THE APPLICATION OF A COMPUTER METHOD TO THE ANALYSIS OF A GRIDDED CIRCULAR CURVED FRAME

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By Shyi-Jiun Leou

SYNOPSIS

The analysis of the gridded circular curved frame can be simplified by using the computer method. Formulas are supplied by applying Castigliano's theorem. They show the relationship between the end moments, end torques and the corresponding rotations and displacements of the member end. A proposed matrix method associated with computer analysis is used in the procedure to solve the problem. The use of this method is illustrated by a detailed numerical example.

INTRODUCTION

To meet the requirements of the modern trend in architecture and the clearance and space of some special structures, it seems that there will be an increasing use of gridded curved frameworks.

Because of the relative influence of deformation between curved girders and cross beams throughout the whole framework, the analysis of the gridded curved frame is a difficult task.

In this report, the computer method will be used to analyze such a framework. It allows the greatest possible simplification in the computation required. The member can have any curvaturs in the plan, including segmental, but particular attention will be paid to the common circular-arc shape. Formulas are presented for frames of constant cross section that give the relation between the end moments, torques, and shears, and the corresponding rotations and displacements of the end sections. These relations have been derived from the expression for the total strain energy due to bending and torsion by applying Castigliano's theorem. From these relationships the stiffness matrix of individual members can be written and the stiffness matrix for the whole structure follows. Through computation with the electronic computer the end forces can be obtained. These end forces are combined with the fixed-end forces to obtain final end forces.

STIFFNESS MATRIX OF AN ARC-SEGMENT

A beam curved in plan with end "a" free, and end "b" fixed, is shown in Fig. 1. Assume that the loading and translation are all perpendicular to the plane of the arc-segment.

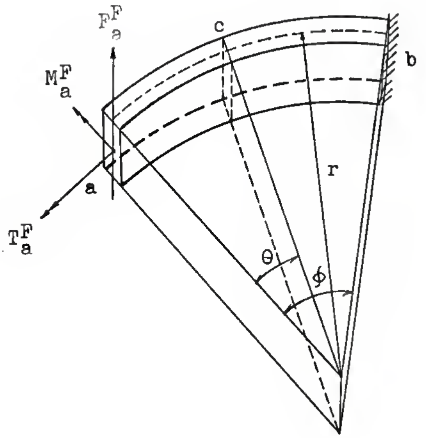


Fig. 1. A beam curved in plan with one fixed and one free end

At end "a" (Fig. 1), a moment M_a^F, a torque T_a^F, and a vertical force F_a^F, are acting on the beam as indicated by the vectors.

The properties of segment ab can be described by the partitioned matrix equation,

$$\begin{Bmatrix} P_a \\ \hline P_b \end{Bmatrix} = \left\{\begin{array}{c|c} K_{ab}^a & K_{ab}^b \\ \hline K_{ba}^a & K_{ba}^b \end{array}\right\} \begin{Bmatrix} D_a \\ \hline D_b \end{Bmatrix}, \qquad \text{------------------------------(1)}$$

where

$$P_a = \begin{Bmatrix} F_a^F \\ M_a^F \\ T_a^F \end{Bmatrix}, \qquad D_a = \begin{Bmatrix} D_{za} \\ R_{ra} \\ R_{ta} \end{Bmatrix},$$

$$P_b = \begin{Bmatrix} F_b^F \\ M_b^F \\ T_b^F \end{Bmatrix}, \qquad \text{and} \qquad D_b = \begin{Bmatrix} D_{zb} \\ R_{rb} \\ R_{tb} \end{Bmatrix}.$$

D_{za} indicates displacement in the direction perpendioular to the plane at end "a". R_{ra} and R_{ta} indicate rotation about radius and tangential axes at end "a" respectively.

Then the K's in the stiffness matrix can be obtained as follows: At any section an angular distance θ from "a" the forces can be expressed as

$$\begin{cases} V_\theta = F_a^F, \\ M_\theta = F_a^F r\sin\theta + M_a^F\cos\theta - T_a^F\sin\theta, \\ T_\theta = F_a^F r(1-\cos\theta) + M_a^F\sin\theta + T_a^F\cos\theta. \end{cases} \qquad \text{------------------------(2)}$$

A sign convention that can be applied to the end of any span is shown in Fig. 2.

The moments are positive when, looking outwards from the center of curvature, they produce a clockwise rotation of the section.

The torques are positive when, looking along the tangent to the beam in a counterclockwise sense, they cause a clockwise rotation of the seotion.

The vertical forces are positive when directed upwards.

Fig. 2. A sign convention that can be applied to the end of any span

The total strain energy stored in the beam dus to the acting forces, neglecting the effect of vertical shear, will be

$$U = \int_0^{\phi} \frac{M_\theta^2 r d\theta}{2EI_r} + \int_0^{\phi} \frac{T_\theta^2 r d\theta}{2GJ}$$

$$= \frac{r}{EI_r}\int_0^{\phi} \left[\frac{M_\theta^2}{2} + \frac{KT_\theta^2}{2}\right] d\theta, \qquad \text{------------------------------(3)}$$

whsre

$$K = \frac{EI_r}{GJ} .$$

EI_r and GJ are the flexible rigidity and torsional rigidity respectively.

The vertioal displacement D_{za} at end "a" in the direction of F_a^F is, by Castigliano's theorem, $D_{za} = \frac{\partial U}{\partial F_a^F}$.

Then

$$\frac{\partial U}{\partial F_a^F} = \frac{r}{EI_r}\int_0^{\phi} \Big[F_a^F r^2 \sin^2\theta + rM_a^F \sin\theta\cos\theta - rT_a^F \sin^2\theta$$

$$+ KF_a^F r^2(1-\cos\theta)^2 + KrM_a^F \sin\theta - KrM_a^F \sin\theta\cos\theta$$

$$+ KrT_a^F \cos\theta - KrT_a^F \cos^2\theta\Big] d\theta.$$

Integrate and get

$$D_{za} = \frac{r^3}{4EI_r} F_a^F \left[2\phi - \sin 2\phi + K(6\phi + \sin 2\phi - 8 \sin\phi)\right]$$

$$+ \frac{r^2}{4EI_r} M_a^F\left[2 \sin^2\phi + 2K(2 - 2\cos\phi - \sin^2\phi)\right]$$

$$+ \frac{r^2}{4EI_r} T_a^F\left[\sin 2\phi - 2\phi + K(4\sin\phi - 2\phi - \sin 2\phi)\right]. \qquad (4)$$

The rotation R_{ra} at "a" in the direction of M_a^F is $R_{ra} = \frac{\partial U}{\partial M_a^F}$.

Then

$$\frac{\partial U}{\partial M_a^F} = \frac{r}{EI_r}\int_0^\phi \left[F_a^F r \sin\theta\cos\theta + M_a^F\cos^2\theta - T_a^F\sin\theta\cos\theta + KF_a^F r \sin\theta\right.$$

$$\left. - KF_a^F r \sin\theta \cos\theta + KM_a^F\sin^2\theta + KT_a^F\sin\theta \cos\theta\right] d\theta.$$

Integrate and get

$$R_{ra} = \frac{r^2}{4EI_r} F_a^F\left[2 \sin^2\phi + 2K(2 - 2\cos\phi - \sin^2\phi)\right]$$

$$+ \frac{r}{4EI_r} M_a^F\left[2\phi + \sin 2\phi + K(2\phi - \sin 2\phi)\right]$$

$$+ \frac{r}{4EI_r} T_a^F\left[2(K - 1)\sin^2\phi\right] . \qquad (5)$$

The rotation R_{ta} at end "a" in the direction of T_a^F is $R_{ta} = \frac{\partial U}{\partial T_a^F}$.

Then

$$\frac{\partial U}{\partial T_a^F} = \frac{r}{EI_r}\int_0^\phi \left[- F_a^F r \sin^2\theta - M_a^F\sin\theta\cos\theta + T_a^F\sin^2\theta + KF_a^F r\cos\theta\right.$$

$$\left. - KF_a^F r\cos^2\theta + KM_a^F\sin\theta\cos\theta + KT_a^F\cos^2\theta\right] d\theta.$$

Integrate and get

$$R_{ta} = \frac{r^2}{4EI_r} F_a^F\left[\sin 2\phi - 2\phi + K(4\sin\phi - 2\phi - \sin 2\phi)\right] + \frac{r}{4EI_r} M_a^F\left[2(K-1)\sin^2\phi\right]$$

$$+ \frac{r}{4EI_r} T_a^F\left[2\phi - \sin 2\phi + K(2\phi + \sin 2\phi)\right] . \qquad (6)$$

If the following substitutions are made in the above equations,

$$a_{11} = \frac{r^3}{4EI_r}\left[2\phi - \sin 2\phi + K(6\phi + \sin 2\phi - 8\sin\phi)\right] .$$

$$a_{12} = \frac{r^2}{4EI_r}\left[2\sin^2\phi + 2K(2 - 2\cos\phi - \sin^2\phi)\right] = a_{21}.$$

$$a_{13} = \frac{r^2}{4EI_r}\left[\sin 2\phi - 2\phi + K(4\sin\phi - 2\phi - \sin 2\phi)\right] = a_{31}.$$

$$a_{22} = \frac{r}{4EI_r}\left[2\phi + \sin 2\phi + K(2\phi - \sin 2\phi)\right].$$

$$a_{23} = \frac{r}{4EI_r}\left[2(K - 1)\sin^2\phi\right] = a_{32}.$$

$$a_{33} = \frac{r}{4EI_r}\left[2\phi - \sin 2\phi + K(2\phi + \sin 2\phi)\right].$$

Equation (4), (5) and (6) can now be expressed in the form of a matrix:

$$\begin{Bmatrix} D_{za} \\ R_{ra} \\ R_{ta} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial U}{\partial F_a^F} \\ \frac{\partial U}{\partial M_a^F} \\ \frac{\partial U}{\partial T_a^F} \end{Bmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{Bmatrix} F_a^F \\ M_a^F \\ T_a^F \end{Bmatrix}, \quad \text{-----(7)}$$

or

$$\begin{Bmatrix} D_{za} \\ R_{ra} \\ R_{ta} \end{Bmatrix} = (A) \begin{Bmatrix} F_a^F \\ M_a^F \\ T_a^F \end{Bmatrix}. \quad \text{-----(8)}$$

Therefore

$$\begin{Bmatrix} F_a^F \\ M_a^F \\ T_a^F \end{Bmatrix} = (A)^{-1} \begin{Bmatrix} D_{za} \\ R_{ra} \\ R_{ta} \end{Bmatrix}. \quad \text{-----(9)}$$

Equation (9) shows that $(A)^{-1}$ is a transformation matrix between displacements and forces at end "a", but this is what the matrix K_{ab}^a in Eq. (1) stands for. So we get

$$K_{ab}^a = (A)^{-1}.$$

If we rewrite Eq. (2) in the matrix form, we get

$$\begin{Bmatrix} V_\theta \\ M_\theta \\ T_\theta \end{Bmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ r\sin\theta & \cos\theta & -\sin\theta \\ r(1-\cos\theta) & \sin\theta & \cos\theta \end{pmatrix} \begin{Bmatrix} F_a^F \\ M_a^F \\ T_a^F \end{Bmatrix}. \text{ ---------------------(10)}$$

Then the stresses at end "b" can be obtained by substituting the values of F_a^F, M_a^F and T_a^F into Eq. (10) and by changing the angle θ into ϕ, or

$$\begin{Bmatrix} F_b^F \\ M_b^F \\ T_b^F \end{Bmatrix} = - \begin{pmatrix} 1 & 0 & 0 \\ r\sin\phi & \cos\phi & -\sin\phi \\ r(1-\cos\phi) & \sin\phi & \cos\phi \end{pmatrix} (A)^{-1} \begin{Bmatrix} D_{za} \\ R_{ra} \\ R_{ta} \end{Bmatrix}. \text{ ----------------(11)}$$

Thus

$$K_{ba}^a = - \begin{pmatrix} 1 & 0 & 0 \\ r\sin\phi & \cos\phi & -\sin\phi \\ r(1-\cos\phi) & \sin\phi & \cos\phi \end{pmatrix} (A)^{-1}. \text{ ---------------------(12)}$$

To determine the values of K_{ba}^b and K_{ab}^b, we keep end "a" of the curved beam in Fig. 1. fixed, and end "b" free. Then a set of stress equations for the section at an angular distance θ from end "b" can be obtained, and are

$$\begin{Bmatrix} V_\theta \\ M_\theta \\ T_\theta \end{Bmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -r\sin\theta & \cos\theta & \sin\theta \\ r(1-\cos\theta) & -\sin\theta & \cos\theta \end{pmatrix} \begin{Bmatrix} F_b^F \\ M_b^F \\ T_b^F \end{Bmatrix}. \text{ ---------------------(13)}$$

Similarly, following the steps from Eq. (3) to Eq. (7), we get

$$\begin{Bmatrix} D_{zb} \\ R_{rb} \\ R_{tb} \end{Bmatrix} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \begin{Bmatrix} F_b^F \\ M_b^F \\ T_b^F \end{Bmatrix}, \text{ ------------------------------(14)}$$

or

$$\begin{Bmatrix} D_{zb} \\ R_{rb} \\ R_{tb} \end{Bmatrix} = (B) \begin{Bmatrix} F_b^F \\ M_b^F \\ T_b^F \end{Bmatrix}.$$

Therefore

$$\begin{Bmatrix} F_b^F \\ M_b^F \\ T_b^F \end{Bmatrix} = (B)^{-1} \begin{Bmatrix} D_{zb} \\ R_{rb} \\ R_{tb} \end{Bmatrix}. \text{------------------------------}(15)$$

Each element of the B matrix in Eq. (14) is equal to each corresponding element of A matrix in Eq. (7), but the signs are not all the same. The elements b_{11}, b_{13}, b_{22}, b_{31} and b_{33} have the same sign as the element a_{11}, a_{13}, a_{22}, a_{31} and a_{33}, but b_{12}, b_{21}, b_{23} and b_{32} have the opposite sign to a_{12}, a_{21}, a_{23} and a_{32}.

Comparing Eq. (15) with Eq. (1), we know that

$$K_{ba}^{b} = (B)^{-1} . \text{-----------------------------------}(16)$$

The stresses at end "a" can be obtained by substituting F_b^F, M_b^F, T_b^F into Eq. (13) and by changing the angle θ into ϕ.

So we get

$$\begin{Bmatrix} F_a^F \\ M_a^F \\ T_a^F \end{Bmatrix} = - \begin{pmatrix} 1 & 0 & 0 \\ -r\sin\phi & \cos\phi & \sin\phi \\ r(1-\cos\phi) & -\sin\phi & \cos\phi \end{pmatrix} (B)^{-1} \begin{Bmatrix} D_{zb} \\ R_{rb} \\ R_{tb} \end{Bmatrix} . \text{-----------}(17)$$

Thus

$$K_{ab}^{b} = - \begin{pmatrix} 1 & 0 & 0 \\ -r\sin\phi & \cos\phi & \sin\phi \\ r(1-\cos\phi) & -\sin\phi & \cos\phi \end{pmatrix} (B)^{-1} . \text{-----------------}(18)$$

As an example the stiffness matrix of a whole structure is presented as follows:

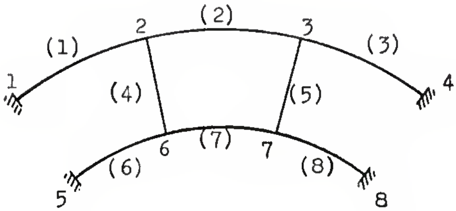


Fig. 3. Plan view of a frame with curved and straight members

Figure 3 indicates a frame with curved and straight members meeting at joints 1, 2, ... , 8. Numbers in parentheses in Fig. 3 indicate the numbering system for the bars. The relationship between member end forces and elastic member end displacements can be indicated by Eq.(19).

$$\{P\} = [K] \{D\} . \tag{19}$$

Vector P in Eq. (19) represents member end forces. Its transpose is

$$\{P\}^T = \lfloor P_1^1, P_2^1, P_2^4, P_2^2, P_3^2, P_3^5, P_3^3, P_4^3, P_5^6, P_6^6, P_6^4, P_6^7, P_7^7, P_7^5, P_7^8, P_8^8 \rfloor . \tag{20}$$

Symbol P_1^1 indicates the forces at end 1(the subscript) of member (1) (the superscript).

For each value of P on the right side of Eq. (20) there will be three components. For example,

$$P_3^2 = \begin{Bmatrix} F_3^2 \\ M_3^2 \\ T_3^2 \end{Bmatrix} . \tag{21}$$

On the right side of Eq.(21) components indicate vertical force, moment and torque at end 3 of member (2).

Vector D in Eq.(19) represents member end displacements. Its transpose is

$$\{D\}^T = \lfloor D_1^1, D_2^1, D_2^4, D_2^2, D_3^2, D_3^5, D_3^3, D_4^3, D_5^6, D_6^6, D_6^4, D_6^7, D_7^7, D_7^5, D_7^8, D_8^8 \rfloor . \text{ -----------(22)}$$

For each value of D on the right side of Eq.(22) there will be three components. For example,

$$D_2^4 = \begin{Bmatrix} D_{z2}^4 \\ R_{r2}^4 \\ R_{t2}^4 \end{Bmatrix} . \text{ -----------------------------------(23)}$$

On the right side of Eq.(23), components indicate vertical displacement, rotation about the radius and rotation about the tangent, respectively, at end 2 of member (4).

The stiffness matrix K in Eq.(19) is an end displacement-end force transformation matrix. It is shown in Eq.(24) in which for example K_{23}^2 is the stiffness matrix for member (2)(the superscript), and it relates the displacements at end 2(the first subscript) and the forces, which are induced by the displacements at end 2, at end 3(the second lower subscript).

$$
[K] = \begin{array}{c} \\ P_1^1 \\ P_2^1 \\ P_2^4 \\ P_2^2 \\ P_3^2 \\ P_3^5 \\ P_3^3 \\ P_4^3 \\ P_5^6 \\ P_6^6 \\ P_6^4 \\ P_6^7 \\ P_7^7 \\ P_7^5 \\ P_7^8 \\ P_8^8 \end{array}
\begin{array}{c} \begin{array}{cccccccccccccccc} D_1^1 & D_2^1 & D_2^4 & D_2^2 & D_3^2 & D_3^5 & D_3^3 & D_4^3 & D_5^6 & D_6^6 & D_6^4 & D_6^7 & D_7^7 & D_7^5 & D_7^8 & D_8^8 \end{array} \\
\left[\begin{array}{cccccccccccccccc}
K_{11}^1 & K_{21}^1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
K_{12}^1 & K_{22}^1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & K_{22}^4 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & K_{62}^4 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & K_{22}^2 & K_{32}^2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & K_{23}^2 & K_{33}^2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & K_{33}^5 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & K_{73}^5 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & K_{33}^3 & K_{43}^3 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & K_{34}^3 & K_{44}^3 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & K_{55}^6 & K_{65}^6 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & K_{56}^6 & K_{66}^6 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & K_{26}^4 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & K_{66}^4 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & K_{66}^7 & K_{76}^7 & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & K_{67}^7 & K_{77}^7 & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & K_{37}^5 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & K_{77}^5 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & K_{77}^8 & K_{87}^8 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & K_{78}^8 & K_{88}^8
\end{array} \right] \end{array}
\quad \text{------(24)}
$$

FIXED-END STRESSES DUE TO UNIFORM LOAD

The end stresses for a beam curved in plan with both ends fixed and loaded uniformly with q per unit length of beam will now be determined.

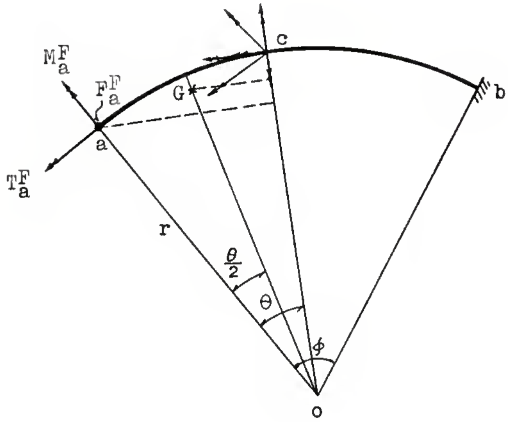


Fig. 4. Beam curved in plan with both ends fixed.

From Fig. 4, the vertical reaction F_a^F at end "a" is equal to

$$F_a^F = \frac{qr\phi}{2} ,$$

and the distance OG from the center of curvature to the center of gravity of arc ac is

$$OG = \frac{r \sin\frac{\theta}{2}}{\frac{\theta}{2}} .$$

At a distance $r\theta$ from end "a",

$$M_\theta = M_a^F\cos\theta - T_a^F\sin\theta + F_a^F r\sin\theta - qr\theta \frac{r\sin\frac{\theta}{2}}{\frac{\theta}{2}}\sin\frac{\theta}{2} ;$$

$$M_\theta = M_a^F\cos\theta - T_a^F\sin\theta + \frac{qr^2\phi}{2}\sin\theta - qr^2(1-\cos\theta) ; \text{-------------}(25)$$

$$T_\theta = M_a^F\sin\theta + T_a^F\cos\theta + F_a^F r(1-\cos\theta) - qr^2\theta(1-\frac{\sin\frac{\theta}{2}}{\frac{\theta}{2}}\cos\frac{\theta}{2}) ;$$

$$T_\theta = M_a^F\sin\theta + T_a^F\cos\theta + \frac{qr^2\phi}{2}(1-\cos\theta) - qr^2(\theta - \sin\theta). \text{-----------}(26)$$

Applying a unit moment $M_a' = 1$ at "a". Then, on the section at an angular distance θ from end "a", $M_\theta' = \cos\theta$ and $T_\theta = \sin\theta$.

As the beam is fixed at end "a",

$$\int_0^\phi M_\theta' M_\theta \frac{rd\theta}{EI_r} + \int_0^\phi T_\theta' T_\theta \frac{rd\theta}{GJ} = R_{ra} = 0 ,$$

or

$$\int_0^\phi \frac{r}{EI_r}\left[M_a^F\cos\theta - T_a^F\sin\theta + \frac{qr^2\phi}{2}\sin\theta - qr^2(1-\cos\theta)\right]\cos\theta\, d\theta$$

$$+ \int_0^\phi \frac{r}{GJ}\left[M_a^F\sin\theta + T_a^F\cos\theta + \frac{qr^2\phi}{2}(1-\cos\theta) - qr^2(\theta-\sin\theta)\right]\sin\theta\, d\theta = 0.$$

Integrate and get

$$M_a^F\left[\phi(K+1) - (K-1)\sin\phi\cos\phi\right] + T_a^F(K-1)\sin^2\phi + qr^2\left[\phi(K+1) + K\phi(\cos\phi + 1)\right.$$

$$\left. - \frac{1}{2}\phi(K-1)\sin^2\phi - 2(K+1)\sin\phi - (K-1)\sin\phi\cos\phi\right] = 0. \text{-------------}(27)$$

If a unit torque $T_a'' = 1$ is applied at end "a", on the section at an angular distance θ from end "a", $M_\theta'' = \sin\theta$ and $T_\theta'' = \cos\theta$. As the beam is fixed at end "a",

$$\int_0^\phi M_\theta'' M_\theta \frac{rd\theta}{EI_r} + \int_0^\phi T_\theta'' T_\theta \frac{rd\theta}{GJ} = R_{ta} = 0 ,$$

or

$$\int_o^\phi \frac{r}{EI_r}\left[M_a^F\cos\theta - T_a^F\sin\theta + \frac{qr^2\phi}{2}\sin\theta - qr^2(1-\cos\theta)\right] (-\sin\theta)d\theta$$

$$+\int_o^\phi \frac{r}{GJ}\left[M_a^F\sin\theta + T_a^F\cos\theta + \frac{qr^2\phi}{2}(1-\cos\theta) - qr^2(\theta - \sin\theta)\right]\cos\theta d\theta = 0.$$

Integrate and get

$$M_a^F(K-1)\sin^2\phi + T_a^F\left[\phi(K+1) + (K-1)\sin\phi\cos\phi\right] - qr^2\left[\frac{1}{2}\phi^2(K+1)\right.$$
$$\left. + \frac{1}{2}\phi(K-1)\sin\phi\cos\phi - (K-1)\sin^2\phi + K\phi\sin\phi - 2(K+1)(1-\cos\phi)\right] = 0. \quad \text{------(28)}$$

From Eq. (27) and (28), the values of M_a^F and T_a^F can be obtained for any known values of the uniform loading q and the angle ϕ. It will be convenient to make the following substitutions in the above equations:

$$\begin{cases} A = \phi(K+1), \\ B = (K-1)\sin^2\phi, \\ C = (K-1)\sin\phi\cos\phi, \\ D = \phi(K+1) + K\phi(\cos\phi + 1) - \frac{1}{2}\phi(K-1)\sin^2\phi, \\ \quad - 2(K+1)\sin\phi - (K-1)\sin\phi\cos\phi, \\ E = \frac{1}{2}\phi^2(K+1) + \frac{1}{2}\phi(K-1)\sin\phi\cos\phi - (K-1)\sin^2\phi \\ \quad + K\phi\sin\phi - 2(K+1)(1-\cos\phi). \end{cases} \quad \text{------(29)}$$

Eq. (27) and (28) can now be expressed in the forms

$$(A-C)M_a^F + BT_a^F + qr^2D = 0, \text{ and} \quad \text{------(30)}$$

$$BM_a^F + (A+C)T_a^F - qr^2E = 0. \quad \text{------(31)}$$

Solving for M_a^F and T_a^F gives

$$M_a^F = -\frac{qr^2\left[D(A+C) + BE\right]}{A^2 - C^2 - B^2}, \text{ and} \quad \text{------(32)}$$

$$T_a^F = \frac{qr^2\left[E(A-C) + BD\right]}{A^2 - C^2 - B^2}. \quad \text{------(33)}$$

FIXED-END STRESSES DUE TO A CONCENTRATED LOAD

The end stresses for a beam curved in plan when both ends are fixed and loaded with a concentrated load located at an angular distance α from end "a" will now be determined.

In Fig. 5, let the displacements at end "a" due to end stresses only be $\left\{D_{FNS}\right\} = \begin{Bmatrix} D_{za} \\ D_{ra} \\ D_{ta} \end{Bmatrix}$, and let the displacements at end "a" due to the acting concentrated load P only be $\left\{D_P\right\}$.

Then according to the fixed-end condition at "a", we get

$$\left\{D_{FNS}\right\} + \left\{D_P\right\} = 0. \qquad (34)$$

By Eq.(8), we get

$$(A) \begin{Bmatrix} F_a^F \\ M_a^F \\ T_a^F \end{Bmatrix} = -\left\{D_P\right\}, \qquad (35)$$

or

$$\begin{Bmatrix} F_a^F \\ M_a^F \\ T_a^F \end{Bmatrix} = -(A)^{-1}\left\{D_P\right\}. \qquad (36)$$

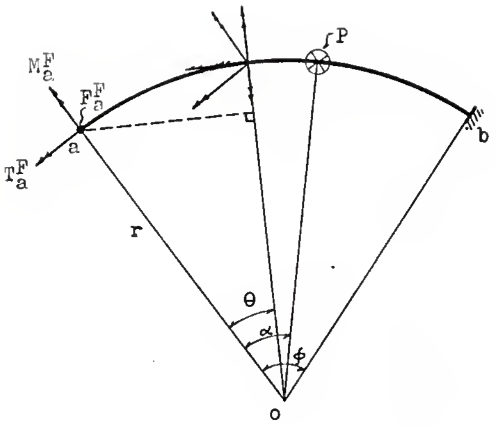


Fig. 5. Plan view of a curved beam with both ends fixed with a concentrated load P acting downward. $0 \leq \theta \leq \alpha$

From Fig. 5, we find that the total stresses on the section at an angular distance θ from end "a", when $0 \leq \theta \leq \alpha$, will be

$$\left.\begin{aligned} V_\theta &= F_a^F, \\ M_\theta &= M_a^F\cos\theta - T_a^F\sin\theta + F_a^F r\sin\theta, \\ T_\theta &= M_a^F\sin\theta + T_a^F\cos\theta + F_a^F r(1-\cos\theta). \end{aligned}\right\} \text{------------(37)}$$

From Fig. 6, the total stresses on the section at an angular distance θ from end "a", when $\alpha \leq \theta \leq \phi$, will be

$$\left.\begin{aligned} V_\theta &= F_a^F - P, \\ M_\theta &= M_a^F\cos\theta - T_a^F\sin\theta + F_a^F r\sin\theta - Pr\sin(\theta - \alpha), \\ T_\theta &= M_a^F\sin\theta + T_a^F\cos\theta + F_a^F r(1-\cos\theta) - Pr\left[1 - \cos(\theta-\alpha)\right] . \end{aligned}\right\} \text{--------(38)}$$

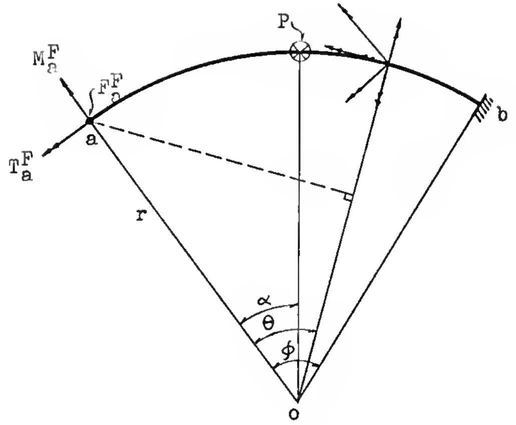


Fig. 6. Plan view of a curved beam with both ends fixed with a concentrated load P acting downward. $\alpha \leq \theta \leq \phi$

If the effect of shear is neglected, the total strain energy is

$$U = \int_0^{\alpha} \frac{M_\theta^2}{2EI_r}\, r d\theta + \int_0^{\alpha} \frac{T_\theta^2}{2GJ}\, r d\theta + \int_{\alpha}^{\phi} \frac{M_\theta^2}{2EI_r}\, r d\theta + \int_{\alpha}^{\phi} \frac{T_\theta^2}{2GJ}\, r d\theta \;. \quad \text{-----}(39)$$

Applying a unit force $F_a' = 1$ at "a". Then, on the section at an angular distance θ from the end "a", $M_\theta' = r\sin\theta$ and $T_\theta' = r(1-\cos\theta)$ when $0 \leq \theta \leq \phi$. As the beam is fixed at end "a",

$$\frac{\partial U}{\partial F_a^F} = D_{za} = \int_0^{\alpha} M_\theta' M_\theta \frac{r d\theta}{EI_r} + \int_0^{\alpha} T_\theta' T_\theta \frac{r d\theta}{GJ}$$

$$+ \int_{\alpha}^{\phi} M_\theta' M_\theta \frac{r d\theta}{EI_r} + \int_{\alpha}^{\phi} T_\theta' T_\theta \frac{r d\theta}{GJ} = 0,$$

or

$$\int_0^{\alpha} \frac{r^2}{EI_r} \left[M_a^F\cos\theta - T_a^F\sin\theta + F_a^F r\sin\theta\right] \sin\theta d\theta$$

$$+ \int_0^{\alpha} \frac{r^2}{GJ} \left[M_a^F\sin\theta + T_a^F\cos\theta + F_a^F r(1-\cos\theta)\right] (1-\cos\theta)d\theta$$

$$+ \int_{\alpha}^{\phi} \frac{r^2}{EI_r} \left[M_a^F\cos\theta - T_a^F\sin\theta + F_a^F r\sin\theta - Pr\sin(\theta-\alpha)\right] \sin\theta d\theta$$

$$+ \int_{\alpha}^{\phi} \frac{r^2}{GJ} \left[M_a^F\sin\theta + T_a^F\cos\theta + F_a^F r(1-\cos\theta) - Pr\ 1-\cos(\theta-\alpha)\right] (1-\cos\theta)d\theta = 0.$$

Collecting terms, we get

$$\frac{r^2}{EI_r} F_a^F\Big[\int_0^{\alpha}(r\sin^2\theta + Kr - 2Kr\cos\theta + Kr\cos^2\theta)d\theta$$

$$+ \int_{\alpha}^{\phi}(r\sin^2\theta + Kr - 2Kr\cos\theta + Kr\cos^2\theta)d\theta\Big]$$

$$+ \frac{r^2}{EI_r} M_a^F\Big[\int_0^{\alpha}(\sin\theta\cos\theta + K\sin\theta - K\sin\theta\cos\theta)d\theta$$

$$+ \int_{\alpha}^{\phi}(\sin\theta\cos\theta + K\sin\theta - K\sin\theta\cos\theta)d\theta\Big]$$

$$+ \frac{r^2}{EI_r} T_a^F\Big[\int_0^{\alpha}(-\sin^2\theta + K\cos\theta - K\cos^2\theta)d\theta$$

$$+ \int_{\alpha}^{\phi}(-\sin^2\theta + K\cos\theta - K\cos^2\theta)d\theta\Big]$$

$$= - \frac{r^2}{EI_r}\Big[\int_{\alpha}^{\phi}(-Pr\sin\theta\sin(\theta-\alpha) - KPr + KPr\cos(\theta-\alpha)$$

$$+ KPr\cos\theta - KPr\cos(\theta-\alpha)\cos\theta d\theta\Big] . \qquad (40)$$

Apply a unit moment $M_a'' = 1$ at "a". Then, on the section at an angular distance θ from the end "a", $M_\theta'' = \cos\theta$ and $T_\theta'' = \sin\theta$ when $0 \leq \theta \leq \phi$.

As the beam is fixed at end "a",

$$\frac{\partial U}{\partial M_a^F} = R_{ra} = \int_0^{\alpha} M_\theta'' M_\theta \frac{r d\theta}{EI_r} + \int_0^{\alpha} T_\theta'' T_\theta \frac{r d\theta}{GJ}$$

$$+ \int_{\alpha}^{\phi} M_\theta'' M_\theta \frac{r d\theta}{EI_r} + \int_{\alpha}^{\phi} T_\theta'' T_\theta \frac{r d\theta}{GJ} = 0,$$

or

$$\int_0^{\alpha} \frac{r}{EI_r} \left[M_a^F\cos\theta - T_a^F\sin\theta + F_a^F r\sin\theta\right] \cos\theta d\theta$$

$$+ \int_0^{\alpha} \frac{r}{GJ}\left[M_a^F\sin\theta + T_a^F\cos\theta + F_a^F r(1-\cos\theta)\right] \sin\theta d\theta$$

$$+ \int_{\alpha}^{\phi} \frac{r}{EI_r}\left[M_a^F\cos\theta - T_a^F\sin\theta + F_a^F r\sin\theta - Pr\sin(\theta-\alpha)\right] \cos\theta d\theta$$

$$+ \int_{\alpha}^{\phi} \frac{r}{GJ}\left\{ M_a^F\sin\theta + T_a^F\cos\theta + F_a^F r(1-\cos\theta) - Pr\left[1-\cos(\theta-\alpha)\right] \right\}\sin\theta d\theta = 0.$$

Collecting terms, we get

$$\frac{r}{EI_r}F_a^F\left[\int_0^{\alpha}(r\sin\theta\cos\theta + Kr\sin\theta - Kr\sin\theta\cos\theta)d\theta + \int_{\alpha}^{\phi}(r\sin\theta\cos\theta + Kr\sin\theta - Kr\sin\theta\cos\theta)d\theta\right]$$

$$+ \frac{r}{EI_r}M_a^F \left[\int_0^{\alpha} (\cos^2\theta + K\sin^2\theta)d\theta + \int_{\alpha}^{\phi} (\cos^2\theta + K\sin^2\theta)d\theta\right]$$

$$+ \frac{r}{EI_r}T_a^F \left[\int_0^{\alpha} (-\sin\theta\cos\theta + K\sin\theta\cos\theta)d\theta + \int_{\alpha}^{\phi} (-\sin\theta\cos\theta + K\sin\theta\cos\theta)d\theta\right]$$

$$= - \frac{r}{EI_r} \int_{\alpha}^{\phi} \left[-Pr\sin(\theta-\alpha)\cos\theta - KPr\sin\theta + KPr\cos(\theta-\alpha)\sin\theta\right]d\theta. \quad (41)$$

Apply a unit torque $T_a''' = 1$ at end "a". Then, on the section at an angular distance θ from the end "a", $M_\theta''' = -\sin\theta$ and $T_\theta''' = \cos\theta$ when $0 \le \theta \le \phi$. As the beam is fixed at end "a",

$$\frac{\partial U}{\partial T_a^F} = R_{ta} = \int_0^{\alpha} M_\theta''' M_\theta \frac{rd\theta}{EI_r} + \int_0^{\alpha} T_\theta''' T_\theta \frac{rd\theta}{GJ} + \int_{\alpha}^{\phi} M_\theta''' M_\theta \frac{rd\theta}{EI_r} + \int_{\alpha}^{\phi} T_\theta''' T_\theta \frac{rd\theta}{GJ} = 0,$$

or

$$\int_0^{\alpha} \frac{r}{EI_r} \left[M_a^F \cos\theta - T_a^F \sin\theta + F_a^F r\sin\theta\right] (-\sin\theta)d\theta$$

$$+ \int_0^{\alpha} \frac{r}{GJ} \left[M_a^F \sin\theta + T_a^F \cos\theta + F_a^F r(1-\cos\theta)\right] \cos\theta d\theta$$

$$+ \int_{\alpha}^{\phi} \frac{r}{EI_r} \left[M_a^F \cos\theta - T_a^F \sin\theta + F_a^F r\sin\theta - Pr\sin(\theta-\alpha)\right] (-\sin\theta)d\theta$$

$$+ \int_{\alpha}^{\phi} \frac{r}{GJ} \left\{ M_a^F \sin\theta + T_a^F \cos\theta + F_a^F r(1-\cos\theta) - Pr\left[1-\cos(\theta-\alpha)\right] \right\} \cos\theta d\theta = 0.$$

Collecting terms, we get

$$-\frac{r}{EI_r} F_a^F \left[\int_0^{\alpha}(-r\sin^2\theta + Kr\cos\theta - Kr\cos^2\theta)d\theta + \int_{\alpha}^{\phi}(r\sin^2\theta + Kr\cos\theta - Kr\cos^2\theta)d\theta\right]$$

$$+ \frac{r}{EI_r} M_a^F \left[\int_0^{\alpha}(-\sin\theta\cos\theta + K\sin\theta\cos\theta)d\theta + \int_{\alpha}^{\phi}(-\sin\theta\cos\theta + K\sin\theta\cos\theta)d\theta\right]$$

$$+ \frac{r}{EI_r} T_a^F \left[\int_0^{\alpha}(\sin^2\theta + K\cos^2\theta)d\theta + \int_{\alpha}^{\phi}(\sin^2\theta + K\cos^2\theta)d\theta\right]$$

$$= - \frac{r}{EI_r} \int_{\alpha}^{\phi} \left[Pr\sin(\theta-\alpha)\sin\theta - Pr\cos\theta + Pr\cos(\theta-\alpha)\cos\theta\right] d\theta. \qquad (42)$$

Eq. (40), (41) and (42) can be written in a matrix form as

$$(Q) \begin{Bmatrix} F_a^F \\ M_a^F \\ T_a^F \end{Bmatrix} = - \begin{Bmatrix} G \\ H \\ S \end{Bmatrix}, \qquad (43)$$

where $\{G, H, S\}^T$ corresponds to the vector D_p in Eq. (35).

Comparing Eq. (43) with Eq. (35), we know that

$$(Q) = (A). \qquad (44)$$

So we get

$$\begin{Bmatrix} F_a^F \\ M_a^F \\ T_a^F \end{Bmatrix} = -(A)^{-1} \begin{Bmatrix} G \\ H \\ S \end{Bmatrix}, \qquad (45)$$

where

$$G = -\frac{r^2}{EI_r} \int_{\alpha}^{\phi} \Big[-Pr\sin\theta\sin(\theta-\alpha) - KPr + KPr\cos(\theta-\alpha) + KPr\cos\theta - KPr\cos(\theta-\alpha)\cos\theta\Big] d\theta,$$

$$H = -\frac{r}{EI_r} \int_{\alpha}^{\phi} \Big[-Pr\sin(\theta-\alpha)\cos\theta - KPr\sin\theta + KPr\cos(\theta-\alpha)\sin\theta\Big] d\theta,$$

$$S = -\frac{r}{EI_r} \int_{\alpha}^{\phi} \Big[Pr\sin(\theta-\alpha)\sin\theta - Pr\cos\theta + Pr\cos(\theta-\alpha)\cos\theta\Big] d\theta.$$

Integrate and get

$$G = -\frac{Pr^3}{4EI_r}\Big\{ (1-K)\cos\alpha(\sin2\phi-\sin2\alpha) + (K-1)\sin\alpha(\cos2\phi-\cos2\alpha) - 2(\phi-\alpha)\Big[(K+1)\cos\alpha + 2K\Big] + 4K\sin(\phi-\alpha) + 4K(\sin\phi-\sin\alpha)\Big\}, \quad \text{------(46)}$$

$$H = -\frac{Pr^2}{4EI_r}\Big[2(\phi-\alpha)(K+1)\sin\alpha + 4K(\cos\phi-\cos\alpha) - \cos\alpha(\cos2\phi-\cos2\alpha)(K-1) - \sin\alpha(\sin2\phi-\sin2\alpha)(K-1)\Big], \quad \text{------(47)}$$

$$S = -\frac{Pr^2}{4EI_r}\Big[2(\phi-\alpha)(K+1)\cos\alpha - 4K(\sin\phi-\sin\alpha) + \cos\alpha(\sin2\phi-\sin2\alpha)(K-1) - \sin\alpha(\cos2\phi-\cos2\alpha)(K-1)\Big]. \quad \text{------(48)}$$

Rewrite Eq.(38) and get

$$\begin{Bmatrix} V_\theta \\ M_\theta \\ T_\theta \end{Bmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ r\sin\theta & \cos\theta & -\sin\theta \\ r(1-\cos\theta) & \sin\theta & \cos\theta \end{pmatrix} \begin{Bmatrix} F_a^F \\ M_a^F \\ T_a^F \end{Bmatrix} - \begin{Bmatrix} P \\ Pr\sin(\theta-\alpha) \\ Pr[1-\cos(\theta-\alpha)] \end{Bmatrix}. \quad \text{--(49)}$$

The fixed-end stresses at end "b" can be obtained by substituting the values of M_a^F, T_a^F, F_a^F into Eq.(49), and changing θ to ϕ. Thus we get

$$\begin{Bmatrix} F_b^F \\ M_b^F \\ T_b^F \end{Bmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ r\sin\phi & \cos\phi & -\sin\phi \\ r(1-\cos\phi) & \sin\phi & \cos\phi \end{pmatrix} (A)^{-1} \begin{Bmatrix} G \\ H \\ S \end{Bmatrix} + \begin{Bmatrix} P \\ Pr\sin(\phi-\alpha) \\ Pr[1-\cos(\phi-\alpha)] \end{Bmatrix}. \quad \text{------(50)}$$

STIFFNESS MATRIX OF A STRAIGHT MEMBER

A straight beam with end "a" free and end "b" fixed is shown in Fig. 7. Assume that the loading and deformation are all in the Z direction.

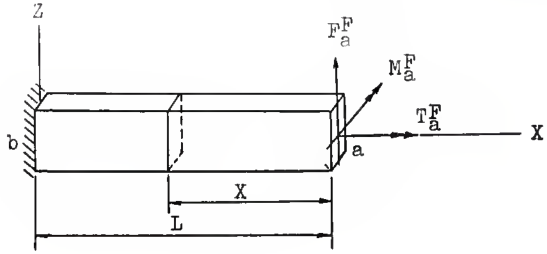


Fig. 7. Straight beam with one fixed and one free end

The properties of segment ab can be described by the partitioned matrix equation,

$$\begin{Bmatrix} P_a \\ \hline P_b \end{Bmatrix} = \left[\begin{array}{c|c} \phi^a_{ab} & \phi^b_{ab} \\ \hline \phi^a_{ba} & \phi^b_{ba} \end{array}\right] \begin{Bmatrix} D_a \\ \hline D_b \end{Bmatrix} \quad \text{------------------------(51)}$$

where

$$P_a = \begin{Bmatrix} F^F_a \\ M^F_a \\ T^F_a \end{Bmatrix}, \qquad D_a = \begin{Bmatrix} D_{za} \\ R_{ya} \\ R_{xa} \end{Bmatrix},$$

$$P_b = \begin{Bmatrix} F^F_a \\ M^F_a \\ T^F_a \end{Bmatrix}, \quad \text{and} \quad D_b = \begin{Bmatrix} D_{zb} \\ R_{yb} \\ R_{xb} \end{Bmatrix}.$$

D_z, R_y and R_x indicate the displacement in the Z direction and rotations about Y and X axis respectively.

Then the ϕ's in the stiffness matrix can be obtained as follows. At any section of a distance x from end "a" the stresses can be expressed as

$$\begin{Bmatrix} V_x \\ M_x \\ T_x \end{Bmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -x & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{Bmatrix} F_a^F \\ M_a^F \\ T_a^F \end{Bmatrix}, \quad \text{------------------------------(52)}$$

The total strain energy stored in the beam due to the acting forces, neglecting the effect of vertical shear, will be:

$$U = \int_0^L \frac{M_x^2 dx}{2EI} + \int_0^L \frac{T_x^2 dx}{2GJ},$$

$$= \frac{1}{2EI}\left[\int_0^L M_x^2 dx + K\int_0^L T_x^2 dx\right],$$

$$= \frac{1}{2EI}\left\{\int_0^L [(M_a^F)^2 - 2M_a^F F_a^F x + (F_a^F x)^2] dx + K\int_0^L (T_a^F)^2 dx\right\},$$

$$= \frac{1}{2EI}\left[(M_a^F)^2 L - M_a^F F_a^F L^2 + \frac{1}{3}(F_a^F)^2 L^3 + KL(T_a^F)^2\right]. \quad \text{---------(53)}$$

By Castigliano's theorem

$$D_{za} = \frac{\partial U}{\partial F_a^F} = \frac{1}{2EI}\left[-M_a^F L^2 + \frac{2}{3}L^3 F_a^F\right], \quad \text{------------------------(54)}$$

$$R_{ya} = \frac{\partial U}{\partial M_a^F} = \frac{1}{2EI}\left[2LM_a^F - L^2 F_a^F\right], \quad \text{------------------------------(55)}$$

$$R_{xa} = \frac{\partial U}{\partial T_a^F} = \frac{1}{2EI} 2KLT_a^F . \quad \text{-------------------------------------(56)}$$

Eq.(54), (55) and (56) can be expressed in the form of a matrix equation

$$\begin{Bmatrix} D_{za} \\ R_{ya} \\ R_{xa} \end{Bmatrix} = \begin{pmatrix} \frac{L^3}{3EI} & -\frac{L^2}{2EI} & 0 \\ -\frac{L^2}{2EI} & \frac{L}{EI} & 0 \\ 0 & 0 & \frac{L}{GJ} \end{pmatrix} \begin{Bmatrix} F_a^F \\ M_a^F \\ T_a^F \end{Bmatrix}, \quad \text{-------------------(57)}$$

or

$$\begin{Bmatrix} F_a^F \\ M_a^F \\ T_a^F \end{Bmatrix} = \begin{pmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 \\ \frac{6EI}{L^2} & \frac{4EI}{L} & 0 \\ 0 & 0 & \frac{GJ}{L} \end{pmatrix} \begin{Bmatrix} D_{za} \\ R_{ya} \\ R_{xa} \end{Bmatrix} . \quad \text{------------(58)}$$

So we get

$$\phi_{ab}^{a} = \begin{pmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 \\ \frac{6EI}{L^2} & \frac{4EI}{L} & 0 \\ 0 & 0 & \frac{GJ}{L} \end{pmatrix} . \quad \text{------------------(59)}$$

To get the stresses at end "b", substitute the value of Eq.(58) into Eq.(52) and let x=L. Then

$$\begin{Bmatrix} F_b^F \\ M_b^F \\ T_b^F \end{Bmatrix} = - \begin{pmatrix} 1 & 0 & 0 \\ -L & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \phi_{ab}^{a} \begin{Bmatrix} D_{za} \\ R_{ya} \\ R_{xa} \end{Bmatrix} , \quad \text{------------(60)}$$

or

$$\begin{Bmatrix} F_b^F \\ M_b^F \\ T_b^F \end{Bmatrix} = - \begin{pmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 \\ -\frac{6EI}{L^2} & -\frac{2EI}{L} & 0 \\ 0 & 0 & \frac{GJ}{L} \end{pmatrix} \begin{Bmatrix} D_{za} \\ R_{ya} \\ R_{xa} \end{Bmatrix} .$$

So we get

$$\phi_{ba}^{a} = - \begin{pmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 \\ -\frac{6EI}{L^2} & -\frac{2EI}{L} & 0 \\ 0 & 0 & \frac{GJ}{L} \end{pmatrix} . \quad \text{------------------(61)}$$

To determine the values of ϕ^{b}_{ba} and ϕ^{b}_{ab}, we keep the end "a" of the beam ab in Fig. 7. fixed and let the end "b" free.

At any section of a distance x from end "b" the stresses can be expressed as in Eq.(62), or

$$\begin{cases} V_x = F^F_b, \\ M_x = M^F_b + F^F_b x, \\ T_x = T^F_b, \end{cases} \quad \text{------------(62)}$$

or

$$\begin{Bmatrix} V_x \\ M_x \\ T_x \end{Bmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ x & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{Bmatrix} F^F_b \\ M^F_b \\ T^F_b \end{Bmatrix}. \quad \text{------------(63)}$$

Similarly following the steps from Eq.(53) to Eq.(57), we can get

$$\begin{Bmatrix} D_{zb} \\ R_{yb} \\ R_{xb} \end{Bmatrix} = \begin{pmatrix} \frac{L^3}{3EI} & \frac{L^2}{2EI} & 0 \\ \frac{L^2}{2EI} & \frac{L}{EI} & 0 \\ 0 & 0 & \frac{L}{GJ} \end{pmatrix} \begin{Bmatrix} F^F_b \\ M^F_b \\ T^F_b \end{Bmatrix}, \quad \text{------------(64)}$$

or

$$\begin{Bmatrix} F^F_b \\ M^F_b \\ T^F_b \end{Bmatrix} = \begin{pmatrix} \frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 \\ -\frac{6EI}{L^2} & \frac{4EI}{L} & 0 \\ 0 & 0 & \frac{GJ}{L} \end{pmatrix} \begin{Bmatrix} D_{zb} \\ R_{yb} \\ R_{xb} \end{Bmatrix}. \quad \text{------------(65)}$$

Thus

$$\phi^{b}_{ba} = \begin{pmatrix} \frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 \\ -\frac{6EI}{L^2} & \frac{4EI}{L} & 0 \\ 0 & 0 & \frac{GJ}{L} \end{pmatrix}. \quad \text{------------(66)}$$

To get the stresses at end "a", substitute the value of Eq.(65) into Eq.(63) and let x=L, then

$$\begin{Bmatrix} F_a^F \\ M_a^F \\ T_a^F \end{Bmatrix} = - \begin{pmatrix} 1 & 0 & 0 \\ L & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \phi_{ba}^{b} \begin{Bmatrix} D_{zb} \\ R_{yb} \\ R_{xb} \end{Bmatrix} . \qquad \text{-----------------(67)}$$

Thus

$$\phi_{ab}^{b} = - \begin{pmatrix} \frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 \\ \frac{6EI}{L^2} & -\frac{2EI}{L} & 0 \\ 0 & 0 & \frac{GJ}{L} \end{pmatrix} . \qquad \text{-----------------(68)}$$

NUMERICAL EXAMPLE

The gridded framework, with straight members and members curved in plan, shown in Fig. 8. is fixed at points 1,4,5 and 8. Points 2,3,6 and 7 are free to rotate and to translate perpendicular to the plane of the frame. The frame could be girders of a bridge. The cross sections A-A, B-B and C-C of Fig. 8. are shown in Fig. 9 and Fig. 10 respectively. The analysis of stresses will be separated into two parts. One is the calculation of stresses due to dead load and the other the caloulation of stresses due to live load. When we do the calculation of stresses due to live load, the method of influence lines will be used in this report.

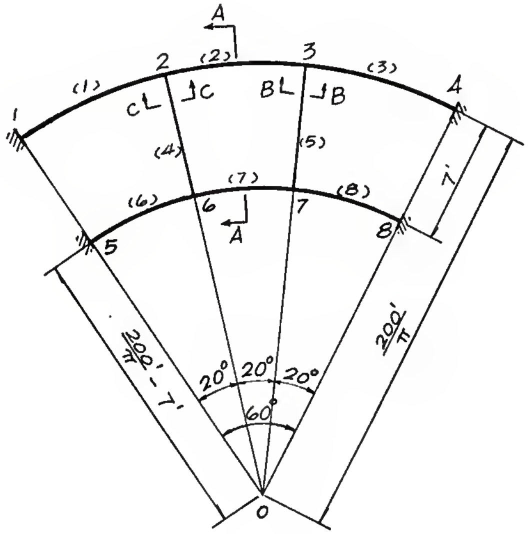


Fig. 8. Plan view of a frame with curved and straight members

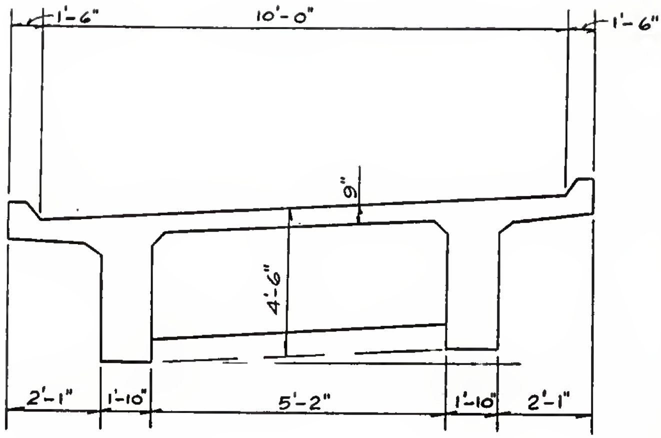


Fig. 9. Section A-A in Fig. 8.

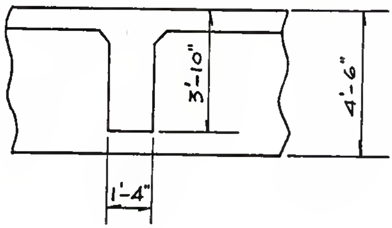


Fig. 10. Section B-B and C-C in Fig. 8.

NOTATION AND ASSUMPTIONS

The following notation and assumptions are used in the calculations:

Modulus of elasticity of steel	$E_s = 29{,}000{,}000\ \text{lb/in}^2$
Ratio of E_s to E_c	$n = 9$
Modulus of elasticity of concrete	$E_c = 3{,}220{,}000\ \text{lb/in}^2$
Moment of inertia of curved beams	$I_r = 863{,}370\ \text{in}^4$
Moment of inertia of straight beams	$I_t = 228{,}750\ \text{in}^4$
Uniform dead load for curved members	$q_c = 1.78$ kips/ft
Uniform dead load for straight members	$q_s = 1.24$ kips/ft

The torsion factor of curved beams is*

$$J_c = \frac{b^3 d^3}{3.58(b^2 + d^2)} = \frac{(22)^3(54)^3}{3.58\left[(22)^2 + (54)^2\right]} = 137{,}000\ \text{in}^4.$$

The torsion factor of straight beams is

$$J_s = \frac{b^3 d^3}{3.58(b^2 + d^2)} = \frac{(16)^3(46)^3}{3.58\left[(16)^2 + (46)^2\right]} = 47{,}000\ \text{in}^4.$$

If

$$\frac{E}{G} = 2.25^*,$$

where E = modulus of elasticity in moment,

G = modulus of elasticity in shear,

then the value of K for curved beams is

$$K = \frac{EI_r}{GJ_c} = 2.25\ \frac{863370}{137000} = 14.2.$$

---

* Refer to reference 11 in the Bibliography.

DETERMINATION OF STRESSES DUE TO DEAD LOAD

The Slope-Deflection procedure in the computer method of stress analysis will be used in this report. First the fixed-end stresses will be determined in part A. Then these fixed-end stresses will be used to evaluate the member end stresses in part B.

A. The calculation of the fixed-end stresses:

In this example we have $\phi = \frac{\pi}{3}$ and K = 14.2 for curved members.

To evaluate the fixed-end moments and torques in Eq.(32) and (33) for each curved member, we shall use the electronic computer. FORGO source program 1 is used for this purpose and is shown on pages 61 and 62. Answers appear below on this page.

For the convenience of computer calculation, some substitutions have been made in the source program. The values BMF and TMF in the source program are used instead of the values M_a^F and T_a^F in Eq.(32) and (33). A,B,C,D, and E are the constants in Eq.(29). We shall use q as a positive value since we have considered that q is negative as we derived Eq.(32) and Eq.(33).

ANSWERS OF PROGRAM 1

A	B	C	D	E
5.30580110	1.54410670	4.24239830	.01103570	-.01567320

BENDING-MOMENT	TORSION-MOMENT	R
-75.369[illegible]	-.34668919	63.66197600

A	B	C	D	E
5.30580110	1.54410670	4.24239830	.01103570	-.01567320

BENDING-MOMENT	TORSION-MOMENT	R
-59.76572460	-.27463982	56.66197600

Now with the data given in the answers of program 1, the fixed-end stresses for each member can be listed.

In the list we use the notation M^F_{21} which will indicate the fixed-end moment of member (1) at end 2(the first subscript refers to the end of a member and the second subscript refers to the number of bars) and ths same subscription will be used for fixed-end torques and forces.

Because members (1), (2) and (3) have the same curvature and length, the fixed-end strssses due to uniform load should be the same. So we get

M^F_{11} = - 75.369 ft-kips, M^F_{21} = + 75.369 ft-kips,
T^F_{11} = - 0.347 ft-kips, T^F_{21} = - 0.347 ft-kips,
F^F_{11} = + 19.778 kips, F^F_{21} + + 19.778 kips;

M^F_{22} = - 75.369 ft-kips, M^F_{32} = + 75.369 ft-kips,
T^F_{22} = - 0.347 ft-kips, T^F_{32} = - 0.347 ft-kips,
F^F_{22} = + 19.778 kips, F^F_{32} = + 19.778 kips;

M^F_{33} = - 75.369 ft-kips, M^F_{43} = + 75.369 ft-kips,
T^F_{33} = - 0.347 ft-kips, T^F_{43} = - 0.347 ft-kips,
F^F_{33} = + 19.778 kips, F^F_{43} = + 19.778 kips.

Because members (6), (7) and (8) have the same curvature and length, the fixed-end stresses due to uniform load should be the sams. So we get

M^F_{56} = - 59.706 ft-kips, M^F_{66} = + 59.706 ft-kips,
T^F_{56} = - 0.275 ft-kips, T^F_{66} = - 0.275 ft-kips,
F^F_{56} = + 17.603 kips, F^F_{66} = + 17.603 kips;

M^F_{67} = - 59.706 ft-kips, M^F_{77} = + 59.706 ft-kips,
T^F_{67} = - 0.275 ft-kips, T^F_{77} = - 0.275 ft-kips,
F^F_{67} = + 17.603 kips, F^F_{77} = + 17.603 kips;

M^F_{78} = - 59.706 ft-kips, M^F_{88} = + 59.706 ft-kips,
T^F_{78} = - 0.275 ft-kips, T^F_{88} = - 0.275 ft-kips,
F^F_{78} = + 17.603 kips, F^F_{88} = + 17.603 kips.

Because members (4) and (5) have the same length, the fixed-end strsses should be the same. So we gst

M^F_{24} = + 5.06 ft-kips, M^F_{64} + - 5.06 ft-kips,
T^F_{24} = 0, T^F_{64} = 0,
F^F_{24} = + 4.34 kips, F^F_{64} = + 4.34 kips;

M^F_{35} = + 5.06 ft-kips, M^F_{75} = - 5.06 ft-kips,
T^F_{35} = 0, T^F_{75} = 0,
F^F_{35} = + 4.34 kips, F^F_{75} = + 4.34 kips.

B. Use the determined fixed-end stresses to evaluate the member end stresses.

This part of the calculation will be separated into 6 steps.

1. General approach.

Define the statics matrix A expressing the balancing forces $\{Q\}$ at the joint in terms of the member end stresses $\{P\}$. Then by definition,

$$\{Q\} = [A]\,\{P\}\,. \quad \text{------------------------(69)}$$

Define the geometry matrix B expressing values of elastic member end rotations and displacements $\{E\}$ in terms of joint rotations and displacements $\{X\}$. Then by definition,

$$\{E\} = [B]\,\{X\}\,, \quad \text{------------------------(70)}$$

where

$$[B] = [A]^{T}. \quad \text{----------------------------(71)}$$

Define the stiffness matrix K expressing the member end stresses $\{P\}$ in terms of the elastic member end rotations and displacements $\{E\}$. Then by definition,

$$\{P\} = [K]\,\{E\}\,. \quad \text{------------------------(72)}$$

Substituting Eq.(70) and (71) into Eq.(72),

$$\{P\} = [K]\,[A]^{T}\{X\} = [G]\,\{X\}\,. \quad \text{---------(73)}$$

Substituting Eq.(73) into Eq.(69),

$$\{Q\} = [A]\,[K]\,[A]^{T}\{X\} = [H]\,\{X\}\,. \quad \text{------------(74)}$$

Therefore

$$\{X\} = [H]^{-1}\{Q\}\,. \quad \text{------------------------(75)}$$

2. Draw the Q-X and P-E diagrams.

In Fig. 11, the Q-X diagram shows the joint balancing forces and corresponding deformations. All the forces are shown in the positive direction.

In Fig. 12, the P-E diagram shows the member end stresses and corresponding member end deformations. To keep the picture clear only the end stresses and corresponding deformations at end 2 of members (1), (2) and (4) are presented. Actually at each end of each member there will be three pairs of stress-deformation expressions. All the member end stresses are shown in the positive direction. Also, since members are perpendicular to each other at joints the torque of the straight bar is in the direction of the moment of the curved bar.

In Fig. 13, the diagram shows the equilibrium condition of balancing forces and end stresses at joint. According to this condition the sum of balancing forces and end stresses at a joint has to be equal to zero. Therefore, the balancing forces will be the sum of the end stresses at member ends. For example,

$$F_2 = F_2^1 + F_2^4 + F_2^2, \quad \text{------------------------}(76)$$

$$M_2 = M_2^1 + T_2^4 + M_2^2, \quad \text{------------------------}(77)$$

$$\cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot \;\; ,$$

at joint 2 in Fig. 13.

It is noted that the values to which F_2^1, M_2^1, . . . , refer in Eq.(76) and (77) are the end stresses at the joints. They are not the member end stresses. The drawing of the equilibrium diagram is an important procedure by which we can establish matrix A in Eq.(69).

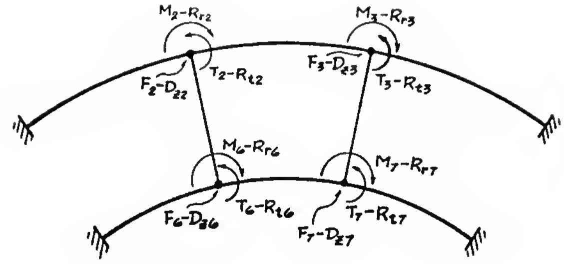


Fig. 11. Q-X diagram

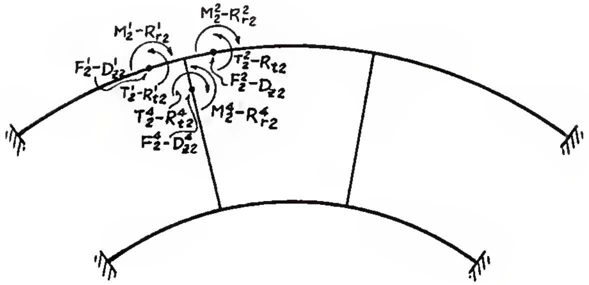


Fig. 12. P-E diagram

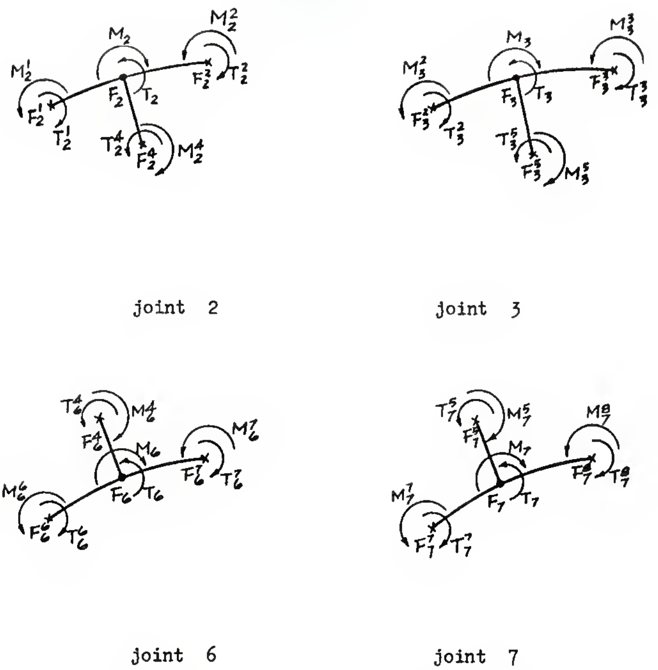


Fig. 13. Equilibrium diagrams of joints 2,3,6 and 7

3. Establish the matrices A,B and K.

To establish matrix A, we inspect every joint and write ths equilibrium equations as in Eq.(76) and (77) for each balancing force component. Then matrix A can be written. For example, the entries of the first row in Eq. (78) are all zero, except the fourth, seventh and tenth which are equal to one. So we can write $F_2 = F_2^1 + F_2^4 + F_2^2$. This is also what Eq.(76) indicates. Matrix B is simply the transpose of matrix A.

If we partition the matrices A and B, we can rewrite them in the following form

$$[A] = [A_1 \mid A_2], \qquad [B] = \begin{bmatrix} B_1 \\ \hline B_2 \end{bmatrix}.$$

$[A_1]$ = ------(78)

	F^1_1	M^1_1	T^1_1	F^1_2	M^1_2	T^1_2	F^4_2	M^4_2	T^4_2	F^2_2	M^2_2	T^2_2	F^2_3	M^2_3	T^2_3	F^5_3	M^5_3	T^5_3	F^3_3	M^3_3	T^3_3	F^3_4	M^3_4	T^3_4
F_2	0.	0.	0.	1.	0.	0.	1.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
M_2	0.	0.	0.	0.	1.	0.	0.	0.	1.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
T_2	0.	0.	0.	0.	0.	1.	0.	1.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
F_3	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	1.	0.	0.	1.	0.	0.	0.	0.	0.
M_3	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	1.	0.	1.	0.	0.	0.	0.
T_3	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	1.	0.	0.	0.	1.	0.	0.	0.
F_6	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
M_6	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
T_6	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
F_7	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
M_7	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
T_7	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

$$
[A_2] = \begin{array}{c} \\ F_2 \\ M_2 \\ T_2 \\ F_3 \\ M_3 \\ T_3 \\ F_6 \\ M_6 \\ T_6 \\ F_7 \\ M_7 \\ T_7 \end{array}
\begin{array}{cccccccccccccccccccccccc}
F_5^6 & M_5^6 & T_5^6 & F_6^6 & M_6^6 & T_6^6 & F_6^4 & M_6^4 & T_6^4 & F_6^7 & M_6^7 & T_6^7 & F_7^7 & M_7^7 & T_7^7 & F_7^5 & M_7^5 & T_7^5 & F_7^8 & M_7^8 & T_7^8 & F_8^8 & M_8^8 & T_8^8 \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 1. & 0. & 0. & 1. & 0. & 0. & 1. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 1. & 0. & 0. & 0. & 1. & 0. & 1. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 1. & 0. & 1. & 0. & 0. & 0. & 1. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 1. & 0. & 0. & 1. & 0. & 0. & 1. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 1. & 0. & 0. & 0. & 1. & 0. & 1. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 1. & 0. & 1. & 0. & 0. & 0. & 1. & 0. & 0. & 0.
\end{array} \qquad \text{------(79)}
$$

$$
\left[B_1\right] = \begin{array}{c} \\ D^1_{z1} \\ R^1_{r1} \\ R^1_{t1} \\ D^1_{z2} \\ R^1_{r2} \\ R^1_{t2} \\ D^4_{z2} \\ R^4_{r2} \\ R^4_{t2} \\ D^2_{z2} \\ R^2_{r2} \\ R^2_{t2} \\ D^2_{z3} \\ R^2_{r3} \\ R^2_{t3} \\ D^5_{z3} \\ R^5_{r3} \\ R^5_{t3} \\ D^3_{z3} \\ R^3_{r3} \\ R^3_{t3} \\ D^3_{z4} \\ R^3_{r4} \\ R^3_{t4} \end{array}
\begin{array}{c}
\begin{array}{cccccccccccc} D_{z2} & R_{r2} & R_{t2} & D_{z2} & R_{r2} & R_{t2} & D_{z2} & R_{r2} & R_{t2} & D_{z2} & R_{r2} & R_{t2} \end{array} \\
\left[\begin{array}{cccccccccccc}
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
1. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 1. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 1. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
1. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 1. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 1. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
1. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 1. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 1. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 1. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 1. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 1. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 1. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 1. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 1. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 1. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 1. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 1. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0.
\end{array}\right]
\end{array}
\quad \text{------(80)}
$$

$[B_2]$ =

	D_{z2}	R_{r2}	R_{t2}	D_{z2}	R_{r2}	R_{t2}	D_{z2}	R_{r2}	R_{t2}	D_{z2}	R_{r2}	R_{t2}
D_{z5}^{6}	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
R_{r5}^{6}	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
R_{t5}^{6}	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
D_{z6}^{6}	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.
R_{r6}^{6}	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.
R_{t6}^{6}	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.
D_{z6}^{4}	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.
R_{r6}^{4}	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.
R_{t6}^{4}	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.
D_{z6}^{7}	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.
R_{r6}^{7}	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.
R_{t6}^{7}	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.
D_{z7}^{7}	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.
R_{r7}^{7}	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.
R_{t7}^{7}	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.
D_{z7}^{5}	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.
R_{r7}^{5}	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.
R_{t7}^{5}	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.
D_{z7}^{8}	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.
R_{r7}^{8}	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.
R_{t7}^{8}	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.
D_{z8}^{8}	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
R_{r8}^{8}	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
R_{t8}^{8}	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

-------(81)

A_1, A_2, B_1 and B_2 all represent the matrices and they are shown in Eq.(78), (79), (80) and (81) respectively. It is noted that the notation of forces or deformations listed on the top and on the left side of the matrix is not included in the matrix. It is only for reference. For example, matrix A is a member end stress and balancing forces transformation matrix. So we list the notation of member end stressss on the top and balancing forces on the left side of the matrix to show to whioh pair of stresses and forces each entry of the matrix refers.

To establish the matrix K, we first have to determine the submatrices K_{11}^1 K_{21}^1, . . ., etc. in Eq.(24).

The values of K_{11}^1 and K_{55}^6 are determined in program 2 which starts on page 62. Answers appear on page 43. In this example $K_{11}^1 = K_{22}^2 = K_{33}^3$, and $K_{55}^6 = K_{66}^7 = K_{77}^8$.

The values of K_{12}^1 and K_{56}^6 are determined in program 3 which starts on page 65. Answers appear on page 44. In this example $K_{12}^1 = K_{23}^2 = K_{34}^3$, and $K_{56}^6 = K_{67}^7 = K_{78}^8$

The values of K_{22}^1 and K_{66}^6 are determined in program 4 which starts on page 67. Answers appear on page 44. In this example $K_{22}^1 = K_{33}^2 = K_{44}^3$, and $K_{66}^6 = K_{77}^7 = K_{88}^8$.

The values of K_{21}^1 and K_{65}^6 are determined in program 5 which starts on page 69. Answers appear on page 44. In this example $K_{21}^1 = K_{32}^2 = K_{43}^3$, and $K_{65}^6 = K_{76}^7 = K_{87}^8$.

The values of K_{22}^4 and K_{26}^4 are determined in program 6 which starts on page 71. Answers appear on page 45. In this example $K_{22}^4 = K_{33}^5$, and $K_{26}^4 = K_{37}^5$.

The values of K_{66}^4 and K_{62}^4 are determined in program 7 which starts on page 73. Answers appear on page 45. In this example $K_{66}^4 = K_{77}^5$, and

$K^4_{62} = K^5_{73}$.

After getting these submatrices, we can insert them into Eq.(24) and get the matrix K.

For convenience in writing the source program, some changes have been made in the notations in the derived formulas as they are used in the source programs.

In program 2, ϕ, K and $[A]^{-1}$ in Eq.(9) have been changed into X, GK and [H], respectively.

In program 3, ϕ and K in Eq.(12) have been changed into X and GK, respectively.

In program 4, ϕ, K, [B] and $[B]^{-1}$ in Eq.(16) have been changed into X, GK, [A] and [H], respectively.

In program 5, ϕ, K and $[B]^{-1}$ in Eq.(18) have been changed into X, GK and $[A]^{-1}$, respectively.

The answers of program 2-7 are listed as follows:

ANSWERS OF PROGRAM 2

STIFFNESS-MATRIX INV.(A) IN EQ.(9) R

63.661976

$$\begin{bmatrix} 20449.9000 & -227054.2800 & 14197.4900 \\ -227054.2800 & 3266355.0000 & -278175.0500 \\ 14197.4900 & -278175.0500 & 92871.4480 \end{bmatrix} = K^1_{11}$$

STIFFNESS-MATRIX INV.(A) IN EQ.(9) R

56.661976

$$\begin{bmatrix} 29003.9020 & -286620.0200 & 17922.0810 \\ -286620.0300 & 3669879.6000 & -312540.7200 \\ 17922.0810 & -312540.7200 & 104344.7600 \end{bmatrix} = K^6_{55}$$

ANSWERS OF PROGRAM 3

STIFFNESS-MATRIX IN EQ.(12)

R

63.661976

$$\begin{pmatrix} -20449.9000 & 227054.2800 & -14197.4900 \\ -227052.5100 & 1779295.9000 & -15968.6430 \\ -14197.2000 & 15966.6400 & -46637.3990 \end{pmatrix} = K^{1}_{12}$$

STIFFNESS-MATRIX IN EQ.(12)

R

56.661976

$$\begin{pmatrix} -29003.9020 & 286620.0200 & -17922.0810 \\ -286617.7800 & 1999109.6000 & -17941.4300 \\ -17921.7130 & 17939.1100 & -52398.9710 \end{pmatrix} = K^{6}_{56}$$

ANSWERS OF PROGRAM 4

STIFFNESS-MATRIX IN EQ.(16)

R

63.661976

$$\begin{pmatrix} 20449.9000 & 227054.2800 & 14197.4900 \\ 227054.2800 & 3266355.0000 & 278175.0500 \\ 14197.4900 & 278175.0500 & 92871.4480 \end{pmatrix} = K^{1}_{22}$$

STIFFNESS-MATRIX IN EQ.(16)

R

56.661976

$$\begin{pmatrix} 29003.9020 & 286620.0200 & 17922.0810 \\ 286620.0300 & 3669879.6000 & 312540.7200 \\ 17922.0810 & 312540.7200 & 104344.7600 \end{pmatrix} = K^{6}_{66}$$

ANSWERS OF PROGRAM 5

STIFFNESS-MATRIX IN EQ.(18)

R

63.661976

$$\begin{pmatrix} -20449.9000 & -227054.2800 & -14197.4900 \\ 227052.5100 & 1779295.9000 & 15968.6430 \\ -14197.2000 & -15966.6400 & -46637.3990 \end{pmatrix} = K^{1}_{21}$$

STIFFNESS-MATRIX IN EQ.(18)

R

56.661976

$$\begin{pmatrix} -29003.9020 & -286620.0200 & -17922.0810 \\ 286617.7800 & 1999109.6000 & 17941.4300 \\ -17921.7130 & -17939.1100 & -52398.9710 \end{pmatrix} = K^{6}_{65}$$

ANSWERS OF PROGRAM 6

STIFFNESS-MATRIX FOR NEAR END

$$\begin{bmatrix} 178954.0800 & 626339.2900 & 0.0000 \\ 626339.2900 & 2922916.7000 & 0.0000 \\ 0.0000 & 0.0000 & 66728.3960 \end{bmatrix} = K^4_{22}$$

STIFFNESS-MATRIX FOR FAR END

$$\begin{bmatrix} -178954.0800 & -626339.2900 & -0.0000 \\ 626339.2900 & 1461458.3000 & -0.0000 \\ -0.0000 & -0.0000 & -66728.3960 \end{bmatrix} = K^4_{26}$$

ANSWERS OF PROGRAM 7

STIFFNESS-MATRIX FOR NEAR END

$$\begin{bmatrix} 178954.0800 & -626339.2900 & 0.0000 \\ -626339.2900 & 2922916.7000 & 0.0000 \\ 0.0000 & 0.0000 & 66728.3960 \end{bmatrix} = K^4_{66}$$

STIFFNESS-MATRIX FOR FAR END

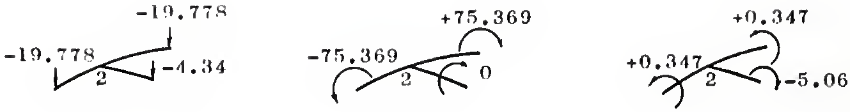
$$\begin{bmatrix} -178954.0800 & 626339.2900 & -0.0000 \\ -626339.2900 & 1461458.3000 & -0.0000 \\ -0.0000 & -0.0000 & -66728.3960 \end{bmatrix} = K^4_{62}$$

4. Calculate matrices G and H^{-1}.

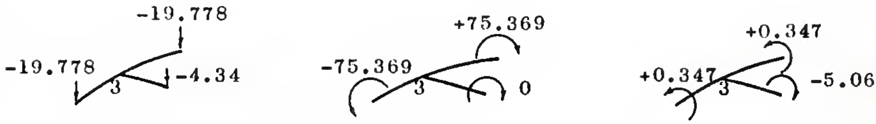
Through matrix multiplication and inversion, the matrices G and H^{-1} can be easily evaluated. Programs 8 and 9 shown on pages 74 and 75 are examples of matrix multiplication and inversion.

5. Determine the vector Q.

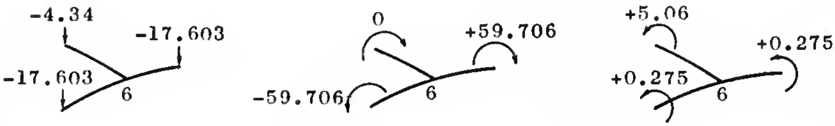
Using the fixed-end stresses at member ends listed on pages 32 and 33, vector Q can be determined from information shown in Fig. 14.



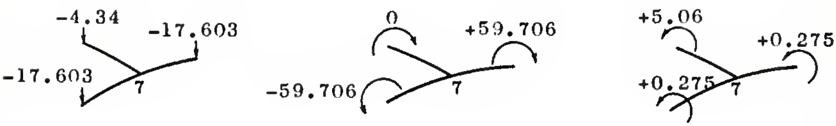
F_2 = -43.896 kips M_2 = 0 T_2 = -4.366 ft-kips



F_3 = -43.896 kips M_3 = 0 T_3 = -4.366 ft-kips



F_6 = -39.546 kips M_6 = 0 T_6 = +5.61 ft-kips



F_7 = -39.546 kips M_7 = 0 T_7 = +5.61 ft-kips

Fig. 14. Balancing forces at joints 2,3,6 and 7

Therefore the vector Q is

$$\{Q\} = \begin{Bmatrix} F_2 \\ M_2 \\ T_2 \\ F_3 \\ M_3 \\ T_3 \\ F_6 \\ M_6 \\ T_6 \\ F_7 \\ M_7 \\ T_7 \end{Bmatrix} = \begin{Bmatrix} -43.896 \\ 0.0 \\ -4.366 \\ -43.896 \\ 0.0 \\ -4.366 \\ -39.546 \\ 0.0 \\ +5.61 \\ -39.546 \\ 0.0 \\ +5.61 \end{Bmatrix} .$$

6. Calculate the member end stresses as indicated in vector P in Eq.(73).

Substitute the value of matrix H^{-1} and vector Q into Eq.(75). Vector X can then be determined. Then substitute the value of matrix G and vector X into Eq.(73). The vector P can be thus determined.

Here we have used a source program of matrix times vector. It is shown in program 10 on page 76.

Vector P was partitioned in the form of $\{P\} = \begin{Bmatrix} P_1 \\ \hline P_2 \end{Bmatrix}$.

Combining the fixed-end stresses with the member end stresses, vector P, we get the final member end stresses which are listed in Tables 1 and 2 on pages 85 and 86.

DRAW INFLUENCE LINE

In this section, we shall draw the influence line for each stress component of each section shown in Fig. 15. If more sections were to be taken the influence lines would be smoother. However, we will define the ordinates only at sections shown in Fig. 15 when we draw the influence lines later. The change of values between sections will be assumed linear.

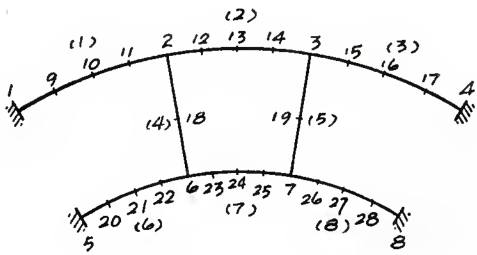


Fig. 15. Plan view of a framework with curved and straight members

First the fixed-end stresses will be determined in part A. Secondly the influence line will be drawn in part B.

A. The calculation of the fixed-end stresses.

Before determining the fixed-end stresses, the constants Q,H and S in Eq.(46), (47) and (48) were evaluated in program 11 starting on page 77. Answers appear on page 49. Then substituting the values of Q, H and S into Eq.(45), we get the fixed-end stresses at end "a". They are determined by program 12 starting on page 80. Similarly if the values of Q,H and S are substituted into Eq.(50) the fixed-end stresses at end "b" can be determined. This is performed by use of program 13 starting on page 82. Answers of program 12 and 13 appear on page 50 and 51.

```
ANSWERS OF PROGRAM 11
              R      63.66197600

   G                H                S                    Z
                                                       .08726646
 -.000140844240  -.000009378848  -.000006671969
              R      63.66197600

   G                H                S                    Z
                                                       .17453293
 -.000066033581  -.000003845325  -.000001600150
              R      63.66197600

   G                H                S                    Z
                                                       .26179939
 -.000017110796  -.000000864254  -.000000061282
              R      56.66197600

   G                H                S                    Z
                                                       .08726646
 -.000099305638  -.000007429725  -.000005285392
              R      56.66197600

   G                H                S                    Z
                                                       .17453293
 -.000046558573  -.000003046185  -.000001267605
              R      56.66197600

   G                H                S                    Z
                                                       .26179939
 -.000012064381  -.000000684644  -.000000048546
     STOP   END OF PROGRAM AT  STATEMENT 0060 + 00 LINES
```

```
ANSWERS OF PROGRAM 12

            R     63.66197600
                 FIXED-END STRESSES

    F               M               T               Z
 .84546821    -3.20061630     .01030862      .08726646

            R     63.66197600
                 FIXED-END STRESSES

    F               M               T               Z
 .50000071    -2.87813180     .01644585      .17453293

            R     63.66197600
                 FIXED-END STRESSES

    F               M               T               Z
 .15455155    -1.07916620     .00820781      .26179939

            R     56.66197600
                 FIXED-END STRESSES

    F               M               T               Z
 .84546832    -2.84868920     .00917506      .08726646

            R     56.66197600
                 FIXED-END STRESSES

    F               M               T               Z
 .50000081    -2.56166610     .01463761      .17453293

            R     56.66197600
                 FIXED-END STRESSES

    F               M               T               Z
 .15455148     -.96050480     .00730520      .26179939
  STOP  END OF PROGRAM AT  STATEMENT 0050 + 00 LINES
```

ANSWERS OF PROGRAM 13

```
          R     63.66197600
               FIXED-END STRESSES

   F            M            T             Z
 .15453180    1.07909800   .00822890    .08726646

          R     63.66197600
               FIXED-END STRESSES

   F            M            T             Z
 .49999930    2.87811500   .01644997    .17453293

          R     63.66197600
               FIXED-END STRESSES

   F            M            T             Z
 .84544850    3.20024310   .01027509    .26179939

          R     56.66197600
               FIXED-END STRESSES

   F            M            T             Z
 .15453170     .96044500   .00732360    .08726646

          R     56.66197600
               FIXED-END STRESSES

   F            M            T             Z
 .4999 920    2.56164840   .01464101    .17453293

          R     56.66197600
               FIXED-END STRESSES

   F            M            T             Z
 .84544860    2.84835840   .00914525    .26179939
  STOP  END OF PROGRAM AT  STATEMENT 0060 + 00 LINES
```

B. Draw the influence line.

This will be separated into four stepe.

1. Determine the vector Q.

Using the fixed-end streeses determined in programs 12 and 13, we can follow the same procedure to determine the vector Q under a unit load ae we determined the vector Q under uniform load on page 46. For every acting position of the unit load, there will be one corrssponding vector Q. Results are listed in Table 3-8 starting on page 87.

2. Calculate final member end etresses.

Substituting the vector Q into Eq.(75), we get values of vector X. Then from Eq.(73), we get values of vector P, the member end stresses. Combining the member end stresses with ths fixed-end stresses, we get the final member end stresses. They are listed in Table 9-12 starting on page 90.

3. Calculate the stresses at a eection between joints.

The stresees at a eection between joints can be determined by substituting final member end strssses into Eq.(10),(13) and (52). Reeults are listed in Tables 13,14,15 and 16, in which the notation referring to each stress component is as it is defined earlier in the report. For example, F_{18} simply represents the vertical force at section 18 shown in Fig. 15. However here we did not use any number of bar as the subscript with the notation as in Table 13-16 starting on page 98.

4. Draw the influence line.

When we draw the influence line ws indicate only the ordinate above the beam. Although data are available to draw an influence for each stress component of each section, we will draw only the influence line for section 13 as an example. For the convenience of drawing the influence line all the curved beams have been lengthened and have become imaginary straight beams.

Figures 16, 17 and 18 show the influence lines of vertical force, moment and torsion at section 13, respectively. The values of coordinates of thess lines can be found from Tables 13, 14, 15 and 16. For example, to find coordinates of the influence line for vertical force at section 13, we turn to page 98 where is the Table 13. In the left most column of this table, we find F_{13}. Then values appearing in the same row with F_{13} are vertical forces at section 13 due to a unit load acting at each different section in turn.

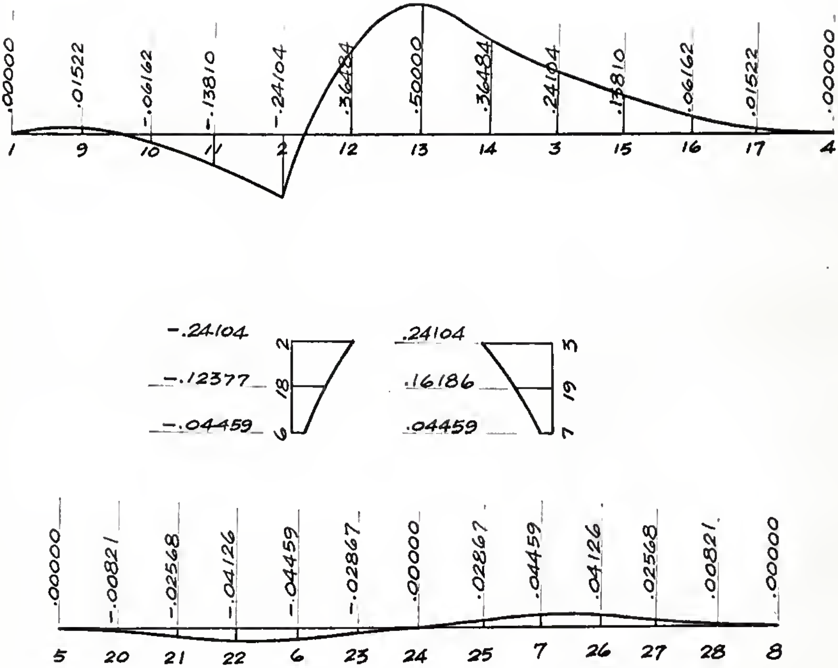


Fig. 16. Influence line for vertical force at section 13

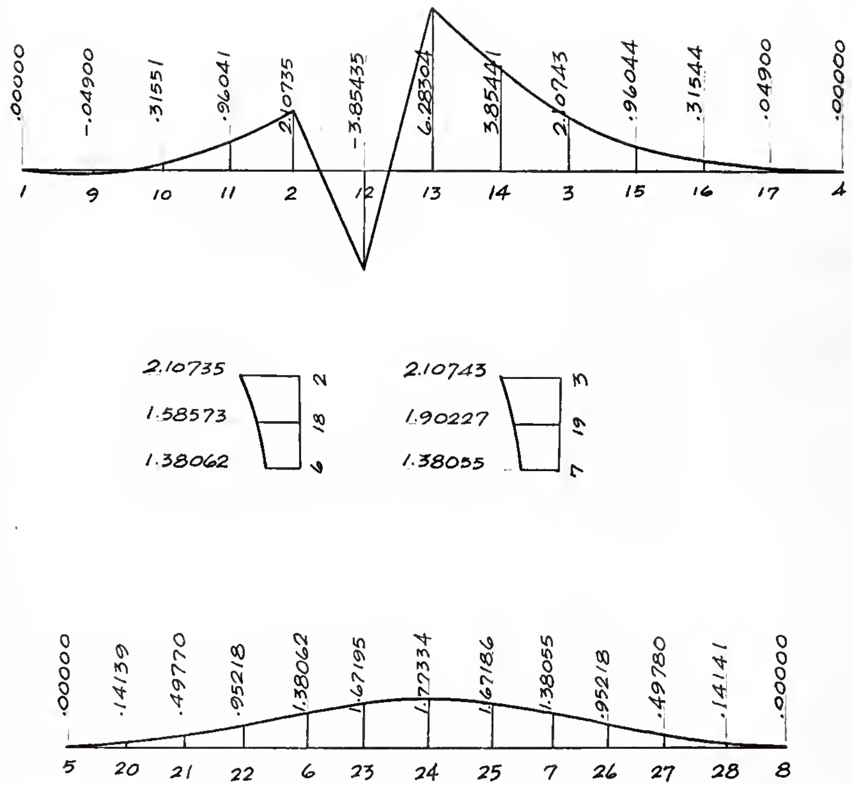


Fig. 17. Influence line for moment at section 13

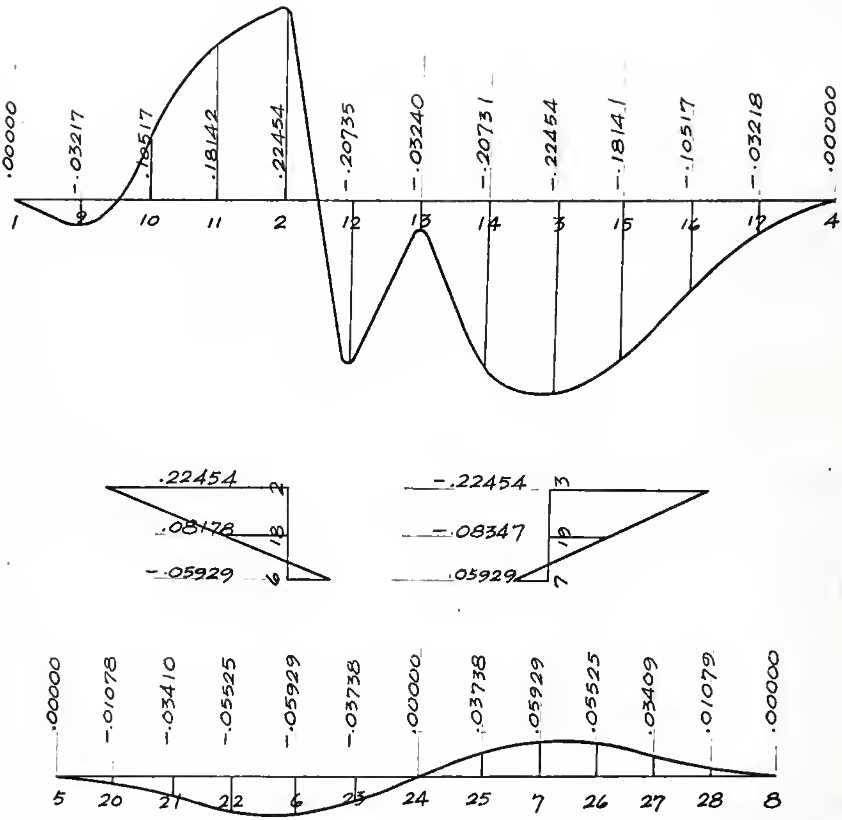


Fig. 18. Influence line for torsion at section 13

CONCLUSION

All the formulas derived in this report are intended for general application to any kind of structure with members which are straight or curved in plan. Any more complicated structure can be analysed by the same procedure. In case the number of unknowns in the stiffness matrix exceeds the capacity of the memory storage of the computer available, the patitioning method is suggested. However if the members of a structure do not meet orthogonally the analysis of stresses will become more complicated.

ACKNOWLEDGEMENT

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APPENDIX

APPENDIX

FORGO SOURCE PROGRAM.

```
C  C  PROGRAM 1   LEOU  11/5/66
C     CAL. OF FIXED-END MOMENTS OF A CURVED-BEAM UNDER UNIF. LOAD
      PI=3.1415926536
      X=PI/9.
      GK=14.2
      A=X*(GK+1.)
      B=(GK-1.)*SINF(X)*SINF(X)
      C=(GK-1.)*SINF(X)*COSF(X)
      D=A+GK*X*(COSF(X)+1.)-B*X/2.-2.*(GK+1.)*SINF(X)-C
      E=(A+C)*X/2.-B+GK*X*SINF(X)-2.*(GK+1.)*(1.-COSF(X))
      Q=1.78
      I=1
      R=200./PI
   10 BN=-Q*R**2*(D*(A+C)+B*E)
      TN=+Q*R**2*(E*(A-C)+B*D)
      CD=A**2-B**2-C**2
      BMF=BN/CD
      TMF=TN/CD
   15 FORMAT(/6X,1HA,11X,1HB,11X,1HC,11X,1HD,11X,1HE)
      PUNCH 15
   16 FORMAT(5F12.8)
      PUNCH 16,A,B,C,D,E
   17 FORMAT(//5X,14HBENDING-MOMENT,7X,14HTORSION-MOMENT,13X,1HR/)
```

```
      PUNCH 17
   18 FORMAT(3F20.8)
      PUNCH 18,BMF,TMF,R
      IF(I-2)20,30,30
   20 R=R-7.
      I=I+1
      GO TO 10
   30 STOP
      END



C  C  PROGRAM 2   LEOU  12/4/66
C     CAL. OF STIFFNESS MATRIX INV.(A) IN EQ.(9)
      DIMENSION A(10,10),S(10),T(10),H(10,10)
      INDEX=1
      PI=3.1415926536
      X=PI/9.
      EI=3220000.*863370./144000.
      R=200./PI
      READ,N
    5 Q=R/(4.*EI)
      SR=R*Q
      CR=R*SR
      GK=14.2
      X2=2.*X
      S2=SINF(X2)
```

```
      S1=SINF(X)
      SS=S1**2
      C1=COSF(X)
      A(1,1)=CR*(X2-S2+GK*(6.*X+S2-8.*S1))
      A(1,2)=SR*(2.*SS+2.*GK*(2.-2.*C1-SS))
      A(2,1)=A(1,2)
      A(1,3)=SR*(S2-X2+GK*(4.*S1-X2-S2))
      A(3,1)=A(1,3)
      A(2,2)=Q*(X2+S2+GK*(X2-S2))
      A(2,3)=Q*(2.*(GK-1.)*SS)
      A(3,2)=A(2,3)
      A(3,3)=Q*(X2-S2+GK*(X2+S2))
      NN=N-1
      DO 6 I=1,3
      DO 6 J=1,3
    6 H(I,J)=A(I,J)
      H(1,1)=1./H(1,1)
      DO 110 M=1,NN
      K=M+1
   50 DO 60 I=1,M
      S(I)=0
      DO 60 J=1,M
   60 S(I)=S(I)+H(I,J)*H(J,K)
      D=0
      DO 70 I=1,M
   70 D=D+H(K,I)*S(I)
      D=-D+H(K,K)
```

```
      H(K,K)=1./D
      DO 80 I=1,M
   80 H(I,K)=-S(I)*H(K,K)
      DO 90 J=1,M
      T(J)=0
      DO 90 I=1,M
   90 T(J)=T(J)+H(K,I)*H(I,J)
      DO 100 J=1,M
  100 H(K,J)=-T(J)*H(K,K)
      DO 110 I=1,M
      DO 110 J=1,M
  110 H(I,J)=H(I,J)-S(I)*H(K,J)
   15 FORMAT(/5X,34HSTIFFNESS-MATRIX INV.(A) IN EQ. 9.,19X,1HR//)
      PUNCH 15
  160 PUNCH 8,R
    8 FORMAT(52X,F12.6)
  170 PUNCH 9,((H(I,J),J=1,N),I=1,N)
    9 FORMAT(3F16.4)
      IF(INDEX-2) 25,30,30
   25 INDEX=INDEX+1
      R=R-7.
      GO TO 5
   30 STOP
      END
C     DATA OF PROGRAM 2
    3
```

```
C  C  PROGRAM 3   LFOU  12/4/66
C     CAL. OF STIFFNESS MATRIX IN EQ.(12)
      DIMENSION B(3,3),AINV(3,3),C(3,3),BQ(3,3)
      INDEX=1
      PI=3.1415926536
      X=PI/9.
      R=200./PI
  101 READ 1,INA,IMA,IMB
    1 FORMAT(3I8)
  102 READ 48,((AINV(I,J),J=1,IMA),I=1,INA)
   48 FORMAT(3F16.4)
    5 S1=SINF(X)
      C1=COSF(X)
      B(1,1)=-1.0
      B(1,2)=-0.
      B(1,3)=-0.
      B(2,1)=-R*S1
      B(2,2)=-C1
      B(2,3)=+S1
      B(3,1)=-R*(1.-C1)
      B(3,2)=-S1
      B(3,3)=-C1
      DO 121 I=1,3
      DO 121 J=1,3
  121 BQ(I,J)=B(I,J)
      DO 7 I=1,INA
```

```
      DO 7 J=1,IMB
      SUM=0.
      DO 6 K=1,IMA
    6 SUM=BQ(I,K)*AINV(K,J)+SUM
      C(I,J)=SUM
    7 CONTINUE
   15 FORMAT(/5X,27HSTIFFNESS-MATRIX IN EQ. 12.,19X,1HR//)
      PUNCH 15
  160 PUNCH 8,R
    8 FORMAT(52X,F12.6)
  170 PUNCH 9,((C(I,J),J=1,IMB),I=1,INA)
    9 FORMAT(3F16.4)
      IF(INDEX-2) 25,30,30
   25 INDEX=INDEX+1
      R=R-7.
      GO TO 102
   30 STOP
      END

C     DATA OF PROGRAM 3
       3         3          3
      20449.9000    -227054.2800      14197.4900
    -227054.2800    3266355.0000    -278175.0500
      14197.4900    -278175.0500      92871.4480
      29003.9020    -286620.0200      17922.0810
    -286620.0300    3669879.6000    -312540.7200
      17922.0810    -312540.7200     104344.7600
```

```
C  C  PROGRAM 4   LEOU  12/4/66
C     CAL. OF STIFFNESS MATRIX IN EQ.(16)
      DIMENSION A(10,10),S(10),T(10),H(10,10)
      INDEX=1
      PI=3.1415926536
      X=PI/9.
      EI=3220000.*863370./144000.
      R=200./PI
      READ,N
    5 Q=R/(4.*EI)
      SR=R*Q
      CR=R*SR
      GK=14.2
      X2=2.*X
      S2=SINF(X2)
      S1=SINF(X)
      SS=S1**2
      C1=COSF(X)
      A(1,1)=CR*(X2-S2+GK*(6.*X+S2-8.*S1))
      A(1,2)=-SR*(2.*SS+2.*GK*(2.-2.*C1-SS))
      A(1,3)=SR*(S2-X2+GK*(4.*S1-X2-S2))
      A(2,1)=A(1,2)
      A(3,1)=A(1,3)
      A(2,2)=Q*(X2+S2+GK*(X2-S2))
      A(2,3)=-Q*(2.*(GK-1.)*SS)
      A(3,2)=A(2,3)
      A(3,3)=Q*(X2-S2+GK*(X2+S2))
```

```
      NN=N-1
      DO 6 I=1,3
      DO 6 J=1,3
    6 H(I,J)=A(I,J)
      H(1,1)=1./H(1,1)
      DO 110 M=1,NN
      K=M+1
   50 DO 60 I=1,M
      S(I)=0
      DO 60 J=1,M
   60 S(I)=S(I)+H(I,J)*H(J,K)
      D=0
      DO 70 I=1,M
   70 D=D+H(K,I)*S(I)
      D=-D+H(K,K)
      H(K,K)=1./D
      DO 80 I=1,M
   80 H(I,K)=-S(I)*H(K,K)
      DO 90 J=1,M
      T(J)=0
      DO 90 I=1,M
   90 T(J)=T(J)+H(K,I)*H(I,J)
      DO 100 J=1,M
  100 H(K,J)=-T(J)*H(K,K)
      DO 110 I=1,M
      DO 110 J=1,M
  110 H(I,J)=H(I,J)-S(I)*H(K,J)
```

```
   15 FORMAT(/5X,34HSTIFFNESS-MATRIX INV.(A) IN EQ. 9.,19X,1HR//)
      PUNCH 15
  160 PUNCH 8,R
    8 FORMAT(52X,F12.6)
  170 PUNCH 9,((H(I,J),J=1,N),I=1,N)
    9 FORMAT(3F16.4)
      IF(INDEX-2) 25,30,30
   25 INDEX=INDEX+1
      R=R-7.
      GO TO 5
   30 STOP
      END
C     DATA OF PROGRAM 4
    3
```

```
C C   PROGRAM 5   LEOU  12/4/66
C     CAL. OF STIFFNESS MATRIX IN EQ. (18)
      DIMENSION B(3,3),AINV(3,3),C(3,3),BQ(3,3)
      INDEX=1
      PI=3.1415926536
      X=PI/9.
      R=200./PI
  101 READ 1,INA,IMA,IMB
    1 FORMAT(3I8)
  102 READ 48,((AINV(I,J),J=1,IMA),I=1,INA)
```

```
 48 FORMAT(3F16.4)
  5 S1=SINF(X)
    C1=COSF(X)
    B(1,1)=-1.0
    B(1,2)=-0.
    B(1,3)=-0.
    B(2,1)=+R*S1
    B(2,2)=-C1
    B(2,3)=-S1
    B(3,1)=-R*(1.-C1)
    B(3,2)=+S1
    B(3,3)=-C1
    DO 121 I=1,3
    DO 121 J=1,3
121 BQ(I,J)=B(I,J)
    DO 7 I=1,INA
    DO 7 J=1,IMB
    SUM=0.
    DO 6 K=1,IMA
  6 SUM=BQ(I,K)*AINV(K,J)+SUM
    C(I,J)=SUM
  7 CONTINUE
 15 FORMAT(/5X,27HSTIFFNESS-MATRIX IN EQ. 12.,19X,1HR//)
    PUNCH 15
160 PUNCH 8,R
  8 FORMAT(52X,F12.6)
170 PUNCH 9,((C(I,J),J=1,IMB),I=1,INA)
```

```
    9 FORMAT(3F16.4)
      IF(INDEX-2) 25,30,30
   25 INDEX=INDEX+1
      R=R-7.
      GO TO 102
   30 STOP
      END
C     DATA OF PROGRAM 5
       3         3         3
      20449.9000      227054.2800      14197.4900
     227054.2600     3266355.0000     278175.0500
      14197.4900      278175.0500      92871.4480
      29003.9020      286620.0200      17922.0810
     286620.0300     3669879.6000     312540.7200
      17922.0810      312540.7200     104344.7600


C  C  PROGRAM 6    LEOU  11/26/66
C     CAL. OF STIFFNESS MATRICES OF STRAIGHT BEAM.IN EQ.(59),(61)
      DIMENSION F(3,3),G(3,3)
      EI=3220000.*228750./144000.
      SL=7.
      GJ=3220000.*47000./(2.25*144000.)
      F(1,1)=12.*EI/(SL**3)
      F(1,2)=6.*EI/(SL**2)
      F(1,3)=0.
      F(2,1)=F(1,2)
      F(2,2)=4.*EI/SL
      F(2,3)=0.
```

```
      F(3,1)=0.
      F(3,2)=0.
      F(3,3)=GJ/SL
      G(1,1)=-12.*EI/(SL**3)
      G(1,2)=-6.*EI/(SL**2)
      G(1,3)=-0.
      G(2,1)=-G(1,2)
      G(2,2)=+2.*EI/SL
      G(2,3)=-0.
      G(3,1)=-0.
      G(3,2)=-0.
      G(3,3)=-GJ/SL
   10 FORMAT(/10X,29HSTIFFNESS-MATRIX FOR NEAR END/)
      PUNCH 10
   16 FORMAT(3F16.4)
      PUNCH 16,((F(I,J),J=1,3),I=1,3)
   21 FORMAT(//9X,28HSTIFFNESS-MATRIX FOR FAR END/)
      PUNCH 21
      PUNCH 16,((G(I,J),J=1,3),I=1,3)
      STOP
      END
```

```
C  C  PROGRAM 7   LFOU  11/26/66
C     CAL. OF STIFFNESS MATRICES OF STRAIGHT BEAM. IN EQ.(66),(68)
      DIMENSION F(3,3),G(3,3)
      EI=3220000.*228750./144000.
      SL=7.
      GJ=3220000.*47000./(2.25*144000.)
      F(1,1)=12.*EI/(SL**3)
      F(1,2)=-6.*EI/(SL**2)
      F(1,3)=0.
      F(2,1)=F(1,2)
      F(2,2)=4.*EI/SL
      F(2,3)=0.
      F(3,1)=0.
      F(3,2)=0.
      F(3,3)=GJ/SL
      G(1,1)=-12.*EI/(SL**3)
      G(1,2)=6.*EI/(SL**2)
      G(1,3)=-0.
      G(2,1)=-G(1,2)
      G(2,2)=+2.*EI/SL
      G(2,3)=-0.
      G(3,1)=-0.
      G(3,2)=-0.
      G(3,3)=-GJ/SL
   10 FORMAT(/10X,29HSTIFFNESS-MATRIX FOR NEAR END/)
      PUNCH 10
```

```
   16 FORMAT(3F16.4)
      PUNCH 16,((F(I,J),J=1,3),I=1,3)
   21 FORMAT(//9X,28HSTIFFNESS-MATRIX FOR FAR END/)
      PUNCH 21
      PUNCH 16,((G(I,J),J=1,3),I=1,3)
      STOP
      END
```

```
C  C  PROGRAM 8  MATRIX MULTIPLICATION   LECU  11/25/66
      DIMENSION A(12,24),B(24,12),C(12,12)
  100 READ 1,INA,IMA,IMB
    1 FORMAT(3I8)
      READ 4,((A(I,J),J=1,IMA),I=1,INA)
      READ 2,((B(I,J),J=1,IMB),I=1,IMA)
    4 FORMAT(6F11.8)
    2 FORMAT(6F12.2)
      DO 7 I=1,INA
      DO 7 J=1,IMB
      SUM=0.
      DO 6 K=1,IMA
    6 SUM=A(I,K)*B(K,J)+SUM
      C(I,J)=SUM
    7 CONTINUE
      PUNCH 3,((C(I,J),J=1,IMB),I=1,INA)
    3 FORMAT(6F12.2)
```

```
      STOP
      END
C     DATA OF PROGRAM 8
      12        24        12



C C   PROGRAM 9  MATRIX INVERSION BY PARTITIONING   LEOU  11/2/66
      DIMENSION A(20,20),B(19),C(19)
    4 FORMAT(6F12.2)
      READ,N
      NN=N-1
   10 READ 4,((A(I,J),J=1,N),I=1,N)
      A(1,1)=1./A(1,1)
      DO 110 M=1,NN
      K=M+1
   50 DO 60 I=1,M
      B(I)=0
      DO 60 J=1,M
   60 B(I)=B(I)+A(I,J)*A(J,K)
      D=0
      DO 70 I=1,M
   70 D=D+A(K,I)*B(I)
      D=-D+A(K,K)
      A(K,K)=1./D
      DO 80 I=1,M
   80 A(I,K)=-B(I)*A(K,K)
```

```
      DO 90 J=1,M
      C(J)=0
      DO 90 I=1,M
   90 C(J)=C(J)+A(K,I)*A(I,J)
      DO 100 J=1,M
  100 A(K,J)=-C(J)*A(K,K)
      DO 110 I=1,M
      DO 110 J=1,M
  110 A(I,J)=A(I,J)-B(I)*A(K,J)
      DO 170 I=1,N
  170 PUNCH 5,(A(I,J),J=1,N)
    5 FORMAT(4E16.8)
      END
C     DATA OF PROGRAM 9
   12
```

```
C  C  PROGRAM 10  MATRIX TIMES VECTOR   LEOU  11/29/66
      DIMENSION A(48,12),VEC(12),PROD(48)
  100 READ 1,INA,IMA
    1 FORMAT(2I8)
    2 FORMAT(6F12.2)
    3 FORMAT(F20.15)
      READ 2,((A(I,J),J=1,IMA),I=1,INA)
      READ 3,(VEC(I),I=1,IMA)
```

```
      DO 115 I=1,INA
      SUM=0.0
      DO 116 K=1,IMA
  116 SUM=A(I,K)*VEC(K)+SUM
      PROD(I)=SUM
  115 CONTINUE
    5 FORMAT(20X,32HPRODUCT OF A MATRIX AND A VECTOR//)
      PUNCH 5
      PUNCH 6,(PROD(I),I=1,INA)
    6 FORMAT(30X,F10.5)
      STOP
      END
C     DATA OF PROGRAM 10
      48        12



C  C  PROGRAM 11   LECU  11/25/66
C     CAL. OF CONSTANTS G,H,S IN EQ.(46),(47) AND (48)
      PI=3.1415926536
      INDEX=1
      X=PI/9.
      P=1.
      GK=14.2
      R=200./PI
    5 Z=PI/36.
      DZ=PI/36.
```

```
      SX=SINF(X)
      X2=2.*X
      SX2=SINF(X2)
      CX=COSF(X)
      CX2=COSF(X2)
      Y=GK+1.
      YN=GK-1.
   10 SZ=SINF(Z)
      Z2=2.*Z
      CZ=COSF(Z)
      SZ2=SINF(Z2)
      CZ2=COSF(Z2)
      XZ=X-Z
      SXZ=SINF(X-Z)
      EI=3220000.*863370./144000.
      T=(P*R**2)/(4.*EI)
      CR=R*T
   20 GA=-YN*CZ*(SX2-SZ2)
      GB=SZ*YN*(CX2-CZ2)
      GC=-2.*XZ*(Y*CZ+2.*GK)
      GD=4.*GK*SXZ+4.*GK*(SX-SZ)
      G=(GA+GB+GC+GD)*CR
      HA=2.*XZ*Y*SZ
      HB=4.*GK*(CX-CZ)
      HC=-CZ*(CX2-CZ2)*YN
      HD=-SZ*(SX2-SZ2)*YN
      H=T*(HA+HB+HC+HD)
```

```
      SA=2.*XZ*Y*CZ
      SB=-4.*GK*(SX-SZ)
      SC=CZ*(SX2-SZ2)*YN
      SD=-SZ*(CX2-CZ2)*YN
      S=T*(SA+SB+SC+SD)
   25 FORMAT(13X,1HR,F15.8)
      PUNCH 25,R
   30 FORMAT(//4X,1HG,15X,1HH,15X,1HS,19X,1HZ)
      PUNCH 30
   35 FORMAT(50X,F14.8)
      PUNCH 35,Z
   40 FORMAT(3F16.12)
      PUNCH 40,G,H,S
      IF(Z-PI/12.) 50,54,54
   50 Z=Z+DZ
      GO TO 10
   54 CONTINUE
      IF(INDEX-2) 55,60,60
   55 INDEX=INDEX+1
      R=R-7.
      GO TO 5
   60 STOP
      END
```

```
C   C   PROGRAM 12    LECU   11/29/66
C       CAL. OF FIXED-END STRESSES IN EQ.(45)
        DIMENSION A(12,12),VEC(12),PROD(12)
        PI=3.1415926536
        DZ=PI/36.
        N=1
        R=200./PI
    100 READ 1,INA,IMA
      1 FORMAT(2I8)
      2 FORMAT(3F16.4)
      3 FORMAT(3F16.12)
     16 Z=PI/36.
        INDEX=1
      8 READ 2,((A(I,J),J=1,IMA),I=1,INA)
      9 READ 3,(VEC(I),I=1,IMA)
        DO 115 I=1,INA
        SUM=0.0
        DO 116 K=1,IMA
    116 SUM=A(I,K)*VEC(K)+SUM
        PROD(I)=SUM
    115 CONTINUE
      4 FORMAT(/13X,1HR,F15.8)
        PUNCH 4,R
      5 FORMAT(17X,18HFIXED-END STRESSES)
        PUNCH 5
     40 FORMAT(/6X,1HF,13X,1HM,13X,1HT,15X,1HZ)
        PUNCH 40
```

```
    6 FORMAT(3F14.8,F14.8)
      PUNCH 6,(PROD(I),I=1,INA),Z
      IF(INDEX-3)20,30,30
   20 INDEX=INDEX+1
      Z=Z+DZ
      GO TO 9
   30 IF(N-2)15,50,50
   15 N=N+1
      R=R-7.
      GO TO 16
   50 STOP
      END
C     DATA OF PROGRAM 12
        3         3
     -20449.9000    +227054.2800     -14197.4900
    +227054.2800   -3266355.0000    +278175.0500
     -14197.4900    +278175.0500     -92871.4480
  -.000140844240  -.000009378848  -.000006671969
  -.000066033581  -.000003845325  -.000001600150
  -.000017110796  -.000000864254  -.000000061282
     -29003.9020    +286620.0200     -17922.0810
    +286620.0300   -3669879.6000    +312540.7200
     -17922.0810    +312540.7200    -104344.7600
  -.000099305638  -.000007429725  -.000005285392
  -.000046558573  -.000003046185  -.000001267605
  -.000012064381  -.000000684644  -.000000048546
```

```
C  C  PROGRAM 13   LECU  12/7/66
C     CAL. OF FIXED-END STRESSES IN EQ.(50)
      DIMENSION B(3,3),AINV(3,3),C(3,3),BQ(3,3),D(3,3),F(3),G(3),H(3)
      DIMENSION VEC(3),PROD(3)
      INDEX=1
      P=1.
      PI=3.1415926536
      X=PI/9.
      R=200./PI
  101 READ 1,INA,IMA,IMB
    1 FORMAT(3I8)
   26 Z=PI/36.
      DZ=PI/36.
  102 READ 48,((AINV(I,J),J=1,IMA),I=1,INA)
   48 FORMAT(3F16.4)
    5 S1=SINF(X)
      C1=COSF(X)
      XZ=X-Z
      SXZ=SINF(XZ)
      CXZ=COSF(XZ)
      B(1,1)=+1.0
      B(1,2)=+0.
      B(1,3)=+0.
      B(2,1)=+R*S1
      B(2,2)=+C1
      B(2,3)=-S1
      B(3,1)=+R*(1.-C1)
```

```
      B(3,2)=+S1
      B(3,3)=+C1
      DO 121 I=1,3
      DO 121 J=1,3
  121 BQ(I,J)=B(I,J)
      DO 7 I=1,INA
      DO 7 J=1,IMB
      SUM=0.
      DO 6 K=1,IMA
    6 SUM=BQ(I,K)*AINV(K,J)+SUM
      C(I,J)=SUM
    7 CONTINUE
      DO 130 I=1,3
      DO 130 J=1,3
  130 D(I,J)=C(I,J)
   10 READ 15,(VEC(I),I=1,IMA)
   15 FORMAT(3F16.12)
      DO 115 I=1,INA
      SUM=0.0
      DO 116 K=1,IMA
  116 SUM=D(I,K)*VEC(K)+SUM
      PROD(I)=SUM
  115 CONTINUE
      DO 120 I=1,3
  120 F(I)=PROD(I)
      G(1)=P
      G(2)=P*R*SXZ
```

```
      G(3)=P*R*(1.-CXZ)
      DO 135 I=1,3
  135 H(I)=F(I)+G(I)
   16 FORMAT(/13X,1HR,F15.8)
      PUNCH 16,R
   17 FORMAT(17X,18HFIXED-END STRESSES)
      PUNCH 17
   41 FORMAT(/6X,1HF,13X,1HM,13X,1HT,15X,1HZ)
      PUNCH 41
   20 FORMAT(3F14.8,F14.8)
      PUNCH 20,(H(I),I=1,INA),Z
      IF(Z-PI/12.) 30,40,40
   30 Z=Z+DZ
      GO TO 5
   40 CONTINUE
      IF(INDEX-2) 45,60,60
   45 INDEX=INDEX+1
      R=R-7.
      GO TO 26
   60 STOP
      END
C     DATA OF PROGRAM 13
      3         3         3
      20449.9000    -227054.2800      14197.4900
    -227054.2800    3266355.0000    -278175.0500
      14197.4900    -278175.0500      92871.4480
  -.000140844240  -.000009378848  -.000006671969
  -.000066033581  -.000003845325  -.000001600150
  -.000017110796  -.000000864254  -.000000061282
      29003.9020    -286620.0200      17922.0810
    -286620.0300    3669879.6000    -312540.7200
      17922.0810    -312540.7200     104344.7600
  -.000099305638  -.000007429725  -.000005285392
  -.000046558573  -.000003046185  -.000001267605
  -.000012064381  -.000000684644  -.000000048546
```

Table 1. Final member end stresses of the structure in Fig. 8 due to uniform load.

	$\{P_1\}$ + fixed end stresses = final member end stresses
F_1^1	47.65 + 19.778 = + 67.428 kips
M_1^1	-767.50 - 75.369 = - 842.829 ft-kips
T_1^1	56.73 - 0.347 = + 56.383 ft-kips
F_2^1	-47.65 + 19.778 = - 27.872 kips
M_2^1	-296.85 + 75.369 = - 221.481 ft-kips
T_2^1	26.26 - 0.347 = + 25.913 ft-kips
F_2^4	3.75 + 4.340 = + 8.090 kips
M_2^4	20.64 + 5.060 = + 25.700 ft-kips
T_2^4	6.10 + 0.000 = + 6.100 ft-kips
F_2^2	- .00 + 19.778 = + 19.778 kips
M_2^2	290.75 - 75.369 = + 215.381 ft-kips
T_2^2	-51.27 - 0.347 = - 51.617 ft-kips
F_3^2	.00 + 19.778 = + 19.778 kips
M_3^2	-290.75 + 75.370 = - 215.381 ft-kips
T_3^2	-51.27 - 0.347 = - 51.617 ft-kips
F_3^5	3.75 + 4.340 = + 8.090 kips
M_3^5	20.64 + 5.060 = + 25.700 ft-kips
T_3^5	-6.10 + 0.000 = - 6.100 ft-kips
F_3^3	-47.65 + 19.778 = - 27.872 kips
M_3^3	296.85 - 75.369 = + 221.481 ft-kips
T_3^3	26.26 - 0.347 = + 25.913 ft-kips
F_4^3	47.65 + 19.780 = + 67.428 kips
M_4^3	767.50 + 75.369 = + 842.829 ft-kips
T_4^3	56.73 - 0.347 = + 56.383 ft-kips

Table 2. Final member end stresses of the structure in Fig. 8 due to uniform load.

	$\{P_2\}$	+ fixed end stresses	= final member end stresses
F_5^6	35.79	+ 17.603	= + 53.403 kips
M_5^6	-543.80	- 59.706	= - 603.536 ft-kips
T_5^6	34.56	- 0.275	= + 34.285 ft-kips
F_6^6	-35.79	+ 17.603	= - 18.197 kips
M_6^6	-170.81	+ 59.706	= - 111.144 ft-kips
T_6^6	31.21	- 0.275	= + 30.925 ft-kips
F_6^4	-3.75	+ 4.340	= + 0.570 kips
M_6^4	5.59	- 5.060	= + 0.550 ft-kips
T_6^4	-6.10	+ 0.000	= - 6.100 ft-kips
F_6^7	- .00	+ 17.603	= + 17.603 kips
M_6^7	176.92	- 59.706	= + 117.244 ft-kips
T_6^7	-31.19	- 0.275	= - 31.475 ft-kips
F_7^7	.00	+ 17.603	= + 17.603 kips
M_7^7	-176.92	+ 59.706	= - 117.244 ft-kips
T_7^7	-31.19	- 0.275	= - 31.475 ft-kips
F_7^5	-3.75	+ 4.340	= + 0.570 kips
M_7^5	5.59	- 5.060	= + 0.550 ft-kips
T_7^5	6.10	+ 0.000	= + 6.100 ft-kips
F_7^8	-35.79	+ 17.603	= - 18.197 kips
M_7^8	170.81	- 59.706	= + 111.144 ft-kips
T_7^8	31.21	- 0.275	= + 30.925 ft-kips
F_8^8	35.79	+ 17.603	= + 53.403 kips
M_8^8	543.80	+ 59.706	= + 603.536 ft-kips
T_8^8	34.56	- 0.275	= + 34.285 ft-kips

TABLE 3. MATRIX Q--UNIT LOAD AT POINTS 9,10,11,2

	UNIT LOAD AT POINT			
	9	10	11	2
F_2	-.15453180	-.50000000	-.84544850	-1.00000000
M_2	-1.07909800	-2.87811500	-3.20024310	0.00000000
T_2	.00822890	.01644997	.01027509	0.00000000
F_3	0.00000000	0.00000000	0.00000000	0.00000000
M_3	0.00000000	0.00000000	0.00000000	0.00000000
T_3	0.00000000	0.00000000	0.00000000	0.00000000
F_6	0.00000000	0.00000000	0.00000000	0.00000000
M_6	0.00000000	0.00000000	0.00000000	0.00000000
T_6	0.00000000	0.00000000	0.00000000	0.00000000
F_7	0.00000000	0.00000000	0.00000000	0.00000000
M_7	0.00000000	0.00000000	0.00000000	0.00000000
T_7	0.00000000	0.00000000	0.00000000	0.00000000

TABLE 4. MATRIX Q--UNIT LOAD AT POINTS 12,13,14,3

	UNIT LOAD AT POINT			
	12	13	14	3
F_2	-.84546821	-.50000000	-.15455155	0.00000000
M_2	3.20061630	2.87812000	1.07916620	0.00000000
T_2	.01030862	.01644585	.00820781	0.00000000
F_3	-.15453180	-.50000000	-.84544850	-1.00000000
M_3	-1.07909800	-2.87812000	-3.20024310	0.00000000
T_3	.00822890	.01644997	.01027509	0.00000000
F_6	0.00000000	0.00000000	0.00000000	0.00000000
M_6	0.00000000	0.00000000	0.00000000	0.00000000
T_6	0.00000000	0.00000000	0.00000000	0.00000000
F_7	0.00000000	0.00000000	0.00000000	0.00000000
M_7	0.00000000	0.00000000	0.00000000	0.00000000
T_7	0.00000000	0.00000000	0.00000000	0.00000000

TABLE 5. MATRIX Q--UNIT LOAD AT POINTS 15,16,17,18

	UNIT LOAD AT POINT			
	15	16	17	18
F_2	0.00000000	0.00000000	0.00000000	-.50000000
M_2	0.00000000	0.00000000	0.00000000	-.87500000
T_2	0.00000000	0.00000000	0.00000000	0.00000000
F_3	-.84546821	-.50000000	-.15455155	0.00000000
M_3	3.20061630	2.87812000	1.07916620	0.00000000
T_3	.01030862	.01644585	.00820781	0.00000000
F_6	0.00000000	0.00000000	0.00000000	-.50000000
M_6	0.00000000	0.00000000	0.00000000	.87500000
T_6	0.00000000	0.00000000	0.00000000	0.00000000
F_7	0.00000000	0.00000000	0.00000000	0.00000000
M_7	0.00000000	0.00000000	0.00000000	0.00000000
T_7	0.00000000	0.00000000	0.00000000	0.00000000

TABLE 6. MATRIX Q--UNIT LOAD AT POINTS 19,20,21,22

	UNIT LOAD AT POINT			
	19	20	21	22
F_2	0.00000000	0.00000000	0.00000000	0.00000000
M_2	0.00000000	0.00000000	0.00000000	0.00000000
T_2	0.00000000	0.00000000	0.00000000	0.00000000
F_3	-.50000000	0.00000000	0.00000000	0.00000000
M_3	-.87500000	0.00000000	0.00000000	0.00000000
T_3	0.00000000	0.00000000	0.00000000	0.00000000
F_6	0.00000000	-.15453170	-.50000000	-.84544860
M_6	0.00000000	-.96044500	-2.56165000	-2.84835840
T_6	0.00000000	.00732360	.01464100	.00914525
F_7	-.50000000	0.00000000	0.00000000	0.00000000
M_7	.87500000	0.00000000	0.00000000	0.00000000
T_7	0.00000000	0.00000000	0.00000000	0.00000000

TABLE 7. MATRIX Q--UNIT LOAD AT POINTS 6,23,24,25

	UNIT LOAD AT POINT			
	6	23	24	25
F_2	0.00000000	0.00000000	0.00000000	0.00000000
M_2	0.00000000	0.00000000	0.00000000	0.00000000
T_2	0.00000000	0.00000000	0.00000000	0.00000000
F_3	0.00000000	0.00000000	0.00000000	0.00000000
M_3	0.00000000	0.00000000	0.00000000	0.00000000
T_3	0.00000000	0.00000000	0.00000000	0.00000000
F_6	-1.00000000	-.84546832	-.50000000	-.15455148
M_6	0.00000000	2.84868920	2.56165000	.96050480
T_6	0.00000000	.00917510	.01463761	.00730520
F_7	0.00000000	-.15453170	-.50000000	-.84544860
M_7	0.00000000	-.96044500	-2.56165000	-2.84835840
T_7	0.00000000	.00732360	.01464100	.00914525

TABLE 8. MATRIX Q--UNIT LOAD AT POINTS 7,26,27,28

	UNIT LOAD AT POINT			
	7	26	27	28
F_2	0.00000000	0.00000000	0.00000000	0.00000000
M_2	0.00000000	0.00000000	0.00000000	0.00000000
T_2	0.00000000	0.00000000	0.00000000	0.00000000
F_3	0.00000000	0.00000000	0.00000000	0.00000000
M_3	0.00000000	0.00000000	0.00000000	0.00000000
T_3	0.00000000	0.00000000	0.00000000	0.00000000
F_6	0.00000000	0.00000000	0.00000000	0.00000000
M_6	0.00000000	0.00000000	0.00000000	0.00000000
T_6	0.00000000	0.00000000	0.00000000	0.00000000
F_7	-1.00000000	-.84546832	-.50000000	-.15455148
M_7	0.00000000	2.84868920	2.56165000	.96050480
T_7	0.00000000	.00917510	.01463761	.00730520

TABLE 9. FINAL MEMBER END STRESSES

UNIT LOAD AT POINTS 9,10,11,2,12,13

	UNIT LOAD AT POINT					
	9	10	11	2	12	13
F_1^1	.97255	.89568	.78215	.64993	.51628	.38767
M_1^1	-4.74157	-7.83342	-9.38622	-9.69314	-9.13661	-7.90639
T_1^1	.05070	.19010	.36607	.49988	.54712	.52594
F_2^1	.02745	.10432	.21785	.35008	-.51628	-.38767
M_2^1	-.21906	-1.01026	-2.53080	-4.87174	-2.46867	-.83152
T_2^1	-.02643	-.03488	.06966	.35026	.62862	.72155
F_2^4	-.01223	-.04269	-.07975	-.10904	-.11888	-.11233
M_2^4	.01797	.04569	.05203	.02768	-.01683	-.08219
T_2^4	-.00299	.00009	.02681	.09277	.15780	.17696
F_2^2	-.01522	-.06162	-.13810	-.24104	.63516	.50000
M_2^2	.22205	1.01018	2.50399	4.77897	2.31087	.65457
T_2^2	.00846	-.01081	-.12168	-.37793	-.61178	-.63936
F_3^2	.01522	.06162	.13810	.24104	.36484	.50000
M_3^2	.12558	.38881	.61230	.62823	.27344	-.65457
T_3^2	-.02547	-.09875	-.21187	-.35396	-.52064	-.63936
F_3^5	-.00565	-.02107	-.04339	-.06942	-.09436	-.11233
M_3^5	-.02362	-.07759	-.13472	-.16840	-.14626	-.08218
T_3^5	-.01124	-.04070	-.08081	-.12354	-.16056	-.17696
F_3^3	-.00957	-.04055	-.09471	-.17161	-.27048	-.38767
M_3^3	-.11434	-.34812	-.53149	-.50469	-.11288	.83152
T_3^3	.04909	.17634	.34660	.52236	.66691	.72155
F_4^3	.00957	.04055	.09471	.17161	.27048	.38767
M_4^3	.33252	1.27032	2.68008	4.38952	6.22349	7.90638
T_4^3	.02970	.10903	.21969	.34062	.45036	.52594

CONTINUATION OF TABLE 9

	UNIT LOAD AT POINT					
	9	10	11	2	12	13
F_5^6	.01234	.04271	.07911	.10757	.11772	.11233
M_5^6	-.29205	-1.00995	-1.87921	-2.59851	-2.93084	-2.87971
T_5^6	-.01240	-.03935	-.06534	-.07698	-.06906	-.04656
F_6^6	-.01234	-.04271	-.07911	-.10757	-.11772	-.11233
M_6^6	.03098	.10797	.21039	.33076	.44903	.51323
T_6^6	.06936	.23647	.43380	.59349	.66503	.64482
F_6^4	.01223	.04269	.07975	.10904	.11888	.11233
M_6^4	-.10359	-.34453	-.61028	-.79093	-.81530	-.70412
T_6^4	.00299	-.00009	-.02681	-.09277	-.15780	-.17696
F_6^7	.00011	.00001	-.00064	-.00146	-.00115	-0.00000
M_6^7	-.03396	-.10788	-.18358	-.23799	-.29123	-.33627
T_6^7	.03423	.10806	.17648	.19744	.15027	.05930
F_7^7	-.00011	-.00001	.00064	.00146	.00115	0.00000
M_7^7	.04144	.13808	.24527	.31952	.34737	.33628
T_7^7	-.02093	-.06470	-.10087	-.09913	-.03767	.05930
F_7^5	.00565	.02107	.04339	.06942	.09436	.11233
M_7^5	-.01594	-.06994	-.16901	-.31757	-.51424	-.70412
T_7^5	.01124	.04070	.08081	.12354	.16056	.17696
F_7^8	-.00554	-.02106	-.04403	-.07089	-.09551	-.11233
M_7^8	-.05268	-.17877	-.32608	-.44306	-.50793	-.51323
T_7^8	.03687	.13463	.26988	.41670	.55191	.64482
F_8^8	.00554	.02106	.04403	.07089	.09551	.11233
M_8^8	.16945	.62220	1.25200	1.93260	2.51697	2.87970
T_8^8	.00230	.00660	.00838	.00219	-.01854	-.04656

TABLE 10. FINAL MEMBER END STRESSES

UNIT LOAD AT POINTS 14,3,15,16,17,18

	UNIT LOAD AT POINT					
	14	3	15	16	17	18
F_1^1	.27048	.17161	.09470	.04055	.00957	.44215
M_1^1	-6.22345	-4.38952	-2.68006	-1.27032	-.33258	-6.03375
T_1^1	.45036	.34062	.21969	.10903	.02971	.39245
F_2^1	-.27048	-.17161	-.09470	-.04055	-.00957	-.44215
M_2^1	.11281	.50469	.53152	.34812	.11434	-3.82314
T_2^1	.66689	.52236	.34660	.17634	.04910	-.00266
F_2^4	-.09436	-.06942	-.04339	-.02108	-.00565	.56592
M_2^4	-.14628	-.16840	-.13473	-.07759	-.02362	1.19219
T_2^4	.16056	.12354	.08081	.04070	.01124	.00405
F_2^2	.36484	.24104	.13810	.06162	.01522	-.12377
M_2^2	-.27337	-.62822	-.61232	-.38881	-.12559	2.94409
T_2^2	-.52061	-.35396	-.21187	-.09875	-.02548	-.31453
F_3^2	.63516	-.24104	-.13810	-.06162	-.01522	.12377
M_3^2	-2.31095	-4.77896	-2.50391	-1.01017	-.22210	-.17917
T_3^2	-.61173	-.37793	-.12166	-.01081	.00846	-.23618
F_3^5	-.11888	-.10903	-.07975	-.04269	-.01224	.01044
M_3^5	-.01685	.02768	.05206	.04569	.01795	.04690
T_3^5	-.15779	-.09277	-.02680	-.00009	.00298	-.02532
F_3^3	-.51628	.35007	.21785	.10432	.02745	-.13421
M_3^3	2.46874	4.87174	2.53071	1.01026	.21912	.20448
T_3^3	.62859	.35026	.06961	-.03488	-.02641	.18929
F_4^3	.51628	.64993	.78215	.89568	.97255	.13421
M_4^3	9.13649	9.69313	9.38639	7.83341	4.74136	2.79480
T_4^3	.54711	.49988	.36606	.19010	.05073	.26745

CONTINUATION OF TABLE 10

	UNIT LOAD AT POINT					
	14	3	15	16	17	18
F_5^6	.09551	.07089	.04403	.02106	.00554	.30251
M_5^6	-2.51695	-1.93260	-1.25199	-.62220	-.16949	-4.18353
T_5^6	-.01853	.00219	.00838	.00660	.00230	.27819
F_6^6	-.09551	-.07089	-.04403	-.02106	-.00554	-.30251
M_6^6	.50790	.44306	.32609	.17877	.05269	-1.83615
T_6^6	.55190	.41670	.26988	.13463	.03688	.13571
F_6^4	.09436	.06942	.04339	.02108	.00565	.43408
M_6^4	-.51424	-.31757	-.16902	-.06994	-.01594	-.73074
T_6^4	-.16056	-.12354	-.08081	-.04070	-.01124	-.00405
F_6^7	.00115	.00146	.00064	-.00001	-.00011	-.13157
M_6^7	-.34734	-.31952	-.24528	-.13808	-.04145	2.71521
T_6^7	-.03767	-.09913	-.10087	-.06470	-.02094	-.27997
F_7^7	-.00115	-.00146	-.00064	.00001	.00011	.13157
M_7^7	.29121	.23800	.18361	.10788	.03395	-.09752
T_7^7	.13026	.19744	.17649	.10806	.03423	-.21598
F_7^5	.11888	.10903	.07975	.04269	.01224	-.01044
M_7^5	-.81527	-.79093	-.61030	-.34453	-.10360	.02618
T_7^5	.15779	.09277	.02680	.00009	-.00298	.02532
F_7^8	-.11772	-.10757	-.07911	-.04271	-.01235	-.12113
M_7^8	-.44900	-.33077	-.21041	-.10796	-.03097	.07220
T_7^8	.66501	.59349	.43380	.23647	.06937	.18981
F_8^8	.11772	.10757	.07911	.04271	.01235	.12113
M_8^8	2.93082	2.59851	1.87921	1.00995	.29211	2.34446
T_8^8	-.06906	-.07698	-.06535	-.03935	-.01240	.21085

TABLE 11. FINAL MEMBER END STRESSES

UNIT LOAD AT POINTS 19,20,21,22,6,23

	UNIT LOAD AT POINT					
	19	20	21	22	6	23
F_1^1	.14659	.02236	.07521	.13555	.18094	.19691
M_1^1	-3.17143	-.25630	-.86872	-1.58509	-2.16366	-2.44253
T_1^1	.27938	.03364	.11506	.21176	.29050	.32618
F_2^1	-.14659	-.02236	-.07521	-.13555	-.18094	-.19691
M_2^1	-.11600	-.23453	-.78202	-1.38944	-1.80724	-1.88061
T_2^1	.25937	-.02980	-.09977	-.17726	-.22766	-.22710
F_2^4	-.01527	.03057	.10089	.17681	.22553	.22558
M_2^4	-.00339	.07292	.24461	.43692	.56892	.58197
T_2^4	.04646	.00640	.01391	.00520	-.03504	-.07639
F_2^2	.16186	-.00821	-.02568	-.04126	-.04459	-.02867
M_2^2	.06955	.22813	.76811	1.38424	1.84228	1.95700
T_2^2	-.25598	-.04311	-.14484	-.25966	-.34126	-.35487
F_3^2	-.16186	.00821	.02568	.04126	.04459	.02867
M_3^2	-3.67716	-.05037	-.21227	-.49120	-.87693	-1.33601
T_3^2	-.40466	-.00599	-.02803	-.07103	-.13821	-.22577
F_3^5	.55058	.00039	.00679	.02638	.06459	.12302
M_3^5	1.15441	.00712	.03791	.10392	.21191	.35575
T_3^5	-.05368	.00340	.01372	.03040	.05176	.07367
F_3^3	-.38872	-.00860	-.03247	-.06764	-.10918	-.15169
M_3^3	2.85584	.04697	.19855	.46080	.82517	1.26234
T_3^3	.12526	-.00113	-.00988	-.03289	-.07370	-.12997
F_4^3	.38872	.00860	.03247	.06764	.10918	.15169
M_4^3	5.82304	.14267	.51705	1.02848	1.57670	2.07220
T_4^3	.39794	.01800	.06604	.13299	.20621	.27277

CONTINUATION OF TABLE 11

	UNIT LOAD AT POINT					
	19	20	21	22	6	23
F_5^6	.10553	.95837	.85010	.70468	.55410	.42294
M_5^6	-2.18641	-3.90090	-5.90906	-6.46161	-6.13498	-5.46658
T_5^6	.15678	.06375	.23299	.44213	.59874	.64469
F_6^6	-.10553	.04163	.14990	.29532	.44589	-.42294
M_6^6	.06305	-.21382	-.99286	-2.48978	-4.76845	-2.83897
T_6^6	.23986	-.10177	-.29878	-.42985	-.35779	-.18137
F_6^4	.01527	-.03057	-.10089	-.17681	-.22553	-.22558
M_6^4	-.10352	.14108	.46162	.80073	1.00982	.99710
T_6^4	-.04646	-.00640	-.01391	-.00520	.03504	.07639
F_6^7	.09026	-.01106	-.04901	-.11851	-.22036	.64952
M_6^7	-.01659	.22022	1.00677	2.49498	4.73341	2.76258
T_6^7	-.13634	-.03930	-.16284	-.37088	-.65204	-.81573
F_7^7	-.09026	.01106	.04901	.11851	.22036	.35148
M_7^7	-1.78021	-.00604	-.05194	-.17462	-.40045	-.77154
T_7^7	-.17463	-.00059	-.02384	-.09984	-.25319	-.49558
F_7^5	.44942	-.00039	-.00679	-.02638	-.06459	-.12302
M_7^5	-.80036	-.00441	.00966	.08073	.24023	.50539
T_7^5	.05368	-.00340	-.01372	-.03040	-.05176	-.07367
F_7^8	-.35916	-.01067	-.04222	-.09213	-.15577	-.22846
M_7^8	2.60153	.00944	.06566	.20502	.45221	.84522
T_7^8	.09999	.00500	.01418	.01910	.01297	-.00980
F_8^8	.35916	.01067	.04222	.09213	.15577	.22846
M_8^8	4.54996	.19966	.76128	1.59939	2.59827	3.62984
T_8^8	.24357	.02854	.10847	.22676	.36544	.50081

TABLE 12. FINAL MEMBER END STRESSES

UNIT LOAD AT POINTS 24,25,7,26,27,28

	UNIT LOAD AT POINT					
	24	25	7	26	27	28
F_1^1	.18465	.15169	.10918	.06764	.03247	.00860
M_1^1	-2.39595	-2.07215	-1.57670	-1.02849	-.51705	-.14269
T_1^1	.31731	.27277	.20621	.13299	.06604	.01801
F_2^1	-.18465	-.15169	-.10918	-.06764	-.03247	-.00860
M_2^1	-1.66060	-1.26234	-.82518	-.46078	-.19855	-.04699
T_2^1	-.18765	-.12997	-.07370	-.03288	-.00988	-.00113
F_2^4	.18465	.12302	.06459	.02638	.00679	.00039
M_2^4	.49559	.35574	.21190	.10391	.03790	.00713
T_2^4	-.08580	-.07367	-.05176	-.03040	-.01372	-.00340
F_2^2	-0.00000	.02867	.04459	.04126	.02568	.00821
M_2^2	1.74640	1.33601	.87694	.49119	.21227	.05038
T_2^2	-.30794	-.22577	-.13821	-.07103	-.02803	-.00599
F_3^2	0.00000	-.02867	-.04459	-.04126	-.02568	-.00821
M_3^2	-1.74640	-1.95694	-1.84227	-1.38427	-.76811	-.22815
T_3^2	-.30794	-.35487	-.34126	-.25966	-.14484	-.04312
F_3^5	.18466	.22558	.22554	.17681	.10089	.03057
M_3^5	.49560	.58197	.56892	.43693	.24462	.07293
T_3^5	.08580	.07639	.03504	-.00520	-.01391	-.00640
F_3^3	-.18465	-.19690	-.18094	-.13555	-.07521	-.02236
M_3^3	1.66060	1.88056	1.80724	1.38947	.78202	.23455
T_3^3	-.18765	-.22709	-.22766	-.17726	-.09978	-.02981
F_4^3	.18465	.19690	.18094	.13555	.07521	.02236
M_4^3	2.39595	2.44246	2.16365	1.58513	.86871	.25632
T_4^3	.31731	.32617	.29050	.21176	.11506	.03365

CONTINUATION OF TABLE 12

	UNIT LOAD AT POINT					
	24	25	7	26	27	28
F_5^6	.31534	.22846	.15577	.09213	.04222	.01067
M_5^6	-4.60897	-3.62984	-2.59827	-1.59937	-.76128	-.19970
T_5^6	.60347	.50081	.36544	.22676	.10847	.02855
F_6^6	-.31534	-.22846	-.15577	-.09213	-.04222	-.01067
M_6^6	-1.57382	-.84527	-.45221	-.20501	-.06566	-.00945
T_6^6	-.06829	-.00981	.01297	.01911	.01418	.00500
F_6^4	-.18465	-.12302	-.06459	-.02638	-.00679	-.00039
M_6^4	.79699	.50537	.24022	.08072	.00966	-.00440
T_6^4	.08580	.07367	.05176	.03040	.01372	.00340
F_6^7	.50000	.34148	.22036	.11851	.04901	.01106
M_6^7	1.48802	.77161	.40045	.17461	.05194	.00605
T_6^7	-.72870	-.49557	-.25319	-.09983	-.02384	-.00060
F_7^7	.50000	.64852	-.22036	-.11851	-.04901	-.01106
M_7^7	-1.48802	-2.76267	-4.73341	-2.49487	-1.00677	-.22030
T_7^7	-.72870	-.80568	-.65204	-.37087	-.16284	-.03932
F_7^5	-.18466	-.22558	-.22554	-.17681	-.10089	-.03057
M_7^5	.79699	.99706	1.00983	.80076	.46162	.14108
T_7^5	-.08580	-.07639	-.03504	.00520	.01391	.00640
F_7^8	-.31534	-.42294	.44590	.29532	.14990	.04164
M_7^8	1.57382	2.83906	4.76845	2.48967	.99286	.21390
T_7^8	-.06829	-.18138	-.35779	-.42989	-.29878	-.10177
F_8^8	.31534	.42294	.55410	.70468	.85010	.95836
M_8^8	4.60897	5.46658	6.13497	6.46170	5.90906	3.90071
T_8^8	.60347	.64469	.59874	.44212	.23299	.06379

TABLE 13. STRESSES AT A SECTION BETWEEN JOINTS

UNIT LOAD AT POINTS 9,10,11,2,12,13

	UNIT LOAD AT POINT					
	9	10	11	2	12	13
F_9	.02745	.89568	.78215	.64993	.51628	.38767
M_9	-.67073	-2.85049	-5.04264	-6.09368	-6.28494	-5.77115
T_9	.09071	-.27636	-.26390	-.18938	-.12620	-.07123
F_{10}	.02745	.89568	.78215	.64993	.51628	.38767
M_{10}	-.52378	2.15413	-.66069	-2.44785	-3.38545	-3.59199
T_{10}	.03856	-.30677	-.51292	-.56232	-.54842	-.48004
F_{11}	.02745	.10432	.78215	.64993	.51628	.38767
M_{11}	-.37284	-1.58828	3.72629	1.21662	-.46018	-1.38550
T_{11}	-.00059	.07858	-.37907	-.61607	-.71632	-.69736
F_{12}	.01522	-.06162	-.13810	-.24104	.36484	.50000
M_{12}	-.13607	.66538	1.73882	3.45631	-5.88207	3.48206
T_{12}	-.02409	.06235	.06356	-.01837	.21775	-.45875
F_{13}	.01522	-.06162	-.13810	-.24104	.36484	.50000
M_{13}	-.04900	.31551	.96041	2.10735	-3.85435	6.28304
T_{13}	-.03217	.10517	.18142	.22454	-.20735	-.03240
F_{14}	.01522	-.06162	-.13810	-.24104	.36484	.50000
M_{14}	.03843	-.03675	.17470	.74235	-1.79729	-3.48206
T_{14}	-.03263	.11734	.23098	.34896	-.45410	-.45875
F_{15}	.00957	-.04055	-.09471	-.17161	-.27048	-.38767
M_{15}	.17119	-.58716	-1.08517	-1.50048	-1.67134	-1.38552

CONTINUATION OF TABLE 13

	UNIT LOAD AT POINT					
	9	10	11	2	12	13
T_{15}	-.03661	.13550	.27601	.43481	.58901	.69736
F_{16}	.00957	-.04055	-.09471	-.17161	-.27048	-.38767
M_{16}	.22683	-.82172	-1.63060	-2.48484	-3.21707	-3.59202
T_{16}	-.01924	.07399	.15744	.26081	.37558	.48004
F_{17}	.00957	-.04055	-.09471	-.17161	-.27048	-.38767
M_{17}	.28074	-1.05004	-2.16362	-3.45030	-4.73832	-5.77118
T_{17}	.00292	-.00773	-.00822	.00168	.02824	.07123
F_{18}	-.01223	-.04269	-.07975	-.10904	-.11888	-.11233
M_{18}	.06078	.19511	.33116	.40932	.39925	.31097
T_{18}	-.00299	.00009	.02681	.09277	.15780	.17696
F_{19}	-.00565	-.02107	-.04339	-.06942	-.09436	-.11233
M_{19}	-.00385	-.00385	.01715	.07457	.18400	.31098
T_{19}	-.01124	0.00000	-.08081	-.12354	-.16056	-.17696
F_{20}	-.01234	.04271	.07911	.10757	.11772	.11233
M_{20}	.22884	-.79176	-1.47569	-2.05069	-2.33232	-2.30996
T_{20}	.03515	-.11801	-.21182	-.27997	-.29885	-.27315
F_{21}	-.01234	.04271	.07911	.10757	.11772	.11233
M_{21}	.16397	-.56754	-1.06093	-1.48726	-1.71605	-1.72263
T_{21}	.05230	-.17736	-.32257	-.43444	-.47561	-.44921
F_{22}	-.01234	.04271	.07911	.10757	.11772	.11233

CONTINUATION OF TABLE 13

	UNIT LOAD AT POINT					
	9	10	11	2	12	13
M_{22}	.09785	-.33900	-.63810	-.91251	-1.08671	-1.12219
T_{22}	.06374	-.21694	-.39675	-.53921	-.59798	-.57342
F_{23}	-.00011	.00001	-.00064	-.23799	-.00115	0.00000
M_{23}	.03622	-.11684	-.20142	-.97848	-.30890	-.34016
T_{23}	-.03115	.09825	.15967	-.03556	.12407	.02977
F_{24}	-.00011	.00001	-.00064	-.23799	-.00115	0.00000
M_{24}	.03826	-.12491	-.21773	-2.14695	-.32421	-.34146
T_{24}	-.02790	.08769	.14137	-.17202	.09643	0.00000
F_{25}	-.00011	.00001	-.00064	-.23799	-.00115	0.00000
M_{25}	.04000	-.13203	-.23239	-3.29908	-.33706	-.34016
T_{25}	-.02449	.07648	.12172	-.40980	.06755	-.02975
F_{26}	.00554	-.02106	-.04403	-.07089	-.09551	-.11233
M_{26}	.08303	-.29383	-.56580	-.82778	-1.02577	-1.12221
T_{26}	-.03094	.11400	.23094	.36121	.48495	.57341
F_{27}	.00554	-.02106	-.04403	-.07089	-.09551	-.11233
M_{27}	.11277	-.40665	-.80121	-1.20619	-1.53580	-1.72265
T_{27}	-.02239	.08341	.17125	.27241	.37311	.44921
F_{28}	.00554	-.02106	-.04403	-.07089	-.09551	-.11233
M_{28}	.14165	-.51637	-1.03053	-1.57543	-2.03414	-2.30998
T_{28}	-.01128	.04311	.09128	.15096	.21724	.27314

TABLE 14. STRESSES AT A SECTION BETWEEN JOINTS

UNIT LOAD AT POINTS 14,3,15,16,17,18

	UNIT LOAD AT POINT					
	14	3	15	16	17	18
F_9	.27048	.17161	.09470	.04055	.00957	.44215
M_9	-4.73826	-3.45032	-2.16357	-1.05000	-.28080	-3.59172
T_9	-.02824	-.00167	.00821	.00772	.00293	-.02780
F_{10}	.27048	.17161	.09470	.04055	.00957	.44215
M_{10}	-3.21701	-2.48487	-1.63060	-.82168	-.22689	-1.12236
T_{10}	-.37557	-.26081	-.15744	-.07400	-.01924	-.23363
F_{11}	.27048	.17161	.09470	.04055	.00957	.44215
M_{11}	-1.67127	-1.50050	-1.08523	-.58711	-.17125	1.35555
T_{11}	-.58900	-.43482	-.27602	-.13551	-.03662	-.22345
F_{12}	.36484	.24104	.13810	.06162	.01522	-.12377
M_{12}	1.79736	.74243	.17472	-.03682	-.03844	2.27356
T_{12}	-.45407	-.34897	-.23097	-.11733	-.03264	-.08672
F_{13}	.36484	.24104	.13810	.06162	.01522	-.12377
M_{13}	3.85441	2.10743	.96044	.31544	.04900	1.58573
T_{13}	-.20731	-.22454	-.18141	-.10517	-.03218	.08178
F_{14}	.36484	.24104	.13810	.06162	.01522	-.12377
M_{14}	5.88213	3.45640	1.73884	.66531	.13606	.88583
T_{14}	.21780	.01838	-.06356	-.06235	-.02410	.18969
F_{15}	-.51628	.35007	.78215	.10432	.02745	-.13421
M_{15}	-.46002	6.76504	-3.72613	1.58828	.37289	-.55746

CONTINUATION OF TABLE 14

	UNIT LOAD AT POINT					
	14	3	15	16	17	18
T_{15}	.71629	.85833	-.37913	.07858	-.00056	.17388
F_{16}	-.51628	.35007	.78215	.10432	.02745	-.13421
M_{16}	-3.38528	8.60685	.66085	2.15420	.52383	-1.31516
T_{16}	.54840	1.52949	-.51296	.24198	.03859	.09212
F_{17}	-.51628	.35007	.78215	.89568	.02745	-.13421
M_{17}	-6.28478	10.38317	5.04281	2.85048	.67078	-2.06285
T_{17}	.12620	2.35861	-.26393	-.27636	.09075	-.05537
F_{18}	-.09436	-.06942	-.04339	-.02108	-.00565	.56592
M_{18}	.18398	.07457	.01714	-.00381	-.00385	-.78853
T_{18}	.16056	.12354	.08081	.04070	.01124	.00405
F_{19}	-.11888	-.10903	-.07975	-.04269	-.01224	.01044
M_{19}	.39923	.40929	.33119	.19511	.06079	.01036
T_{19}	-.15779	-.09277	-.02680	-.00009	.00298	-.02532
F_{20}	.09551	.07089	.04403	.02106	.00554	.30251
M_{20}	-2.03409	-1.57535	-1.03052	-.51640	-.14169	-2.69794
T_{20}	-.21723	-.15097	-.09128	-.04311	-.01129	-.02226
F_{21}	.09551	.07089	.04403	.02106	.00554	.30251
M_{21}	-1.53575	-1.20612	-.80120	-.40668	-.11281	-1.19181
T_{21}	-.37309	-.27241	-.17125	-.08342	-.02240	-.19209
F_{22}	.09551	.07089	.04403	.02106	.00554	.30251

CONTINUATION OF TABLE 14

	UNIT LOAD AT POINT					
	14	3	15	16	17	18
M_{22}	-1.02572	-.82770	-.56579	-.29386	-.08306	.32339
T_{22}	-.48493	-.36121	-.23094	-.11400	-.03095	-.23001
F_{23}	.00115	.00146	.00064	-.00001	-.00011	-.13157
M_{23}	-.33706	-.30245	-.23239	-.13196	-.04001	2.07953
T_{23}	-.06755	-.12629	-.12173	-.07649	-.02450	-.07063
F_{24}	.00115	.00146	.00064	-.00001	-.00011	-.13157
M_{24}	-.32421	-.28309	-.21774	-.12485	-.03827	1.42803
T_{24}	-.09642	-.15185	-.14138	-.08770	-.02791	.08252
F_{25}	.00115	.00146	.00064	-.00001	-.00011	-.13157
M_{25}	-.30889	-.26156	-.20143	-.11678	-.03623	.76565
T_{25}	-.12406	-.17563	-.15968	-.09825	-.03117	.17829
F_{26}	-.11772	-.10757	-.07911	-.04271	-.01235	-.12113
M_{26}	-1.08660	-.91246	-.63810	-.33908	-.09789	-.54281
T_{26}	.59796	.53921	.39675	.21695	.06374	.16926
F_{27}	-.11772	-.10757	-.07911	-.04271	-.01235	-.12113
M_{27}	-1.71593	-1.48721	-1.06093	-.56762	-.16406	-1.15369
T_{27}	.47560	.43444	.32257	.17736	.05231	.09519
F_{28}	-.11772	-.10757	-.07911	-.04271	-.01235	-.12113
M_{28}	-2.33221	-2.05064	-1.47568	-.79183	-.22898	-1.75578
T_{28}	.29886	.27997	.21182	.11801	.03515	-.03184

TABLE 15. STRESSES AT A SECTION BETWEEN JOINTS

UNIT LOAD AT POINTS 19,20,21,22,6,23

	UNIT LOAD AT POINT					
	19	20	21	22	6	23
F_9	.14659	.02236	.07521	.13555	.18094	.19691
M_9	-2.37036	-.13419	-.45814	-.84541	-1.17680	-1.36911
T_9	.03742	.01659	.05713	.10564	.14465	.15976
F_{10}	.14659	.02236	.07521	.13555	.18094	.19691
M_{10}	-1.55124	-.01106	-.04407	-.09930	-.18098	-.28527
T_{10}	-.13380	.01025	.03520	.06440	.08537	.08753
F_{11}	.14659	.02236	.07521	.13555	.18094	.19691
M_{11}	-.72032	.11215	.37033	.64756	.81621	.80075
T_{11}	-.23298	.01466	.04945	.08833	.11311	.11004
F_{12}	.16186	-.00821	-.02568	-.04126	-.04459	-.02867
M_{12}	.98968	.18547	.63533	1.17267	1.61760	1.82141
T_{12}	-.20973	-.02505	-.08356	-.14802	-.19020	-.18990
F_{13}	.16186	-.00821	-.02568	-.04126	-.04459	-.02867
M_{13}	1.90227	.14139	.49770	.95218	1.38062	1.67195
T_{13}	-.08347	-.01078	-.03410	-.05525	-.05929	-.03738
F_{14}	.16186	-.00821	-.02568	-.04126	-.04459	-.02867
M_{14}	2.80039	.09624	.35630	.72444	1.13312	1.50977
T_{14}	.12185	-.00041	.00319	.01795	.05046	.10154
F_{15}	-.38872	-.00860	-.03247	-.06764	-.10918	-.15169
M_{15}	.67724	-.00083	.01850	.08661	.22267	.42721

CONTINUATION OF TABLE 15

	UNIT LOAD AT POINT					
	19	20	21	22	6	23
T_{15}	.27952	.00088	-.00040	-.00899	-.02795	-.05620
F_{16}	-.38872	-.00860	-.03247	-.06764	-.10918	-.15169
M_{16}	-1.50651	-.04862	-.16170	-.28824	-.38153	-.41117
T_{16}	.24331	-.00127	-.00666	-.01779	-.03489	-.05550
F_{17}	-.38872	-.00860	-.03247	-.06764	-.10918	-.15169
M_{17}	-3.67880	-.09604	-.34066	-.66089	-.98282	-1.24642
T_{17}	.01691	-.00759	-.02859	-.05923	-.09446	-.12787
F_{18}	-.01527	.03057	.10089	.17681	.22553	.22558
M_{18}	.05006	-.03408	-.10851	-.18192	-.22044	-.20756
T_{18}	.04646	.00640	.01391	.00520	-.03504	-.07639
F_{19}	.55058	.00039	.00679	.02638	.06459	.12302
M_{19}	-.77262	.00576	.01415	.01159	-.01416	-.07482
T_{19}	-.05368	.00340	.01372	.03040	.05176	.07367
F_{20}	.10553	.04163	.85010	.70468	.55410	.42294
M_{20}	-1.67060	-.84339	-1.70873	-2.99555	-3.42744	-3.41331
T_{20}	-.01162	.03741	-.09961	.02923	.18124	.25699
F_{21}	.10553	.04163	.85010	.70468	.55410	.42294
M_{21}	-1.14208	-.63785	2.50460	.49330	-.69382	-1.33407
T_{21}	-.13443	-.02726	-.06486	-.08003	.00130	.04971
F_{22}	.10553	.04163	.14990	.70468	.55410	.42294

CONTINUATION OF TABLE 15

	UNIT LOAD AT POINT					
	19	20	21	22	6	23
M_{22}	-.60487	-.42746	-1.75539	3.97840	2.04509	.75533
T_{22}	-.21070	-.07377	-.17879	.11521	.06030	.02444
F_{23}	.09026	-.01106	-.04901	-.11851	-.22036	.35148
M_{23}	.44110	.16819	.77510	1.93256	3.68400	-6.02804
T_{23}	-.11781	-.02234	-.08504	-.17757	-.28453	.39960
F_{24}	.09026	-.01106	-.04901	-.11851	-.22036	.35148
M_{24}	.89543	.11488	.53753	1.35543	2.60655	-4.30417
T_{24}	-.05945	-.00998	-.02773	-.03401	-.00988	-.05151
F_{25}	.09026	-.01106	-.04901	-.11851	-.22036	.35148
M_{25}	1.34294	.06069	.29587	.76798	1.50926	-2.54755
T_{25}	.03828	-.00232	.00866	.05870	.16982	-.35066
F_{26}	-.35916	-.01067	-.04222	-.09213	-.15577	-.22846
M_{26}	.80923	-.04372	-.14433	-.25240	-.31990	-.28537
T_{26}	.24891	.00350	.01075	.01703	.01875	.01464
F_{27}	-.35916	-.01067	-.04222	-.09213	-.15577	-.22846
M_{27}	-.98922	-.09656	-.35321	-.70790	-1.08957	-1.41379
T_{27}	.24105	-.00262	-.01098	-.02490	-.04279	-.05954
F_{28}	-.35916	-.01067	-.04222	-.09213	-.15577	-.22846
M_{28}	-2.78015	-.14865	-.55941	-1.15801	-1.85095	-2.53145
T_{28}	.07647	-.01333	-.05082	-.10636	-.17118	-.23180

TABLE 16. STRESSES AT A SECTION BETWEEN JOINTS

UNIT LOAD AT POINTS 24,25,7,26,27,28

	UNIT LOAD AT POINT					
	24	25	7	26	27	28
F_9	.18465	.15169	.10918	.06764	.03247	.00860
M_9	-1.38996	-1.24639	-.98289	-.66087	-.34068	-.09600
T_9	.15201	.12788	.09446	.05923	.02859	.00759
F_{10}	.18465	.15169	.10918	.06764	.03247	.00860
M_{10}	-.37338	-.41113	-.38159	-.28821	-.16171	-.04858
T_{10}	.07503	.05551	.03488	.01779	.00666	.00128
F_{11}	.18465	.15169	.10918	.06764	.03247	.00860
M_{11}	.64603	.42724	.22261	.08663	.01848	-.00079
T_{11}	.08693	.05621	.02794	.00899	.00040	-.00088
F_{12}	0.00000	.02867	.04459	.04126	.02568	.00821
M_{12}	1.76659	1.50968	1.13306	.72444	.35639	.09626
T_{12}	-.15456	-.10152	-.05045	-.01795	-.00320	.00041
F_{13}	0.00000	.02367	.04459	.04126	.02568	.00821
M_{13}	1.77334	1.67186	1.38055	.95218	.49780	.14141
T_{13}	-0.00000	.03738	.05929	.05525	.03409	.01079
F_{14}	0.00000	.02867	.04459	.04126	.02568	.00821
M_{14}	1.76659	1.82131	1.61754	1.17268	.63542	.18549
T_{14}	.15455	.18990	.19019	.14802	.08357	.02506
F_{15}	-.18465	-.19690	-.18094	-.13555	-.07521	-.02236
M_{15}	.64610	.80069	.81626	.64753	.37044	.11219

CONTINUATION OF TABLE 16

	UNIT LOAD AT POINT					
	24	25	7	26	27	28
T_{15}	-.08694	-.11003	-.11312	-.08832	-.04946	-.01467
F_{16}	-.18465	-.19690	-.18094	-.13555	-.07521	-.02236
M_{16}	-.37331	-.28526	-.18094	-.09933	-.04396	-.01102
T_{16}	-.07503	-.08752	-.08538	-.06439	-.03521	-.01025
F_{17}	-.18465	-.19690	-.18094	-.13555	-.07521	-.02236
M_{17}	-1.38988	-1.36905	-1.17675	-.84544	-.45803	-.13415
T_{17}	-.15201	-.15975	-.14466	-.10564	-.05713	-.01659
F_{18}	.18465	.12302	.06459	.02638	.00679	.00039
M_{18}	-.15069	-.07483	-.01417	.01158	.01414	.00577
T_{18}	-.08580	-.07367	-.05176	-.03040	-.01372	-.00340
F_{19}	.18466	.22558	.22554	.17681	.10089	.03057
M_{19}	-.15071	-.20756	-.22047	-.18191	-.10850	-.03407
T_{19}	.08580	.07639	.03504	-.00520	-.01391	-.00640
F_{20}	.31534	.22846	.15577	.09213	.04222	.01067
M_{20}	-3.08675	-2.53144	-1.85098	-1.15807	-.55934	-.14874
T_{20}	.26747	.23180	.17118	.10637	.05081	.01334
F_{21}	.31534	.22846	.15577	.09213	.04222	.01067
M_{21}	-1.54103	-1.41378	-1.08959	-.70796	-.35314	-.09664
T_{21}	.06542	.05955	.04279	.02490	.01097	.00262
F_{22}	.31534	.22846	.15577	.09213	.04222	.01067

CONTINUATION OF TABLE 16

	UNIT LOAD AT POINT					
	24	25	7	26	27	28
M_{22}	.01641	-.28536	-.31992	-.25246	-.14425	-.04381
T_{22}	-.00115	-.01464	-.01875	-.01704	-.01075	-.00351
F_{23}	.50000	.34148	.22036	.11851	.04901	.01106
M_{23}	4.01508	2.49824	1.50922	.76790	.29585	.06070
T_{23}	-.48843	-.35280	-.16981	-.05868	-.00865	.00231
F_{24}	.50000	.34148	.22036	.11851	.04901	.01106
M_{24}	6.51158	4.20585	2.60651	1.35534	.53751	.11488
T_{24}	-.02882	-.06010	.00989	.03402	.02773	.00998
F_{25}	.50000	.34148	.22036	.11851	.04901	.01106
M_{25}	-4.01508	5.88145	3.68396	1.93247	.77508	.16820
T_{25}	-.48843	.38032	.28453	.17757	.08504	.02234
F_{26}	-.31534	-.42294	.44590	.70468	.14990	.04164
M_{26}	.01650	.75541	6.98353	-3.97832	1.75539	.42759
T_{26}	.00114	-.02444	.15531	.11518	-.17879	-.07376
F_{27}	-.31534	-.42294	.44590	.70468	.14990	.04164
M_{27}	-1.54094	-1.33399	9.14546	-.49322	2.50456	.63803
T_{27}	-.06541	-.04970	.85952	-.08005	.00721	-.02724
F_{28}	-.31534	-.42294	.44590	.70468	.85010	.04164
M_{28}	-3.08666	-3.41323	11.23778	2.99564	1.70873	.84361
T_{28}	-.26746	-.25697	1.74947	.02921	-.09961	.03745

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by

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Since a problem in the field of gridded frameworks can be reduced to the solution of a set of simultaneous linear equations, the difficulty in solving these problems involves the solution of a set of simultaneous equations. If any of the members of the framework are curved in plan, many trigonometric terms will appear in the coefficients of these equations. This makes the analyses of the problems in this field very complicated. Efforts to solve this kind of problems have been made by many authors. Most of them have tried to use the moment distribution method to avoid solving a set of simultaneous equations. However, the allowed simplifications will be much reduced if the structure is of the gridded type. Since the moment distribution method itself will become tedious if it is used to solve the problems of the gridded type.

In this report, advantage will be taken of electronic computer procedures to solve the simultaneous equations and to do the analysis work. It allows the greatest possible simplification in the computation required. The member can have any curvature in the plan, including segmental, but particular attention will be paid to the common, circular-arc shape.

Formulas are presented for frames of constant cross section that give the relationship between the end moments, end torques and shears and the corresponding rotations and displacements of the end sections. These relations have been derived from the expression for the total strain energy due to bending and torsion by means of Castigliano's theorem. From these relations the stiffness matrix of each single member can be written and the stiffness matrix for the whole structure follows. Through computation with the electronic computer the end forces can be obtained. These end forces are combined with the fixed-end forces to obtained final end forces.