha)v_L}\right)$ respectively. Substituting $\bar{p}_1^*$ and $\bar{p}_2^*$ into Equation (2.2), $\hat{p}_1^*$ and $\hat{p}_2^*$ into Equation (2.4), we can obtain that $\bar{\Pi}\left(m+\frac{c}{v_L}\right) = (1+\delta)(m+S-M)\lambda nv_L + \delta(1-\lambda)(m+S-M)nv_H + \lambda nc + \delta nc\left(\lambda\frac{v_L}{v_H}+1-\lambda\right) - \left(\sqrt{\lambda n}+\delta\sqrt{n}\right)C$, and $\hat{\Pi}\left(\frac{c+2mv_L-\delta(1-\lambda)f}{(2-\alpha)v_L}\right) = \frac{\alpha\lambda n\left[\left(1+\delta\left(1-\frac{\lambda}{2}\right)\right)(c+2mv_L)-\delta(1-\lambda)f\right]}{2(2-\alpha)}$. By comparing $\bar{\Pi}\left(m+\frac{c}{v_L}\right)$ and $\hat{\Pi}\left(\frac{c+2mv_L-\delta(1-\lambda)f}{(2-\alpha)v_L}\right)$ in terms of c, we obtain that $\bar{\Pi}\left(m+\frac{c}{v_L}\right) > \hat{\Pi}\left(\frac{c+2mv_L-\delta(1-\lambda)f}{(2-\alpha)v_L}\right)$ if and only if

$$c > \frac{\left(\sqrt{\lambda}+\delta\right)\frac{C}{\sqrt{n}} + \frac{\alpha\lambda\left[2\left(1+\delta\left(1-\frac{\lambda}{2}\right)\right)mv_L - \delta(1-\lambda)f\right]}{2(2-\alpha)} - (m+S-M)[(1+\delta)\lambda v_L + \delta(1-\lambda)v_H]}{\lambda\left(1-\frac{\alpha\left(1+\delta\left(1-\frac{\lambda}{2}\right)\right)}{2(2-\alpha)}\right) + \delta\left(\lambda\frac{v_L}{v_H}+1-\lambda\right)} \equiv c_1,$$

where c_1 decreases in n.

In Region (ii), the equilibrium expected total discounted profits of the UCC and the platform are $\bar{\Pi}(m+\frac{c-h}{v_H})$ and $\hat{\Pi}\left(\frac{c+2mv_L-h}{(2-\alpha)v_L}\right)$ respectively. Substituting $\bar{p}_1^*$ and $\bar{p}_2^*$ into Equation (2.2), $\hat{p}_1^*$ and $\hat{p}_2^*$ into Equation (2.4), we can obtain that $\bar{\Pi}\left(m+\frac{c-h}{v_H}\right) = (m+S-M)((1+\delta)\lambda n v_L + (1-\lambda)n v_H) + \delta\lambda n c + (c-h)n\left(\lambda\frac{v_L}{v_H}+1-\lambda\right) - \left(\sqrt{n}+\delta\sqrt{\lambda n}\right)C$, and $\hat{\Pi}\left(\frac{c+2mv_L-h}{(2-\alpha)v_L}\right) = \frac{\alpha\lambda n[(1+\delta)(c+2mv_L)-h]}{2(2-\alpha)}$. By comparing $\bar{\Pi}\left(m+\frac{c-h}{v_H}\right)$ and $\hat{\Pi}\left(\frac{c+2mv_L-h}{(2-\alpha)v_L}\right)$ in terms of c and δ, we that $\bar{\Pi}\left(m+\frac{c-h}{v_H}\right) > \hat{\Pi}\left(\frac{c+2mv_L-h}{(2-\alpha)v_L}\right)$ if and only if

$$c > \frac{\left(1+\delta\sqrt{\lambda}\right)\frac{C}{\sqrt{n}} + h\left(\lambda\frac{v_L}{v_H}+1-\lambda\right) + \frac{\alpha\lambda[(1+\delta)2mv_L - h]}{2(2-\alpha)} - (m+S-M)[(1+\delta)\lambda v_L + (1-\lambda)v_H]}{\delta\lambda + (\lambda\frac{v_L}{v_H}+1-\lambda) - \frac{\alpha\lambda(1+\delta)}{2(2-\alpha)}} \equiv c_2$$

and $\delta > \frac{\frac{\lambda\alpha}{2(2-\alpha)} - \left(\lambda\frac{v_L}{v_H}+1-\lambda\right)}{\lambda\left(1-\frac{\alpha}{2(2-\alpha)}\right)} \equiv \delta_1$; or $c < c_2$ and $\delta < \delta_1$.

In Region (iii), the equilibrium expected total discounted profits of the UCC and the platform are $\bar{\Pi}(m+\frac{c-h}{v_L})$ and $\hat{\Pi}\left(\frac{c+2mv_L-h}{(2-\alpha)v_L}\right)$ respectively. Substituting $\bar{p}_1^*$ and $\bar{p}_2^*$ into Equation (2.2), we can obtain that $\bar{\Pi}\left(m+\frac{c-h}{v_L}\right) = (1+\delta)(m+S-M)\lambda n v_L + \lambda n((1+\delta)c-h) - (1+\delta)\sqrt{\lambda n}C$. By comparing $\bar{\Pi}\left(m+\frac{c-h}{v_L}\right)$ and $\hat{\Pi}\left(\frac{c+2mv_L-h}{(2-\alpha)v_L}\right)$, we obtain the following results.

1. $\bar{\Pi}\left(m+\frac{c-h}{v_L}\right) > \hat{\Pi}\left(\frac{c+2mv_L-h}{(2-\alpha)v_L}\right)$ if and only if $c > \frac{(1+\delta)\frac{\sqrt{\lambda}C}{\sqrt{n}} + \frac{\alpha\lambda[2(1+\delta)mv_L - h]}{2(2-\alpha)} + \lambda h - (m+S-M)(1+\delta)\lambda v_L}{(1+\delta)\lambda\left(1-\frac{\alpha}{2(2-\alpha)}\right)} \equiv c_3$, where c_3 decreases in n.

2. $\bar{\Pi}\left(m+\frac{c-h}{v_L}\right) > \hat{\Pi}\left(\frac{c+2mv_L-h}{(2-\alpha)v_L}\right)$ if and only if $h < \frac{(1+\delta)(m+S-M)\lambda v_L + (1+\delta)\lambda c - (1+\delta)\frac{\sqrt{\lambda}C}{\sqrt{n}} - \frac{\alpha\lambda(1+\delta)(c+2mv_L)}{2(2-\alpha)}}{\lambda\left(1-\frac{\alpha}{2(2-\alpha)}\right)} \equiv h_1$, where h_1 increases in n.

□

Proof of Theorem 2.7. In Region (i), according to Theorems 2.4 and 2.5, we can obtain that $n_1 = \lambda n$, $n_2 = n$, $n_{p,1} = \frac{\lambda n}{2}$, and $n_{p,2} = \frac{\lambda n}{2}$. Thus, we have

$\bar{\Delta}_\psi = (\lambda n - \sqrt{\lambda n} + n - \sqrt{n})\psi$ and $\hat{\Delta}_\psi = \frac{5}{4}\lambda n\psi$. By comparing $\bar{\Delta}_\psi$ and $\hat{\Delta}_\psi$ in terms of n, we obtain that $\bar{\Delta}_\psi > \hat{\Delta}_\psi$ if and only if $n > \left(\frac{1+\sqrt{\lambda}}{1-\lambda/4}\right)^2$. Similarly, the results for Regions (ii) and (iii) can be determined. $\square$

Proof of Theorem 2.8. Define $m_7 = \frac{(2-\alpha)\left[(M-S)\lambda n v_L - \lambda nc + \sqrt{\lambda n}C\right] + \frac{\alpha\lambda nc}{2}}{2(1-\alpha)\lambda n v_L}$,
$m_8 = \frac{(M-S)(\lambda n v_L + (1-\lambda)n v_H) - \left(\lambda\frac{v_L}{v_H} + 1 - \lambda\right)nc + \sqrt{n}C + \frac{\alpha\lambda nc}{2(2-\alpha)}}{\frac{2(1-\alpha)\lambda n v_L}{2-\alpha} + (1-\lambda)n v_L}$, and
$m_9 = \frac{(M-S)(1-\lambda)n v_H + \left[\lambda\left(1-\frac{v_L}{v_H}\right) - (1-\lambda)\right]nc + (\sqrt{n}-\sqrt{\lambda n})C}{(1-\lambda)n v_H}$. One can see that $\max\{m_8, m_9\}$ decreases with S. Define $\theta = \min\left\{\frac{\tilde{n}_p}{\tilde{n}_s}, 1\right\}$, where $\tilde{n}_p$ and $\tilde{n}_s$ are the rational beliefs about the number of carriers who purchase capacity from the platform and who sell capacity on the platform, respectively.

Similar to the proofs of Lemma 2.1, we can derive the optimal decision of each carrier i as follows.

1. Each carrier i with $v_i = v_H$ uses the UCC's service if $\bar{p} \le m + \frac{c}{v_H}$, or delivers on his own if $\bar{p} > m + \frac{c}{v_H}$.

2. Each carrier i with $v_i = v_L$ uses the UCC's service if $\bar{p} \le m + \frac{c}{v_L} - \theta[(1-\alpha)\hat{p} - m]$ and $\hat{p} \ge \bar{p}$, or purchases capacity from the platform if $\bar{p} > \hat{p}$ and $\hat{p} < \frac{c + (1+\theta)m v_L}{[1+\theta(1-\alpha)]v_L}$, or delivers on his own if $\bar{p} > m + \frac{c}{v_L} - \theta[(1-\alpha)\hat{p} - m]$ and $\hat{p} > \frac{c+(1+\theta)m v_L}{[1+\theta(1-\alpha)]v_L}$. Note that carrier i is indifferent between purchasing capacity from the platform and delivering on his own if $\hat{p} = \frac{c+(1+\theta)m v_L}{[1+\theta(1-\alpha)]v_L}$.

According to the assumption $(1-\alpha)\hat{p} - m < (\frac{1}{v_L} - \frac{1}{v_H})c$, we can obtain that $m + \frac{c}{v_L} - \theta[(1-\alpha)\hat{p} - m] > m + \frac{c}{v_H}$. Similar to the proof of Lemma 2.1, we can obtain that the optimal choice of the consolidator is among the following:

1. Choose $\bar{p}^* > \frac{c+2m v_L}{(2-\alpha)v_L}$ and $\hat{p}^* = \frac{c+2m v_L}{(2-\alpha)v_L}$. Under these prices, each carrier i with $v_i = v_H$ delivers on his own, and each carrier i with $v_i = v_L$ is indifferent between delivering on his own (and selling his remaining capacity to the platform) and purchasing capacity on the platform. The consolidator's profit is $\frac{\alpha\lambda n(c+2m v_L)}{2(2-\alpha)}$.

2. Choose $\bar{p}^* = m + \frac{c}{v_L}$ and $\hat{p}^* \ge m + \frac{c}{v_L}$. Under these prices, each carrier i with $v_i = v_H$ delivers on his own, and each carrier i with $v_i = v_L$ uses the UCC's service. The consolidator's profit is $(m + S - M)\lambda n v_L + \lambda nc - \sqrt{\lambda n}C$.

3. Choose $\bar{p}^* = m + \frac{c}{v_H}$ and $\hat{p}^* \ge m + \frac{c}{v_L}$. Under these prices, each carrier i uses the UCC's service. The consolidator's profit is $(m + S - M)(\lambda n v_L + (1-\lambda)n v_H) + (\lambda\frac{v_L}{v_H} + 1 - \lambda)nc - \sqrt{n}C$.

It is optimal for the consolidator to choose the choice that leads to a largest profit. Comparing the consolidator's profit under the above three choices, we can obtain the results in Theorem 2.8.

$\square$

Proof of Theorem 2.9. Define $m_{10} = M - S + \frac{\left(\sqrt{n}-\sqrt{\lambda n}\right)C+\lambda nc-\left(\lambda\frac{v_L}{v_H}+1-\lambda\right)(c-h)n}{(1-\lambda)nv_H}$, $m_{11} = M - S + \frac{\left(\sqrt{\lambda(1-\lambda)+\lambda n}-\lambda\sqrt{n}\right)C+\lambda^2 nf-\lambda(1-\lambda)\left(1+\frac{v_H}{v_L}\right)nc}{\lambda(1-\lambda)n(v_L+v_H)}$, and $m_{12} = M - S + \frac{\left(\sqrt{\lambda(2-\lambda)n}-\sqrt{\lambda n}\right)C+\left(\lambda\frac{v_L}{v_H}+1-\lambda\right)(c+f)\lambda n-\left(1+(1-\lambda)\frac{v_H}{v_L}\right)\lambda nc}{\lambda(1-\lambda)nv_L}$. The proof is similar to the proof of Theorem 2.4 and thus omitted. □

Appendix B

Proofs of Chapter 3

Recall that we make the following assumptions in this chapter:

Assumption 1. If $\pi_{i,t}(d'_{i,t}) = \pi_{i,t}(d''_{i,t})$ and $d'_{i,t} < d''_{i,t}$, then the retailer always chooses $d'_{i,t}$.

Assumption 2. $p + u_o \leq V < p + u_b + (1-\theta)c$.

Assumption 3. $(1-\theta)c/2 < u_o - u_b < \min\{\theta c, (1-\theta)c\}$.

Assumption 4. $\theta + \frac{(1-\theta)c}{a} < \underline{\lambda} < 1 - \frac{(1-\theta)c}{a}$ and $a > 2c/\theta^2$.

Given the probabilities $P\{m\}$ and $P\{s|m\}$, we can derive the probability distribution of s as $P\{s=0\} = 1-(1-\theta)d_{i,t}$ and $P\{s=1\} = (1-\theta)d_{i,t}$, according to Bayes' Theorem.

Proof of Lemma 3.1. In the model without consumer reviews, the retailer decides a product description level $d_{b,t}$ for the offline channel. The consumers with perceived signal $s=1$ will never purchase the product according to Equation (3.1) and Assumption 6. The consumers with perceived signal $s=0$ purchase the product if $E[U|s=0] \geq u_b$, i.e., $V - \frac{c(1-\theta)(1-d_{b,t})}{1-(1-\theta)d_{b,t}} - p \geq u_b$, according to Equation (3.1). By solving this inequality for $d_{b,t}$, we obtain that $d_{b,t} \geq \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_b-c)}$, under which these consumers with perceived signal $s=0$ will purchase the product. Note that the retailer can only choose a $d_{b,t} \leq \bar{d} < 1$, thus,

1. if $\frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_b-c)} \leq \bar{d}$, then the retailer can attract the consumers with perceived signal $s=0$ to purchase the product by choosing a $d_{b,t} \in [\frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_b-c)}, \bar{d}]$, or not attract these consumers to purchase the product by choosing a $d_{b,t} < \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_b-c)}$ which will reuslt in zero profit and always be dominated by the former choice. Thus the latter is omitted in the following discussions.

2. if $\frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_b-c)} > \bar{d}$, then the retailer can never attract the consumers with perceived signal $s = 0$ to purchase the product.

By solving the above inequalities for V, under Assumption 6, we obtain the following cases:

1. if $V \geq p + u_b + \frac{(1-\theta)(1-\bar{d})c}{1-(1-\theta)\bar{d}} = \hat{V}$ and $\bar{d} \leq \frac{(1-\theta)c-(u_o-u_b)}{(1-\theta)[c-(u_o-u_b)]} = \hat{d}$, or $\bar{d} > \hat{d}$, the retailer can attract the consumers with perceived signal $s = 0$ to purchase the product by choosing a $d_{b,t} \in [\frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_b-c)}, \bar{d}]$, and generate a profit $\pi_{b,t}(d_{b,t}) = p \times n \times (1-(1-\theta)d_{b,t})$. One can see that $\pi_{b,t}(d_{b,t})$ decreases in $d_{b,t}$, so it is optimal for the retailer to choose $d^*_{b,t} = \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_b-c)}$ to maximize her profit. By substituting $d^*_{b,t} = \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_b-c)}$ into $\pi_{b,t}(d_{b,t})$, we obtain that $\pi^*_{b,t} = \frac{\theta npc}{p-V+u_b+c}$.

2. if $V < \hat{V}$ and $\bar{d} \leq \hat{d}$, the retailer can never attract the consumers with perceived signal $s = 0$ to purchase the product, which means $\pi_{b,t}(d_{b,t}) = 0$. Thus, according to Assumption 1, the retailer is optimal to choose $d^*_{b,t} = 0$ with $\pi^*_{b,t} = 0$.

□

Proof of Lemma 3.2. In the model without consumer reviews, the retailer decides a product description level $d_{o,t}$ for the online channel. Similar to the proof of Lemma 3.1, we can obtain that the consumers with perceived signal $s = 1$ will never purchase the product, and the consumers with $s = 0$ will purchase the product if $d_{o,t} \geq \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}$. The retailer can attract these consumers with perceived signal $s = 0$ to purchase the product by choosing a $d_{o,t} \geq \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}$ and generate a profit $\pi_{o,t}(d_{o,t}) = p \times n \times (1 - (1-\theta)d_{o,t})$. One can see that $\pi_{o,t}(d_{o,t})$ decreases in $d_{o,t}$, so it is optimal for the retailer to choose $d^*_{o,t} = \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}$ to maximize her profit. By substituting $d^*_{o,t} = \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}$ into $\pi_{o,t}(d_{o,t})$, we obtain that $\pi^*_{o,t} = \frac{\theta npc}{p-V+u_o+c}$. □

Proof of Lemma 3.3. In the model without consumer reviews, the retailer decides a product description level $d_{omni,t}$ for both channels. The offline consumers will purchase the product if $E[U|s] \geq u_b$ and online consumers will purchase the product if $E[U|s] \geq u_o$, where $E[U|s]$ is derived in Equation (3.1). Similar to the proofs of Lemma 3.1 and 3.2, we can obtain that the consumers with perceived signal $s = 1$ will never purchase the product, the offline consumers with perceived signal $s = 0$ will purchase the product if $d_{omni,t} \geq \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_b-c)}$, and the online consumers with perceived signal $s = 0$ will purchase the product if $d_{omni,t} \geq \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}$. According to Assumption 3, we can obtain $\frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)} > \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_b-c)}$. The optimal choice of the retailer is among the following two:

1. attract both the offline and online consumers with perceived signal $s = 0$ to purchase the product by choosing a $d_{omni,t} \geq \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}$, and generate a profit

$$\begin{aligned} \pi_{omni,t}(d_{omni,t}) &= p \times (n \times \phi_{b,t}(d_{omni,t}) + n \times \phi_{o,t}(d_{omni,t})) \\ &= p \times (n \times P\{s=0\} + n \times P\{s=0\}) \\ &= 2pn(1-(1-\theta)d_{omni,t}). \end{aligned} \quad \text{(B.1)}$$

One can see that $\pi_{omni,t}(d_{omni,t})$ decreases in $d_{omni,t}$, so it is optimal for the retailer to choose $d^*_{omni,t} = \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}$ to maximize her profit. By substituting $d^*_{omni,t} = \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}$ into Equation (B.1), we obtain that $\pi^*_{omni,t} = \frac{2\theta npc}{p-V+u_o+c}$.

2. attract the offline consumers with perceived signal $s = 0$ to purchase the product by choosing a $d_{omni,t} \in [\frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_b-c)}, \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)})$, and generate a profit

$$\begin{aligned} \pi_{omni,t}(d_{omni,t}) &= p \times (n \times \phi_{b,t}(d_{omni,t}) + n \times \phi_{o,t}(d_{omni,t})) \\ &= p \times n \times P\{s=0\} \\ &= pn(1-(1-\theta)d_{omni,t}). \end{aligned} \quad \text{(B.2)}$$

One can see that $\pi_{omni,t}(d_{omni,t})$ decreases in $d_{omni,t}$, so it is optimal for the retailer to choose $d^*_{omni,t} = \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_b-c)}$ to maximize her profit. By substituting $d^*_{omni,t} = \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_b-c)}$ into Equation (B.2), we obtain that $\pi^*_{omni,t} = \frac{\theta npc}{p-V+u_b+c}$.

By comparing the optimal profits of the retailer under choices 1 and 2, we can obtain that $\frac{2\theta npc}{p-V+u_o+c} > \frac{\theta npc}{p-V+u_b+c}$ according to assumptions 6 and 3. Thus choice 1 is optimal for the retailer, that is $d^*_{omni,t} = \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}$ and $\pi^*_{omni,t} = \frac{2\theta npc}{p-V+u_o+c}$. □

Proof of Theorem 3.4. According to Lemmas 3.1 and 3.2, we determine the optimal profit of the retailer who operates the offline and online channels separately $\pi^*_{dual,t} = \pi^*_{b,t} + \pi^*_{o,t}$ as follows.

1. $\pi^*_{dual,t} = \frac{\theta npc}{p-V+u_o+c}$ if $\bar{d} \leq \hat{d}$ and $V \geq \hat{V}$.
2. $\pi^*_{dual,t} = \frac{\theta npc}{p-V+u_o+c} + \frac{\theta npc}{p-V+u_b+c}$ otherwise.

According to Lemma 3.3, the optimal profit of the retailer who integrates the offline and online channels is $\pi^*_{omni,t} = \frac{2\theta npc}{p-V+u_o+c}$. One can see that $\pi^*_{omni,t} > \pi^*_{dual,t}$ if $\bar{d} \leq \hat{d}$ and $V \geq \hat{V}$. Otherwise, according to Assumption 3, we can prove that $\pi^*_{omni,t} < \pi^*_{dual,t}$. □

Proof of Lemma 3.5. In the model with consumer reviews, the retailer decides a product description level $d_{o,2}$ in period 2 for the online channel. According to Equation (3.2), period 2 consumers with perceived signal $s = 0$ purchase the product if $E[U|s = 0] \geq u_o$, i.e., $V - \frac{c(1-\theta)(1-d_{o,2})}{1-(1-\theta)d_{o,2}} + \psi(\lambda) - p \geq u_o$, and period 2 consumers with perceived signal $s = 1$ purchase the product if $E[U|s = 1] \geq u_o$, i.e., $V - c + \psi(\lambda) - p \geq u_o$. Solving these two inequalities for V and $d_{o,2}$, we obtain the following cases:

1. if $V \geq p + u_o + (1 - \theta)c - \psi(\lambda)$, the retailer can attract all the period 2 consumers to purchase the product by choosing any $d_{o,2}$. In this case, $\pi_{o,2}(d_{o,2}) = p \times n$, so it's optimal for the retailer to choose $d^{\dagger}_{o,2} = 0$ with $\pi^{\dagger}_{o,2} = pn$.
2. if $p + u_o - \psi(\lambda) \leq V < p + u_o + (1 - \theta)c - \psi(\lambda)$, the retailer can attract the period 2 consumers with perceived signal $s = 0$ to purchase the product by choosing a $d_{o,2} \geq \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V+\psi(\lambda)-p-u_o-c)}$, and generate a profit $\pi_{o,2}(d_{o,2}) = pn(1-(1-\theta)d_{o,2})$. One can see that $\pi_{o,2}(d_{o,2})$ decreases in $d_{o,2}$, so it is optimal for the retailer to choose $d^{\dagger}_{o,2} = \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V+\psi(\lambda)-p-u_o-c)}$ to maximize her profit. By substituting $d^{\dagger}_{o,2} = \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V+\psi(\lambda)-p-u_o-c)}$ into $\pi_{o,2}(d_{o,2})$, we obtain that $\pi^{\dagger}_{o,2} = \frac{\theta npc}{p-V+u_o+c-\psi(\lambda)}$.
3. if $V < p + u_o - \psi(\lambda)$, the retailer can never attract any period 2 consumer to purchase the product which means $\pi_{o,2}(d_{o,2}) = 0$. Thus it's optimal for the retailer to choose $d^{\dagger}_{o,2} = 0$ with $\pi^{\dagger}_{o,2} = 0$.

Combining Assumption 6 and the conditions of V in the above 3 cases, we can obtain the results in Lemma 3.5 accordingly. ☐

Proof of Lemma 3.6. In the model with consumer reviews, the retailer decides a product description level $d_{o,1}$ in period 1 for the online channel. The period 1 consumers with perceived signal $s = 1$ will never purchase the product according to Assumption 6. The period 1 consumers with perceived signal $s = 0$ purchase the product if $E[U|s = 0] \geq u_o$, i.e., $V - \frac{c(1-\theta)(1-d_{o,1})}{1-(1-\theta)d_{o,1}} - p \geq u_o$. By solving this inequality for $d_{o,1}$, we obtain that $d_{o,1} \geq \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}$. Thus the retailer has two choices in period 1: not attract period 1 consumers to purchase the product by choosing a $d_{o,1} < \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}$; or attract period 1 consumers with perceived signal $s = 0$ to purchase the product by choosing a $d_{o,1} \geq \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}$.

We first analyze the case that $d_{o,1} < \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}$, in which the profit is $\pi_{o,1}(d_{o,1}) = 0$ in period 1. In this case, no consumer review is generated so the expected impact of consumer reviews on period 2 consumers is $\psi(\lambda) = 0$. Thus, in period 2, the consumers have the same expected utilities as that in the model without consumer reviews, and the retailer will make the same optimal decisions as that in the model without consumer reviews. According to Lemma 3.2, it is optimal for the retailer to choose $d^{\dagger}_{o,2} = \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}$ in period 2 with a profit $\pi^{\dagger}_{o,2} = \frac{\theta npc}{p-V+u_o+c}$. In this case, $\Pi_o = \pi_{o,1}(d_{o,1}) + \pi^{\dagger}_{o,2} = \frac{\theta npc}{p-V+u_o+c}$, which is independent of $d_{o,1}$. Thus it's optimal for the retailer to choose $d^{\dagger}_{o,1} = 0$ with $\Pi^{\dagger}_o = \frac{\theta npc}{p-V+u_o+c}$.

We then analyze the case that $d_{o,1} \geq \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}$. In this case, period 1 consumers with perceived signal $s = 0$ will purchase the product, and the retailer generate a profit in period 1 as follows:

$$\begin{aligned}\pi_{o,1}(d_{o,1}) &= p \times n \times \phi_{o,1}(d_{o,1}) \\ &= p \times n \times P\{s = 0\} \\ &= pn(1-(1-\theta)d_{o,1}).\end{aligned}$$

The actual misfit degree m of these period 1 consumers who purchase the product has the following distributions:

$$\begin{aligned}P\{m = 0|s = 0\} &= \frac{P\{s=0|m=0\}P\{m=0\}}{P\{s=0\}} = \frac{\theta}{1-(1-\theta)d_{o,1}}, \\ P\{m = 1|s = 0\} &= \frac{P\{s=0|m=1\}P\{m=1\}}{P\{s=0\}} = \frac{(1-\theta)(1-d_{o,1})}{1-(1-\theta)d_{o,1}}.\end{aligned}$$

With probability η, the period 1 consumers with $U = V - p - cm \geq u_o$ will write positive reviews, and the period 1 consumers with $U = V - p - cm < u_o$ will write negative reviews. According to Assumption 6, these two inequalities indicate that with probability η, the period 1 consumers with $m = 0$ will write positive reviews, while the consumers with $m = 1$ will write negative reviews. Thus, the expected number of positive consumer reviews is $\eta \times n \times P\{s = 0\} \times P\{m = 0|s = 0\} = \eta\theta n$ and the expected number of negative consumer reviews is $\eta \times n \times P\{s = 0\} \times P\{m = 1|s = 0\} = \eta(1-\theta)(1-d_{o,1})n$, so the expected fraction of positive consumer reviews among all the consumer reviews is $\lambda = \frac{\eta\theta n}{\eta\theta n+\eta(1-\theta)(1-d_{o,1})n} = \frac{\theta}{1-(1-\theta)d_{o,1}}$, and the expected impact of consumer reviews on period 2 consumers is $\psi(\lambda) = a(\lambda - \underline{\lambda}) = a\left(\frac{\theta}{1-(1-\theta)d_{o,1}} - \underline{\lambda}\right)$. According to Lemma 3.5, we have the following three cases.

1. $\pi^{\dagger}_{o,2} = 0$ if $\psi(\lambda) \leq u_o - u_b - (1-\theta)c$, or $u_o - u_b - (1-\theta)c < \psi(\lambda) < 0$ and $V < p + u_o - \psi(\lambda)$. Substituting $\psi(\lambda)$ into these inequalities and solve them for $d_{o,1}$, we can obtain $d_{o,1} < \frac{1}{1-\theta} + \frac{\theta a}{(1-\theta)(V-p-u_o-a\underline{\lambda})}$. In this case, $\Pi_o = \pi_{o,1}(d_{o,1}) + \pi^{\dagger}_{o,2} = pn(1-(1-\theta)d_{o,1})$.

2. $\pi^{\dagger}_{o,2} = \frac{\theta npc}{p-V+u_o+c-\psi(\lambda)}$ if $u_o - u_b - (1-\theta)c < \psi(\lambda) < 0$ and $V \geq p + u_o - \psi(\lambda)$, or $0 \leq \psi(\lambda) \leq u_o - u_b$, or $u_o - u_b < \psi(\lambda) < (1-\theta)c$ and $V < p + u_o + (1-\theta)c - \psi(\lambda)$. Substituting $\psi(\lambda)$ into these inequalities and solve them for $d_{o,1}$, we can obtain $\frac{1}{1-\theta} + \frac{\theta a}{(1-\theta)(V-p-u_o-a\underline{\lambda})} \leq d_{o,1} < \frac{1}{1-\theta} + \frac{\theta a}{(1-\theta)(V-p-u_o-(1-\theta)c-a\underline{\lambda})}$. In this case, $\Pi_o = \pi_{o,1}(d_{o,1}) + \pi^{\dagger}_{o,2} = pn(1-(1-\theta)d_{o,1}) + \frac{\theta npc}{p-V+u_o+c-\psi(\lambda)} = pn(1-(1-\theta)d_{o,1}) + \frac{(1-(1-\theta)d_{o,1})\theta npc}{(1-(1-\theta)d_{o,1})(a\underline{\lambda}+p-V+u_o+c)-\theta a}$.

3. $\pi^{\dagger}_{o,2} = pn$ if $u_o - u_b < \psi(\lambda) < (1-\theta)c$ and $V \geq p + u_o + (1-\theta)c - \psi(\lambda)$, or $\psi(\lambda) \geq (1-\theta)c$. Substituting $\psi(\lambda)$ into these inequalities and solve them

for $d_{o,1}$, we can obtain $d_{o,1} \geq \frac{1}{1-\theta} + \frac{\theta a}{(1-\theta)(V-p-u_o-(1-\theta)c-a\underline{\lambda})}$. In this case, $\Pi_o = \pi_{o,1}(d_{o,1}) + \pi^{\dagger}_{o,2} = pn(1-(1-\theta)d_{o,1}) + pn$.

Since $d_{o,1} \geq \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}$, the above three cases may or may not happen depending on their corresponding conditions of $d_{o,1}$. Specifically,

(1) if $\frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)} \leq \frac{1}{1-\theta} + \frac{\theta a}{(1-\theta)(V-p-u_o-a\underline{\lambda})}$, which can be rewritten as $V \geq p + u_o + (1-\theta)c - \frac{c(a(\underline{\lambda}-\theta)-(1-\theta)c)}{a-c}$, the optimal choice of the retailer is among the following three:

(i) choose a $d_{o,1} \in [\frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}, \frac{1}{1-\theta} + \frac{\theta a}{(1-\theta)(V-p-u_o-a\underline{\lambda})})$ which satisfies the condition in the above case 1, with a profit $\Pi_o = pn(1-(1-\theta)d_{o,1})$. One can see that Π_o decreases in $d_{o,1}$, so it is optimal for the retailer to choose $d^{\dagger}_{o,1} = \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}$ to maximize her profit. In this case, $\Pi^{\dagger}_o = \frac{\theta npc}{p-V+u_o+c}$.

(ii) choose a $d_{o,1} \in [\frac{1}{1-\theta} + \frac{\theta a}{(1-\theta)(V-p-u_o-a\underline{\lambda})}, \frac{1}{1-\theta} + \frac{\theta a}{(1-\theta)(V-p-u_o-(1-\theta)c-a\underline{\lambda})})$ which satisfies the condition in the above case 2, with a profit $\Pi_o = pn(1-(1-\theta)d_{o,1}) + \frac{(1-(1-\theta)d_{o,1})\theta npc}{(1-(1-\theta)d_{o,1})(a\underline{\lambda}+p-V+u_o+c)-\theta a}$. According to Assumption 4, we can prove that Π_o increases in $d_{o,1}$, so it is optimal for the retailer to choose $d^{\dagger}_{o,1} = \frac{1}{1-\theta} + \frac{\theta a}{(1-\theta)(V-p-u_o-(1-\theta)c-a\underline{\lambda})}$ to maximize her profit. In this case, $\Pi^{\dagger}_o = \frac{\theta npa}{p-V+u_o+(1-\theta)c+a\underline{\lambda}} + pn$.

(iii) choose a $d_{o,1} \in [\frac{1}{1-\theta} + \frac{\theta a}{(1-\theta)(V-p-u_o-(1-\theta)c-a\underline{\lambda})}, 1]$ which satisfies the condition in the above case 3, with a profit $\Pi_o = pn(1-(1-\theta)d_{o,1}) + pn$. One can see that Π_o decreases in $d_{o,1}$, so it is optimal for the retailer to choose $d^{\dagger}_{o,1} = \frac{1}{1-\theta} + \frac{\theta a}{(1-\theta)(V-p-u_o-(1-\theta)c-a\underline{\lambda})}$ to maximize her profit. In this case, $\Pi^{\dagger}_o = \frac{\theta npa}{p-V+u_o+(1-\theta)c+a\underline{\lambda}} + pn$.

Comparing the above optimal profits $\Pi^{\dagger}_o$ of the retailer under the above 3 choices, we can prove that $\frac{\theta npa}{p-V+u_o+(1-\theta)c+a\underline{\lambda}} + np > \frac{\theta npc}{p-V+u_o+c}$ according to Assumption 4. Thus choice(iii) is optimal for the retailer, that is $d^{\dagger}_{o,1} = \frac{1}{1-\theta} + \frac{\theta a}{(1-\theta)(V-p-u_o-(1-\theta)c-a\underline{\lambda})}$ and $\Pi^{\dagger}_o = \frac{\theta npa}{p-V+u_o+(1-\theta)c+a\underline{\lambda}} + pn$.

Recall that the optimal profit in the above case that $d_{o,1} < \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}$ is $\Pi^{\dagger}_o = \frac{\theta npc}{p-V+u_o+c}$. According to Assumption 4, we can prove that $\frac{\theta npa}{p-V+u_o+(1-\theta)c+a\underline{\lambda}} + pn > \frac{\theta npc}{p-V+u_o+c}$. Therefore, the case that $d_{o,1} < \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}$ is always dominated by the current case and thus omitted.

(2) if $\frac{1}{1-\theta} + \frac{\theta a}{(1-\theta)(V-p-u_o-a\underline{\lambda})} < \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)} \leq \frac{1}{1-\theta} + \frac{\theta a}{(1-\theta)(V-p-u_o-(1-\theta)c-a\underline{\lambda})}$, which can be rewritten as $p + u_o + (1-\theta)c - \frac{ca(\underline{\lambda}-\theta)}{a-c} \leq V < p + u_o + (1-\theta)c - \frac{c(a(\underline{\lambda}-\theta)-(1-\theta)c)}{a-c}$, the optimal choice of the retailer is among the following two:

(i) choose a $d_{o,1} \in [\frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}, \frac{1}{1-\theta} + \frac{\theta a}{(1-\theta)(V-p-u_o-(1-\theta)c-a\underline{\lambda})})$ which satisfies the condition in the above case 2, with a profit $\Pi_o = pn(1-(1-\theta)d_{o,1}) + \frac{(1-(1-\theta)d_{o,1})\theta npc}{(1-(1-\theta)d_{o,1})(a\underline{\lambda}+p-V+u_o+c)-\theta a}$. According to Assumption 4, we can prove that Π_o increases in $d_{o,1}$, so it is optimal for the retailer to choose $d^{\dagger}_{o,1} = \frac{1}{1-\theta} + \frac{\theta a}{(1-\theta)(V-p-u_o-(1-\theta)c-a\underline{\lambda})}$ to maximize her profit. In this case, $\Pi^{\dagger}_o = \frac{\theta npa}{p-V+u_o+(1-\theta)c+a\underline{\lambda}} + pn$.

(ii) choose a $d_{o,1} \in [\frac{1}{1-\theta} + \frac{\theta a}{(1-\theta)(V-p-u_o-(1-\theta)c-a\underline{\lambda})}, 1]$ which satisfies the condition in the above case 3, with a profit $\Pi_o = pn(1-(1-\theta)d_{o,1}) + pn$. One can see that Π_o decreases in $d_{o,1}$, so it is optimal for the retailer to choose $d^{\dagger}_{o,1} = \frac{1}{1-\theta} + \frac{\theta a}{(1-\theta)(V-p-u_o-(1-\theta)c-a\underline{\lambda})}$ to maximize her profit. In this case, $\Pi^{\dagger}_o = \frac{\theta npa}{p-V+u_o+(1-\theta)c+a\underline{\lambda}} + pn$.

Note that the optimal description level $d^{\dagger}_{o,1}$ and profit $\Pi^{\dagger}_o$ are the same under the above two choices, that is $d^{\dagger}_{o,1} = \frac{1}{1-\theta} + \frac{\theta a}{(1-\theta)(V-p-u_o-(1-\theta)c-a\underline{\lambda})}$ and $\Pi^{\dagger}_o = \frac{\theta npa}{p-V+u_o+(1-\theta)c+a\underline{\lambda}} + pn$.

(3) if $\frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)} > \frac{1}{1-\theta} + \frac{\theta a}{(1-\theta)(V-p-u_o-(1-\theta)c-a\underline{\lambda})}$, which can be rewritten as $V < p + u_o + (1-\theta)c - \frac{ca(\underline{\lambda}-\theta)}{a-c}$, the optimal choice of the retailer is choosing a $d_{o,1} \in [\frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}, 1]$ which satisfies the condition in the above case 3, with a profit $\Pi_o = pn(1-(1-\theta)d_{o,1}) + pn$. One can see that Π_o decreases in $d_{o,1}$, so it is optimal for the retailer to choose $d^{\dagger}_{o,1} = \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}$ to maximize her profit. In this case, $\Pi^{\dagger}_o = \frac{\theta npc}{p-V+u_o+c} + pn$.

In summary, the retailer's optimal decision $d^{\dagger}_{o,1}$ and profit $\Pi^{\dagger}_o$ are as follows:

1. If $V \geq p+u_o+(1-\theta)c-\frac{ca(\underline{\lambda}-\theta)}{a-c} = \bar{V}_1$, then $d^{\dagger}_{o,1} = \frac{1}{1-\theta}+\frac{\theta a}{(1-\theta)(V-p-u_o-(1-\theta)c-a\underline{\lambda})}$ and $\Pi^{\dagger}_o = \frac{\theta npa}{p-V+u_o+(1-\theta)c+a\underline{\lambda}} + pn$.
2. If $V < \bar{V}_1$, then $d^{\dagger}_{o,1} = \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}$ and $\Pi^{\dagger}_o = \frac{\theta npc}{p-V+u_o+c} + pn$.

Combining Assumption 6 and the conditions of V in that case, we can obtain the results in Lemma 3.6 accordingly. □

Proof of Theorem 3.7. According to Lemma 3.2, we can determine the optimal online total profit of the retailer who operates the offline and online channels separately over the two periods under the model without consumer reviews as $\Pi^*_o = \pi^*_{o,1} + \pi^*_{o,2} = \frac{2\theta npc}{p-V+u_o+c}$. Comparing it with $\Pi^{\dagger}_o$ in Lemma 3.6, we find that

1. if $\underline{\lambda} \geq \bar{\lambda}$ and $V \geq \bar{V}_1$, $\Pi^{\dagger}_o = \frac{\theta npa}{p-V+u_o+(1-\theta)c+a\underline{\lambda}}+pn$. By solving $\frac{2\theta npc}{p-V+u_o+c} < \frac{\theta npa}{p-V+u_o+(1-\theta)c+a\underline{\lambda}} + pn$ for V, we obtain that $V < p + u_o + (1-\theta)c -$

$\frac{\theta c - a(\theta+\underline{\lambda}) + \sqrt{(a(\theta+\underline{\lambda})-\theta c)^2 + 4\theta c a(\underline{\lambda}-\theta)}}{2} = \bar{V}_2$. Thus, $\Pi_o^* < \Pi_o^\dagger$ if $V < \bar{V}_2$, and $\Pi_o^* \geq \Pi_o^\dagger$ otherwise.

2. if $\underline{\lambda} \geq \bar{\lambda}$ and $V < \bar{V}_1$, or $\underline{\lambda} < \bar{\lambda}$, $\Pi_o^\dagger = \frac{\theta npc}{p-V+u_o+c} + pn$. According to Assumption 6, we have $\frac{\theta npc}{p-V+u_o+c} < pn$, and thus $\Pi_o^* < \Pi_o^\dagger$.

Note that $\bar{V}_2 > \bar{V}_1$, so Theorem 3.7 follows. □

Proof of Lemma 3.8. In the model with consumer reviews, the retailer decides a product description level $d_{omni,2}$ for both channels in period 2. The period 2 offline consumers will purchase the product if $E[U|s] \geq u_b$ and period 2 online consumers will purchase the product if $E[U|s] \geq u_o$, where $E[U|s]$ is derived in Equation (3.2). Similar to the proofs of Lemma 3.5, by solving these inequalities for V, we can determine the conditions under which different types of consumers will purchase the product, and obtain the following cases:

1. if $V \geq p + u_o + (1-\theta)c - \psi(\lambda)$, the retailer can attract the period 2 offline consumers with perceived signal $s = 0$ and period 2 online consumers with perceived signal $s = 0$ to purchase the product by choosing any $d_{omni,2}$. In this case, the retailer's profit is

$$\begin{aligned}\pi_{omni,2}(d_{omni,2}) &= p \times (n \times \phi_{b,2}(d_{omni,2}) + n \times \phi_{o,2}(d_{omni,2})) \\ &= p \times (n \times P\{s=0\} + n \times P\{s=0\}) \\ &= 2pn(1-(1-\theta)d_{omni,2}).\end{aligned}$$

   One can see that $\pi_{omni,2}(d_{omni,2})$ decreases in $d_{omni,2}$, so it is optimal for the retailer to choose $d^\dagger_{omni,2} = 0$ to maximize her profit. In this case, $\pi^\dagger_{omni,2} = 2pn$.

2. if $p + u_o - \psi(\lambda) \leq V < p + u_o + (1-\theta)c - \psi(\lambda)$, the optimal choice of the retailer is among the following two:

   (a) attract both the period 2 offline consumers with perceived signal $s = 0$ and the period 2 online consumers with perceived signal $s = 0$ to purchase the product by choosing a $d_{omni,2} \geq \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V+\psi(\lambda)-p-u_o-c)}$, and generate a profit

$$\begin{aligned}\pi_{omni,2}(d_{omni,2}) &= p \times (n \times \phi_{b,2}(d_{omni,2}) + n \times \phi_{o,2}(d_{omni,2})) \\ &= p \times (n \times P\{s=0\} + n \times P\{s=0\}) \\ &= 2pn(1-(1-\theta)d_{omni,2}).\end{aligned}$$

   One can see that $\pi_{omni,2}(d_{omni,2})$ decreases in $d_{omni,2}$, so it is optimal for the retailer to choose $d^\dagger_{omni,2} = \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V+\psi(\lambda)-p-u_o-c)}$ to maximize her profit. In this case, $\pi^\dagger_{omni,2} = \frac{2\theta npc}{p-V+u_o+c-\psi(\lambda)}$.

   (b) attract the period 2 offline consumers with perceived signal $s = 0$ to purchase the product by choosing a $d_{omni,2} \in [\frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V+\psi(\lambda)-p-u_b-c)}, \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V+\psi(\lambda)-p-u_o-c)})$, and generate a profit

$$\begin{aligned}\pi_{omni,2}(d_{omni,2}) &= p \times (n \times \phi_{b,2}(d_{omni,2}) + n \times \phi_{o,2}(d_{omni,2})) \\ &= p \times n \times P\{s = 0\} \\ &= pn(1 - (1-\theta)d_{omni,2}).\end{aligned}$$

One can see that $\pi_{omni,2}(d_{omni,2})$ decreases in $d_{omni,2}$, so it is optimal for the retailer to choose $d^{\dagger}_{omni,2} = \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V+\psi(\lambda)-p-u_b-c)}$ to maximize her profit. In this case, $\pi^{\dagger}_{omni,2} = \frac{\theta npc}{p-V+u_b+c-\psi(\lambda)}$.

Comparing the optimal profits of the retailer under choices (a) and (b), we can prove that $\frac{2\theta npc}{p-V+u_o+c-\psi(\lambda)} > \frac{\theta npc}{p-V+u_b+c-\psi(\lambda)}$ according to assumptions 6 and 3. Thus choice (a) is optimal for the retailer in this case, that is $d^{\dagger}_{omni,2} = \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V+\psi(\lambda)-p-u_o-c)}$ and $\pi^{\dagger}_{omni,2} = \frac{2\theta npc}{p-V+u_o+c-\psi(\lambda)}$.

3. if $p + u_b - \psi(\lambda) \le V < p + u_o - \psi(\lambda)$, the optimal choice of the retailer is attracting the period 2 offline consumers with perceived signal $s = 0$ to purchase the product by choosing a $d_{omni,2} \in [\frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V+\psi(\lambda)-p-u_b-c)}, \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V+\psi(\lambda)-p-u_o-c)})$, and generate a profit

$$\begin{aligned}\pi_{omni,2}(d_{omni,2}) &= p \times (n \times \phi_{b,2}(d_{omni,2}) + n \times \phi_{o,2}(d_{omni,2})) \\ &= p \times n \times P\{s = 0\} \\ &= pn(1 - (1-\theta)d_{omni,2}).\end{aligned}$$

   One can see that $\pi_{omni,2}(d_{omni,2})$ decreases in $d_{omni,2}$, so it is optimal for the retailer to choose $d^{\dagger}_{omni,2} = \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V+\psi(\lambda)-p-u_b-c)}$ to maximize her profit. In this case, $\pi^{\dagger}_{omni,2} = \frac{\theta npc}{p-V+u_b+c-\psi(\lambda)}$.

4. if $V < p + u_b - \psi(\lambda)$, the retailer can never attract any period 2 consumer to purchase the product which means $\pi_{omni,2}(d_{omni,2}) = 0$. Thus, it's optimal for the retailer to choose $d^{\dagger}_{omni,2} = 0$ with $\pi^{\dagger}_{omni,2} = 0$.

Combining Assumption 6 and the conditions of V in the above 4 cases, we can obtain the results in Lemma 3.8 accordingly. □

Proof of Lemma 3.9. In the model with consumer reviews, the retailer decides a product description level $d_{omni,1}$ for both channels in period 1. The period 1 offline consumers will purchase the product if $E[U|s] \ge u_b$ and the period 1 online consumers will purchase the product if $E[U|s] \ge u_o$. Similar to the proofs of Lemma 3.6, by solving these inequalities for $d_{omni,1}$, we can determine the conditions under which different types of consumers will purchase the product, and obtain that the optimal choice of the retailer is among the following three:

1. attract both the period 1 online consumers with perceived signal $s = 0$ and the period 1 offline consumers with perceived signal $s = 0$ to purchase the product by choosing a $d_{omni,1} \ge \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}$. Similar to the proof of Lemma 3.6, by considering the expected impact of consumer reviews on period 2 consumers and the optimal profit $\pi^{\dagger}_{omni,2}$ the retailer can generate in period 2, we can determine the retailer's optimal decision and profit in period 1 under this choice as follows:

(a) If $V \geq p + u_o + (1-\theta)c - \frac{ca(\underline{\lambda}-\theta)}{a-c} = \bar{V}_1$, then $d^{\dagger}_{omni,1} = \frac{1}{1-\theta} + \frac{\theta a}{(1-\theta)(V-p-u_o-(1-\theta)c-a\underline{\lambda})}$ and $\Pi^{\dagger}_{omni} = \frac{2\theta npa}{p-V+u_o+(1-\theta)c+a\underline{\lambda}} + 2pn$.

(b) If $V < \bar{V}_1$, then $d^{\dagger}_{omni,1} = \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}$ and $\Pi^{\dagger}_{omni} = \frac{2\theta npc}{p-V+u_o+c} + 2pn$.

2. attract the period 1 offline consumers with perceived signal $s = 0$ to purchase the product by choosing a $d_{omni,1} \in [\frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_b-c)}, \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)})$. Similar to the proof of Lemma 3.6, by considering the expected impact of consumer reviews on period 2 consumers and the optimal profit $\pi^{\dagger}_{omni,2}$ the retailer can generate in period 2, we can determine the retailer's optimal decision and profit in period 1 under this choice. It turns out that the optimal profits under this choice are no larger than the profits under the above choice 1, which means that this choice 2 is dominated by the above choice 1, and thus not optimal.

3. not attract period 1 consumers to purchase the product by choosing a $d_{omni,1} < \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_b-c)}$, with a profit $\pi_{omni,1}(d_{omni,1}) = 0$ in period 1. Under this choice, no consumer review is generated so the expected impact of consumer reviews on period 2 consumers is $\psi(\lambda) = 0$. Thus, in period 2, the consumers have the same expected utilities as that in the model without consumer reviews, and the retailer will make the same optimal decisions as that in the model without consumer reviews. According to Lemma 3.3, it is optimal for the retailer to choose $d^{\dagger}_{omni,2} = \frac{1}{1-\theta} + \frac{\theta c}{(1-\theta)(V-p-u_o-c)}$ in period 2 with a profit $\pi^{\dagger}_{omni,2} = \frac{2\theta npc}{p-V+u_o+c}$. Therefore, $\Pi_{omni} = \pi_{omni,1}(d_{omni,1}) + \pi^{\dagger}_{omni,2} = \frac{2\theta npc}{p-V+u_o+c}$, which is independent of $d_{o,1}$. Thus, it's optimal for the retailer to choose $d^{\dagger}_{omni,1} = 0$ with $\Pi^{\dagger}_{omni} = \frac{2\theta npc}{p-V+u_o+c}$. It is easy to see that this profit is less than the optimal profits of the retailer under choice 1, which means that this choice 3 is dominated by the above choice 1 and thus not optimal.

In conclusion, the results under the above choice 1 are optimal. Combining Assumption 6 and the conditions of V in that choice, we can obtain the results in Lemma 3.9 accordingly. □

Proof of Corollary 3.10. The proof is similar to the proof of Theorem 3.7 and thus omitted. □

Proof of Theorem 3.11. According to Lemmas 3.1 and 3.6, we can determine the optimal total profit of the retailer who operates the offline and online channels separately over 2 periods $\Pi^{\dagger}_{dual} = \pi^{*}_{b,1} + \pi^{*}_{b,2} + \Pi^{\dagger}_{o}$ as follows.

1. $\Pi^{\dagger}_{dual} = \frac{\theta npc}{p-V+u_o+c} + np$ if $\bar{d} \leq \hat{d}$, $\underline{\lambda} > \hat{\lambda}$, and $V < \bar{V}_1$; or $\bar{d} \leq \hat{d}$, $\underline{\lambda} \leq \hat{\lambda}$, and $V < \hat{V}$.

2. $\Pi_{dual}^{\dagger} = \frac{\theta npa}{a\underline{\lambda}+p-V+u_o+(1-\theta)c} + np$ if $\bar{d} \leq \hat{d}$, $\underline{\lambda} > \hat{\lambda}$, and $\bar{V}_1 \leq V < \hat{V}$.

3. $\Pi_{dual}^{\dagger} = \frac{2\theta npc}{p-V+u_b+c} + \frac{\theta npa}{a\underline{\lambda}+p-V+u_o+(1-\theta)c} + np$ if $\bar{d} \leq \hat{d}$, $\underline{\lambda} > \hat{\lambda}$, and $V \geq \hat{V}$; or $\bar{d} \leq \hat{d}$, $\bar{\lambda} \leq \underline{\lambda} \leq \hat{\lambda}$, and $V \geq \bar{V}_1$; or $\bar{d} > \hat{d}$, $\underline{\lambda} > \bar{\lambda}$, and $V \geq \bar{V}_1$.

4. $\Pi_{dual}^{\dagger} = \frac{2\theta npc}{p-V+u_b+c} + \frac{\theta npc}{p-V+u_o+c} + np$ if $\bar{d} \leq \hat{d}$, $\bar{\lambda} \leq \underline{\lambda} \leq \hat{\lambda}$, and $\hat{V} \leq V < \bar{V}_1$; or $\bar{d} \leq \hat{d}$, $\underline{\lambda} \leq \bar{\lambda}$, and $V \geq \hat{V}$; or $\bar{d} > \hat{d}$, $\underline{\lambda} > \bar{\lambda}$, and $V < \bar{V}_1$; or $\bar{d} > \hat{d}$ and $\underline{\lambda} \leq \bar{\lambda}$.

Similar to the proof of Theorem 3.4, by comparing $\Pi_{dual}^{\dagger}$ with $\Pi_{omni}^{\dagger}$ in Lemma 3.9 in each cases, we can obtain the results in Theorem 3.11. □

Proof of Corollary 3.12. The proof is similar to the proof of Lemmas 3.1 and 3.2, and thus omitted. □

Proof of Corollary 3.13. The proof is similar to the proof of Theorem 3.4 and thus omitted. □

Appendix C

Proofs of Chapter 4

We make the following assumptions in the chapter:

Assumption 5. $t \leq \min\{V - p_c, V - p_{s,m}, V - p_{s,c}\}$.

Assumption 6. $V_H - V \in [t, 2t]$.

Assumption 7. $p_c > \bar{p} \equiv \max\{(1 + \frac{2}{\theta})t, 2(V_H - V) - t, 3t - \theta(V_H - V), V_H - V + \frac{3t}{2\theta - 1}, V_H - V + \frac{[(1-\alpha_m)+(1-\theta)(1-\alpha_c)]t}{\theta(1-\alpha_m)}\}$.

Proof of Lemma 4.1. If the seller only sells her product through the marketplace c, according to Equations (4.1) and (4.2), we can obtain a high-type consumer's utility of purchasing a product from the seller and the marketplace c are as follows.

$$u_{s,c} = V - tx - p_{s,c}$$
$$u_c = V_H - tx - p_c$$

A low-type consumer's utility of purchasing a product from the seller and the marketplace c are as follows.

$$u_{s,c} = V - tx - p_{s,c}$$
$$u_c = V - tx - p_c$$

According to the above equations, we can determine the conditions under which a high-type consumer at location x will purchase a product: If $p_{s,c} \geq p_c + V - V_H$, then the consumer purchases the product of the marketplace c; If $p_{s,c} < p_c + V - V_H$, then the consumer purchases the product of the seller. The conditions under which a low-type consumer at location x will purchase a product are: If $p_{s,c} \geq p_c$, then the consumer purchases the product of the marketplace c; If $p_{s,c} < p_c$, then the consumer purchases the product of the seller. The seller has the following options to decide the price of her product.

1. The seller chooses a price $p_{s,c} \geq p_c$, under which all the consumers will purchase the product of the marketplace c. The seller's expected profit is $\pi_{s,c} = 0$.

2. The seller chooses a price $p_{s,c} \in [p_c + V - V_H, p_c)$, under which the high-type consumers will purchase the product of the marketplace c, and the low-type consumers will purchase the product of the seller. The seller's expected profit is $\pi_{s,c} = (1-\theta)(1-\alpha)np_{s,c}$, which is linearly increasing in $p_{s,c}$. To maximize her profit, the seller will choose $p_{s,c} = p_c$, with an expected profit $\pi_{s,c} = (1-\theta)(1-\alpha)np_c$.

3. The seller chooses a price $p_{s,c} < p_c + V - V_H$, under which all the consumers will purchase the product of the seller. The seller's expected profit is $\pi_{s,c} = (1-\alpha)np_{s,c}$, which is linearly increasing in $p_{s,c}$. To maximize her profit, the seller will choose $p_{s,c} = p_c + V - V_H$, with an expected profit $\pi_{s,c} = (1-\alpha)n(p_c + V - V_H)$.

The seller will set the price to maximize her profit by comparing the profits in the above options, and Lemma 4.1 follows. □

Proof of Lemma 4.2. If the seller only sells her product through the marketplace m, according to Equations (4.1) and (4.2), we can obtain a high-type consumer's utility of purchasing a product from the seller and the marketplace c are as follows.

$$u_{s,m} = V - t(1-x) - p_{s,m}$$
$$u_c = V_H - tx - p_c$$

A low-type consumer's utility of purchasing a product from the seller and the marketplace c are as follows.

$$u_{s,m} = V - t(1-x) - p_{s,m}$$
$$u_c = V - tx - p_c$$

According to the above equations, we can determine the conditions under which a high-type consumer at location x will purchase a product: If $x \le \frac{1}{2} - \frac{p_c - p_{s,m} + V - V_H}{2t}$, then the consumer purchases the product of the marketplace c; If $x > \frac{1}{2} - \frac{p_c - p_{s,m} + V - V_H}{2t}$, then the consumer purchases the product of the seller. The conditions under which a low-type consumer at location x will purchase a product are: If $x \le \frac{1}{2} - \frac{p_c - p_{s,m}}{2t}$, then the consumer purchases the product of the marketplace c; If $x > \frac{1}{2} - \frac{p_c - p_{s,m}}{2t}$, then the consumer purchases the product of the seller. The seller has the following options to decide the price of her product. In summary, the seller's demand is $D_{s,m} = \theta n[1 - \mathbf{F}(\max\{0, \min\{1, \frac{1}{2} - \frac{p_c - p_{s,m} + V - V_H}{2t}\}\})] + (1-\theta)n[1 - \mathbf{F}(\max\{0, \min\{1, \frac{1}{2} - \frac{p_c - p_{s,m}}{2t}\}\})]$, where $\mathbf{F}(\cdot)$ is the cumulative distribution function of the distribution of x. The seller has the following options to decide the price of her product.

1. The seller chooses a price $p_{s,m} > p_c + t$, under which $D_{s,m} = 0$ and $\pi_{s,m} = 0$.

2. The seller chooses a price $p_{s,m} \in (p_c + t + V - V_H, p_c + t]$, under which $D_{s,m} = \frac{(1-\theta)n}{2t}(p_c - p_{s,m} + t)$ and $\pi_{s,m} = \frac{(1-\theta)(1-\alpha_m)n}{2t} p_{s,m}(p_c - p_{s,m} + t)$, which is concave in $p_{s,m}$. According to Assumption 7, to maximize her profit, the seller will choose $p^*_{s,m} = p_c + t + V - V_H$, with an expected profit $\pi_{s,m} = \frac{(1-\theta)(1-\alpha_m)n}{2t}(V_H - V)(p_c + t + V - V_H)$.

3. The seller chooses a price $p_{s,m} \in (p_c - t, p_c + t + V - V_H]$, under which $D_{s,m} = \frac{\theta n}{2t}(p_c + V - V_H + t - p_{s,m}) + \frac{(1-\theta)n}{2t}(p_c - p_{s,m} + t)$ and $\pi_{s,m} = \frac{\theta(1-\alpha_m)n}{2t}(p_c + V - V_H + t - p_{s,m})p_{s,m} + \frac{(1-\theta)(1-\alpha_m)n}{2t} p_{s,m}(p_c - p_{s,m} + t)$, which is concave in $p_{s,m}$. According to Assumption 7, to maximize her profit, the seller will choose $p^*_{s,m} = p_c - t$, with an expected profit $\pi_{s,m} = (1 - \alpha_m)n(p_c - t)(1 - \frac{\theta(V_H - V)}{2t})$.

4. The seller chooses a price $p_{s,m} \in (p_c - t + V - V_H, p_c - t]$, under which $D_{s,m} = \frac{\theta n}{2t}(p_c + V - V_H + t - p_{s,m}) + (1 - \theta)n$ and $\pi_{s,m} = \frac{\theta(1-\alpha_m)n}{2t}(p_c + V - V_H + t - p_{s,m})p_{s,m} + (1 - \theta)(1 - \alpha_m)np_{s,m}$, which is concave in $p_{s,m}$. According to Assumption 7, to maximize her profit, the seller will choose $p^*_{s,m} = p_c - t + V - V_H$, with an expected profit $\pi_{s,m} = (1 - \alpha_m)n(p_c + V - V_H - t)$ if $\theta > 0.5$, or choose $p^*_{s,m} = p_c - t$, with an expected profit $\pi_{s,m} = (1 - \alpha_m)n(p_c - t)[1 - \frac{\theta}{2t}(V_H - V)]$ if $\theta \leq 0.5$.

5. The seller chooses a price $p_{s,m} \leq p_c - t + V - V_H$, under which $D_{s,m} = n$ and $\pi_{s,m} = (1 - \alpha_m)np_{s,m}$, which is linearly increasing in $p_{s,m}$. To maximize her profit, the seller will choose $p^*_{s,m} = p_c - t + V - V_H$ with an expected profit $\pi_{s,m} = (1 - \alpha_m)n(p_c - t + V - V_H)$.

The seller will set the price to maximize her profit by comparing the profits in the above options, and Lemma 4.2 follows. □

Proof of Lemma 4.3. If the seller sells her product through both marketplaces c and m, according to Equations (4.1) and (4.2), we can obtain a high-type consumer's utility of purchasing a product from the seller and the marketplace c are as follows.

$$
\begin{aligned}
u_{s,c} &= V - tx - p_s \\
u_{s,m} &= V - t(1 - x) - p_s \\
u_c &= V_H - tx - p_c
\end{aligned}
$$

A low-type consumer's utility of purchasing a product from the seller and the marketplace c are as follows.

$$
\begin{aligned}
u_{s,c} &= V - tx - p_s \\
u_{s,m} &= V - t(1 - x) - p_s \\
u_c &= V - tx - p_c
\end{aligned}
$$

According to the above equations, we can determine the conditions under which a high-type consumer at location x will purchase a product: If $p_s \geq p_c + V - V_H$ and $p_s \geq p_c + V - V_H - t(1-2x)$, then the consumer purchases the product of the marketplace c; If $p_s < p_c + V - V_H$ and $t(1-2x) \geq 0$, then the consumer purchases the product of the seller from the marketplace c; If $p_s < p_c + V - V_H - t(1-2x)$ and $t(1-2x) < 0$, then the consumer purchases the product of the seller from the marketplace m. The conditions under which a low-type consumer at location x will purchase a product are: If $p_s \geq p_c$ and $p_s \geq p_c - t(1-2x)$, then the consumer purchases the product of the marketplace c; If $p_s < p_c$ and $t(1-2x) \geq 0$, then the consumer purchases the product of the seller from the marketplace c; If $p_s < p_c - t(1-2x)$ and $t(1-2x) < 0$, then the consumer purchases the product of the seller from the marketplace m. The seller has the following options to decide the price of her product.

1. The seller chooses a price $p_s \geq p_c$, under which the high-type consumers will purchase the product of the seller from the marketplace m if $x > \frac{1}{2} - \frac{p_c - p_s + V - V_H}{2t}$ or purchase the product of the marketplace c if $x \leq \frac{1}{2} - \frac{p_c - p_s + V - V_H}{2t}$, and the low-type consumers will purchase the product of the seller from the marketplace m if $x > \frac{1}{2} - \frac{p_c - p_s}{2t}$ or purchase the product of the marketplace c if $x \leq \frac{1}{2} - \frac{p_c - p_s}{2t}$. Thus, the seller's demand is $D_s = \theta n[1 - \mathbf{F}(\max\{0, \min\{1, \frac{1}{2} - \frac{p_c - p_s + V - V_H}{2t}\}\})] + (1-\theta)n[1 - \mathbf{F}(\max\{0, \min\{1, \frac{1}{2} - \frac{p_c - p_s}{2t}\}\})]$.

2. The seller chooses a price $p_s \in [p_c + V - V_H, p_c)$, under which the high-type consumers will purchase the product of the seller from the marketplace m if $x > \frac{1}{2} - \frac{p_c - p_s + V - V_H}{2t}$ or purchase the product of the marketplace c if $x \leq \frac{1}{2} - \frac{p_c - p_s + V - V_H}{2t}$, and the low-type consumers will purchase the product of the seller from the marketplace m if $x > \frac{1}{2}$ or purchase the product of the seller from the marketplace c if $x \leq \frac{1}{2}$. Thus, the seller's demand is $D_s = \theta n[1 - \mathbf{F}(\max\{0, \min\{1, \frac{1}{2} - \frac{p_c - p_s + V - V_H}{2t}\}\})] + \frac{(1-\theta)n}{2} + \frac{(1-\theta)n}{2}$.

3. The seller chooses a price $p_s < p_c + V - V_H$, under which the high-type consumers will purchase the product of the seller from the marketplace m if $x > \frac{1}{2}$ or purchase the product of the seller from the marketplace c if $x \leq \frac{1}{2}$, and the low-type consumers will purchase the product of the seller from the marketplace m if $x > \frac{1}{2}$ or purchase the product of the seller from the marketplace c if $x \leq \frac{1}{2}$. Thus, the seller's demand is $D_s = \frac{n}{2} + \frac{n}{2}$.

More specifically, the seller can choose from the following options to decide her price to maximize the profit.

1. The seller chooses a price $p_s \geq p_c + t$, under which the seller's demand is $D_s = 0$ and $\pi_s = 0$.

2. The seller chooses a price $p_s \in [p_c, p_c + t)$, under which the seller's demand is $D_s = \frac{(1-\theta)n}{2t}(p_c - p_s + t)$ and $\pi_s = \frac{(1-\theta)(1-\alpha_m)n}{2t} p_s(p_c - p_s + t)$, which

is concave in p_s. According to Assumption 7, to maximize her profit, the seller will choose $p_s = p_c$, with an expected profit $\pi_s = \frac{(1-\theta)(1-\alpha_m)np_c}{2}$.

3. The seller chooses a price $p_s \in [p_c+t+V-V_H, p_c)$, under which the seller's demand is $D_s = \frac{(1-\theta)n}{2} + \frac{(1-\theta)n}{2}$ and $\pi_s = \frac{(1-\theta)(1-\alpha_m)np_s}{2} + \frac{(1-\theta)(1-\alpha_c)np_s}{2}$, which is linearly increasing in p_s. To maximize her profit, the seller will choose $p_s = p_c$ with an expected profit $\pi_s = \frac{(1-\theta)(1-\alpha_m)np_c}{2} + \frac{(1-\theta)(1-\alpha_c)np_c}{2}$.

4. The seller chooses a price $p_s \in [p_c + V - V_H, p_c + t + V - V_H)$, under which the seller's demand is $D_s = \frac{\theta n}{2t}(p_c + V - V_H + t - p_s) + \frac{(1-\theta)n}{2} + \frac{(1-\theta)n}{2}$ and $\pi_s = \frac{\theta(1-\alpha_m)n}{2t}(p_c + V - V_H + t - p_s)p_s + \frac{(1-\theta)(1-\alpha_m)np_s}{2} + \frac{(1-\theta)(1-\alpha_c)np_s}{2}$, which is concave in p_s. According to Assumption 7, to maximize her profit, the seller will choose $p_s = p_c + V - V_H$, with an expected profit $\pi_s = \frac{n(p_c+V-V_H)[1-\alpha_m+(1-\theta)(1-\alpha_c)]}{2}$.

5. The seller chooses a price $p_s \le p_c+V-V_H$, under which the seller's demand is $D_s = \frac{n}{2} + \frac{n}{2}$ and $\pi_s = \frac{(1-\alpha_m)np_s}{2} + \frac{(1-\alpha_c)np_s}{2}$, which is linearly increasing in p_s. To maximize her profit, the seller will choose $p_s = p_c + V - V_H$, with an expected profit $\pi_s = \frac{(1-\alpha_m)n(p_c+V-V_H)}{2} + \frac{(1-\alpha_c)n(p_c+V-V_H)}{2}$.

The seller will set the price to maximize her profit by comparing the profits in the above five options, and Lemma 4.3 follows. □

Proof of Theorem 4.4. The seller will decide to sell her product through the marketplace(s) that can maximize her profit, by comparing the profits in Lemmas 4.1, 4.2, and 4.3. □

Proof of Theorem 4.5. According to Theorem 4.4 and Assumption 7, the marketplace c can choose from the following options to decide its product price p_c to maximize its profit.

1. If $\alpha_c \le \alpha_m$, the marketplace c can set any price $p_c > (1+\frac{2}{\theta})t$. In this case, the seller only sells through the marketplace c and $p^*_{s,c} = p_c + V - V_H$. The marketplace c has $D_c = \frac{\theta n}{2}$ and $\pi_c = \alpha_c n(p_c + V - V_H) + \frac{\theta np_c}{2}$, which is linearly increasing in p_c. To maximize its profit, the marketplace c will choose the highest p^*_c.

2. If $\alpha_c > \alpha_m$, the marketplace c can set any price $p_c \le V_H - V + \frac{2(1-\alpha_m)t}{\alpha_c-\alpha_m}$. In this case, the seller sells through both marketplaces and $p^*_s = p_c + V - V_H$. The marketplace c has $D_c = \frac{\theta n}{2}$ and $\pi_c = \frac{\alpha_c n(p_c+V-V_H)}{2} + \frac{\theta np_c}{2}$, which is linearly increasing in p_c. To maximize its profit, the marketplace c will choose $p^*_c = V_H - V + \frac{2(1-\alpha_m)t}{\alpha_c-\alpha_m}$.

□

Proof of Corollary 4.6. The proof of this theorem is similar to the proof of Theorem 4.5, and thus omitted here. □

Proof of Theorem 4.7. Define $\bar{\alpha}_1 = \frac{[2t-\theta(V_H-V)]^2}{2(1-\theta)t(3t-\theta(V_H-V))}$, $\bar{\alpha}_2 = \frac{4t-(1+\theta)(V_H-V)}{3t-\theta(V_H-V)}$, $\bar{\alpha}_3 = \frac{2t+V-V_H}{(1-\theta)(3t-\theta(V_H-V))}$. The proof of this theorem is similar to the proof of Theorem 4.5, and thus omitted here. □

Bibliography

Abhishek V, Guajardo JA, Zhang Z (2016) Business models in the sharing economy: Manufacturing durable goods in the presence of peer-to-peer rental markets. Working paper, Carnegie Mellon University.

Agarwal R, Ergun Ö (2010) Network design and allocation mechanisms for carrier alliances in liner shipping. *Oper. Res.* **58**(6):1726–1742.

Allen J, Browne M, Woodburn A, Leonardi J (2012) The role of urban consolidation centres in sustainable freight transport. *Transport Rev.* **32**(4):473–490.

Ambrosini C, Routhier J (2004) Objectives, methods and results of surveys carried out in the field of urban freight transport: An international comparison. *Transport Rev.* **24**(1):57–77.

Bai J, So KC, Tang CS, Chen X, Wang H (2018) Coordinating supply and demand on an on-demand service platform with impatient customers. *Manufacturing Service Oper. Management*, forthcoming.

Balasubramanian S (1998) Mail versus Mall: A Strategic Analysis of Competition between Direct Marketers and Conventional Retailers. *Marketing Science* **17**(3):181–195.

Bell D, Gallino S, Moreno A (2014) How to win in an omnichannel world. *MIT Sloan Management Rev.* **56**(1):49–53.

Bell D, Gallino S, Moreno A (2017) Offline showrooms in omnichannel retail: Demand and operational benefits. *Management Science* **64**(4):1629–1651.

Benjaafar S, Bernhard H, Courcoubetis C (2017) Drivers, riders and service providers: The impact of the sharing economy on mobility. *Proceedings of the 12th workshop on the economics of networks, systems and computation*, 27 June 2017, pp.1-6.

Benjaafar S, Kong G, Li X, Courcoubetis C (2018) Peer-to-peer product sharing: Implications for ownership, usage and social welfare in the sharing economy. *Management Sci.* **65**(2):477–493.

Berger J, Heath C (2007) Where consumers diverge from others: Identity signaling and product domains. *Journal of Consumer Research* **34**(2):121–134.

Berman B, Thelen S (2004) A guide to developing and managing a well-integrated multi-channel retail strategy. *International Journal of Retail & Distributon Management* **32**(2):147–156.

Bernstein F, Song J, Zheng X (2008) Bricks-and-mortar vs. clicks-and-mortar: An equilibrium analysis. *European Journal of Operational Research* **187**(3):671–690.

Bickart B, Schindler RM (2001) Internet forums as influential sources of consumer information. *Journal of Interactive Marketing* **15**(3):31–40.

Bimpikis K, Candogan O, Saban D (2016) Spatial pricing in ride-sharing networks. Working paper, Stanford University.

Boudoin D, Morel C, Gardat M (2014) Supply chains and urban logistics platforms, in Gonzalez-Feliu J, Semet F, Routhier J-L (eds), *Sustainable Urban Logistics: Concepts, Methods and Information Systems*, EcoProduction, Springer Berlin Heidelberg, pp. 1–20.

Broner F, Martin A, Ventura J (2010) Sovereign risk and secondary markets. *Amer. Econom. Rev.* **100**(4):1523–1555.

Browne M, Sweet M, Woodburn A, Allen J (2005) Urban Freight Consolidation Centres Final Report. Transport Studies Group, University of Westminster, London.

Cachon GP, Swinney R (2009) Purchasing, pricing, and quick response in the presence of strategic consumers. *Management Sci.* **55**(3):497–511.

Cachon GP, Daniels KM, Lobel R (2017) The role of surge pricing on a service platform with self-scheduling capacity. *Manufacturing Service Oper. Management* **19**(3):368–384.

Cattani K, Gilland W, Heese HS, Swaminathan J (2006) Boiling frogs: Pricing sttrategies for a manufacturer adding a direct channel that competes with the traditional channel. *Production Oper. Management* **15**(1):40–56.

Chen K-Y, Kaya M, Özer Ö (2008) Dual sales channel management with service competition. *Manufacturing Service Oper. Management* **10**(4):654–675.

Chen J, Esteban S, Shum M (2013) When do secondary markets harm firms? *Amer. Econom. Rev.* **103**(7):2911–2934.

Chen Y (2017) Alibaba tests 60 futuristic pop-up stores across China for Singles Day. *Digiday* November 10, 2017.

Chen Y, Xie J (2008) Online consumer review: Word-of-mouth as a new element of marketing communication mix. *Management Science* **54**(3):477–491.

Chiang WyK, Chhajed D, Hess JD (2003) Direct marketing, indirect profits: A strategic analysis of dual-channel supply-chain design. *Management Science* **49**(1):1–20.

Choe T, Scott A, Mauricio G, Jon W (2017) The future of freight: How new technology and new thinking can transform how goods are moved. *Deloitte Insights*.

Chou C (2018) Maserati first automaker to tap Alibaba's smart-store tech. *ALZILA: News from Alibaba group* March 21, 2018.

Cisco Study (2013) Eight out of 10 consumers shop through bits and bytes. *Cisco Study* January 14, 2013.

Cohen M, Zhang R (2017) Coopetition and profit sharing for ride-sharing platforms. Working paper, New York University.

Crainic R, Ricciardi N, Storchi G (2009) Models for evaluating and planning city logistics systems. *Transportation Sci.* **43**(4):432–454.

Dablanc L (2005) French strategic approach to urban consolidation. *Bestufs II First Workshop 13th–14th January 2005*, London.

Dablanc L (2007) Goods transport in large European cities: Difficult to organize, difficult to modernize. *Transportation Res. Part A* **41**(3):280–285.

Dablanc L (2011) *City logistic best practices: A Handbook for Authorities*, SUGAR, Bologna, Italy.

Digimarc Corporation (2015) Survey: 75 Percent of U.S. Adults Want More Product Information And Crave Quick Access to Mobile Research When Shopping. *Digimarc Corporation* October 26, 2015.

Doig JW (2001) *Empire on the Hudson: Entrepreneurial Vision and Political Power at the Port of New York Authority.* Columbia University Press, New York.

Dong L, Rudi N (2004) Who benefits from transshipment? Exogenous vs. endogenous wholesale price. *Management Sci.* **50**:645–657.

Dudarenok AG (2018) China leaps into the brave new world of 'New Retail'. *Asia Times* February 5, 2018.

Forman C, Ghose A, Goldfarb A (2009) Competition between local and electronic markets: How the benefit of buying online depends on where you live. *Management Science* **55**(1):47–57.

Fraiberger S, Sundararajan A (2015) Peer-to-peer rental markets in the sharing economy. Working paper, New York University.

Gallino S, Moreno A (2014) Integration of online and offline channels in retail: The impact of sharing reliable inventory availability information. *Management Science* **60**(6):1434–1451.

Gallino S, Moreno A, Stamatopoulos I (2017) Channel integration, sales dispersion, and inventory management. *Management Science* **63**(9):2813–2831.

Gao F, Su X (2017) Omnichannel retail operations with buy-online-and-pick-up-in-store. *Management Science* **63**(8):2478–2492.

Gao F, Su X (2017) Online and offline information for omnichannel retailing. *Manufacturing Service Oper. Management* **19**(1):84–98.

Gao F, Cui S, Agrawal V (2018) The effect of multi-channel and omni-channel retailing on physical stores. Working paper, Indiana University Bloomington.

Gesing B (2017) Transport capacity sharing, in *Sharing Economy Logistics: Rethinking Logistics with Access over Ownership*, DHL Trend Research, pp. 21–22.

Google Data, U.S. Jan. – June 2015 vs. Jan. – June 2017. *think with Google*.

Greiger M (2003) Electronic marketplaces: A literature review and a call for supply chain management research. *Eur. J. Oper. Res.* **144**(2): 280-294.

Gu Z, Xie Y (2013) Facilitating fit revelation in the competitive market. *Management Science* **59**(5):1196–1212.

Hagiu A, Spulber D (2013) First-party content and coordination in two-sided markets. *Management Science.* **59**(4): 939-949.

Hagiu A, Wright J (2015) Marketplace or reseller? *Management Sci.* **61**(1):184–203.

Hagiu A, Jullien B, Wright J (2020) Creating platforms by hosting rivals.

Handoko D, Lau HC, Cheng SF (2016) Achieving economic and environmental sustainability in urban consolidation center with bi-criteria auction. *IEEE Trans. Automation Science and Engineering* **13**(4):1471–1479.

Harsha P, Subramanian S, Uichanco J (2019) Dynamic pricing of omnichannel inventories. *Manufacturing Service Oper. Management* **21**(1): 47–65.

Huang J, Chen Y (2006) Herding in online product choice. *Psychology & Marketing* **23**(5):413–428.

Holguin-Veras J, Silas M, Polimeni J (2008) An investigation into the attitudinal factors determining participation in cooperative multi-carrier delivery initiatives, in Taniguchi E, Thomson R (eds), *Innovations in City Logistics IV*, Nova Science Publishers, pp. 55–68.

Hu N, Pavlou PA, Zhang J (2006) Can online reviews reveal a product's true quality?: Empirical findings and analytical modeling of online word-of-mouth communication. *Proceedings of the 7th ACM conference on Electronic commerce* 324–330.

Hu M, Zhou Y (2017) Price, wage and fixed commission in on-demand matching. Working paper, University of Toronto.

Ifrach B, Maglaras C, Scarsini M (2015) Bayesian social learning from consumer reviews. Working paper, Stanford University.

Jiang B, Tian L (2016) Collaborative consumption: Strategic and economic implications of product sharing. *Management Sci.* **64**(3):1171–1188.

Klein L (1998) Evaluating the potential of interactive media through a new lens: Search versus experience goods. *Journal of Business Research* **41**(3):195–203.

Kwark Y, Chen J, Raghunathan S (2014) Online product reviews: Implications for retailers and competing manufacturers. *Information Systems Research* **25**(1):93–110.

Lawson M (2016) 5 Ways consumers connect to stores with mobile shopping. *think with Google.*

Lawton C (2008) The war on returns. *Wall Street Journal* (May 8), https://www.wsj.com/articles/SB121020824820975641.

Lee H, Whang S (2001) Winning the last mile of e-commerce. *MIT Sloan Management Rev.* **42**(4):54–62.

Lee H, Whang S (2002) The impact of the secondary market on the supply chain. *Management Sci.* **48**(6):719–731.

Lewis TR, Sappington DEM (1994) Supplying information to facilitate price discrimination. *International Economic Review* **35**(2):309–327.

Li X, Hitt LM (2008) Self-selection and information role of online product reviews. *Information Systems Research* **19**(4) 456–474.

Lim SFWT, Rabinovich E, Rogers DS, Laseter TM (2016) Last-mile supply network distribution in omnichannel retailing: A configuration-based typology. *Foundations and Trends in Technology, Information and Operations Management* **10**(1) 1–87.

Liu Y, Cooper WL, Wang Z (2019) Information provision and pricing in the presence of consumer search costs. *Production and Operations Management* **28**(7):1603–1620.

Liu Y, Feng J, Liao X (2017) When online reviews meet sales volume information: Is more or accurate information always better? *Information System Research* **28**(4):723–743.

Lopez E (2017) Why is the last mile so inefficient? *Supply Chain Dive.*

Mantin B, Krishnan H, Dhar T (2014) The strategic role of third-party marketplaces in retailing. *Production and Operations Management.* **23**(11): 1937-1949.

Mayzlin D, Shin Jiwoong (2011) Uninformative advertising as an invitation to search. *Marketing Science* **30**(4):666–685.

McDermott DR (1975) An alternative framework for urban goods distribution: Consolidation. *Transportation J.* **15**(1):29–39.

Mehra A, Kumar S, Raju JS (2017) Competitive strategies for brick-and-mortar stores to counter "showrooming". *Management Science* **64**(7):3076–3090.

Mendelson H, Tunc T (2007) Strategic spot trading in supply chains. *Management Sci.* **53**(5):742–759.

Milner J, Kouvelis P(2007) Inventory, speculation, and sourcing strategies in the presence of online exchanges. *Manufacturing Service Oper. Management* **9**(3):312–331.

Moe WW, Trusov M (2011) The value of social dynamics in online product ratings forums. *Journal of Marketing Research* **48**(3):444–456.

Nair AA (2018) Your shopping will get easier now: Paytm Mall introduces Alibaba's New Retail model in India. *Yourstory* February 14, 2018.

National Express (2010) National Express International Ltd case. http://www.ne56.com/experiential/ne56109111369BB3F5.html.

Netessine S, Rudi N (2006) Supply chain choice on the Internet. *Management Science* **52**(6):844–864.

Ofek E, Katona Z, Sarvary M (2011) "Bricks and clicks": The impact of product returns on the strategies of multichannel retailers. *Marketing Science* **30**(1):42–60.

Özener OÖ, Ergun Ö (2008) Allocating costs in a collaborative transportation procurement network. *Transportation Sci.* **42**(2):146–165.

Park D, Lee J, Han I (2007) The effect of on-line consumer reviews on consumer purchasing intention: The moderating role of involvement. *International Journal of Electronic Commerce* **11**(4):125–148.

Quak H, Tavasszy L (2011) Customized solutions for sustainable city logistics: The viability of urban freight consolidation centres, in van Nunen JAEE, Huijbregts P, Rietveld P (eds), *Transitions Towards Sustainable Mobility*, Springer Berlin Heidelberg.

Rai S (2018) Paytm Mall Takes Cues From Alibaba in Indian New Retail Experiment. *Bloomberg Technology*, February 14, 2018.

Ranieri L, Digiesi S, Silvestri B, Roccotelli M (2018) A review of last mile logistics innovations in an externalities cost reduction vision. *Sustainability* **10**(3):782.

Rochet JC, Tirole J (2006) Two-sided markets: A progress report. *The RAND J. Econom.* **37**(3):645–667.

Ru J, Wang Y (2010) Consignment contracting: Who should control inventory in the supply chain? *Eur. J. Oper. Res.* **201**(3): 760-769.

Rudi N, Kapur S, Pyke D (2001) A two-location inventory model with transshipment and local decision making. *Management Sci.* **47**:1668–1680.

Ryan JK, Sun D, Zhao X (2012) Competition and coordination in online marketplaces. *Production and Operations Management.* **21**(6): 997-1014.

Shaffer G, Zettelmeyer F (2002) When good news about your rival is good for you: The effect of third-party information on the division of channel profits. *Marketing Science* **21**(3):273–293.

Snapp S (2012) The overestimation of the benefits of 3PL versus private fleets. *Brightwork research.*

Steinerberger S (2015) New bounds for the traveling salesman constant. *Advances Appl. Probability* **47**(1).

Stuart A, Ord K (1994) Kendall's advanced theory of statistics: Volume 1, Distribution theory.

Su X, Zhang F (2008) Strategic customer behavior, commitment, and supply chain performance. *Management Sci.* **54**(10):1759–1773.

Sun M, Tyagi R (2017) Product fit uncertainty and information provision in a distribution channel. Working paper, Boston University, University of California, Irvine.

Taylor T (2018) On-demand service platforms. *Manufacturing Service Oper. Management*, **20**(4):704–720.

Tian L, Jiang B (2018) Effects of consumer-to-consumer product sharing on distribution channel. *Production Oper. Management* **27**(2):350–367.

Ulu C, Honhon D, Alptekinoğlu A (2012) Learning consumer tastes through dynamic assortments. *Operations Research* **60**(4) 833–849.

van Duin R, Quak H, Munuzuri J (2008) Revival of the cost benefit analysis for evaluating the city distribution center concept? In Taniguchi E, Thompson RG (eds), *Innovations in City Logistics*, Nova Publishers, New York, pp. 97–114.

Vivaldini M, Pires S, de Souza F (2012) Improving logistics services through the technology used in fleet management. *J. Information Systems and Technology Management: JISTEM* **9**(3): 541–562.

Wang Y, Jiang L, Shen ZJ (2004) Channel performance under consignment contract with revenue sharing.

Wang S, Zheng S, Xu L, Li D, Meng H (2008) A literature review of electronic marketplace research: Themes, theories and an integrative framework. *Inf. Sys. Front.* **10**(5): 555-571.

Wang C, Lau HC, Lim YF (2015) A rolling horizon auction mechanism and virtual pricing of shipping capacity for urban consolidation centers, in Corman F, VoßS, Negenborn RR (eds), *Computational Logistics*, Springer International Publishing Switzerland, pp. 422-436.

Wang Y, Zhang D, Liu Q, Shen F, Lee LH (2016) Towards enhancing the last-mile delivery: An effective crowd-tasking model with scalable solutions. *Transportation Res. Part E* **93**:279–293.

Wang C, Lim YF, Lau HC (2018) An auction mechanism for last-mile delivery with cost uncertainty. Working paper, Singapore Management University.

Weyl EG (2010) A price theory of multi-sided platforms. *Amer. Econom. Rev.* **100**(4):1642–1672.

Yang W (2018) Beyond Omnichannel: Alibaba's "New Retail" strategy. *Consumer Value Creation*, February 18, 2018.

Yu M, Debo L, Kapuscinski R (2016) Strategic waiting for consumer-generated quality information: Dynamic pricing of new experience goods. *Management Science* **62**(2) 410–435.

Zhu F, Liu Q (2018) Competing with complementors: An empirical look at Amazon.com. *Strategic Management J.* **39**(10): 2618-2642.