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economic power dispatch

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This article presents a new nonlinear convex network flow programming model and algorithm for solving the on-line economic power dispatch with N and N − 1 security. Based on the load flow equations, a new nonlinear convex network flow model for secure economic power dispatch is set up and then transformed into a quadratic programming model, in which the search direction in the space of the flow variables is to be solved. The concept of maximum basis in a network flow graph was introduced so that the constrained quadratic programming model was changed into an unconstrained quadratic programming model which was then solved by the reduced gradient method. The proposed model and its algorithm were examined numerically with an IEEE 30-bus test system on an ALPHA 400 Model 610 machine. Satisfactory results were obtained.

Keywords: Economic dispatch control, Security analysis, N − 1 security constraints, Convex network flow programming, Quadratic programming

I. Introduction

The aim of real power economic dispatch is to minimize the total production cost of supplying all the loads on the power system. Security dispatch implies the continuity of supply service, even in the event of failure of equipment, and must handle several physical constraints. Among these post-outage constraints, the line loading constraints should be met if possible, but may be violated temporarily. Therefore, the security problem is an additional and important aspect of power system operation, and has a profound influence on the overall economic dispatch problem.

It is well known that in real time secure economic power dispatch, the crux of the matter is to provide speed and precision in the calculation. A number of methods have been proposed to solve this problem [1–4]. The usual methods (linear or nonlinear model) require a large number of AC or DC load flow calculations to be carried out, and seem to be inefficient for large scale power systems. Network flow programming (NFP) models of security economic power dispatch were presented in recent years [1,5,6]. NFP is a specialized form of linear programming. It is characterized by simple calculation and rapid convergence. In particular, the concept of an ''$N-1$ constrained zone'' was proposed in reference [1] so that the economic power dispatch with N and $N-1$ security can be solved by NFP. However, because of the adoption of a linear NFP model in [1], the calculation precision is still not fully satisfactory.

In order to solve the combined problem of speed and precision in the calculation of secure economic power dispatch, this article presents a new nonlinear convex network flow programming model of on-line economic power dispatch with N and $N-1$ security, which is solved by using a combination of quadratic programming (QP) and NFP. Based on the load flow equations, a new nonlinear convex network flow model for secure economic power dispatch is set up and then transformed into a QP model, in which the search direction in the space of the flow variables is found. The concept of a maximum basis in the network flow graph is introduced, allowing the constrained QP model to be changed into an unconstrained QP model, which is then solved using the reduced gradient method.

In the article, the fast $N-1$ security analysis method [7,8] is also used to seek out all the binding constraint cases for all possible single line outages, and then an ''$N-1$ constrained zone'' is formed which is coordinated with the convex network flow programming model. As a result, the proposed secure economic dispatch and associated solution method have both high calculation precision and high calculation speed.

The proposed model and algorithm are examined numerically with an IEEE 30-bus test system on an ALPHA 400 Model 610 machine. Satisfactory results are demonstrated and also compared with the results obtained through the conventional methods.

II. Convex NFP model of secure economic power dispatch

II.1 *Calculation of N − 1 security constraints*

Economic power dispatch with $N-1$ security means that the line flows will not exceed the settings of the protective devices for the intact lines when any branch has an outage [1–3]. The usual methods of calculating $N-1$ security

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constraints need a large number of AC or DC load flow calculations to be carried out, and seem to be inefficient for on-line application.

A fast and efficient calculation approach for $N-1$ security constraints, which was proposed in references [1,7,8], is adopted in this article. Based on the fast $N-1$ security analysis, all the cases with binding constraints for all possible single outages can be found. Thus, the maximum value of the violation in line *ij* can be determined by the following equations:

$$\Delta P_{ij\max} = \max_{1\in NL}\left\{\mathrm{P}_{ij}(l) - P_{ij\mathrm{M}}\right\} \quad ij \in NT1 \tag{1}$$

$$\Delta P_{ij\min} = \min_{1\in NL}\left\{\mathrm{P}_{ij}(l) - P_{ij\mathrm{m}}\right\} \quad ij \in NT2 \tag{2}$$

$$P_{ij\mathrm{m}} = -P_{ij\mathrm{M}} \tag{3}$$

where $NT1$ and $NT2$ represent the number of lines which violate their upper and lower bounds, respectively, for line l outage. NL is the set of single line outages. $P_{ij\mathrm{M}}$ is the active power flow constraint on transmission line *ij*.

Therefore, an ''$N-1$ constrained zone'', which is formed by the intersection of the secure zones for single contingencies, can be determined from the following equations.

$$\Delta P_{ij} = -\Delta P_{ij\max} \quad ij \in NT1 \tag{4}$$

$$\Delta P_{ij} = -\Delta P_{ij\min} \quad ij \in NT2 \tag{5}$$

$$-P_{ij\mathrm{M}} \le P_{ij} \le P_{ij\mathrm{M}} \quad ij \in NT - NT1 - NT2 \tag{6}$$

where NT is the total number of transmission lines in the power network.

It can be observed from the ''$N-1$ constrained zone'', equations (4)–(6) that the number of $N-1$ constraints is

$$NT1 + NT2 + (NT - NT1 - NT2) = NT$$

This means that the huge number of $N-1$ security constraints can be reduced to the same scale as N security constraints due to the adoption of ''$N-1$ constrained zone''. Therefore, similar to N security, $N-1$ security can be introduced into the network flow model.

II.2 *Mathematical model*

It is well known that the active power flow equations of a transmission line can be written as follows.

$$P_{ij} = V_i^2 g_{ij} - V_i V_j g_{ij}\cos\theta_{ij} - V_i V_j b_{ij}\sin\theta_{ij} \tag{7}$$

$$P_{ji} = V_j^2 g_{ij} + V_i V_j(-g_{ij}\cos\theta_{ij} + b_{ij}\sin\theta_{ij}) \tag{8}$$

where

P_{ij} is the sending end active power on transmission line *ij*;
P_{ji} is the receiving end active power on transmission line *ij*;
V_i is the node voltage magnitude of node *i*;
θ_{ij} is the difference of node voltage angles between the sending end and receiving end of the line *ij*;
b_{ij} is the susceptance of transmission line *ij*;
g_{ij} is the conductance of transmission line *ij*.

In a high voltage power network, the value of θ_{ij} is very small, and the following approximate equations are easily obtained

$$V \cong 1.0 \text{ p.u.} \tag{9}$$

$$\sin\theta_{ij} \cong \theta_{ij} \tag{10}$$

$$\cos\theta_{ij} \cong 1 - \theta_{ij}^2/2 \tag{11}$$

Substituting equations (9)–(11) into equations (7) and (8), the active power load flow equations of a line can be simplified and deduced as follows.

$$P_{ij} = -b_{ij}\theta_{ij} + g_{ij}\theta_{ij}^2/2 \tag{12}$$

$$P_{ji} = b_{ij}\theta_{ij} + g_{ij}\theta_{ij}^2/2 \tag{13}$$

Let

$$P_{ij}^* = -b_{ij}\theta b_{ij} \tag{14}$$

We can obtain

$$\theta_{ij} = -P_{ij}^*/b_{ij} \tag{15}$$

Substituting equations (14) and (15) into equations (12) and (13), we obtain

$$P_{ij} = P_{ij}^* + \frac{1}{2}\left(-\frac{P_{ij}^*}{b_{ij}}\right)^2 g_{ij} \tag{16}$$

$$P_{ji} = -P_{ij}^* + \frac{1}{2}\left(-\frac{P_{ij}^*}{b_{ij}}\right)^2 g_{ij} \tag{17}$$

The active power loss on transmission line *ij* can be obtained according to equations (16) and (17), i.e.

$$P_{Lij} = P_{ij} + P_{ji} = \left(-\frac{P_{ij}^*}{b_{ij}}\right)^2 g_{ij} \tag{18}$$

$$= P_{ij}^{*2}\frac{\left(R_{ij}^2 + X_{ij}^2\right)}{X_{ij}^2}R_{ij}$$

where

R_{ij} is the resistance of transmission line *ij*;
X_{ij} is the reactance of transmission line *ij*.

Let

$$Z_{ij}^* = \frac{\left(R_{ij}^2 + X_{ij}^2\right)}{X_{ij}^2}R_{ij} \tag{19}$$

the active power loss on the transmission line *ij* can be expressed as follows

$$P_{\mathrm{L}ij} = P_{ij}^{*2} Z_{ij}^* \tag{20}$$

Therefore, the following nonlinear convex network flow programming model, $M-1$, for real power economic dispatch can be set up.

$$\min F = \sum_{i\in NG}\left(a_i P_{\mathrm{G}i}^2 + b_i P_{\mathrm{G}i} + c_i\right) + h\sum_{ij\in NT} P_{ij}^{*2} Z_{ij}^* \tag{21}$$

such that

$$P_{\mathrm{G}i} = P_{\mathrm{D}i} + \sum_{j\to i}\left[P_{ij}^* + \frac{P_{ij}^{*2}}{2b_{ij}^2}g_{ij}\right] \tag{22}$$

$$P_{\mathrm{G}i\mathrm{m}} \le P_{\mathrm{G}i} \le P_{\mathrm{G}i\mathrm{M}} \quad i \in NG \tag{23}$$

$$-P_{ij\mathrm{M}}^* \le P_{ij}^* \le P_{ij\mathrm{M}}^* \quad ij \in NT \tag{24}$$

where

$P_{\mathrm{G}i}$ is the active power of the generator *i*;
$P_{\mathrm{D}i}$ is the active power load on the load bus *i*;
P_{ij} is the flow in the line connected to node *i*, and would have a negative value for a line in which the flow is towards node *i*;

a_i, b_i, c_i are the cost coefficients of the ith generator;
NG is the number of generators in the power network;
NT is the number of transmission lines in the power network;
$P_{ij\mathrm{M}}$ is the active power flow constraint on transmission line ij;
$P_{\mathrm{L}ij}$ is the active power loss on transmission line ij;
Z_{ij}* is called an equivalent impedance of transmission line ij, as shown in equation (19);
h is the weighting coefficient of the transmission losses;
$j \rightarrow i$ represents node j connected to node i through transmission line ij;
Subscripts m and M represent the lower and upper bounds of the constraint.

The second term of the objective function (equation (21)) is a penalty on transmission losses based on the system marginal cost h (in \$ per MWh). Equation (24) is the line security constraint. Equation (23) defines the generator power upper and lower limits. Equation (22) is Kirchoff's first law (i.e. node current law, KCL).

The model $M-1$ is similar to a NFP model in form. In fact, it is a nonlinear convex network flow programming (NLCNFP) model.

In the traditional network flow programming model of economic power dispatch, the total transmission losses are represented approximately as the sum of the products of the line resistance and the square of the transmitted power on the line (i.e. $P_{\mathrm{L}} = \Sigma P_{ij}{}^2 R_{ij}$), and the power loss of an individual line is assumed to be distributed equally to its ends. Thus, the active load $P_{\mathrm{D}i}$ involved half the transmission losses on all the lines connected to node i, which had to be estimated *a priori* from a power flow calculation [1,5,6]. Obviously, the NLCNFP model of economic power dispatch developed here, which is shown in equations (21)–(24), does not require these assumptions, and has higher precision than the traditional NFP model of economic power dispatch.

II.3 *Consideration of KVL*

It is well known that Kirchhoff's second law (i.e. loop voltage law, KVL) has not been considered in the study of secure economic power dispatch using general NFP. This is why there always exists some modeling error when secure economic power dispatch is solved using traditional linear NFP. KVL will be considered in this article.

The voltage drop on the transmission line ij can be approximately expressed as

$$V_{ij} = P^*_{ij} Z^*_{ij} \tag{25}$$

In this way, the voltage equation of the lth loop can be obtained, i.e.

$$\sum_{ij} \left(P^*_{ij} Z^*_{ij}\right) \mu_{ij,l} = 0 \quad l = 1, 2, \cdots, NM \tag{26}$$

where

NM is the number of loops in the network;
$\mu_{ij,l}$ is the element in the related loop matrix, which takes the value 0 or 1.

Introducing the KVL equation into model $M-1$, we can get the following model, $M-2$.

$$\min F = \sum_{i \in NG} \left(a_i P^2_{\mathrm{G}i} + b_i P_{\mathrm{G}i} + c_i\right) + h \sum_{ij \in NT} P^{*2}_{ij} Z^*_{ij}$$

such that

$$P_{\mathrm{G}i} = P_{\mathrm{D}i} + \sum_{j \to i} \left[P^*_{ij} + \frac{P^{*2}_{ij}}{2b^2_{ij}} g_{ij}\right]$$

$$\sum_{ij} \left(P^*_{ij} Z^*_{ij}\right) \mu_{ij,l} = 0 \quad l = 1, 2, \cdots, NM$$

$$P_{\mathrm{G}i\mathrm{m}} \le P_{\mathrm{G}i} \le P_{\mathrm{G}i\mathrm{M}} \quad i \in NG$$

$$-P^*_{ij\mathrm{M}} \le P^*_{ij} \le P^*_{ij\mathrm{M}} \quad ij \in NT$$

In order to obtain the same form as model $M-1$, the following transformation should be carried out.

The Lagrange function can be obtained from equations (21) and (26).

$$F_{\mathrm{L}} = \sum_{i \in NG} \left(a_i P^2_{\mathrm{G}i} + b_i P_{\mathrm{G}i} + c_i\right) + h \sum_{ij \in NT} P^{*2}_{ij} Z^*_{ij} - \lambda_l \sum_{ij} \left(P^*_{ij} Z^*_{ij}\right) \mu_{ij,l} \tag{27}$$

Therefore, NLCNFP model $M-3$ for secure economic power dispatch, which considers KVL, can be written as follows.

$$\min F_{\mathrm{L}} = \sum_{i \in NG} \left(a_i P^2_{\mathrm{G}i} + b_i P_{\mathrm{G}i} + c_i\right) + h \sum_{ij \in NT} P^{*2}_{ij} Z^*_{ij} - \lambda_l \sum_{ij} \left(P^*_{ij} Z^*_{ij}\right) \mu_{ij,l} \tag{28}$$

Subject to equations (22)–(24), where λ_l is the Lagrange multiplier, which can be obtained through minimizing equation (28) with respect to variable P_{ij}*, i.e.

$$2hP^*_{ij} Z^*_{ij} - \lambda_l \sum_{ij} Z^*_{ij} \mu_{ij,l} = 0 \tag{29}$$

$$\lambda_l = 2hP^*_{ij} / \sum_{ij} \mu_{ij,l} \tag{30}$$

$$l = 1, 2, \cdots, NM$$

If $N-1$ security constraints are considered, the NLCNFP model for economic power dispatch with $N-1$ security can be written as follows.

$$\min F_{\mathrm{L}} = \sum_{i \in NG} \left(a_i P^2_{\mathrm{G}i} + b_i P_{\mathrm{G}i} + c_i\right) + h \sum_{ij \in NT} P^{*2}_{ij} Z^*_{ij} - \lambda_l \sum_{ij} \left(P^*_{ij} Z^*_{ij}\right) \mu_{ij,l}$$

Subject to equations (4)–(6), (22) and (23), where equations (4)–(6) are $N-1$ security constraints, which include the N security constraints (equation (24)).

III. Solution of NLCNFP

III.1 *NLCNFP model*

The model $M-3$ of real power economic dispatch with security constraints can be changed into a standard model of NLCNFP, i.e. model $M-4$

$$\min C = \sum_{ij} \mathrm{c}(f_{ij}) \tag{31}$$

such that

$$\sum_{j\in n}(f_{ij}-f_{ji})=r_i \quad i\in n \tag{32}$$

$$L_{ij}\le f_{ij}\le U_{ij} \quad ij\in m \tag{33}$$

where

f_{ij} is the flow on the arc ij in the network;
L_{ij} is the lower bound of the flow on the arc ij in the network;
U_{ij} is the upper bound of flow on the arc ij in the network;
n is the total number of the nodes in the network;
m is the total number of the arcs in the network.

In model $M-4$, the objective function equation (31) corresponds to equation (28) in model $M-3$. The equality constraint equation (32) corresponds to equation (22) in model $M-3$. The inequality constraint equation (33) corresponds to equations (23) and (24) for N security economic dispatch, or equations (4)–(6) and (23) for N and $N-1$ security economic dispatch.

Equation (32) in the model $M-4$ can be written as

$$Af=r \tag{34}$$

where A is a matrix with $n\times(n+m)$, in which every column corresponds to an arc in the network, and every row corresponds to a node in the network.

The matrix A can be divided into a basic sub-matrix and non-basic sub-matrix, which is similar to the convex simplex method, i.e.

$$A=[B,S,N] \tag{35}$$

where the columns of B form a basis, and both S and N correspond to the non-basic arcs. S corresponds to the non-basic arcs in which the flows are within the corresponding constraints. N corresponds to the non-basic arcs in which the flows reach the corresponding bounds.

A similar division can be made for the other variables, i.e.

$$f=[f_B,f_S,f_N]$$

$$g(f)=[g_B,g_S,g_N]$$

$$G(f)=\mathrm{diag}[G_B,G_S,G_N]$$

$$D=[D_B,D_S,D_N]$$

where

g(f) is the first-order gradient of the objective function;
G(f) is the Hessian matrix of the objective function which is a block diagonal matrix;
D is the search direction in the space of the flow variables.

III.2 *Solution of model*

In order to solve model $M-4$, Newton's method can first be used to calculation the search direction in the space of the flow variables. The idea behind Newton's method is that the function being minimized is approximated locally by a quadratic function, and this approximate function is minimized exactly.

Suppose that f is a feasible solution and the search step along the search direction in the space of flow variables $\beta=1$. Then the new feasible solution can be obtained.

$$f'=f+D \tag{36}$$

Substituting equation (36) into equation (31) in the model $M-4$, we obtain

$$C=\mathrm{C}(f')=\mathrm{C}(F+D) \tag{37}$$

Near f we can approximate C by the truncated Taylor series.

$$\mathrm{C}(D)\cong\frac{1}{2}D^{\mathrm{T}}\mathrm{G}(f)D+\mathrm{g}(f)^{\mathrm{T}}D \tag{38}$$

Substituting equation (36) into equation (32) in the model $M-4$, we obtain

$$AD=0 \tag{39}$$

Obviously, $D_{ij}\ge 0$, when $f_{ij}=L_{ij}$ and $D_{ij}\le 0$, when $f_{ij}=U_{ij}$. In this way, the NLCNFP model $M-4$ can be changed into the following QP model $M-5$, in which the search direction in the space of the flow variables is to be solved.

$$\min\mathrm{C}(D)=\tfrac{1}{2}D^{\mathrm{T}}\mathrm{G}(f)D+\mathrm{g}(f)^{\mathrm{T}}D \tag{40}$$

such that

$$AD=0 \tag{41}$$

$$D_{ij}\ge 0,\quad \text{when } f_{ij}=L_{ij} \tag{42}$$

$$D_{ij}\le 0,\quad \text{when } f_{ij}=U_{ij} \tag{43}$$

Model $M-5$ is a special quadratic programming model which has the form of network flow. In order to enhance the calculation speed, we present a new approach, in place of the general quadratic programming algorithm, to solve the model $M-5$. The main calculation steps are as follows.

III.2.1 Neglect temporarily equations (42) and (43)

This means that $L_{ij}<f_{ij}<U_{ij}$ in this case. Thus $D_N=0$ according to the definition of the corresponding non-basic arc.

From equation (41), we know that

$$AD=[B,S,N]\begin{bmatrix}D_B\\ D_S\\ 0\end{bmatrix}=0 \tag{44}$$

From equation (44), we can obtain

$$D_B=-B^{-1}SD_S \tag{45}$$

Then, the search direction in the space of flow variables D can be written as follows.

$$D=\begin{bmatrix}-B^{-1}S\\ I\\ 0\end{bmatrix}D_S=ZD_S \tag{46}$$

Substituting equation (46) into equation (40), we get

$$\min\mathrm{C}(D)=\tfrac{1}{2}(ZD_S)^{\mathrm{T}}\mathrm{G}(f)(ZD_S)+\mathrm{g}(f)^{\mathrm{T}}(ZD_S) \tag{47}$$

Through minimizing equation (47) to variable D_S, the model $M-5$ can be changed into an unconstrained problem, the optimization solution of which can be solved from the following equations.

$$D_N=0 \tag{48}$$

$$BD_B=-SD_S \tag{49}$$

$$(Z^{\mathrm{T}}GZ)D_S=-Z^{\mathrm{T}}g \tag{50}$$

III.2.2 Introduction of equations (42) and (43)
According to equations (48)–(50), D_S can be solved from equation (50), and then D_B can be solved from equation (49). If D_B violates the constraint equations (42) and (43), a new basis must be sought to calculate the new search direction in the space of flow variables. This step will not be terminated until D_B satisfies the constraint equations (42) and (43).

III.2.3 Introduction of maximum basis in network
Obviously, the general repeated calculation of D_B and D_S, which is similar to that of pivoting in linear programming, is not only time-consuming, but also does not improve the value of the objective function. In order to speed up the calculation, we adopt a new method to form a basis in advance, so that D_B and D_S can satisfy the constraints equations (42) and (43). Therefore, the maximum basis in a network, which consists of as many free basic arcs as possible, is introduced in the paper.

The maximum basis in a network can be obtained by solving the following model $M - 6$.

$$\max_B \sum_{ij} d_{ij} A_{ij} \tag{51}$$

where

$$d_{ij} = \begin{cases} 1, & \text{when arc } ij \text{ is a free one, i.e. the flow in arc } ij \text{ is within its bounds} \\ 0, & \text{when arc } ij \text{ is not a free one, i.e. the flow in arc } ij \text{ reaches its upper or lower bounds} \end{cases}$$

$$A_{ij} = \begin{cases} 1, & \text{when arc } ij \text{ is in the basis } B \\ 0, & \text{when arc } ij \text{ is not in the basis } B \end{cases}$$

Suppose basis B is the maximum basis from equation (51), in order to satisfy equation (39), if the flow on a free non-basic arc needs to be adjusted only the flows on the free arcs in basis B need to be adjusted [9].

The introduction of the maximum basis indicates the direction of flow adjustment, i.e. the change of flow is carried out according to the maximum basis. Through selecting the maximum basis, equations (42) and (43) in model $M - 5$ can always be satisfied in the calculation of the search direction in the space of the flow variables. Therefore, the QP model $M - 5$ is equivalent to the unconstrained problem equations (48)–(50).

In order to enhance the calculation speed further, equations (48)–(50) can be solved by the reduced gradient method [10].

IV. Test examples

The proposed economic power dispatch with N and $N - 1$ security, including the model and its algorithm, are examined with the IEEE 30-bus test system on an ALPHA 400 Model 610 computer. The system consists of six generators, 21 loads and 41 transmission/transformation branches. The basic numerical data and parameters are taken from reference [3]. The security constraints of transmission lines and real power constraints of generators used in the paper are listed in Tables 1 and 2, respectively. The test results are given in Tables 3 and 4, where the proposed NLCNFP economic dispatch method with N and $N - 1$ security is identified as NLCNFP, and the other methods are identified by their reference. All values of power in Tables 1–4 are in p.u., and the base value is 100 MVA. Table 5 is the results showing the effect of KVL on solution accuracy and speed.

From Tables 3 and 4, the results by the proposed NLCNFP method are coincident with those obtained from the conventional methods. It can be observed from Tables 3–5 that the

Table 1. Security limits of transmission lines for IEEE 30-bus system

Line	P_{ijM}	Line	P_{ijM}	Line	P_{ijM}
1	1.3000	15	0.6500	29	0.3200
2	1.3000	16	0.6500	30	0.1600
3	0.6500	17	0.3200	31	0.1600
4	1.3000	18	0.3200	32	0.1600
5	1.3000	19	0.3200	33	0.1600
6	0.6500	20	0.1600	34	0.1600
7	0.9000	21	0.1600	35	0.1600
8	0.7000	22	0.1600	36	0.6500
9	1.3000	23	0.1600	37	0.1600
10	0.3200	24	0.3200	38	0.1600
11	0.6500	25	0.3200	39	0.1600
12	0.3200	26	0.3200	40	0.3200
13	0.6500	27	0.3200	41	0.3200
14	0.6500	28	0.3200		

Table 2. Generator data for IEEE 30-bus system

Gen. No	a_i	b_i	c_i	P_{Gim}	P_{GiM}
PG1	37.50	200.00	0.00	0.50	2.00
PG2	175.00	175.00	0.00	0.20	0.80
PG5	625.00	100.00	0.00	0.15	0.50
PG8	83.40	325.00	0.00	0.10	0.35
PG11	250.00	300.00	0.00	0.10	0.30
PG13	250.00	300.00	0.00	0.12	0.40

Table 3. Results and comparison of economic dispatch without $N-1$ security

Gen. no.	NLCNFP	Ref. [3]
P_{G1}	1.7595	1.7626
P_{G2}	0.4884	0.4884
P_{G5}	0.2152	0.2151
P_{G8}	0.2229	0.2215
P_{G11}	0.1227	0.1214
P_{G13}	0.1200	0.1200
Total generation	2.9286	2.9290
Total active power losses	0.0946	0.0948
Total generation cost ($)	802.3986	802.4000

Table 5. Effect of KVL on solution accuracy and speed

Method	NLCNFP without KVL	NLCNFP with KVL
Solution error	0.1860%	0.0226%
Improved accuracy	0.0	0.1634%
Power loss (MW)	7.401	7.034
Reduced loss (MW)	0.0	0.367
Computer type	ALPHA 400/610	ALPHA 400/610
CPU time (s)	0.125	0.140
Added time (s)	0.0	0.015

Table 4. Results and comparison of economic dispatch with $N-1$ security

Gen. no.	NLCNFP	Ref. [1]	Ref. [2]	Ref. [3]
P_{G1}	1.41270	1.40625	1.41108	1.38540
P_{G2}	0.50080	0.60638	0.58172	0.57560
P_{G5}	0.24060	0.25513	0.26183	0.24560
P_{G8}	0.35000	0.30771	0.30114	0.35000
P_{G11}	0.20110	0.17340	0.14871	0.17930
P_{G13}	0.19910	0.16154	0.20208	0.16910
Total generation	2.90434	2.91041	2.90656	2.9050
Total active power losses	0.07034	0.07641	0.07256	0.0711
Total generation cost ($)	813.2135	813.44	813.34	813.74
Computer type	ALPHA 400/610	M-340	IBM 370/168	CDC 7600
CPU time (s)	0.140	0.308	0.45	14.30

proposed approach has very small solution error, 0.0226% compared with the exact method. The solution error will be raised to 0.1860% if KVL is neglected in the proposed NLCNFP method. However, the calculation speed of the proposed approach is far faster than that of the exact method. It means that the proposed NLCNFP method has not only high calculation precision, but also fast calculation speed compared with the conventional methods. Therefore, the problem obtaining the requisite speed and accuracy in the calculation of secure economic power dispatch can be eliminated by using the proposed NLCNFP model and the corresponding algorithm.

V. Conclusions

A new NLCNFP model of on-line economic power dispatch with N and $N-1$ security, which is solved by using a combined method of QP and NFP, has been presented. Based on the load flow equations, a new nonlinear convex network flow model for secure economic power dispatch is set up, and then transformed into a QP model, in which the search direction in the space of the flow variables is to be solved. The concept of maximum basis in a network flow graph was introduced so that the constrained QP model was changed into an unconstrained QP model, which is then solved by the reduced gradient method. Resulting from the adoption of ''$N-1$ constrained zone'' as well as consideration of KVL in the new NLCNFP solution method, the problem of speed and accuracy in the calculation of N and $N-1$ security economic dispatch is solved in the article. The test results and comparison show that the proposed model and its algorithm are feasible and effective.

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VII. References

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