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Commercial wind turbines modeling using single and composite cumulative probability density functions

Othman A. M. Omar [1], Hamdy M. Ahmed [2], Reda A. Elbarkouky[3]
[1,3]Department Physics and Engineering Mathematics, Faculty of Engineering, Ain Shams University, Egypt
[2]Higher Institute of Engineering, El-Shorouk Academy, El-Shorouk City, Egypt

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ABSTRACT

As wind turbines more widely used with newer manufactured types and larger electrical power scales, a brief mathematical modelling for these wind turbines operating power curves is needed for optimal site matching selections. In this paper, 24 commercial wind turbines with different ratings and different manufactures are modelled using single cumulative probability density functions modelling equations. A new mean of a composite cumulative probability density function is used for better modelling accuracy. Invasive weed optimization algorithm is used to estimate different models designing parameters. The best cumulative density function model for each wind turbine is reached through comparing the RMSE of each model. Results showed that Weibull-Gamma composite is the best modelling technique for 37.5% of the reached results.

Corresponding Author:

Othman A. M. Omar,
Departement of Physics and Engineering Mathematics,
Faculty of Engineering, Ain Shams University,
Abdo Basha square, Abbassia, Cairo11517, Egypt.
Email: Osman_ahmed@eng.asu.edu.eg

1. INTRODUCTION

Modelling of renewable energy resources to maximize the best use of them can be made through using best mathematical modelling techniques which give a great effect on more describing the probabilistic nature of these resources. Wind energy is one of most commonly used renewable energy resources today especially in electrical power engineering. Wind power is transferred into electrical power by the mean of wind turbines. Mathematical modelling of wind turbines results in forecasting the generated electrical power from them and also helps to compare different turbines performance on same site location as a tool for optimal site matching. Each wind turbine has a power curve describing the relation between input wind speed and turbine output electrical power [1]. These curves are varied according to different manufacturers and different generated power ratings. The mathematical models which are used to describe the wind turbines operating power curves will be different. Each wind turbine curve has three main points cut-in speed point (2-5 m/s) at which turbine starts to operate and generate power, rated speed point (7-16 m/s) at which wind turbine generates its rated power and cut-out speed (20-30 m/s) at which turbine is shutdown to prevent it from damage. These ranges of cut-in speed, rated speed and cut-out speed are reached from manufacturers' tests. The most commonly used mathematical models to approximate wind speed-power characteristics curve for the turbines are linear functions [2], quadratic functions [3], cubic functions [4], spline functions [5] and logistic functions [6]. The modelling functions are categorized into piecewise continuous functions and single continuous functions. Authors in [7] reached that modelling wind turbine electrical power curve using single continuous functions is better than using piecewise continuous functions modelling where piecewise modelling techniques give higher errors especially at the rated wind speed point. As using logistic functions in this area of modelling results in very good approximations, so authors used a new proposed modification of a 4-parameters logistic function modelling to give better modelling accuracy than the default 4-parameters logistic functions. Authors in [8] made a comprehensive review about wind turbines power curves modelling techniques. Power curves during modelling are categorized into deterministic modelling and probabilistic modelling. Mathematical curve fitting and artificial intelligence techniques are examples of deterministic modeling [9, 10]. Curve fitting models have two subsections one of them is segmented curve fitting models like linear functions, polynomial functions and exponential functions [11-13]. The second subsection is integrated curve fitting models like double exponential-based functions and logistic functions [14, 15]. The accuracy of different models is described through the following commonly used criteria: mean absolute error (MAE), root mean square error (RMSE) and determination coefficient R^2.

Authors in [16] used nine different logistic functions equations to model six commonly used wind turbines. In [17], authors used weibull cumulative probability density function modelling as a new tool to model different six wind turbines with generation power ranges (2-7 MW). Weibull cumulative density function gives lower RMSE than all types of 4-point logistic functions with simpler modelling equation. In this paper as an extension for previous literature, different single cumulative probability density functions and a new approach of a composite cumulative probability density function are used to model larger scale wind speed and power values. To check these models validity, different 24 commercial wind turbines are modelled. The generated power of these wind turbines covers the range (200 KW-7 MW). The used single cumulative probability density functions for modelling are weibull, gamma and log-normal distribution functions. Also a new composite of weibull – gamma cumulative probability density function is generated and compared with single distribution functions used in this paper to reach more accurate modelling of the different wind turbines. Each wind turbine is modelled using all cumulative density functions mentioned in this paper and finally results are compared to reach more accurate model in each case using RMSE comparison.

2. MATHEMATICAL MODELLING WITH CUMULATIVE DENSITY FUNCTIONS

Mathematical modelling is used in this paper to describe wind turbines output power according to valid wind speed inputs. The used single cumulative density function models are weibull, gamma and log-normal. Also a composite cumulative density function model is used. It consists of two terms, the first term is weibull term with a weighting coefficient and the second is gamma term with another weighting coefficient. The general equation to describe the fitted wind turbine output power p_i at input wind speed v_i is described in (1). $(\theta_1, \theta_2, \ldots, \theta_n)$ are the cumulative density function designing variables which are optimally selected for optimal power curve modelling. If (1) is multiplied by the wind turbine rated output power (P_r) it will transferred to (2) where output power P_i unit will be in (KW).

As indicated in Table 1, weibull cumulative density function has two designing variables which are the shaping parameter (k) and the scaling parameter (c). But the standard cumulative gamma density function has only a shaping parameter (a). The log-normal cumulative distribution function has two designing variables which are the distribution mean (μ) and standard deviation (σ). As a way to reach a more accurate fitting curve the weibull – gamma composite distribution function is generated with five designing variables as indicated in Table 1, where w_1 is the weibull cumulative density function weight in the composite. The designing variables of each model can be reached using least square regression [18, 19] or using function optimization techniques [20]. In this paper invasive weed optimization algorithm [21] is used for optimal selection of each cumulative model designing parameters values. The objective function assigned to the optimization algorithm is the RMSE which should be minimized as an indicator for high accuracy modelling. In (3) shows the objective function which will be minimized, where N is the number of measured data points between input wind speeds with spacing interval 1 m/s and turbine corresponding output powers. n is the number of models designing variables. Wind turbines under study have manufacturers' nameplates and measured tests values as on [22].

$$p_i = F(v_i, \theta_1, \theta_2, \ldots, \theta_n) \quad (\text{p.u.}) \tag{1}$$

$$P_i = P_r \; F(v_i, \theta_1, \theta_2, \ldots, \theta_n) \quad (\text{KW}) \tag{2}$$

Minimize:

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(P_{measured} - P_i(v_i, \theta_1, \theta_2, \ldots, \theta_n))^2} \tag{3}$$

Subject to:

$$\theta_{i-min} \leq \theta_1, \theta_2, \ldots, \theta_n \leq \theta_{i-max} \;, \; i = 1,2,3, \ldots, n \tag{4}$$

Table 1. Cumulative probability density functions modelling equations

Cumulative density function model	Equation	θ_1	θ_2	θ_3	θ_4	θ_5
Weibull	$P_i = P_r\,(1 - e^{-\left(\frac{v_i}{c}\right)^k})$	k	c	–	–	–
Gamma	$P_i = P_r \frac{\int_0^{v_i} t^{a-1} e^{-t}.\,dt}{\Gamma(a)}$	a	–	–	–	–
Log-normal	$P_i = P_r \Phi(\frac{\ln(v_i) - \mu}{\sigma})$	μ	σ	–	–	–
Weibull & Gamma Composite	$P_i = P_r\,(w_1\left(1 - e^{-\left(\frac{v_i}{c}\right)^k}\right) + (1 - w_1)\frac{\int_0^{v_i} t^{a-1} e^{-t}.\,dt}{\Gamma(a)})$	k	c	μ	σ	w_1

3. INVASIVE WEED OPTIMIZATION ALGORITHM (IWO)

Invasive weed optimization algorithm will be used to select the optimal values of the designing parameters for each cumulative density function model. The sequential steps of invasive weed optimization technique are [23]:

- Populations Initialization: populations (seeds) with finite numbers are being dispread over d-dimensional search space with random positions. Where d is number of the designing variables.
- Seeds Reproduction: every seed grows to form a new plant and produces newer number of seeds. The number of seeds of each plant depends on their fitness value while as each plant fitness value increases, number of plant seeds also increases.
- Spatial Dispersal: random selections of each plant seeds and adaptations in the algorithm are made in this part. The new seeds are being randomly scattered over the search space using standard normal distribution functions under variance variations. The standard deviation (σ) of the standard normal random functions will be produced within initial value ($\sigma_{initial}$) and a final value ($\sigma_{final)}$ in every step. During simulation, a nonlinear alteration- modulation index (n) is selected to reach certain satisfactory performance. In (5) shows how the standard deviation (σ_{iter}) at each iteration (iter) is calculated, where $iter_{max}$ is the maximum number of iterations.

$$\sigma_{iter} = \frac{(iter_{max} - iter)^n}{{iter_{max}}^n}(\sigma_{initial} - \sigma_{final}) + \sigma_{final} \quad (5)$$

- Competitive Exclusion: in this process and after maximum number of plants is reached, only the plants with lower fitness can pull out and produce seeds, others are being discarded. The process continues in each iteration till maximum iterations are reached and simply the plant with best fitness is the closest to the optimal solution. IWO algorithm flow chart is as indicated in Figure 1.

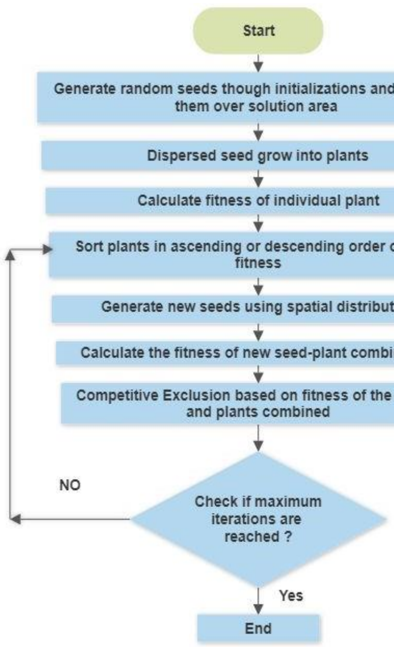


Figure 1. IWO algorithm flow chart

4. RESULTS AND DISCUSSION

Firstly, six commercial wind turbines with electrical output power range from 2 MW to 7 MW are modelled using single and composite cumulative density functions (CDFs) [24, 25]. Different commercial wind turbines with different manufactures are handled. The number of iterations made by IWO is 1000 iterations to get designing parameters values of each cumulative density function model to minimize the RMSE of all models then RMSE of all models are compared. The model with the lowest RMSE is considered as the optimal one. Then, the cumulative density functions models are extended to model wider range of commercial wind turbines to check their validity of modelling. The other commercial wind turbines are categorized into the following ranges in a descending order:
- Wind turbines with generation range from 600 KW to 1000 KW.
- Wind turbines with generation range from 400 KW to 500 KW.
- Wind turbines with generation range from 200 KW to 300 KW.

4.1. Wind turbines with generation range [2 MW-7 MW]

Wind turbines modelled in this section are: Vestas (V80), Siemens (S82), Repower (RE82), Nordex (N90), Siemens (S107) and Vestas (V164). The rating of each wind turbine is as indicated in Table 2. In this section results showed that single weibull cumulative probability density function gives the minimum RMSE for all the six wind turbines, so it considered the optimal model in this generation power range. Weibull- gamma composite gives lower accuracy than single weibull models but it has higher accuracy than both single gamma and single log-normal cumulative density function (CDF) models. Log-normal density function model gives the lowest accuracy. The cumulative density functions models of each wind turbine and actual power curve by manufacturers reached through measured data points are sketched on same axes as shown in Figure 2. Weibull power curves give a very accurate approximations for the actual power curves as indicated in Figure 2, where actual curves and weibull curves are very near to coincide. Error between actual power curves and all models has higher values around the rated speed point of each wind turbine and slightly after cut-in speed point of all wind turbines. In case of all wind turbines under using wiebull modelling curves, error takes very small values for wind speed values higher than wind turbines rated wind speeds.

4.2. Wind turbines with generation range [600 KW-1000 KW]

Wind turbines modelled in this part are An-Bonus 1000/54, Enercon (E53), Leitwind (LTW77-1000), Leitwind (LTW80-850), Leitwind (LTW9-1000) and Vestas (V47) with rated powers shown in Table 3. Through results, weibull cumulative probability density (CDF) gives lowest RMSE and higher accuracy in four out of six of the wind turbines in this generation range. The wind turbines best modelled using weibull CDF are Leitwind (LTW77-1000), Leitwind (LTW80-850), (Leitwind LTW90-1000) and Vestas (V47). Weibull–gamma composite CDF gives lowest RMSE and higher accuracy in two out of six of the wind turbines in this range. The wind turbines best modelled using weibull–gamma composite is An-Bonus 1000/54 and Enercon (E53). Also from results, the single log-normal CDF is the worst model with highest RMSE values. Each wind turbine actual power curve and most accurate CDF model describing it are sketched on same graph as shown in Figure 3.

4.3. Wind turbines with generation range [400 KW-500 KW]

Danish Wind Tech Windane (DWT34), Enercon (E40)/5.4, Turbowinds T400-34, Vestas (V39), Wespa 500/47 and Windflow 45-500 are the six wind turbines modelled in this part using different CDF models. As shown in Table 4, weibull–gamma CDF gives more accurate results with lower RMSE in four out of six of wind turbines in this power range. The wind turbines best modelled using weibull–gamma composite are Enercon (E40)/5.4, Turbowinds T400–34, Vestas (V39) and Windflow 45-500. Single weibull CDF gives lowest RMSE and higher accuracy in two wind turbines which are DWT34 and Wespa 500/47. The wind turbine (DWT34) has actual power curve differ than usual actual power curves where the data points with wind speeds lower than rated wind speed are intermittent and not smooth. Weibull CDF is the best model for this wind turbine actual curve shape. Actual power curves and most accurate CDF model describing each turbine is sketched on same graph as shown in Figure 4.

4.4. Wind turbines with generation range [200 KW-300 KW]

Wind turbines modelled in this section are: Leitwind LTW42 – 250, Windmaster Hmzwm300/25, Vergnet-C275/30, Vestas (V27), Wespa 200/31 and Norwin 29-STALL-225 (Norwin225). The rating of each wind turbine is as indicated in Table 5. All wind turbines in this part have cut-out speed (25 m/s) except Hmzwm300/25. Results showed that single weibull CDF gives the minimum RMSE for 50% of wind turbines under study in this section and weibull – gamma composite gives higher accuracy in the rest 50%.

The two wind turbines Wespa 200 and Norwin225 actual power curves are differ in shape compared to other four wind turbines as they have a maximum power point. Weibull-gamma composite is the best model for these two different curves shapes. Also single gamma CDF model gives higher accuracy than single wiebull CDF for modelling these two power curves with lower RMSE. In case of Hmzwm300/25 wind turbine, the best CDF to verify best approximations is weibull-gamma composite then gamma CDF wih intermediate approximations and better accuracy than weibull CDF in this case. Log-normal CDF models give the lowest accuracy on most of results. Actual power curves and most accurate CDF models describing each case are sketched on same graph as shown in Figure 5.

5. CONCLUSION

Results showed that single and composite CDF modelling for 24 commercial wind turbines with wide range of rated powers and rated wind speeds are good mathematical modelling tools which could be used for optimal site matching selections. In the range [2MW-7MW], single weibull CDF is the optimal model for all wind turbines under study. Weibull-gamma composite CDF modelling accuracy started to increase as turbine rated power decreases. In the range [600KW-1000 KW], weibull-gamma composite CDF gives best modelling results for two out of six of wind turbines in this manufacturing power range. In the manufacturing power range [400KW-500 KW], four wind turbines have optimally modelled with weibull-gamma composite CDF. Both single weibull and weibull-gamma composite have equal share of optimal modelling for wind turbines with rated power range [200 KW-300 KW]. Weibull-gamma composite is the optimal model to describe wind turbines actual power curves which containing maximum power points like Wespa 200 and Norwin225. Weibull-gamma composite CDF proofs its superiority as a mathematical tool to best fit 37.5% of the 24 wind turbines power curves under study.

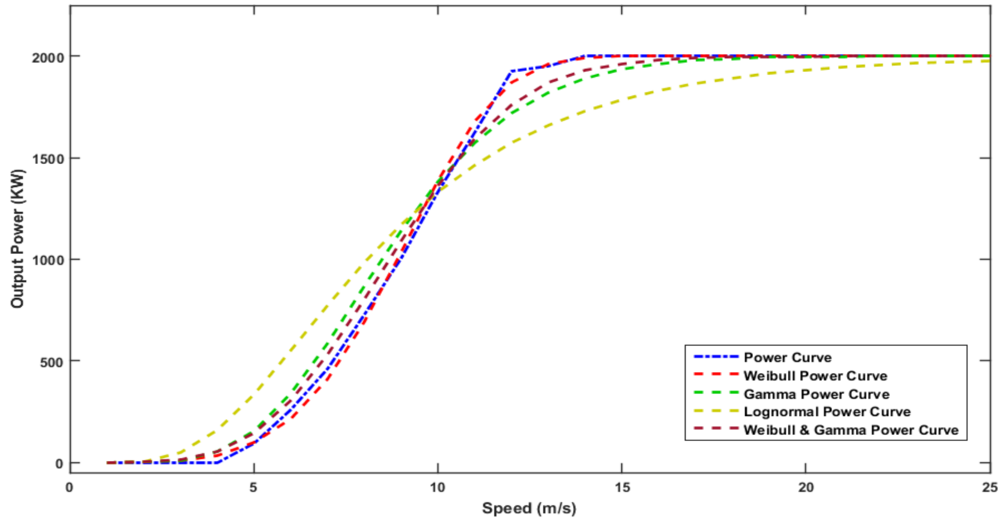
APPENDIX

Table 2. Single and composite cumulative density functions models result for power range [2 MW–7 MW]

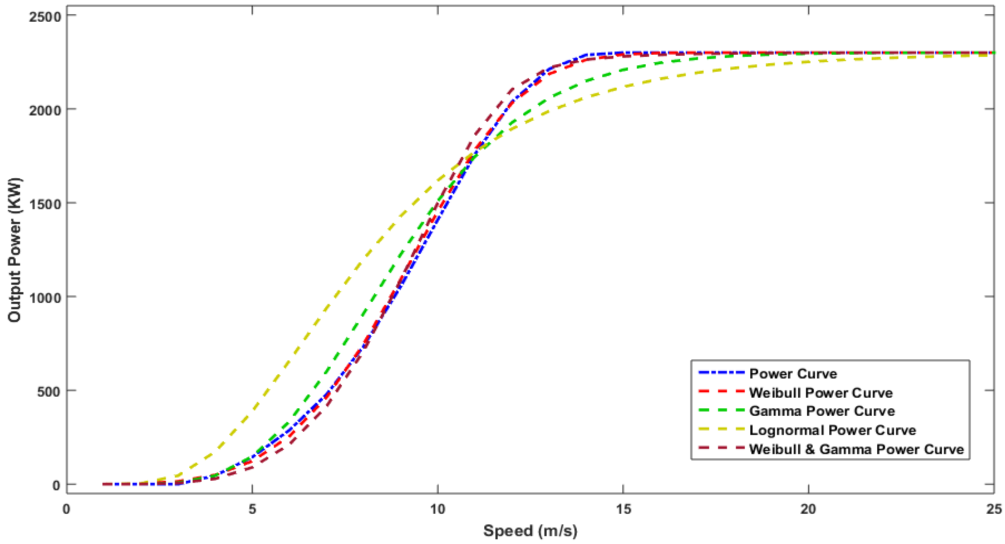
Model	Parameters	V80 (2000KW)	S82 (2300KW)	RE 82 (2050KW)	N90 (2300 KW)	S107 (3600 KW)	V164 (6995 KW)
Weibull	k	4.5786415	4.1929594	4.557548	4.1613744	4.6331647	4.3835995
	c	9.6420951	10	9.4212791	9.9995814	9.6739173	9.4674353
	RMSE	**26.3209**	**17.5502**	**28.5429**	**15.8745**	**42.3526**	**51.2118**
Gamma	a	8.8108292	9.0903382	8.6008824	9.0863184	8.8427663	8.6258787
	RMSE	77.0521	78.6602	70.6185	76.2077	129.9588	290.2388
Log-normal	μ	2.0895277	2.1019618	2.0692422	2.1266333	2.0833491	2.0347969
	σ	0.4979363	0.8466739	0.4084169	0.404273	0.4425731	0.5
	RMSE	177.7675	202.9696	121.4790	137.8745	261.9741	650.0082
Weibull & Gamma Composite	k	3.9559485	5.4499193	3.6516703	4.4999684	5.4750602	4.2696103
	c	9.6805698	9.9910549	9.5393878	9.9524426	9.5816836	9.4758624
	a	8.8530`694	8.7451374	8.6501366	9.2805681	7.4819563	8.8373
	w_1	0.5066826	0.7247037	0.9908485	0.9920755	0.9997306	0.9772746
	RMSE	52.1191	39.5255	31.3164	27.3973	53.9815	65.3657

Table 3. Single and composite cumulative density functions models result for power range [600 KW–1000 KW]

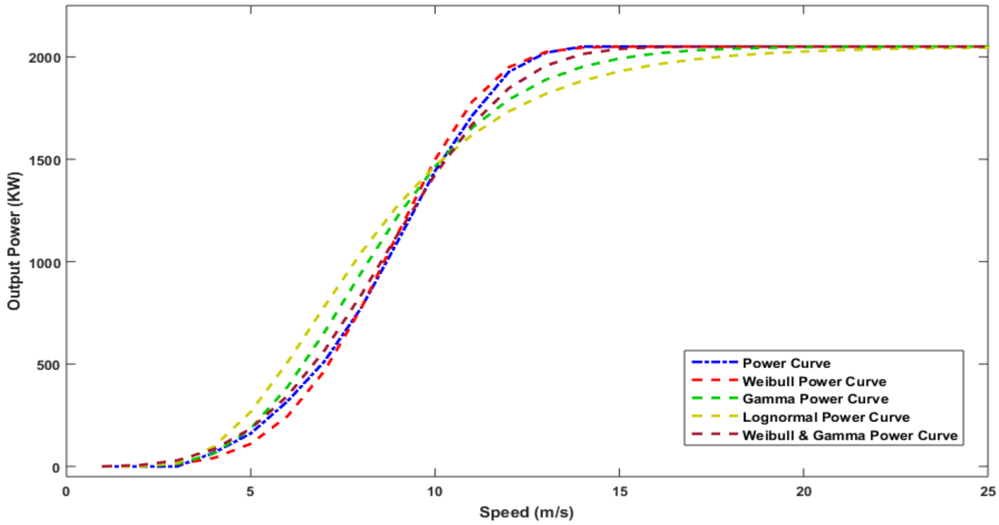
Model	Parameters	V80 (2000KW)	S82 (2300KW)	RE 82 (2050KW)	N90 (2300 KW)	S107 (3600 KW)	V164 (6995 KW)
Weibull	k	3.51399081	3.50978311	3.75144762	5.08338348	4.77729061	4.048118103
	c	10.1682039	9.11191916	7.91437529	7.21305636	6.86532481	9.66602728
	RMSE	12.3277	20.1612	**17.2832**	**12.8621**	**17.4803**	**4.0960**
Gamma	a	9.15195429	8.19993779	7.14830941	6.62738716	6.28817718	8.767931229
	RMSE	27.6991	33.9007	45.4558	43.4489	55.9566	17.2480
Log-normal	μ	2.17761678	2.08182163	1.84885661	1.87383548	1.71654983	2.163028368
	σ	0.27096470	0.20919276	0.48594758	0.19408070	0.49933960	0.128639362
	RMSE	28.6595	33.3016	79.0039	30.5036	83.3064	51.5041
Weibull & Gamma Composite	k	4.67171616	5.19487854	4.57082859	5.39986899	5.49695513	5.486421942
	c	10.3782582	8.99324919	7.84314606	6.88081072	6.91339074	9.602042169
	a	8.71934719	7.90422764	7.13052487	6.88631243	6.07089206	8.529169868
	w_1	0.55869607	0.79811014	0.49833116	0.47644887	0.69884878	0.713736952
	RMSE	**9.9728**	**13.5948**	23.9242	23.9750	18.9921	12.3724



V80 – Actual power curve and CDF power models

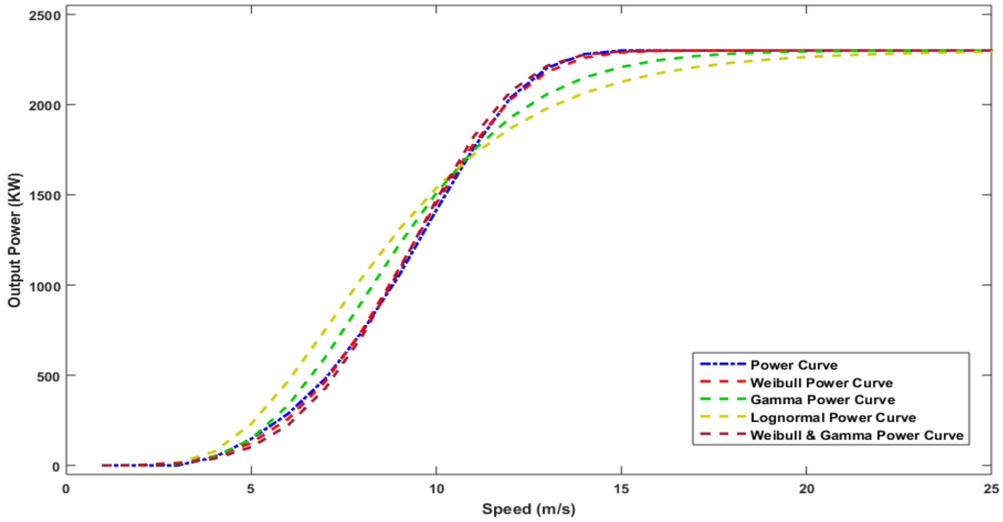


S82 – Actual power curve and CDF power models

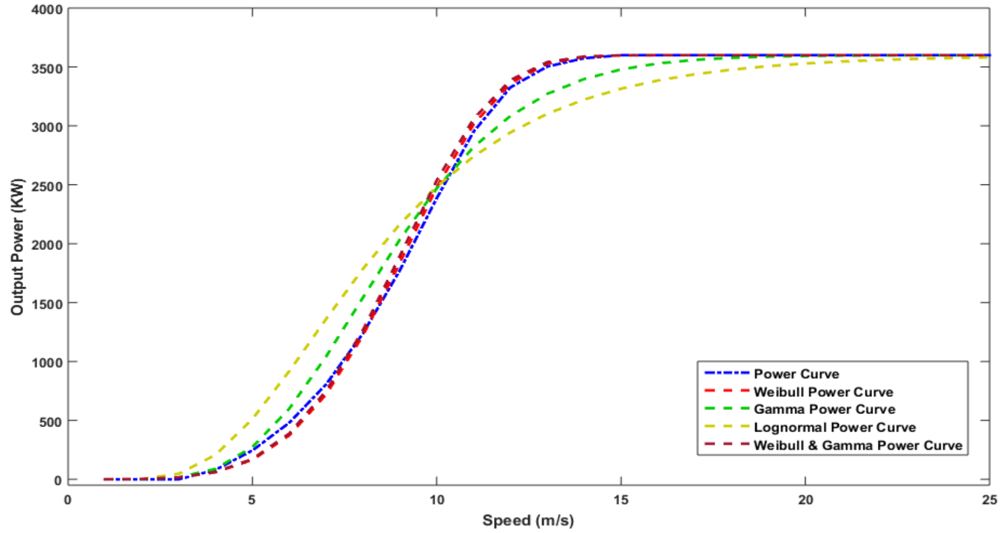


RE82 – Actual power curve and CDF power models

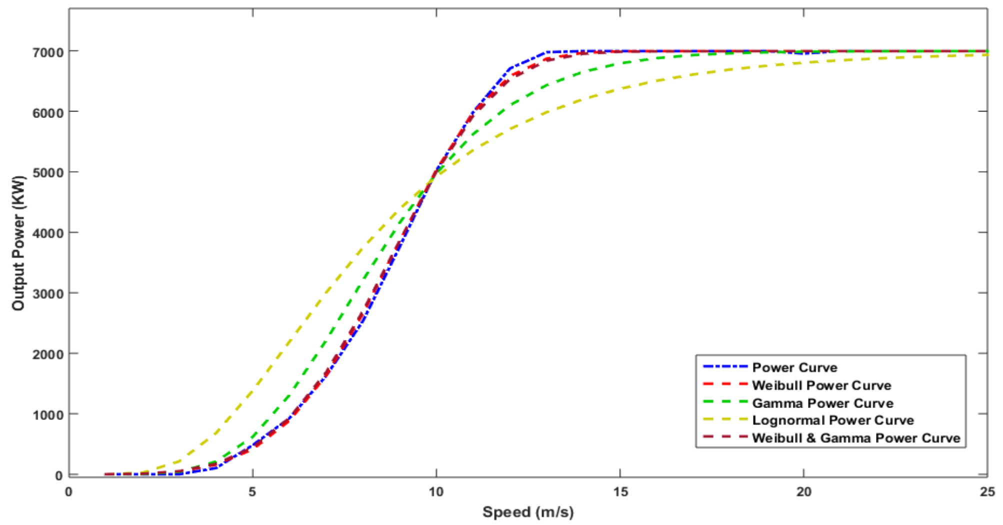
Figure 2. Wind turbines actual power curves and CDF power models with output power range [2 MW-7 MW]



N90 – Actual power curve and CDF power models



S107 – Actual power curve and CDF power models



V164 – Actual power curve and CDF power models

Figure 2. Wind turbines actual power curves and CDF power models with output power range [2 MW-7 MW] (*continue*)

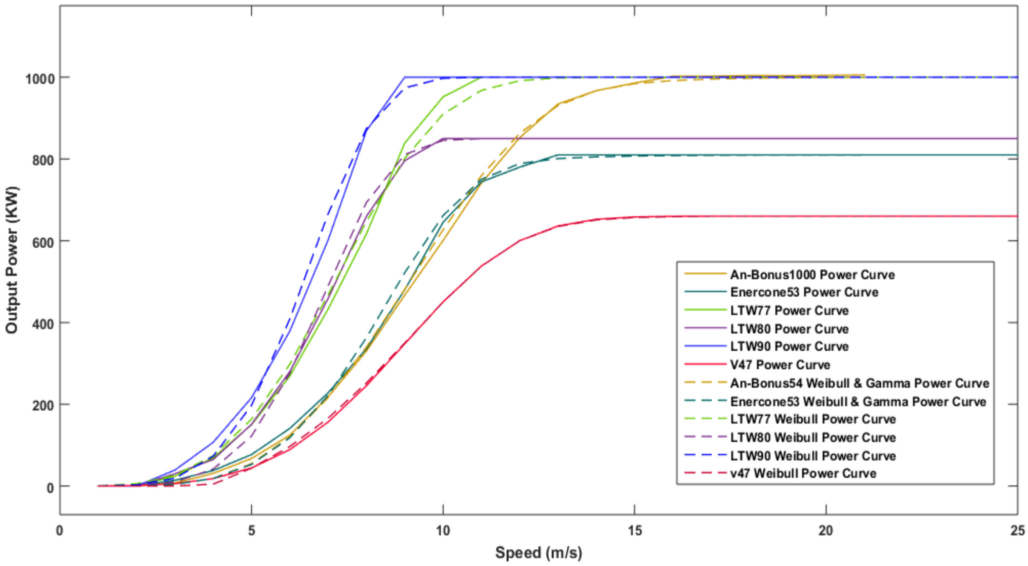


Figure 3. Wind turbines actual power curves and their optimal CDF models for power range [600 KW-1000 KW]

Table 4. Single and composite cumulative density functions models results for power range [400 KW-500 KW]

Model	Parameters	DWT34 (400 KW)	E40 (500 KW)	T400 (400KW)	V39 (500KW)	Wespa500 (500KW)	Windflow45 (500KW)
Weibull	k	5.07686715	5.08680599	5.44584277	3.95460524	5.30386838	3.755277291
	c	10.9005567	9.61051239	9.96184430	10.1185051	8.47204202	9.443184015
	RMSE	**6.8053**	12.9906	12.6138	4.9861	**10.3904**	13.3432
Gamma	a	10.0191590	8.833068618	9.18984033	9.16575333	7.80638918	8.526807326
	RMSE	16.6081	20.4273	14.5984	15.2630	27.1224	22.6648
Log-normal	μ	2.22099442	2.02428298	2.05734850	2.05878172	1.95173058	2.056619633
	σ	0.41683316	0.58071723	0.59738347	0.58954574	0.45376877	0.413980454
	RMSE	31.8266	55.6390	46.4912	56.1114	42.2353	35.8623
Weibull & Gamma Composite	k	5.44983184	5.24910190	4.79628596	4.40221460	4.43987452	4.070479611
	c	10.8146725	9.57014069	10.1764332	10.2524061	8.76400099	9.354395881
	a	10.1737986	8.87547077	8.70697015	8.44657368	7.10032993	8.689891341
	w_1	0.79604839	0.67676089	0.78859391	0.79961165	0.79156114	0.799203941
	RMSE	7.3912	**11.3769**	**5.4756**	**3.9539**	12.2600	**11.1905**

Table 5. Single and composite cumulative density functions models results for power range [200 KW-300 KW]

Model	Parameters	LTW42 (250 KW)	Hmzwm300 (300 KW)	Vergnet275 (275 KW)	V27 (225KW)	Wespa200 (200 KW)	Norwin225 (225KW)
Weibull	k	4.0697689	3.56446705	4.73408344	3.93238423	5.2200093	4.0108782
	c	7.3715829	11.9992984	9.44229627	9.71250815	9.4369866	11.004852
	RMSE	**7.0131**	15.5949	**4.3829**	**1.7841**	7.7425	5.5275
Gamma	a	6.6875637	10.8173660	8.63979768	8.79714262	8.6861505	9.9749357
	RMSE	15.5454	14.4286	14.0277	7.0280	6.4450	4.0539
Log-normal	μ	1.8802266	2.29442812	1.97136886	2.01716517	1.9512223	2.2091740
	σ	0.2010117	0.49575513	0.64734251	0.58253198	0.6993891	0.4367508
	RMSE	10.7015	34.5871	37.5644	23.9209	25.1937	12.3601
Weibull & Gamma Composite	k	3.6365983	5.49392529	3.67535431	4.78120402	5.4180348	3.8489197
	c	11.581524	11.9992975	9.73174163	9.76929944	9.5559157	11.797809
	a	7.0368648	9.99934563	8.32769061	8.33768211	8.3211904	9.6720970
	w_1	0.6875333	0.75	0.69122968	0.74795646	0.7380425	0.3043273
	RMSE	46.4423	**5.4768**	10.9273	2.3653	**5.8144**	**3.8268**

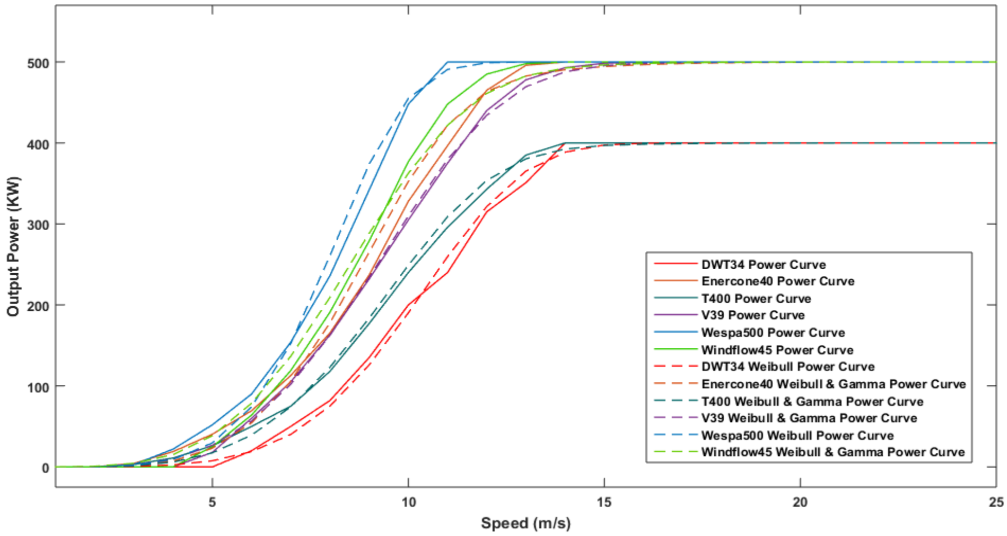


Figure 4. Wind turbines actual power curves and their optimal CDF models for power range [400 KW-500 KW]

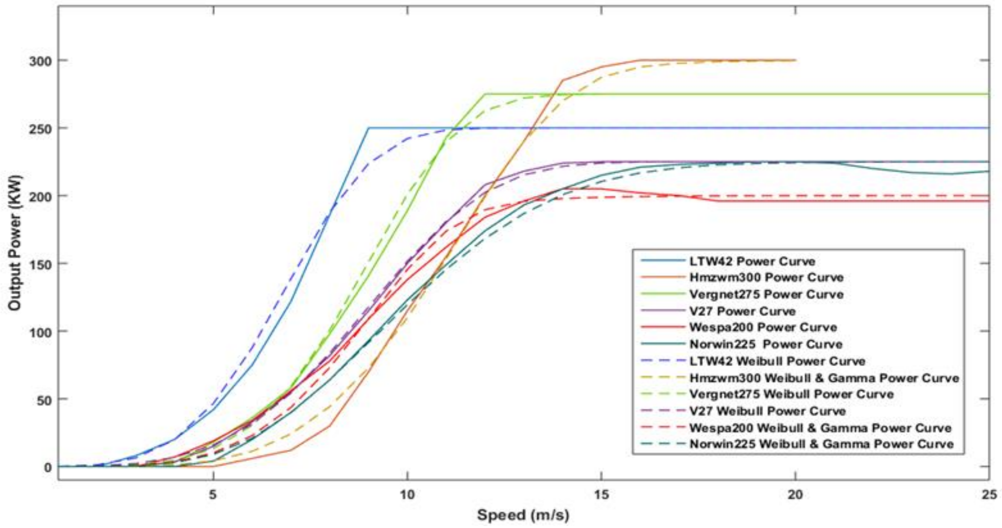


Figure 5. Wind turbines actual power curves and their optimal CDF models for power range [200 KW-300 KW]

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