International Journal of Electrical and Computer Engineering (IJECE)
Vol. 4, No. 4, August 2014, pp. 498~511
ISSN: 2088-8708

A MODEL FOR ASSESSMENT OF TRANSIENT STABILITY OF ELECTRICAL POWER SYSTEM

Ganiyu A. Ajenikoko, Anthony A. Olaomi
Department of Electronic & Electrical Engineering, Ladoke Akintola University of Technology, Ogbomoso, Nigeria

Article Info

Article history:

Received Feb 25, 2014
Revised May 31, 2014
Accepted Jun 16, 2014

Keyword:

Critical clearing time
Electrical power
Generators' synchronism
Infinite bus-bar
Power system stability
Rotor angles
Swing curves
Transient stability

ABSTRACT

The stability of a system is its ability to return to normal or stable operation after having been subjected to some forms of disturbances. A disturbance in a power system is a sudden change or sequence of changes in one or more of the physical quantities. In this paper, the transient reactance of a synchronous machine, mechanical input power, kinetic energy of a rotating body, moment of inertia, angular acceleration, angular displacement and the rotor displacement angles were used as input parameters for the development of the Transient Stability model. The model is validated with a single machine system, a 2-machine system and a multi-machine system. The results of the work showed that the single machine system supplying an infinite bus-bar fluctuates while the 2-machine system remains unstable throughout the period. Generator 3 of the multi-machine system experienced the most violent swing, pulled out of synchronism during the first swing thus making the system to be unstable. The Transient Stability Model developed can be used for effective planning and operation of power systems.

Corresponding Author:

Ganiyu A. Ajenikoko,
Department of Electronic & Electrical Engineering,
Ladoke Akintola University of Technology,
P.M.B. 4000, Ogbomoso, Nigeria
Email: ajeedollar@gmail.com

1. INTRODUCTION

Power systems are designed to operate in the steady and transient states and are characterized by transients or disturbances [5], [6], [7], [9]. A disturbance in a power system is sudden change or sequence of changes in one or more of the physical quantities [2]. Large disturbances often refer to severe disturbances, such as a fault on transmission network, loss of generation or loss of a large load [11], [16], [24], and that the equations describing the power system cannot be linearized for analysis purpose when subjected to large disturbances [21]. The main factor contributing to the transient instability is the insufficient synchronizing torque during the disturbance period in the system [12, 13]. The disturbances may be either electromagnetic such as over-voltage, over-current, abnormal wave shapes or electromechanical transients which are concerned with the stability of the power system. The recovery of a power system subjected to a severe large disturbance is of interest to system planners and operators. Typically the system must be designed and operated in such a way that a number of credible contingencies do not result in failure of quality and continuity of power supply to the loads. This calls for accurate calculation of the system dynamic behavior, which includes the electro-mechanical dynamic characteristics of the rotating machines, generator control, static var compensators, loads, protective system and other controls. The stability of the power system is the ability of the generator to remain in synchronism after a disturbance to the system [10], [14], [15]. The holding together of these generators are affected through the power network and any loss in the degradation of the network security such as outages, blackouts and so on [1], [33].

Power system stability is classified into three types namely: steady-state, transient and long term [3], [36], [38]. Steady-state stability is primarily concerned with the ability of the system generators to remain in synchronism after minor disturbances such as gradual load changes, changes in excitation, line switching and so on. It is also concerned with sudden and large changes in the network conditions such as brought about by faults the most severe of which is the three phase short-circuit [18], [25], [29].

When a fault occurs at the terminals of a synchronous generator, the power output of the machine is greatly reduced as it is supplying mainly, an inductive circuit [18], [22], [26], [29]. However, the input power to the generator from the turbine has no time to change during the short period of the fault and the rotor endeavours to gain speed to store the excess energy [38]. If the fault persists long enough, the rotor angle will increase continuously and synchronism is lost. Hence, the time of operation of the protection and circuit breakers is all important. An autoreclosing circuit breaker opens when the fault is detected and automatically recloses after a prescribed period. If the fault persists, the circuit breaker reopens and then recloses as before. This is repeated once more, when if the fault still persists, the breaker remains open [22], [30], [33].

Transient stability is judged from the nature of the swing curves. If the curves settles at the pre-fault level or some new level after some times, the system is stable. However, if the rotor angle increases continuously with time, the system is unstable [35], [37].

Long-term stability forms the transition between transient stability and steady state stability [27], [31], [32], [34].

When a fault occurs, the system may become unstable in the dynamic process and separate into several parts [4]. Obviously, the system is unreliable under this situation. Hence, it is important and necessary to evaluate the system reliability based on both the dynamic and static behavior of the system. In real systems, transient faults contribute to most of the total faults [8]. The system may also lose stability under transient faults. It is evident that the power quality will be affected and even the load may be shed [15], [17]. The reclosing time together with fault duration are used to include the impacts of both transient and permanent faults [19], [23]. In reliability analysis, when transient stability is considered, the disturbances are referred to as permanent faults [25], [28].

Transient stability analysis can be used for dynamic analysis over time period from few seconds to few minutes depending on the time constant of the dynamic phenomenon modeled. In the past, transient stability has been evaluated using time domain (TD) approach. If the system could survive for the first swing, i.e stable in the first swing it will generally remain stable in the following swings [4], [27]. TD approach is found to be time consuming and inefficient for evaluating stability for a large system where the system component vary dynamically and yet, repeated simulation has to be made. This has encouraged the expansion of various transient stability assessments, such as Extended Equal Area Criterion (EEAC) [35], [37], Direct Method of Lyapunov Function [7], [9], transient Energy Function (TEF) [9], [10], [11], Decision Tree Transient Stability Method [2], Composite Electromechanical Distance (CED) Method [2] and others.

A direct method of transient stability analysis of a multi-machine power system using extension of EAC has been proposed by [2]. [1], [15] have indepth details on EEAC method for multi machine system transient stability. Direct method of Lyapunov function or TEF has been used by [1], [5], [14], [15], [18], [26], [27]. [4] has provided further explanations on the concept of direct method of Lyapunov / energy function

2. CLASSIFICATION OF POWER SYSTEM STABILITY.

It is based on the following [4], [8], [9]:

(a) The physical nature of the resulting mode of instability as indicated by the main system variable in which instability can be observed.

(b) The size of the disturbance considered which influences the method of calculation and prediction of stability.

(c) The devices, processes, and the time span that must be taken into consideration in order to assess stability.

Power system is a highly non-linear system that operates in a constantly changing environment.

According to [5], [3], [2], power system stability can be classified into the following :

2.1. Rotor Angle Stability

It is the ability of interconnected synchronous machines of a power system to remain in synchronism. The stability problem involves the study of the electromechanical oscillations inherent in power systems [1], [7], [11]. According to [7], a fundamental factor in this problem is how the outputs of synchronous machines vary with respect to their rotors oscillations. A brief discussion of synchronous machines characteristics is helpful to develop the basic concepts of stability [23], [25], [29].

A synchronous machine has two essential circuits: the field, which is on the rotors, and the armature, which is on the stator. The field winding is supplied by direct current power while the terminals of the armature provide the load power. The rotating magnetic filed of the field winding induces alternating voltages when the rotor is driven by a prime mover (turbine). The frequency of the induced voltages depends on the speed of the rotor and the number of poles of the machine.The change in electromagnetic torque of a synchronous machines following a perturbation can be resolved into two components [17], [29]:

(a) Synchronizing torque component, in phase with rotor angle deviation
(b) Damping torque component, in phase with the speed deviation.

System stability depends on the existence of both components of torque for each of the synchronous machines. Lack of sufficient synchronizing torque results in a period or non oscillation instability, whereas lack of damping torque in oscillation instability.

As in the case of rotor angle stability, it is useful to classify voltage stability into the following subcategories [5], [7], [9], [11].

(a) Large-disturbance voltage stability refers to the system's ability to maintain steady voltages following large disturbances such as system faults, loss of generation, or circuit contingencies. This ability is determined by the system and load characteristics, and the interactions of both continuous and discrete controls and protections [10], [17]. Determination of large-disturbance voltage stability requires the examination of the nonlinear response of the power system over a period of time sufficient to capture the performance and interactions of such devices as motors, under load transformer tap changers, and generator field-cur-rent limiters. The study period of interest my extend from a few seconds to tens of minutes [11], [13], [27].
(b) Small-disturbance voltage stability refers to the system's ability to maintain steady voltages when subjected to small perturbations such as incremental changes in system load. This form of stability is influenced by the characteristics of loads, continuous controls, and discrete controls at a given instant of time. This concept is useful in determining, at any instants, how the system voltages will respond to small system changes. With appropriate assumptions, system equations can be linearized for analysis thereby allowing computation of valuable sensitivity information as in the case of rotor angle stability [34], [36].

2.2. Voltage Stability:

Voltage Stability refers to the ability of a power system to maintain steady voltages at all buses in the system after being subjected to a disturbance from a given initial operating condition. It depends on the ability to maintain and restore equilibrium between load demand and load supply from the power system. In-stability that may result occurs in the form of a progressive fall or rise of voltage of some buses [9], [10], [17], [19]. A possible outcome of voltages instability is loss of load in an area, or tripping of transmission lines and other elements by their protective systems leading to cascading outages. Loss of synchronism of some generators may result from these outages or from operating conditions that violate field current limit.

(a) Short-term voltage stability involves dynamics of fast acting load components such as induction motors, electronically controlled loads, and HVDC converters. The study period of interest is in the order of several seconds and analysis requires solution of appropriate system differential equations; this is similar to analysis of rotor angle stability. Dynamic modeling of loads is often essential. In contrast to angle stability, short circuits near loads are important. It is recommended that the term transient voltage stability not be used [12], [18], [26].
(b) Long-term voltage stability involves slower acting equipment such as tap-changing transformers, thermostatically controlled loads, and generator current limiters. The study period of interest may extend to several or many minutes, and long-term simulations are required for analysis of system dynamic performance [8], [9], [19]. Stability is usually determined by the resulting outage of equipment, rather than the severity of the initial disturbance. Instability is due to the loss of long-term equilibrium (e.g, when loads try to restore their power beyond the capability of the transmission network and connected generation), post-disturbance stead-state operating point being small-disturbance unstable, or a lack of attraction toward the stable post disturbance equilibrium (e.g, when a remedial action is applied too late.) [41].

2.3 Frequency Stability

Frequency stability refers to the ability of a power system to maintain steady frequency following a severe system upset resulting in a significant imbalance between generation and load. It depends on the ability to maintain/restore equilibrium between system generation and load, with minimum unintentional loss of load [4], [6]. Instability that may result occurs in the form of sustained frequency swings leading to tripping of generating units and/or loads. Severe system upsets generally result in large excursions of frequency, power flows, voltage, and other system variables, thereby invoking the actions of processes, controls, and protections that are not modeled in conventional transient stability or voltage stability studies [19], [26]. These processes may be very slow, such as boiler dynamics, or only triggered for extreme system conditions, such as volts/Hertz protection tripping generators. In large interconnected power systems, this type of situation is most commonly associated with conditions following splitting of systems into islands [12], [24].

3. SINGLE MACHINE INFINITE BUS

According to [40], single machine infinite bus system (SMIB) system is used to demonstrate the fundamental concepts and principles of transient stability when subjected to large disturbances.

To simplify the assessment on transient stability, a classical model of the machine is used. The assumptions made are as follow [4], [39]:

(i) All mechanical power inputs are constant
(ii) Damping or asynchronous power is negligible
(iii) Voltage E behind the transient reactance is constant.
(iv) Loads are represented as constant impedances.

The equation of motion or the swing equation describing the SMIB system is as below [4]:

$$\frac{2H}{\omega_0}\frac{d^2\delta}{dt^2} = P_a = P_m - P_e;\ \frac{d\delta}{dt} = \omega \qquad (1)$$

Where P_m is the constant mechanical power input and P_e is the generator's electrical power output.

4. EQUAL AREA CRITERION (EAC)

The three conditions: Pre-fault, during-fault, post-fault conditions are very significant in the analysis of Equal Area Criterion (EAC). Fault occurrence on one of the transmission lines reduces the electrical power output and accelerates the rotor angle. System kinetic energy eventually builds up until it arrives at the clearing angle δ_{cl}, with acceleration area A_{acc} accumulated. At this instance, the excess of electrical power output decelerates the rotor angle until it reaches a point where the previous stored kinetic energy is totally converted into potential energy, i.e. when the area A_{dec} is equal to area A_{acc} [39], [41].

5. LYAPUNOV THEOREM

Lyapunov's stability theorem [21], [26] states that:

The equilibrium point of a dynamic system dx/dt=f(x) is stable if there exists a continuously differentiable positive definite function V(x) such that dV/dt<0. If the total derivative is negative, then the equilibrium point is said to be asymptotically stable.

Direct method of Lyapunov function only requires the knowledge at the instant when the last operation is carried out [15], [39-41].

The post-fault equation of a simple system is

$$M\frac{d\omega}{dt} = P_m - P_{e\_postfault} = P_m - P_{max\_postfault}\sin\delta \qquad (2)$$

Integrating both sides give the system energy:

$$V = \frac{1}{2}M\omega^2 - P_m(\delta-\delta_{SEP}) - P_{max\_postfault}[\cos(\delta)-\cos(\delta_{SEP})] \qquad (3)$$

The critical energy V_{cr} is evaluated

where $\delta=\delta_{UEP}$, $\omega=0$ as indicated in equation (4).

$$V_{cr} = -P_m(\delta_{UEP}-\delta_{SEP}) - P_{max\_postfault}[\cos(\delta_{UEP})-\cos(\delta_{SEP})] \qquad (4)$$

The stability of the system could be assured if $\delta \leq \delta_{UEP}$ or the system's total energy v is less than the critical energy V_{cr} for $\delta_{SEP} < \delta < \delta_{UEP.}$ If the system exceeds the unstable equilibrium point $\delta_{UEP,}$ the system would continue to build-up the kinetic energy, which gives rise to the velocity. As a result, the rotor angle would accelerate and machine would lose synchronism [15], [40].

6. MULTI-MACHINE SYSTEM

Let us extend the transient stability assessment to the multi-machine system. A classical model of the machines is used in system with mechanical power and electrical power assumed to be constant throughout the transient, and all loads are modeled as constant impedance. The conductance G's and subsceptances B's vary from pre-, during-, to post- fault system configurations. The motion of the *i*-th machine of a multi-machine system reduced to generator internal nodes is described by [27], [36], [37].

$$\frac{d\delta_i}{dt} = \omega_i \,;\, M_I \frac{d\omega_i}{dt} = P_{mi} - P_{ei} \quad i= 1, 2, \ldots, n \tag{5}$$

Where

$P_{ei} = E_i^2 G_{ii} + \sum E_i\, E_j\, E_{ij} \cos(\,\delta_i - \delta_j - \delta_{ij})$

$P_{ei} = E_i^2 G_{ii} + \sum E_i\, E_j\, E_{ij} \cos(\,\delta_{0i} - \delta_{0j} - \delta_{ij})$

For $j = 1, 2, \ldots, n$, $j \neq i$

δ_{0i}, δ_{0j} = initial operating rotor angle.

By solving the non-linear swing equation (1), the transient stability of a power system could be determined. However due to the non-linearity of the differential equations, the solving process is tedious and complicated. Thus the numerical intergration methods have been applied to examine a system's stability. Rotor angle plot is obtained to determine the transient stability. Numerical intergration methods, such as Runge-Kutta methods, are used iteratively to approximate the solution of ordinary differential equations.

7. EXTENDED EQUAL AREA CRITERION (EEAC)

Extended equal-area criterion (EEAC) basically reduces the multi-machine transient stability assessment to the equal-area criterion by decomposing and aggregating the multi-machine system into a two-machine equivalence, and further into a single-machine infinite bus (SMIB) equivalence [6], [39]. In this paper, only a single critical machine, presumed to move apart from the rest, is considered for simplicity.

The multi-machine system is decomposed into a critical machine and (*n*-1) of the remaining machines. The expression of relative motion of the critical machine with respect to the remaining machines in the system is developed in [29], [36].

The following notations were used:

s "critical machines"
a its equivalent, aggregated machine
A the set of all remaining machines

The equivalent inertia coefficients:

Ms=inertia coefficients of the critical machine

$$M_a = \sum_{I \in A} M_i;\; M_{total} = \sum_{i=I}^{n} M_i;\; M = \frac{M_a M_s}{M_{total}} \tag{6}$$

Centre of angles (COA) concept is used to model the equivalent machines and their motions [2]:

δ_s= rotor angle of critical machine

$$\delta_u = M_a^{-1} \sum_{I \in A} M_I \delta_I \tag{7}$$

The motion of the critical machine and the (n-1) remaining machines, which are described by [5]:

$$M_s \frac{d^2\delta_s}{dt^2} = P_{ms} - P_{es}$$

$$\text{For } 1 \in A \tag{8}$$

$$M_l \frac{d^2\delta_l}{dt^2} = P_{ml} - P_{el}$$

The motion of the remaining system A is illustrated by the total sum of all the motion of each remaining machines, which gives the following [5]

$$M_a \frac{d^2\delta_a}{dt^2} = \sum(P_{ml} - P_{el}) \quad \text{for } 1 \in A \tag{9}$$

For further simplification, rotor angle of the remaining machines δ_j are made equivalent to δ_a for $j \in A$.

$$\text{Hence, } \delta_s\text{-}\delta_I = \delta_s\text{-}\delta_a;\ \delta_j\text{-}\delta_I = 0 \text{ for } I, j \in A \tag{10}$$

The electrical power contributed by each system are described as [5], [6]:

$$P_{es} = E_s^2\, Y_{ss} \cos(\theta_{ss}) + \sum_{j\in A, j\neq 1} E_s E_j Y_{sj} \cos(\delta_s - \delta_a - \theta_{sj}) \tag{11}$$

$$P_{el} = E_l^2\, Y_{ll} \cos(\theta_{ll}) + E_l\, E_s Y_{ls} \cos(\delta_a\text{-}\delta_s\text{-}\theta_{ls}) + \sum_{j\in A, j\neq 1} E_l E_j Y_{lj} \cos(\theta_{ls}) \tag{12}$$

To model equivalent SMIB system, the rotor angle is defined as δ=δs-δa [5]
The motion of the equivalent SMIB system is

$$M \frac{d^2\delta}{dt^2} = P_m - P_e \tag{13}$$

Where

$P_m = M_{total}^{-1}(M_a P_{ms} - M_s \sum_{l\in A} P_{ml})$; $P_e = M_{total}^{-1}(M_a P_{es} - M_s \sum_{l\in A} P_{el})$

Now, the equivalent SMIB equation of motion is modeled as follows [5]

$$M \frac{d^2\delta}{dt^2} = P_m - P_e = P_m - [P_c - P_{max} \sin(\delta - v)] \tag{14}$$

Where

$P_e = P_c - P_{max} \sin(\delta\text{-}v) = M_{total}^{-1}(M_a P_{es} - M_s \sum P_{el})$

For $1 \in A$

$P_{max} = M_{total}^{-1}[\ \sum_{l\in A} E_s E_I Y_{sl} \sqrt{(Ms^2} + Ma^2 - 2M_s M_a \cos(2\theta_{sl})]$

$V = \tan^{-1} [\frac{M_{total}}{M_a - M_s} \tan(\theta_{sa})] - \frac{\pi}{2}$

$\tan(\theta_{sa}) = \frac{\sum_{l\in A} B_{sl}}{\sum_{l\in A} G_{sl}} = \frac{\sum_{l\in A} Y_{sl} \sin\theta_{sl}}{\sum_{l\in A} Y_{sl} \cos\theta_{sl}}$

8. REVIEW OF RELATED WORK

In {32}, the improvement of power system transient stability with static synchronous series compensator was presented.

The study applied the static synchronous series compensator (SSSC) to improve transient stability of power system. The mathematical and control strategy of a SSSC is presented to verify the effect of the SSSC on transient stability. The SSSC is presented via variable voltage injection with associate transformer leakage reactance and the voltage source. The series voltage injection model SSSC is modeled into power flow equation, which is used to determine and control the strategy. The work uses machine speed deviation to control it. The swing curve of the three phase faulted power system with and without a SSSC is tested and compared in various cases. The swing curve of system without a SSSC increases monotonically and thus the system can be considered unstable, where as the swing curve of system with a SSSC can be considered stable. SSSC can therefore improve transient stability on power system.

[11] presented a Power Flow and Transient Stability Models of Facts Controllers for voltage and angle stability studies. In the work, transient stability and power flow model of Thyristor Controlled Reactor (TCR) and Voltage Source Inverter (VSI) based Flexible AC transmission System (FACTS) controllers were presented.

[31] presented a method to improve transient stability of power system by Thyristor Controlled Phase Shifter Transformer TCPST). The mathematical model of power system equipped with a TCPST was systematically derived. The parameters of TCPST are modeled into power flow equation and thus it was used to determine control strategy. The swing curves of the three phase faulted power system with and without a

TCPST are tested and compared in various cases. The swing curve of system without a TCPST increases monotonically and thus the system can be considered as unstable whereas the swing curves of system with a TCPST returns to stable equilibrium point. From the simulation results, the TCPST increases transient stability of power system.

In [41], the presentation of a comparative study of the different techniques in assessing transient stability was carried out. The paper discussed the transient stability of a small power system subjected to large disturbances via application of time domain (TD) approach, extended equal area criterion (EEAC) and direct method of Lyapunov function. These three method are used to determine transient stability of a system. Studies have been carried out on the IEEE 14 Bus system and simulation assessment can be conducted on a small power system effectively. In using TD approach, several simulations are required to determine the critical clearing time. EEAC can determine critical clearing angle through a single simulation for any nature of fault, and hence, the system's critical clearing time could be calculated. Direct method of Lyapunov function requires only the knowledge at the last instant of fault clearing to determine transient stability. This method is straightforward but computational requirements to determine the unstable equilibrium point are significant.

9. MODEL DEVELOPMENT

Conssider a classical power system shown below in Figure 1.

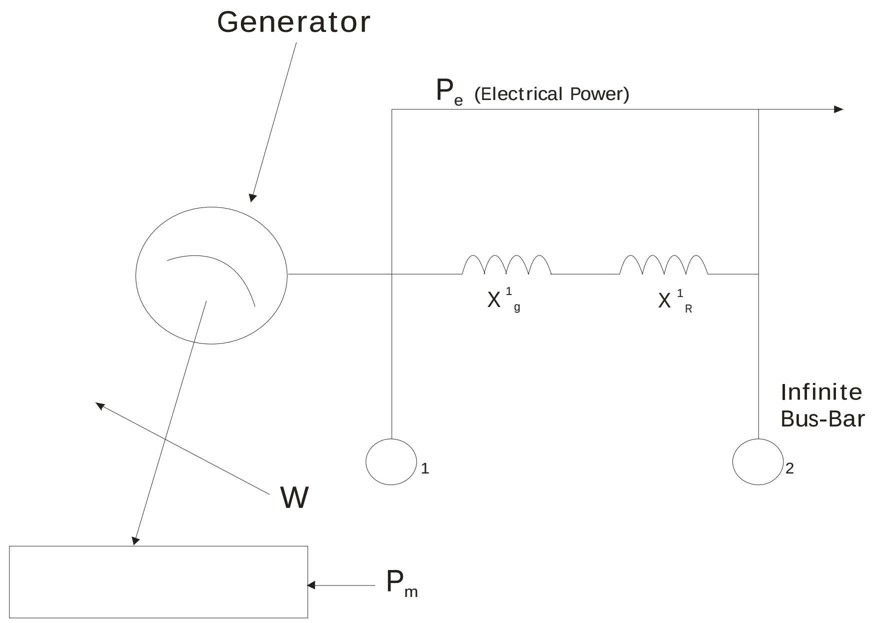

Figure 1. A Classical Power System.

Define $X^1 = X^1_g + X^1_R$ (15)

Where:

X^1_g = transient reactance of synchronous machine.

P_m = mechanical input power.

From Figure 1, in the absence of friction and windage losses,

$P_m = P_e$ in the steady state

The kinetic energy (KE) of a rotating body is given by:

$$K.E = \frac{1}{2} I \boldsymbol{\omega}^2 \text{ joule} \quad (16)$$

Where

I = moment of inertia

$\boldsymbol{\omega}$ = angular velocity in $rad/second$

Or

$$K.E = \frac{1}{2} I\,(\boldsymbol{\omega})\,\boldsymbol{\omega}$$
$$= \frac{1}{2} M\,\boldsymbol{\omega} \text{ joule} \tag{17}$$

Where

M = angular momentum

$M = I\,\boldsymbol{\omega}$

Define H = Inertia constant of the machine

$$H = \frac{\text{stored energy in a machine of Synchronous speed (Mega–Joule)}}{\text{Synchronous machine rating (M V A)}}$$

$$H = \frac{\frac{1}{2} M \omega_S}{G} \tag{18}$$

Where

G = machine rating.

$$M = \frac{2\,GH}{\omega_S} \tag{19}$$

Where

$$\boldsymbol{\omega}_s = 2\pi f\ rad/sec.$$
$$= 360^o f \text{ elect - } deg/sec.$$
$$M = \frac{mVA \times \frac{mJ}{mVA}}{rad/sec}$$
$$= mJ - sec/rad$$
$$M = \frac{2GH}{360\,°f} = \frac{GH}{180°f} = \frac{MVA \times \frac{MJ}{MVA}}{elect. \times sec^{-1}}$$
$$= MJ - sec/deg$$

If a sudden disturbance occurs in the system, the generator experiences an accelerating torque

$$T_a = T_m - T_e \tag{20}$$

Expressing this in terms of power;

$$T_a = T_m - T_e \tag{21}$$

$$P_a = T_a \tag{22}$$
$$= (I\boldsymbol{\alpha})\,\boldsymbol{\omega} = (I\boldsymbol{\omega})\,\boldsymbol{\alpha}$$
$$= M\boldsymbol{\alpha}$$

Where $\boldsymbol{\alpha}$ is the angular acceleration.

Assume that the angular displacement is θ, then

$$\theta = \omega_s + \delta \tag{23}$$

Where δ is the rotor displacement angle.

Differentiate equation (23) with respect to time,

$$\frac{d}{dt}(\theta) = \frac{d}{dt}(w_s + \delta) \tag{24}$$

Differentiate equation (24) with respect to time,

$$\frac{d}{dt}(d\theta) = \frac{d}{dt}\left(\frac{\omega_s + d\delta}{dt}\right)$$

If there is no disturbance in the system,

$$\frac{d}{dt}(\omega_s) = 0$$
$$\frac{d^2\theta}{dt^2} = \frac{d^2\delta}{dt^2} = \boldsymbol{\alpha} \tag{25}$$
$$P_a = \frac{Md^2\delta}{dt^2}$$

$$\frac{Md^2\delta}{dt^2} = P_a = P_m - P_e \tag{26}$$

$$\frac{Md^2\delta}{dt^2} = P_a = P_m - \frac{E'g\ V\sin\delta}{X} \tag{27}$$

Where
Eg[1] = generator internal voltage calculated behind the transient reactance in p.u..
V= voltage at the infinite bus bar.
δ = Rotor displacement angle in radians or degree.
Equation (27) is known as the Transient stability model.
To solve equation (27), the step-by-step approach is used with the following basic assumptions.
(i) The accelerating power (Pa) computed at the beginning of an interval is constant from the middle of the proceeding interval to the interval considered.
(ii) The angular velocity is constant throughout any interval at the value computed for the middle of the interval.
Integrating equation (26) twice over a small interval of time t, yields

$$\Delta\delta = \frac{Pa(\Delta t)}{M} \tag{28}$$

$$\delta = \frac{Pa(\Delta t)2}{2M} \tag{29}$$

$$\Delta t = \sqrt{\frac{2M\Delta\delta}{Pa}} \tag{30}$$

Equations (28) and (30) give the solution of the Transient stability model in terms of change in time and rotor angle respectively. This model is used to assess the stability levels of electrical power system and indicates the behaviour of power systems in the transient state.
Transient stability is judged from the nature of the swing curves which is a plot of the rotor angles against time. If the curves settle at the pre-fault level or some new level after some time, the system is stable. However if the rotor angle increases continuously with time, the system is unstable.

10. SIMULATION

Based on the developed model and the assumptions, a program is written in MATLAB programming language for the computation of the machines rotor angles, electrical power, acceleration power and change in rotor angles. A plot of the swing curve for each of the machine systems is done with this programming language.

11. NUMERICAL RESULTS

The results of the work indicated that for the single machine system supplying an infinite busbar, the rotor angle increases as the time increases. The change in rotor angles increases within the first 0.2500 seconds while it falls after 0.6500 seconds, making the system to be unstable within the first 0.6500 seconds. It becomes stable between 0.6500 seconds and 1.1000 seconds and between 1.100 seconds and 3.250 seconds, it loses its stability. The change in rotor angles as well as the rotor angles increase steadily after 0.70 seconds as the time progresses as shown in Figure 2. The relationship between electrical power and time for a single machine system is shown in Figure 3.

The electrical power and acceleration power for the 2-machine system increase from 0 p.u to 0.1192 p.u and 0 p.u to 0.7200 p.u respectively within the first 0.05 seconds while the rotor angles and change in rotor angles increase from 0 p.u to 6.6672 p.u and 11.46 p.u to 18.1272 p.u respectively as well within the same time frame. The electrical power decrease from 0.1192 p.u to -0.5198 p.u while the acceleration power increases from 0.7200 p.u to 1.5198 p.u after 0.1000 seconds. The rotor angles and change in rotor angles increase steadily as the time progresses as illustrated in Figure 4. While the electrical power and acceleration power fluctuate throughout the time period as shown in Figure 5.

For the multi-machine systems, the electrical power and acceleration power fluctuate as the time progresses with generator 2 with average values of -1.3800 p.u and 2.0900 p.u respectively. Generator 3 has an average electrical power and acceleration power of -1.4900 p.u and 2.1600 p.u respectively while generator 4 has an average electrical and acceleration powers of -0.9700 p.u and 2.0800 p.u respectively with the rotor angles and change in rotor angles increasing continuously with time. for generators 2, 3 and 4. Generator 2 has an average change in rotor angle and rotor angles of 193.61 degrees and 783.01 degrees respectively. Generator 3 has an average change in rotor angles and rotor angles of 268.17 degrees and 473.12 degrees respectively while generator 4 has an average change in rotor angles and rotor angles of 313.16 degrees and 473.12 degrees respectively.

For the multi-machine system, Generator 3 experienced the most violent swing because it is close to the fault, has the largest rotor angle, and a fairly low inertia constant, hence it is expected to be the first to go unstable, hence unreliable. During the first swing generator 3 pulls out of synchronism and it indicate that the system is unstable/ unreliable as shown in Figure 6.

It is also observed from the swing curves that the fault must be cleared within a certain period for the system to be able to regain its stability operating state. The variation between electrical power and time is shown in Figure 7.

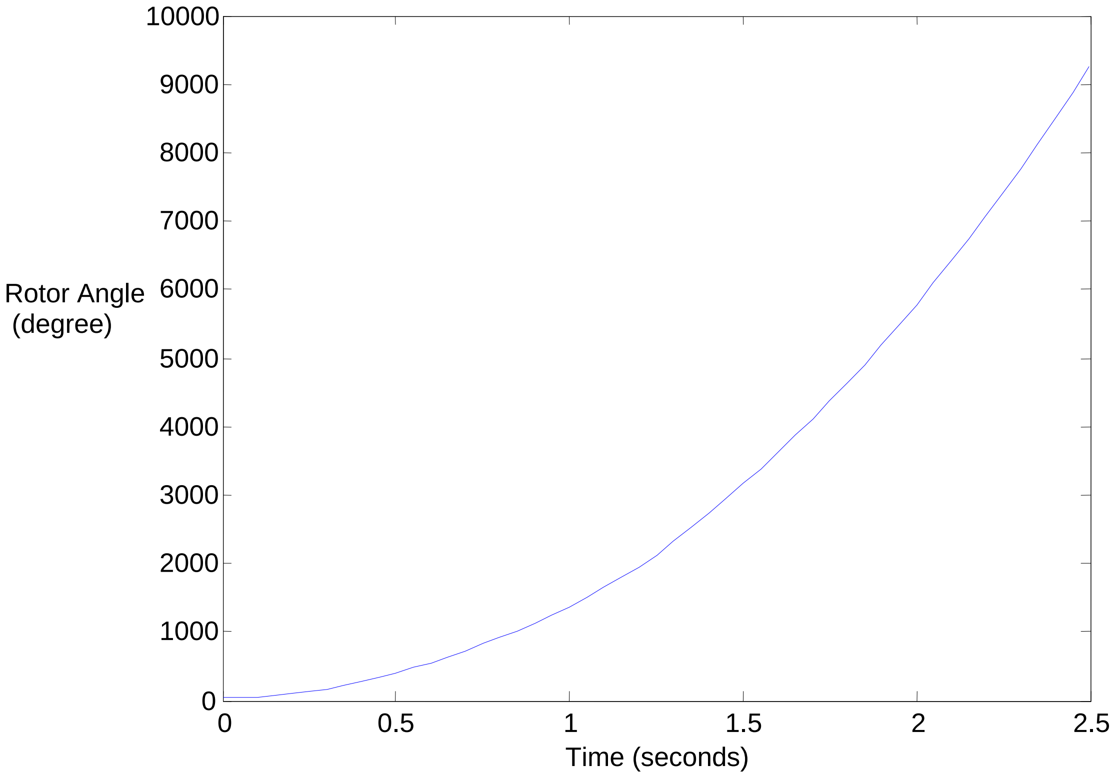

Figure 2. Swing curve of a single machine system.

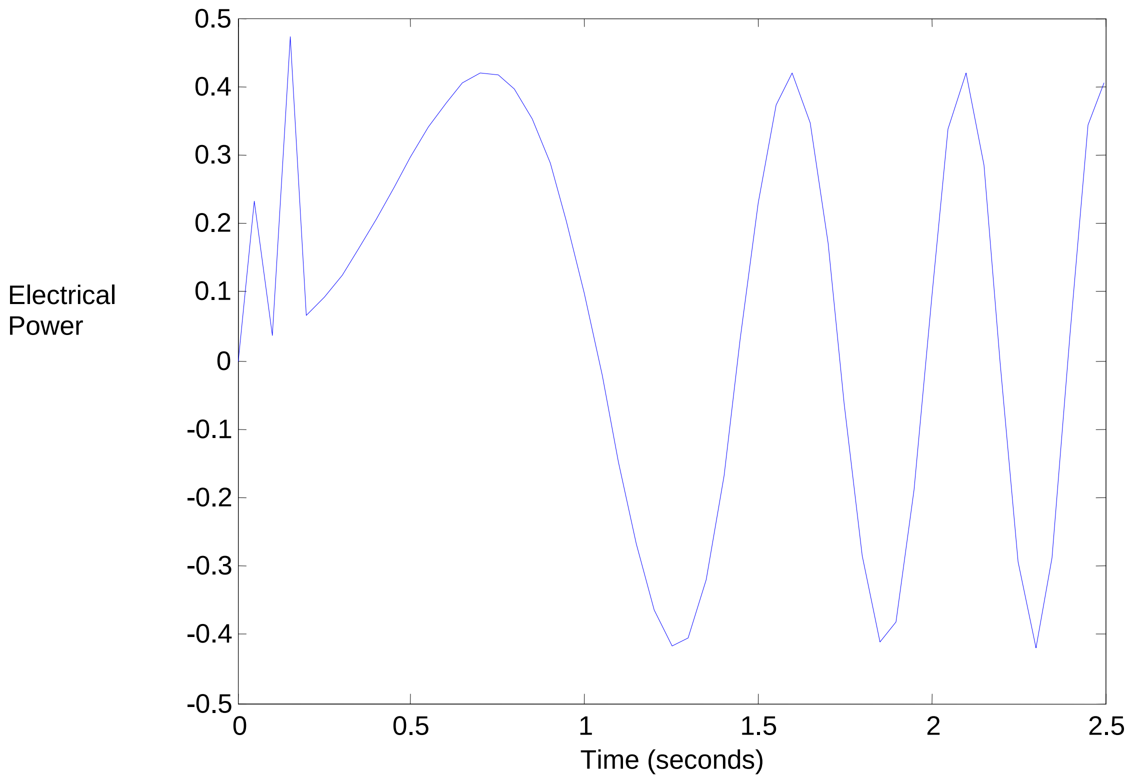

Figure 3. Electrical power against Time for a single machine system.

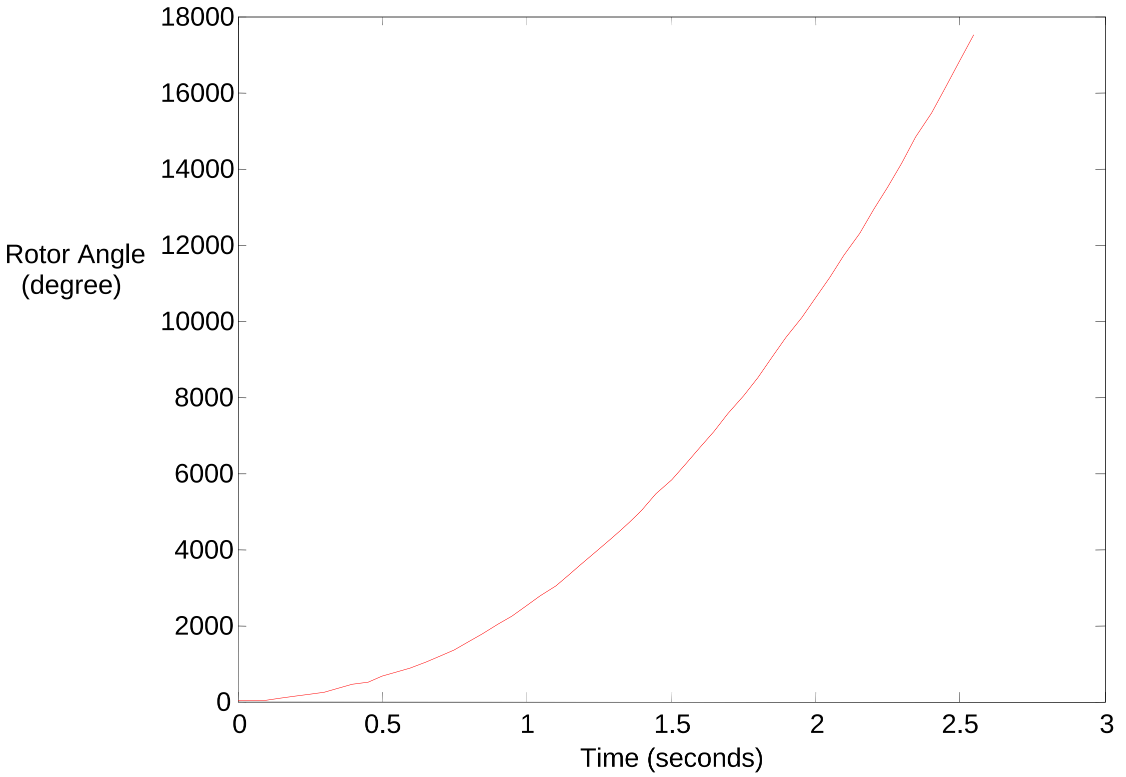

Figure 4. Swing curve of a 2-machine system.

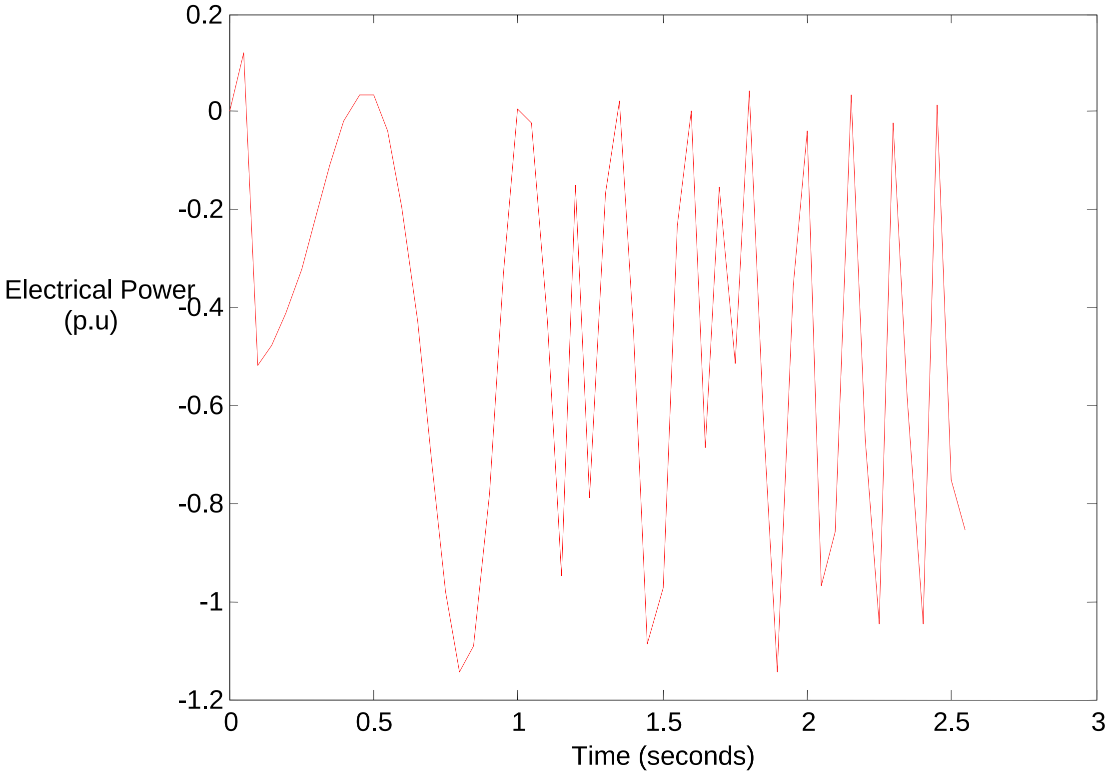

Figure 5. Electrical power against Time for a 2-machine system.

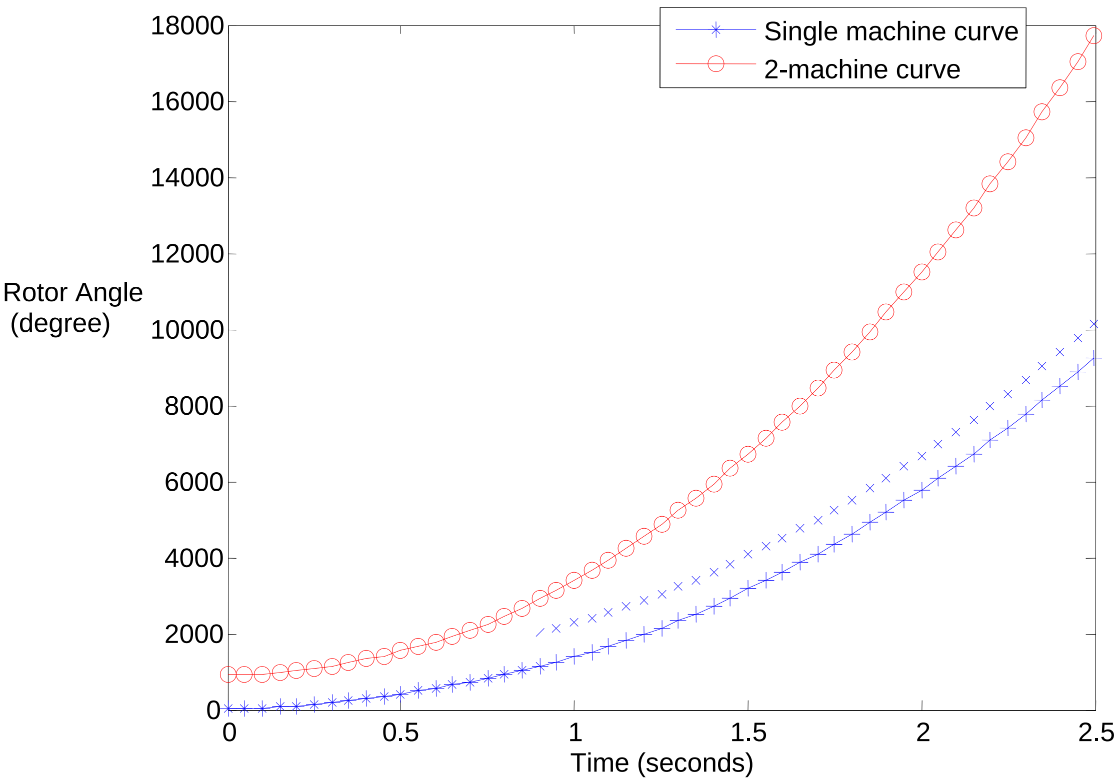

Figure 6. Swing curve for a single machine and a 2-machine system.

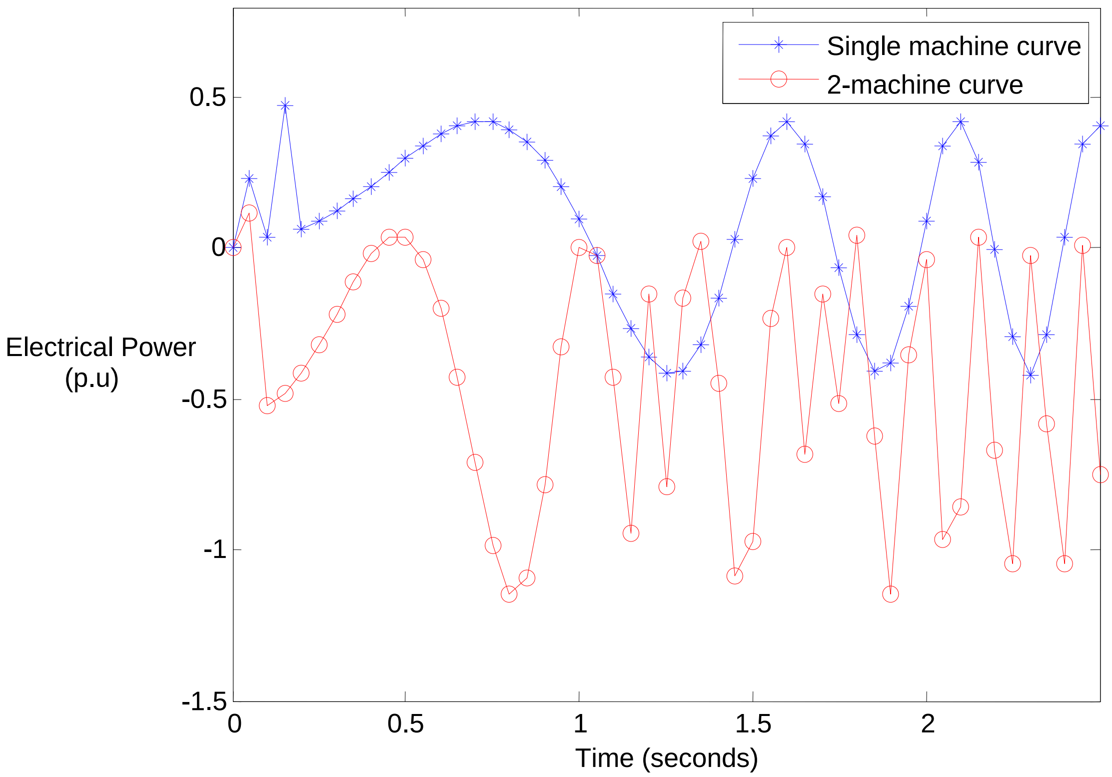

Figure 7. Electrical power against Time for a single machine and a 2-machine system.

12. CONCLUSION

A model has been developed for the transient stability assessment of electrical power system. The computer simulation of a single machine supplying an infinite busbar, a 2-machine system and a multi-machine system have revealed a lot of information about the behavior of power systems in the transient state. During the faulted state, the swing curves increase continuously with time, thus making the system to be unstable. Electrical power and rotor angles which are very important complimentary parameters form the basis upon which the transient stability of power system is judged.

REFERENCES

[1] Ahmed H, El-Abiad E, Nagappan K. “Transient Stability regions of Multi-machine Power Systems”, *IEEE Transactions on Power Apparatus and Systems*, 2011, Vol. PAS-85(2): 169-179.
[2] Ajjarapu V and Christy C. “The Continuation Power Flow: A tool for Steady State Voltage Stability analysis”, *IEEE Trans. Power Systems*, 2010, 7(1): 416-423.
[3] Al-Rawi N.M, Anwar A and Abdul-Majeed D, 2007, “Computer Aided Transient Stability analysis”, *Journal of Computer Science*, 2007, Vol.3:149-153.
[4] Anderson P.M, John A.A and Fouad A.T., “Power System Control and Stability”, John Wiley & Sons, Inc, L0ndon, 2003
[5] Anthony N.M, Fouad A.A, Vijay V. “Power System Transient Stability using individual Machine Energy Functions”, *IEEE Transactions on Circuit and Systems*, 2010, 30(5): 266-276,
[6] Babainejad, S and Keypour, “Analysis of Transient Voltage Stability of a Variable Speed Wind Turbine with Doubly-fed Induction Generator affected by different Electrical Parameters of Induction Generators”, *Trends in Applied Sci*, 2010, 5:251-278.
[7] Begovic M.M and Mills R.Q., “Load identification and Voltage Stability Monitoring”, *IEEE/PES 94 WM 213 -9 PWRS*. New York, 1994.
[8] Byerly R and Kingbark E.W., “Stability of Large Electrical Power Systems”, *Second Edition, IEEE Press*, New York, 2004.
[9] Canizarcs G.A., “On Bifurication; Voltage Collapse and Load Modeling”, *IEEE Trans. Power Systems*; 1995, 10(1): 512-522.
[10] Chetih S and Khiat M, and Chaker A., “Optimal Distribution of the Reactive Power and Voltages Control in Algerian Network using the Genetic Algorithm Method”, Inform Technol J, 7: 1170-1175.,DOL:10.3923/itj2008.1170-1175.

[11] Claudio A.C., “Power Flow and Transient Stability Models of FACTS Controllers for Voltage and Angle Stability Studies”. Third Edition, John Wiley & Sons Inc, London, 2011.
[12] Fink L.H., “Bulk Power System Voltage Phenomena--Voltage Stability and Security”, Third Edition, Khuwer Academic Publishers, New York, 2009.
[13] Fink L.H., “Power System Voltage Voltage Stability and Security”, Fourth Edition, Macmillan, New York, 2010.
[14] Fink L.H., “Power System Voltage Phenomena 111-Voltage Stability and Security”, Fifth Edition, Macmillan, New York, 2011.
[15] Fouad A.A, Stanton S.A., “Transient Stability of a Multi-machine Power System. Part1: Investigation of System. Trajectories; and Part 11: Critical Transient Energy”, *IEEE Transactions on Power Apparatus and Systems*, 2001, Vol. PAS-100(7): 3408-3434.
[16] Gless G.E., “Direct Method of Liapunov Applied to Transient Power Systems Stability”, *IEEE Transactions on Power Apparatus and Systems*, 2006. Vol. PAS-85(2): 159-168.
[17] Hannan M.A and Chan K.W. 2006. “Transient analysis of FACTS and Custom Power Devices using Phasor Dynamics”, *Journal of .Applied sciences*, 2006, 6{10): 1074-1081.
[18] Hill D.J., “Nonlinear Dynamic Load Models with recovery for Voltage Stability Studies”, *IEEE Trans. Power System*. 2011, 9(1):166-176.
[19] Khedkar M.K, Dhole G.M. and Neve V.G., “Transient Stability Analysis by Transient Energy Function Method: and Controlling of Unstable Equilibrium Point Approach”, *IE(I) Journal – EL*, 2004, 85(4):73-87.
[20] Krantz N.U., Gustafsson M.N and Daalder J.E., “VoltageSstability: The Significance of Induction Motor Loads”, Proc. NAPS, 2007, 4(2):394-402.
[21] Krishna, S and Padiyar, K.R., “Transient Stability assessment using Artificial Neural Network”. Proceedings of IEEE International Conference on Industrial Technology, 2009, 2:627-632
[22] Kundur. P., “Power System Stability and Control”, Third Edition, Macmillan, New York, 2004.
[23] Lesicutre B.C, Sauer P.W and Pai M.A, “Transient Algebraic Circuits for Ppower System Dynamic Modelling”, *Journal of Electrical Power and Energy Systems*, 2003, 4(2): 85-93.
[24] Makarov Y.V, Popoiv D.H, Hisken I.A and Hill D.J., “Tap Locking Strategies for Emergency Voltage Control in Power Supply Systems”, Proceedings of the 12th Power Systems Computation Conference, 2006, 309-315.
[25] Mazuzer A.Y. “Fundamentals of Power System Concepts”, *Journal of Electrical Engineering*, 2007, 6(2): 213-247.
[26] Mulukuta S.S (2003): “Synchronous Machines- Their Theory, Stability and Excitation Systems”, Fourth Edition, McGraw- Hill Book Company, New York, 2008
[27] Pai M.A., “Power System stability Analysis by the Direct Method of Lyapunv”, Second Edition, North-Holland Publishing Company, Holland, 2009.
[28] Pavella M and Murthy P.G., “Transient Stability of Power Systems”, Theory and Practice, John Wiley & Sons Inc., New York, 2004.
[29] Popovic D.H, Hiskens I.A and Hill D.J., “Stability Analysis of Induction Motor Nnetworks”, Technical report. University of Sydncy. NSW, Australia, 2009.
[30] Prabha K., “Power System Stability and Control”, Third Edition, McGraw Hill Publishing company, New York, 2004.
[31] Prochanon K., “Improvement of Transient Stability of Power System by Thyristor Controlled Phase Shifter Transformer”, American Journal of Applied Sciences, 2010, 7(1): 1405-1409.
[32] Prochanon K., “Improving Power System Transient Stability with Static Synchronous Series Compensator”, *American Journal of Applied Sciences*, 2011, 8(1): 77-81.
[33] Sanyal, K.K. (2004): “Transient Stability assessment using Neural Network”, *IEEE International Conference on Electric Utility Deregulation, Restructuring and Power Technologies*, Hong Kong, pp 633-637.
[34] Sawhney H and Jeyasurya B, “On-line Transient Stability assessment using Artificial Neural network”, *Large Engineering systems conference on power Engineering*, 2004, 76-80
[35] Silveira A.D.P. “Lotufo C.R and Minussi C.R (2003): “*Transient Stability analysis of Electrical Power System using a Neural Network based on Fuzzy ARTMAP*”, Proc. IEEE Bologna Power Tech Conference, 122-132..
[36] Stagg, G.W and El-Abiab A.H., “Computer Method of Power Systems”, Third Edition, McGraw-Hill Book Company, New York, 2004.
[37] Stevenson, W.D., “Elements of Power System analysis”, Third Edition, Macmillan, New York, 2007
[38] Weed B.M., “Electrical Power Systems”, Third Edition, McGraw- Hill Book Company, New York, 2006
[39] Xue Y., Van T, and Ribbens-pavlla M., “A Simple Direct Method For Fast Transient Stability Assessment of Large Power Systems”, *IEEE Trans on Power Systems*, 1988, 3(2): 400-412..
[40] Xue Y, Van T.C, and Ribbens-pavella M., “Extended Equal Area Criterion Justifications – Generalizations and Applications”, *IEEE Transactions on Power Systems*, 2009, 4(1): 44-52,
[41] Yin C.C, Mohammad A.K, and Michael N., “Transient Stability Assessment of a Small Power System Under the influence of Major Disturbances”, *Journal of Applied Sciences*, 2011, 6(3):68-75.