Using the Weibull distribution to characterise road transport vibration levels

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Abstract

This paper reviews the various ways that have been proposed to characterise road transport vehicle vibrations and recommends a new approach to characterise the vibrations levels during a transport journey. Some 47 road vehicle vibration records, obtained from a broad range of conditions, were analysed and results show that the rms distribution of the vibrations can be accurately modelled with a reduced version of the three-parameter Weibull distribution (shape parameter set to 2). This statistical approach to characterising road vehicle vibrations takes into account the random fluctuations in rms levels that occur naturally during a road journey and can be used to classify the severity of RVV. This offers significant improvement on the simplistic mean rms value that has, so far, been the sole parameter to describe vibration levels during transport. The Weibull location parameter describes the low rms threshold of rms level whereas the Weibull range parameter is proportional to the range of rms level. Results also reveal a strong relationship between the rms mean and the sum of the location and scale parameters. In addition, this enables generation of rms distributions from the mean PDS alone. The modified (fixed-shape) Weibull distribution can be used to faithfully describe the entire statistical distribution of the rms level of a journey or transport mode with just two parameters. This new approach can be used in a practical way for quantifying and comparing transport vibration rms levels for design and testing purposes.

Introduction

It is now well recognised that the levels of vibrations generated by transport vehicles, especially road vehicles, often vary considerably depending, generally, on vehicle type, road type (roughness) and speed. Despite this there is little information available to assist in characterising and classifying the severity of such vibrations during road transport. This is essential for the design and optimisation of products and protective packaging systems to ensure that shipments reach their destination intact while making sure that only the necessary amount of packaging material is used. Currently, road vehicle vibrations (RVV) are broadly characterised by the mean (overall) root-mean-square vibration (usually acceleration) for the entire journey irrespective of the variations in rms levels along the way. Improved characterisation of these potentially damaging vibrations that take into account the random variations in rms level along a journey will make it possible to better understand the nature of vibration levels during particular journeys and make it easier to identify the cause and source of excessive vibrations enabling suitable action to be taken.

This paper reviews the various ways that have been proposed and used to characterise road transport vehicle vibrations and proposes a new approach to characterise the vibrations during a transport journey with an eye on classifying vibration severity and, eventually, developing more sophisticated laboratory test schedules for validating and optimising protective packaging systems.

Background

Today, random vibrations generated by transport vehicles continue to be characterised by the overall (mean) rms level and the corresponding frequency spectrum in the form of the Power Density Spectrum (PDS). This information is used to design and validate the ability products and packaging systems to survive transport. This approach assumes that vibrations that are generated during transport (in particular road transport) are not controllable and that the user of transport services is at the mercy of the transport environment. For instance, there is no guidelines or standard relating to quantifying the levels of vibrations (and shocks) that are to be expected or achieved during road transport. Developed economies generally understand the adverse effects of poorly-maintained (rough) roads in combination with vehicle fitted with poor quality and poorly-maintained suspension systems and do manage road roughness (see International Roughness Index or IRI [1] for instance) and vehicle suspensions (see OECD DEVINE [2] experiment for instance). However, little of this is mandatory or regulated and goods that are transported across countries and continents are more often than not subjected to poor conditions. Because of this, protective packaging systems are often designed for the worse case or worse event scenarios. In order to manage and mitigate excessive vibration levels during transport, an accurate and reliable method to characterise vibrations must be available.

Recognition of the statistical nature of road vehicle vibrations in the context of protective packaging was recognised quite early (see Schlue and Phelps [3] for instance) who conducted a vibration study to analyse the influence of road roughness, and vehicle loading conditions. This study and many like it [4] presented their data as PDS for various statistical levels of occurrences. Hasegawa [5] was the first to recognise the benefit of characterising road vehicle vibrations with a statistical distribution and made an attempt at fitting (very limited) data to various statistical models including the Exponential, Weibull, Poisson and Modified log-normal distributions without any strong conclusion but did suggest that "The Irrational or the Weibull distribution(s) are suited to compare transport test data or to describe the status of changing conditions." This, it is assumed, is meant to refer to the variations in vibration level during a journey.

As recognition that road vehicle vibrations were best describe statistically took hold along with the development of suitable test equipment (namely Random Vibration Controllers), test protocols to synthesize random vibrations that were similar to the motion of transport vehicles were published by ISO, ASTM, ISTA among others and adopted by the distribution packaging industry [6]. These test protocols, which are still in use today use constant root-mean-square (rms) levels in laboratory-based transportation trials. These test levels are based not on a specific statistic (such as the mean or median) of vibration levels encountered during typical road journeys but employ an artificially-elevated rms levels based on an adaptation of the Basquin [7] model for cyclic fatigue.

With the availability of powerful and easy-to-use vibration data recorders in the early 1990s, a significant number of studies were undertaken by numerous researchers to characterise various distribution environments using various vehicle and route types. In the early days, the majority of publications reported PDS and overall rms values with no real attempt at analysing the variation in rms level along particular routes [8, 9, 10, and 11].

Rouillard [12] introduced a novel method for analysing non-stationary random vibrations by representing the random fluctuation in vibration levels with a statistical distribution of the vibration envelope. Subsequently, Rouillard [13, 14] used a reasonably large data set to show that this distribution could be reasonably well described with a modified version of the Rayleigh distribution. This was also used in a technique to synthesize non-stationary (randomly fluctuating rms) random vibrations.

Singh et al. [15] reported rms levels for two types of trucks using statistical bands (top 30% and bottom 70%) which they purport to separate events into high and low components. However, justification for choosing two rms bands with these particular levels is not given. Garcia-Romeu et al. [16] investigated the relationship between rms level and vehicle speed for two types of trucks. In this study, the PDS were segregated (in a fashion similar to that of Singh [15]) but this time using rms bands of top 25% and bottom 75%. A modified four-parameter Weibull distribution (no details given) was used to characterise the rms distributions for the three vehicle speed ranges used namely: 0 – 40 km/h, 40 – 70 km/h and greater than 70 km/h showing a general increase in rms levels with increased speed. The results also showed an increase in rms levels for unloaded leaf-spring vehicles compared to loaded vehicles while the corresponding increase for air-ride vehicle was negligible. Singh et al. [17] studied the vibrations measured from less-than-truckload (LTL) shipments as a function of road surface. This time, the authors separated the data in two rms bands namely to 20% and bottom 90%. The rms levels were further categorised according to two road types: City streets and highways, Parking areas, terminals and unpaved roads. Variations in rms levels during particular journeys were not otherwise investigated.

Lu et al. [18] also studied the variations in rms levels as a function of vehicle speed but this was done for a single truck over some 850 kms of local, prefectural roads and highways. However, the variations in rms levels as a function of speed were not subjected to treatment beyond graphical representation. Garcia -Romeu and Rouillard [19] built upon the work done by Rouillard [13] and Garcia-Romeu at al. [16] to develop a 'Universal or Generic statistical function which they used to describe the statistical distribution of the (moving) rms levels for a number of RVV records obtained from a variety of vehicle and route combinations in Australia and Spain. Their results show that, in some cases, the Generic model yields better agreement than the three-parameter Weibull distribution especially at higher rms levels were, it may be conjectured, contributions from transient and shock events are significant. They suggest that the variations in rms levels (a manifestation of the nonstationarity) can be described mathematically using the distribution parameters. Otari et al. [20] propose a modified Gaussian distribution to model the nonstationarity of RVV acceleration vibration and show how it can be used to describe overall ride quality for two road types (motorway and country road). Kurniawan et al. [21] presented vibration data collected from two and three-wheeled vehicles as statistical distributions, thus acknowledging that variations occur within individual trips. However, they only conclude that the distributions are not Gaussian as predicted by Rouillard [13]. Furthermore, their data exhibit suspiciously large positive skewness values which remained unexplained. Zhou et al. [22], in an approach similar to Garcia-Romeu et al. [16] investigated the effects of truck speed (0–30 km/h, 31–60km/h and 61–90 km/h), road condition (highways, arterial roads, secondary roads and tertiary roads) and load on the vibration levels. The (moving) rms levels, peak acceleration and crest factor (all calculated with an analysis window of two seconds) were presented a cumulative distribution functions and were all fitted with a three-parameter Weibull distribution. Despite this, the Weibull parameters were not published nor used in any way to characterise the vibrations under various conditions.

More recent studies where the variations in vibration levels during individual journeys are recognised and given statistical treatment include Borocz [23] where the distribution of (event) peak acceleration for various road transport scenarios (road condition and vehicle type) are given. In this paper, the statistical distributions are shown to deviate significantly from the Gaussian distributions as previously revealed by Rouillard [12]. In the study of vibration levels for 'express logistics' transportation in South China by Zhou [24], the variations in (event) rms is presented as Cumulative Distribution Functions (CFD) for a range of vehicle type, road type and speed combinations. In this case, the rms distributions are compared with Gaussian functions without any justification.

Another study of multi-modal transport vibrations by Borocz [25] made use of the two-parameter Weibull distribution as the 'best fit' to model the variations in rms levels for various routes. However, the authors do not compare the Weibull functions but restrict their analysis to skewness and kurtosis parameters for various transport modes and routes. Further, their results reveal that the agreement between the rms distributions and the Weibull function is only occasionally reasonable.

In summary, the variation in rms levels during road transport is becoming increasingly acknowledged but its characterisation remains outstanding. In the main, RVV continue to be characterised by the overall rms with some exceptions where the distribution in rms levels is given. In order to enable protective packaging to be optimised, the ability to define an expected or desired range of vibration levels for particular distribution environments is needed. This may also lead to more customised design, testing and validation protocols for specific routes and supply chains.

This paper addresses this by attempting to statistically characterise the rms distribution for a broad range of measured road transport vibration data and quantify severity using statistical parameters.

Methodology

The analysis presented in this paper is based on empirical vibration data believed to represent a broad range of road transport conditions including road types, vehicle types and lading conditions. Vertical acceleration data was collected with the vehicles travelling at ambient speed which was not recorded but assumed to represent normal driving conditions (dictated by speed limits as well as traffic and road conditions). Specific details on each vibration record (measured over two decades in Australia and Spain) are given in Table 1. Sampling rates, not particularly relevant to the computation and analysis of moving rms, ranged between 500 and 2000 Hz which ensured that aliasing was avoided in all cases. Most data sets were sampled continuously whereas three used (regular interval) used time-triggered sampling and five used level-triggered sampling. In these latter cases, the trigger levels were sufficiently low so as not to adversely affect the corresponding rms distribution. In total, the data set represents some 142 hours of vibration records with individual records ranging between 0.23 and 12.4 hours in duration with an average duration of 3 hours.

Especially important, the set contains one vibration record (VU43) from a heavily-loaded vehicle travelling on very well-maintained roads. This can be considered as the lower limit of vibration levels that can be achieved during normal road transport journeys. In addition, one record (VU12) was measured with the aim of establishing the realistic extremes in vibration levels. This was generated by driving a lightly-loaded vehicle with poorly-maintained steel suspensions along a route that contained a significant proportion of very rough roads. These data sets can be considered to represent the extremes in vibration levels that are generated during road transport with the remaining data sets representing everything in between.

Analysis was restricted to vibration levels as represented by the moving rms (using, in this case, a two-second window with maximum overlap - see Rouillard [13]). All data sets were carefully pre-processed to remove any aberrations (handling shocks, pre-and post-transport events etc.) so as to include only genuine vibration data.

Results

The rms distributions of all 47 vibration records are shown in Figure 1 where similarity in the overall shape of the distributions can be discerned.

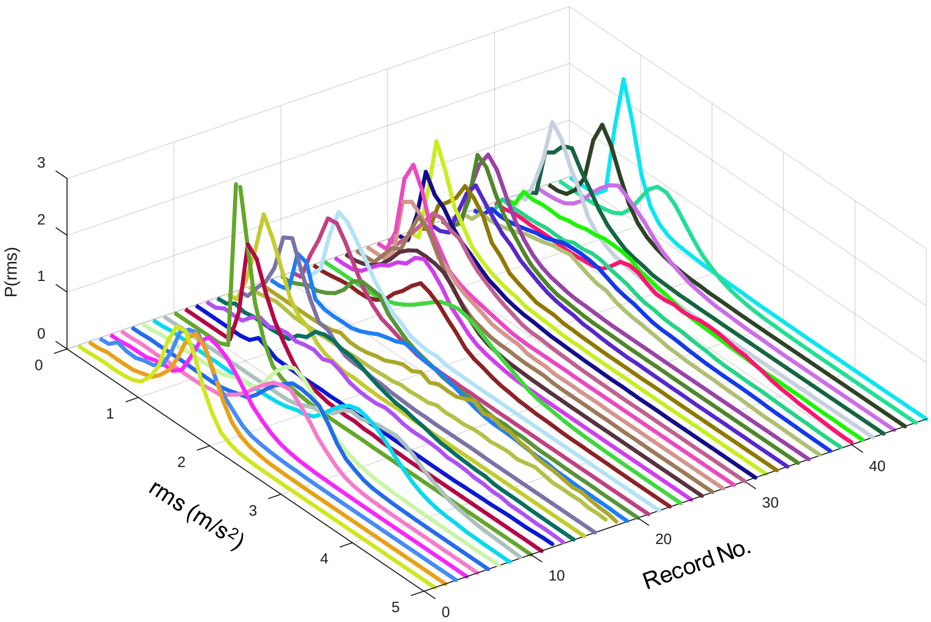

Figure 1. Probability distributions of the moving rms of all 47 vibration records.

Although valuable, the usefulness of graphical representations of the rms distribution is limited and significant benefits can be afforded if these distributions can be modelled with a few parameters such that they can be characterised and compared more easily. Based on the work of Rouillard [13] and Garcia-Romeu [19], the three-parameter Weibull distribution was selected as the most promising function to describe the rms distributions.

$$P(x) = \frac{\beta}{\eta}\left(\frac{x - x_0}{\eta}\right)^{\beta - 1} \cdot \exp\left[-\left(\frac{x - x_0}{\eta}\right)^{\beta}\right] \qquad \forall \begin{cases} x_0 \in \mathbb{R} \\ x_0 \leq x < \infty \\ \beta, \eta \in \mathbb{R}^+ \end{cases} \tag{1}$$

Where β is the shape parameter, η the scale parameter and x_o the location parameter.

The influence of the three parameters on the Weibull distribution are illustrated in Figure 2.

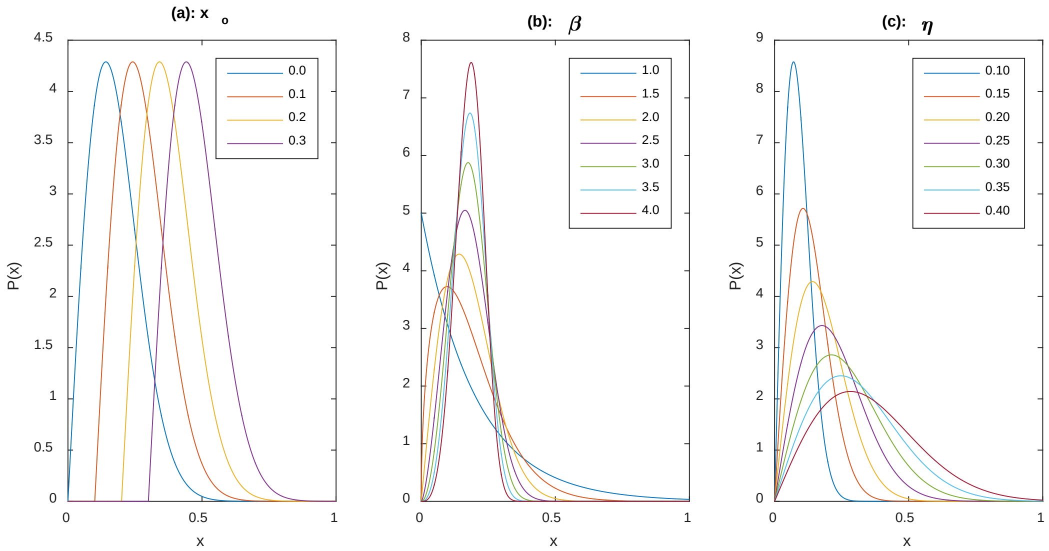

Figure 2. Parameter influence on the Weibull distribution: (a): Location parameter; (b) Shape parameter; (c): Scale parameter.

The location parameter merely introduces a shift along the abscissa whereas the shape parameter distorts the function and the scale parameter affects the overall breadth of the distribution. In a practical sense, the location parameter can be used to represent the low rms threshold except when x_o is less than zero, in which case the low rms threshold is zero. The three-parameter Weibull function was fitted to all 47 records and the goodness of fit evaluated using the Pearson's regression coefficient, R^2. It must be noted that low-level vibrations generated by engine idling, resulting in a secondary peak near zero in the rms distribution, were discounted when determining the parameters of best fit. Three representative examples are shown in Figure 3 and a summary of the statistical analysis on every vibration record is given in Table 2. Note that the low regression coefficient for record VU19 is caused by having to manually correct the Weibull parameters as the automatic least-squared regression (which searched for the maximum R^2) yielded large negative x_o values resulting in a distorted distribution which did not match the important (higher rms levels) section of the measured distribution. One shortcoming of the Weibull model is that it sometimes fails to accurately take into account low rms values. Given that this occurs at very low rms values (below x_o) – Figures 3(b) and 3(c) for example - , which are rarely the cause of product damage, this is not seen as significant.

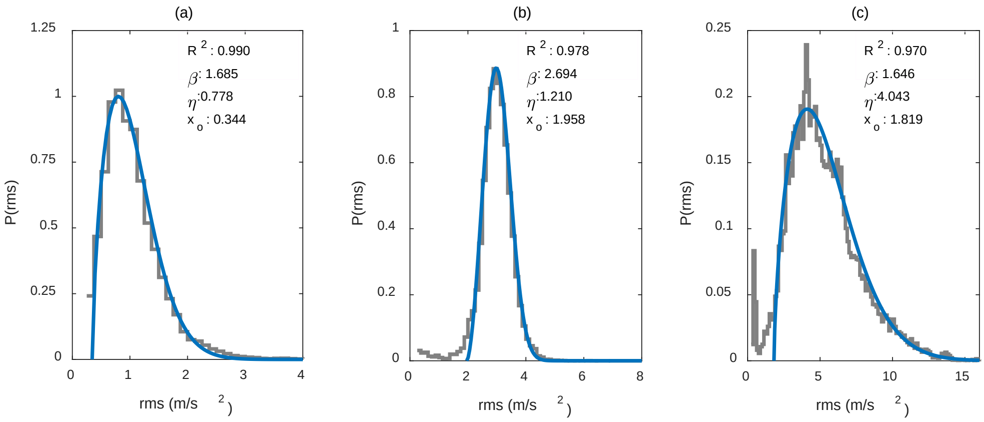

Figure 3. Representative examples ((a): low; (b): moderate and (c): high) of the rms distribution and the best fitting three-parameter Weibull model.

The relationship between the three Weibull parameters and the mean rms for all records are shown graphically in Figure 4. One noteworthy feature of the results is the scattered nature of the shape parameter with respect to rms mean as opposed to the location and scale parameter which exhibit a reasonable correlation with the mean rms (μ). From Figure 2, it can be seen that the influence of the shape parameter on the distribution is not particularly strong and may possibly be set to a constant. The benefit of being able to describe the rms distribution with just two parameters is quite significant as it would enable the entire process to be described with just two parameters with the location parameter quantifying the low rms threshold (except when x_o is less than zero, in which case the low rms threshold is zero) and scale parameter describing the range of rms variation with both parameters having the same units as the measurand namely, rms levels. To that end, the shape parameter was set to a constant (approximate to the mean from the entire data set) namely 2 as shown in Figure 4(b), and the best-fitting Weibull re-computed. In fact, given that the relationship between the sum of the scale and location parameters with respect to the mean rms is clear and strong, the three parameter Weibull distribution, in effect, becomes a function of the mean rms level.

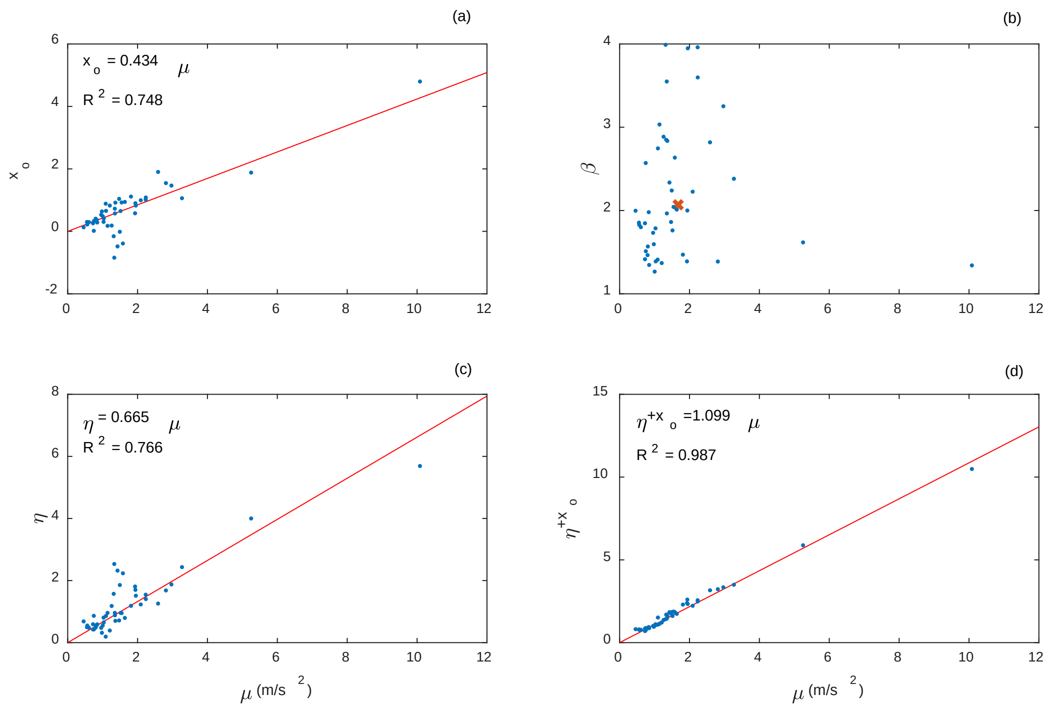

Figure 4. (a): Weibull location parameter vs mean rms; (b): Weibull shape parameter vs mean rms (**x** indicates mean value); (c): Weibull scale parameter vs mean rms; (d): Weibull scale + location parameters vs mean rms.

Figure 5 shows the same three representative examples as those shown in Figure 3 along with the best-fitting fixed-shape parameter Weibull function which clearly show that the difference is negligible with R^2 values well above 0.9. Results for the complete data set are shown in Table 3. The shape parameter of 2 has the effect of removing the tail to the left of the distribution, thereby reducing the occurrence of negative x_o values while providing an accurate description of the distribution. This is a distinct advantage of the fixed shape parameter approach.

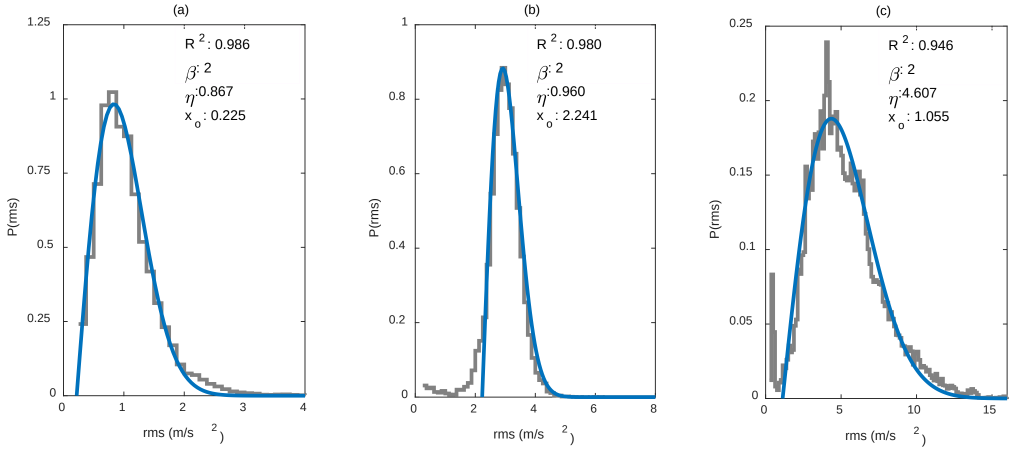

Figure 5. Representative examples ((a): low; (b): moderate and (c): high) of the rms distribution and the best fitting three-parameter Weibull model with the shape parameter set to 2.

Using R^2 as the main goodness of fit measure, the effect of fixing the shape parameter to 2, is shown graphically in Figure 6 with the worst case showing a difference of 5% which, for the benefits of reducing the model to two parameters with physical meaning, is acceptable.

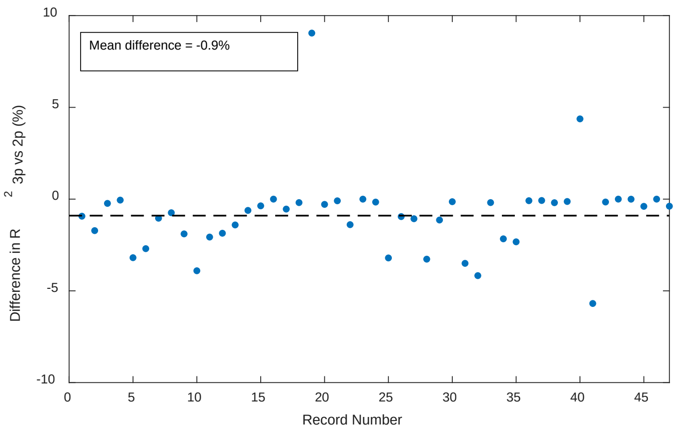

Figure 6. Effect on the regression coefficient, R^2, when setting the Weibull shape parameter to two.

The relationship between the location and scale parameters and the mean rms with the shape parameter set to 2 (2p) are shown in Figure 7. To facilitate comparison, data from the three-parameter (3p) results are included. In the main, it can be said that the strong relationship between the location and scale parameters with the mean rms is not adversely affected by the use of the fixed-shape parameter Weibull model.

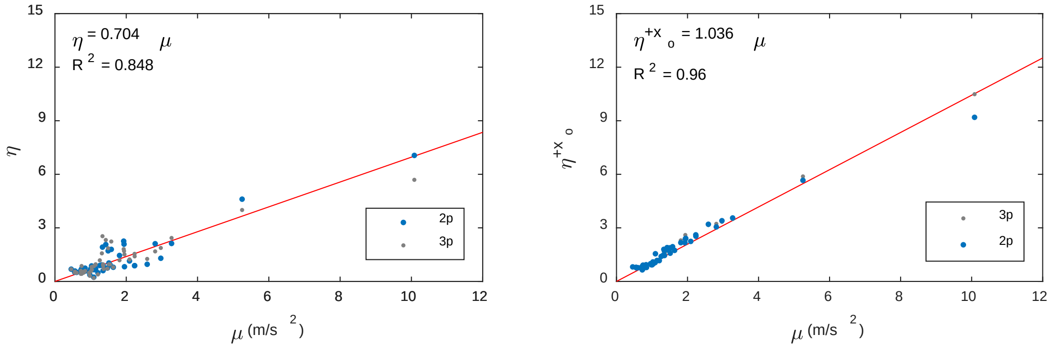

Figure 7. Relationships between the Weibull scale parameter (left) and the scale + location parameters and the mean rms with the Weibull shape parameter set to 2.

It is interesting to note that, when the shape parameter is set to 2, the three-parameter Weibull function approaches the Rayleigh with offset (used by Rouillard [26] to describe the rms distribution of road profiles):

$$P(x)=\frac{2}{\eta^2}(x-x_0)\cdot\exp\left[-\left(\frac{x-x_0}{\eta}\right)^2\right] \qquad \forall \begin{cases} x\in\mathbb{R} \\ x_0\le x<\infty \\ \eta\in\mathbb{R}^+ \end{cases} \tag{2}$$

When η is set to $\sigma\sqrt{2}$ and $x_o = 0$ the distribution reduces to the Rayleigh distribution:

$$P(x)=\frac{x}{\sigma^2}\cdot\exp\left[-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2\right] \qquad \forall \left\{x,\sigma\in\mathbb{R}^+\right. \tag{3}$$

The Rayleigh function is suitable to describe the magnitude distribution of narrow-banded, Gaussian random signals [27] which rarely apply to road vehicle vibrations which are, in the main non-Gaussian. These can be described as a sequence of Gaussian segments of varying rms levels and durations which, when combined, produces a non-stationary process with leptokurtic characteristics as demonstrated by Rouillard [13].

Salient statistical parameters, such as the mean, μ', and standard deviation, σ', of the moving rms for $x > xo$ were also compared to further evaluate the appropriateness of using the fixed shape (2p) Weibull model. These can be calculated from the calculated rms distributions using moments and from the Weibull distribution as follows:

$$\mu'=\eta\,\Gamma\left[1+\frac{1}{\beta}\right]+x_0 \tag{4}$$

$$\sigma' = \sqrt{\eta^2 \left[\Gamma\left(1+\frac{2}{\beta}\right) - \Gamma^2\left(1+\frac{1}{\beta}\right) \right]} \tag{5}$$

These results, shown in Figure 8, further confirm show the suitability of the Weibull model to characterise the statistical distribution of the moving rms of road vehicle vibrations. The model – described here with the location and scale parameters, can be used to determine both the mean and standard deviation of the distribution. The only exception is the record VU12 case (representing the roughest ride in the set which also yields the lowest R^2 values for both three and fixed-shape parameter Weibull fits) where both the three-parameter and the fixed-shape Weibull models underestimate the standard deviation and, to a lesser extent, the mean rms. The cause of this can clearly be seen in Figure 9 which shows the measured rms distribution along with the best-fit of the three-parameter and the fixed-shape Weibull models. In this case, the Weibull models fail to properly account for higher rms levels (above 15 m/s^2) that are likely to be caused by shock and transients.

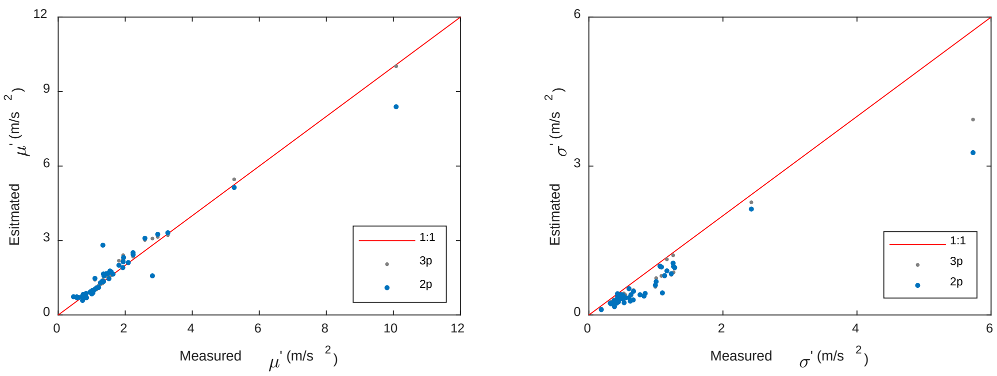

Figure 8. Comparison of rms distribution mean (left) and standard deviation (right) estimates computed from the three-parameter and the fixed-shape Weibull models.

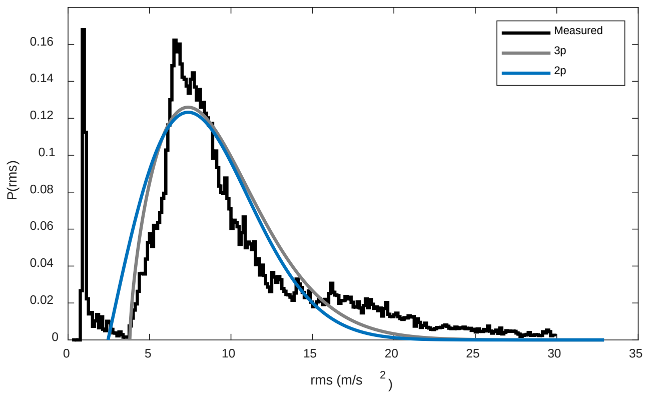

Figure 9. rms distribution of record VU12 (roughest in set) along with best-fit of the three-parameter and the fixed-shape Weibull models.

Discussion

Analysis on all 47 road vehicle vibration records show that the rms distribution of the vibrations can be accurately modelled with a reduced version of the three-parameter Weibull distribution. In this case, the shape parameter is fixed to two (approximately the average value for all 47 vibration records in the data set). This leaves two remaining parameters, namely the location parameter and the scale parameter. The location parameter represents the low threshold of the rms level in the record (except when x_o is less than zero, in which case the low rms threshold is zero) whereas the scale parameter is proportional to the range of the rms in the record. This approach is ideally-suited to describe the level of vibrations within a particular transport environment; it offers significant improvement on the simplistic mean rms value that has, so far, been the sole parameter to describe vibration levels during transport despite not accounting for the non-stationarity (variation in rms level) of typical road vehicle vibrations.

The modified (fixed-shape) Weibull distribution can be used to describe the entire statistical distribution of the rms level of a journey or transport mode with just two parameters. This new approach can be used in a practical way for quantifying and comparing transport vibration rms levels for design and testing purposes. Further, this can be used to establish acceptable levels of vibrations for particular distribution environments and promote a risk-based approach to managing distribution vibrations by placing a financial cost / penalty structure on the levels of vibrations produced during transport. This will, in turn, lead to a reduction in both product waste and excessive protective packaging. This is becoming increasingly important as the difficulties in achieving reasonable recovery and recycling rates of packaging waste material is becoming clearer.

The strong correlation between the location and scale parameters with the mean rms is an added benefit as it enables the rms distribution to be estimated from the mean rms alone. This means that vibration surveys that contain only average PDS (hence the mean rms) can be used to retrospectively generate the rms distribution thus producing a more complete picture of the variations in vibration levels. This, of course, assumes that the measured vibrations are of sufficient length so that the mean rms is representative of the true value. The question as to how long a vibration record needs to be to ensure that this is applicable is beyond the scope of this paper and will, possibly, be the subject of further investigation. However, the findings of Rouillard and Lamb [28] which indicate that analysis on at least 12% of the realised vibrations is sufficient to obtain a complete statistical picture remains a valid proposition.

Conclusions

The paper has shown that a modified version of the Weibull distribution – with the shape parameter fixed to two – can be used to accurately describe the statistical distribution of the moving rms for road vehicle vibrations. This is based on some 47 vibration records obtained from a broad range of vehicles travelling at ambient speed on a variety of roads. The Weibull location parameter describes the low rms threshold of rms level whereas the Weibull range parameter is proportional to the range of rms level. Importantly, for well-sampled data, the existence of a strong relationship between the rms mean and the sum of the location and scale parameters enables generation of rms distributions from the mean PDS alone. This statistical approach to characterising road vehicle vibrations takes into account the random fluctuations in rms levels that occur naturally during a road journey and can be used to classify the severity of RVV. These can be used to support the management and mitigation of RVV as well as for laboratory-based transport trials.

Further works, based on this paper, can include:

- Investigating the possibility of extracting the Weibull distribution parameters from vibration data collected through level-triggered sampling.
- Validation of the model using additional data sets including those from controlled experiments.
- Using the approach in this paper to characterise vibrations from rail, air and sea transport modes would be useful as these are generally highly-nonstationary in nature.
- Applying the two-parameter Weibull model to specify test severities for laboratory-based transport testing.

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Table 1. Summary of vibration data sets used for analysis. (Timer *x s/ ys* indicate regular triggering with *x* representing the measurement duration and *y* the interval both in seconds)

ID	Vehicle Type:	Susp.	Payload [% cap.]	Road Type	Trigger Mode	Sampling Rate [Hz]	Sensor Location	Duration [mins]
VU1	Road Train	Air	>80	Mixed Hwy	Continuous	760	Rear load tray	360
VU2	Road Train	Air	>80	Mixed Hwy	Continuous	760	Rear load tray	366
VU3	Road Train	Air	>80	Mixed Hwy	Continuous	760	Rear load tray	367
VU4	Road Train	Air	>80	Mixed Hwy	Continuous	760	Rear load tray	368
VU5	Road Train	Air	>80	Mixed Hwy	Continuous	760	Rear load tray	369
VU6	Road Train	Air	>80	Mixed Hwy	Continuous	760	Rear load tray	369
VU7	Road Train	Air	>80	Mixed Hwy	Continuous	760	Rear load tray	369
VU8	Road Train	Air	>80	Mixed Hwy	Continuous	760	Rear load tray	369
VU9	Road Train	Air	>80	Mixed Hwy	Continuous	760	Rear load tray	369
VU10	Rigid Truck	Air	0	Motorway	Continuous	500	Rear load tray	21
VU11	Small truck	Air	0	Motorway	Continuous	500	Rear load tray	22
VU12	Utility	Leaf	20	Metro	Continuous	500	Rear axle	14
VU13	Semi trailer	Air	90	Metro	Continuous	1024	Rear axle	34
VU14	Small van	Leaf	50	Metro	Continuous	1024	Rear axle	34
VU15	Small van	Leaf	50	Metro	Continuous	1024	Rear axle	34
VU16	Small van	Leaf	50	Metro	Continuous	1024	Rear axle	34
VU17	Semi trailer	Air	0	Metro	Continuous	1024	Rear axle	34
VU18	Rigid tipper	Air	30	Metro	Continuous	1024	Rear axle	34
VU19	Rigid truck	Leaf	0	Metro	Continuous	1024	Rear axle	34
VU20	Sedan	Coil	20	Metro	Continuous	1024	Rear load tray	34
VU21	Rigid truck	Air	70	Intercity	Timer 8s/60s	1024	Rear load tray	43
VU22	Rigid truck	Air	20	Intercity	Timer 8s/60s	1024	Rear load tray	43
VU23	Rigid truck	Air	100	Intercity	Timer 8s/60s	1024	Rear load tray	43
VU24	Semi Trailer	Air	>80	Intercity	Level 0.2 g	1024	Rear axle	331
VU25	Semi Trailer	Air	>80	Intercity	Level 0.2 g	1024	Rear axle	49
VU26	Semi Trailer	Air	>80	Intercity	Level 0.2 g	1024	Rear axle	327
VU27	B Double	Air	>80	Mixed HWy	Continuous	400	Rear Trailer B	58
VU28	Utility	Leaf	70	Metro	Continuous	2000	Rear axle	28
VU29	Utility	Leaf	70	Metro	Continuous	2000	Rear axle	16
VU30	Rigid Truck	Leaf	20	Metro	Continuous	1000	Rear axle	185
VU31	Rigid Truck	Leaf	30	Metro	Continuous	1000	Rear axle	185
VU32	Rigid Truck	Leaf	20	Metro	Continuous	1000	Rear axle	115
VU33	Rigid Truck	Leaf	30	Metro	Continuous	1000	Rear axle	100
VU34	Rigid Truck	Leaf	30	Mixed Hwy	Continuous	1000	Rear axle	100
VU35	Small van	Leaf	30	Mixed Hwy	Continuous	1000	Rear axle	105
VU36	Small van	Leaf	30	Metro	Continuous	1000	Rear axle	110
VU37	Small van	Leaf	40	Metro	Continuous	1000	Rear axle	185
VU38	Small van	Leaf	40	Metro	Continuous	1000	Rear axle	190
VU39	Small van	Leaf	40	Metro	Continuous	1000	Rear axle	220
VU40	Small van	Leaf	40	Metro	Continuous	1000	Rear axle	240
VU41	Small van	Leaf	40	Mixed Hwy	Continuous	1000	Rear axle	100
VU42	Utility	Leaf	40	Metro	Continuous	1000	Rear axle	205
VU43	Utility	Leaf	40	Metro	Continuous	1000	Rear axle	251
VU44	Semi Trailer	Air	>80	Motorway	Level 0.2 g	2500	Rear axle	134
VU45	B Double	Air	>80	Mixed HWy	Continuous	400	Front Trailer A	130
VU46	Semi Trailer	Air	>80	Motorway	Continuous	2000	Rear axle	650
VU47	Semi Trailer	Air	>80	Motorway	Level 0.2 g	1000	Rear axle	738

Table 2. Summary of statistical analysis and three-parameter Weibull fit to entire data set with those with the lowest and highest mean rms highlighted. μ and σ are the mean and standard deviation of the moving rms.

ID	β	η	x_o	R^2	μ	σ
VU1	2.836	0.702	0.918	0.993	1.365	0.526
VU2	3.549	0.959	0.725	0.986	1.349	0.618
VU3	2.746	0.860	0.655	0.975	1.096	0.662
VU4	1.862	0.715	1.040	0.990	1.472	0.598
VU5	3.598	1.404	1.084	0.996	2.236	0.628
VU6	3.960	1.544	1.017	0.995	2.233	0.763
VU7	3.948	1.511	0.825	0.977	1.950	0.823
VU8	2.819	1.259	1.902	0.987	2.587	1.097
VU9	3.252	1.875	1.466	0.983	2.969	0.992
VU10	1.410	0.193	0.885	0.940	1.088	0.186
VU11	1.369	0.390	0.825	0.980	1.205	0.330
VU12	1.341	5.690	4.797	0.856	10.082	5.731
VU13	1.389	1.808	0.578	0.961	1.928	1.257
VU14	1.470	1.185	1.113	0.966	1.811	1.004
VU15	1.800	0.480	0.296	0.958	0.608	0.394
VU16	1.980	0.568	0.368	0.942	0.833	0.441
VU17	1.617	4.000	1.881	0.951	5.250	2.424
VU18	2.381	2.433	1.062	0.929	3.271	1.061
VU19	2.000	1.700	0.900	0.838	1.938	1.083
VU20	1.761	0.959	0.651	0.937	1.513	0.666
VU21	1.847	0.592	0.257	0.977	0.725	0.400
VU22	1.568	0.536	0.350	0.984	0.808	0.374
VU23	2.043	0.952	0.924	0.974	1.544	0.840
VU24	2.226	1.230	0.995	0.993	2.091	0.599
VU25	3.990	1.572	-0.157	0.982	1.316	0.427
VU26	2.886	1.181	0.185	0.995	1.258	0.436
VU27	3.033	0.958	0.176	0.961	1.141	0.505
VU28	1.266	0.553	0.485	0.971	1.003	0.436
VU29	1.511	0.426	0.328	0.962	0.750	0.337
VU30	1.786	0.806	0.303	0.987	1.027	0.505
VU31	1.346	0.588	0.284	0.954	0.845	0.544
VU32	1.415	0.423	0.275	0.909	0.726	0.415
VU33	2.570	0.863	0.016	0.967	0.747	0.467
VU34	1.389	0.654	0.413	0.984	1.034	0.522
VU35	1.464	0.485	0.405	0.973	0.800	0.416
VU36	1.828	0.549	0.220	0.988	0.561	0.404
VU37	2.240	1.856	-0.014	0.961	1.492	1.130
VU38	2.635	2.234	-0.389	0.954	1.580	1.230
VU39	2.336	2.321	-0.482	0.968	1.426	1.280
VU40	1.387	1.681	1.545	0.921	2.811	1.164
VU41	2.849	2.533	-0.844	0.966	1.333	1.263
VU42	1.854	0.497	0.300	0.977	0.552	0.395
VU43	1.997	0.684	0.129	0.989	0.456	0.422
VU44	1.965	0.881	0.573	0.989	1.352	0.477
VU45	1.732	0.476	0.530	0.976	0.958	0.319
VU46	2.011	0.795	0.942	0.990	1.636	0.428
VU47	1.595	0.317	0.637	0.978	0.979	0.382

Table 3. Summary of statistical analysis and fixed-shape Weibull fit to entire data set.

ID	β	η	x_o	R^2	μ	σ
VU1	2.000	0.535	1.107	0.984	1.365	0.526
VU2	2.000	0.604	1.117	0.970	1.349	0.618
VU3	2.000	0.657	0.893	0.973	1.096	0.662
VU4	2.000	0.751	0.995	0.990	1.472	0.598
VU5	2.000	0.884	1.649	0.964	2.236	0.628
VU6	2.000	0.880	1.729	0.968	2.233	0.763
VU7	2.000	0.823	1.587	0.966	1.950	0.823
VU8	2.000	0.960	2.241	0.980	2.587	1.097
VU9	2.000	1.297	2.103	0.964	2.969	0.992
VU10	2.000	0.235	0.819	0.904	1.088	0.186
VU11	2.000	0.486	0.678	0.959	1.205	0.330
VU12	2.000	7.058	2.134	0.840	$\boxed{10.082}$	5.731
VU13	2.000	2.256	-0.090	0.948	1.928	1.257
VU14	2.000	1.453	0.717	0.960	1.811	1.004
VU15	2.000	0.506	0.261	0.954	0.608	0.394
VU16	2.000	0.572	0.362	0.942	0.833	0.441
VU17	2.000	4.607	1.055	0.946	5.250	2.424
VU18	2.000	2.126	1.428	0.928	3.271	1.061
VU19	2.000	2.087	0.292	0.913	1.938	1.083
VU20	2.000	1.031	0.549	0.934	1.513	0.666
VU21	2.000	0.622	0.219	0.976	0.725	0.400
VU22	2.000	0.621	0.236	0.970	0.808	0.374
VU23	2.000	0.938	0.942	0.973	1.544	0.840
VU24	2.000	1.146	1.096	0.991	2.091	0.599
VU25	2.000	0.924	0.544	0.950	1.316	0.427
VU26	2.000	0.893	0.505	0.985	1.258	0.436
VU27	2.000	0.710	0.463	0.951	1.141	0.505
VU28	2.000	0.700	0.229	0.939	1.003	0.436
VU29	2.000	0.499	0.216	0.951	0.750	0.337
VU30	2.000	0.867	0.225	0.986	1.027	0.505
VU31	2.000	0.742	0.040	0.921	0.845	0.544
VU32	2.000	0.515	0.131	0.871	0.726	0.415
VU33	2.000	0.704	0.207	0.965	0.747	0.467
VU34	2.000	0.809	0.175	0.963	1.034	0.522
VU35	2.000	0.587	0.257	0.951	0.800	0.416
VU36	2.000	0.581	0.178	0.987	0.561	0.404
VU37	2.000	1.712	0.167	0.960	1.492	1.130
VU38	2.000	1.795	0.149	0.952	1.580	1.230
VU39	2.000	2.064	-0.170	0.967	1.426	1.280
VU40	2.000	1.920	-0.124	0.961	1.333	1.263
VU41	2.000	2.111	0.942	0.911	2.811	1.164
VU42	2.000	0.518	0.273	0.976	0.552	0.395
VU43	2.000	0.685	0.128	0.989	$\boxed{0.456}$	0.422
VU44	2.000	0.891	0.560	0.989	1.352	0.477
VU45	2.000	0.518	0.474	0.972	0.958	0.319
VU46	2.000	0.792	0.946	0.990	1.636	0.428
VU47	2.000	0.365	0.568	0.974	0.979	0.382