A Comparative Study of Two Methods for Uncertainty Analysis in Power System State Estimation

A. K. Al-Othman and M. R. Irving

***Abstract*—This letter presents a comparative study between two methods for estimating the uncertainty interval in power system state estimation. Constrained nonlinear and linear formulations are proposed to estimate the tightest possible upper and lower bounds on the states. The study compares the performance of these methods in terms of estimating the bounds of the uncertainty interval. In addition, an assessment of time performance for both methods is carried out with varying measurement redundancy levels.**

***Index Terms*—State estimation and measurement uncertainty.**

I. Introduction

Uncertainty in power system state estimation is mainly due to measurement inaccuracy and the network mathematical model used. For instance, meter inaccuracies and communication errors are major sources of measurement uncertainty. Parameter approximations in modeling of the Pi-equivalent, such as line resistance, reactance, and shunt capacitance, also contribute to the uncertainty in state estimation. Unfortunately, the magnitudes of such errors and approximations are not known, which, in turn, lead to uncertainty in the estimates obtained in state estimation. Practically, error statistics are difficult to characterize. In such circumstances, it is desirable to provide not just a single "optimal" estimate of each state variable but also an uncertainty range within which we can be assured that the "true" state variable may lie with high confidence. This letter compares two different inequality-constrained formulations for estimating the uncertainty interval in power system state estimation. The uncertainty in measurements is assumed to be known and bounded. Nonlinear and linear approaches are used to obtain the tightest possible upper and lower bounds of the states. A six-bus test system is used to check the ability of both methods in accurately and efficiently estimating the uncertainty interval for power system state estimation problems.

II. Proposed Problem Formulation

In power system state estimation, inequality constraints are usually needed in optimization to deal with uncertainties. In [1], an inequality constraint is employed, in a least absolute value (LAV) estimator, for the pseudo measurements since they are not measured, but they are known to vary within a bounded interval. An inequality constraints LAV estimator based on penalty functions was formulated, in [2], to estimate the states of external systems. An unknown-but-bounded model was used in [3] with a reformulated constrained weighted least squares (WLS), to handle unmeasured loads in the system. Such model is due to Schweppe [4], who assumed that measurements errors are unknown but fall within a bounded range. This letter, however, introduces two double inequality-constrained formulations to estimate the uncertainty interval of the state variables accordingly.

A. Estimation of State Bounds With a Nonlinear Method

Uncertainty intervals of the state variables can be determined by the solution of a series of appropriately formulated optimization problems. Each measurement, with its associated uncertainty, can be represented by upper and lower limits. These constraint limits define the tolerances on the measurements (i.e., the range of values within which the true value of the measured quantity must lie). Minimizing a particular state variable of interest, subject to all the measurement inequality constraints, provides the lower bound on that state variable. Similarly, maximizing that state variable, again subject to all the measurement inequalities, provides the upper bound for that state. In mathematical form

$$\min_{\underline{x}} x_i \qquad \text{subject to} \quad \underline{Z}^l \leq h(\underline{x}) \leq \underline{Z}^u \tag{1}$$

where $\underline{Z}^l$ is the lower bound of the measurement vector, and $\underline{Z}^u$ is the upper bound, with

$$\underline{Z}^l = \underline{Z} - \underline{\tau} \tag{2}$$

$$\underline{Z}^u = \underline{Z} + \underline{\tau} \tag{3}$$

where $\underline{\tau}$ is the transducer tolerance. The tolerance describes the deterministic uncertainty of each measurement. It represents the overall accuracy of the meter and can usually be provided by the manufacturer. Different values for the elements of positive and negative tolerances are permissible, so a transducer can be specified to have asymmetric accuracy if required (e.g., an accuracy from -3% to $+5\%$ of the nominal value).

B. Estimation of State Bounds With a Linear Method

Alternatively, (1) may be linearized about a suitable point $\underline{\hat{x}}$ (which, in this case, can be provided by the WLS estimate), and then, a series of linear programs may be solved to obtain updates dx_i to the uncertainty bounds on the state variables. For instance, the incremental change to the lower bound for the ith state can be computed by solving the following linear programming (LP) problem:

$$\min_{\Delta \underline{x}} dx_i \qquad \text{subject to} \quad \Delta \underline{z}^l \leq J \Delta \underline{x} \leq \Delta \underline{z}^u \tag{4}$$

where J is the Jacobian of $h(\underline{x})$ evaluated at $\underline{\hat{x}}$, and $\Delta \underline{z}^l$ and $\Delta \underline{z}^u$ are vectors of the incremental changes to the measurement of the lower and upper bounds, respectively, which are computed in the following form:

$$\Delta \underline{z}^l = \underline{z}^l - h(\underline{\hat{x}}) \tag{5}$$

$$\Delta \underline{z}^u = \underline{z}^u - h(\underline{\hat{x}}). \tag{6}$$

Once $d\underline{x}^+$ and $d\underline{x}^-$ (vectors of upper and lower updates) are known, the bounds on $\underline{\hat{x}}$ are simply found as

$$\underline{x}^+ = \underline{\hat{x}} + d\underline{x}^+ \tag{7}$$

$$\underline{x}^- = \underline{\hat{x}} + d\underline{x}^- \tag{8}$$

where $\underline{\hat{x}}$ is the estimate obtained by unconstrained WLS.

III. Results and Analysis

This section presents some typical results obtained by applying the proposed methods to the six-bus test network. The computation of all state variables will be shown to illustrate the concepts. However, for improved computational efficiency, only the variables of present interest to the power system operator would need to be computed.

The nonlinear problems have been solved by the function *fmincon* incorporated in the MATLABf 6.1 optimization toolbox. The linear programs have been solved by the function *linprog*.

Manuscript received February 26, 2004; revised September 10, 2004. Paper no. PESL-00019-2004.

The authors are with the Brunel Institute of Power Systems, Brunel University, Uxbridge UB8 3PH, U.K.

Digital Object Identifier 10.1109/TPWRS.2005.846163

TABLE I
ESTIMATED STATE VARIABLES OF THE SIX-BUS SYSTEM

Bus #	LP^- (lower bound)		LP^+ (upper bound)		SQP^- (lower bound)		SQP^+ (upper bound)	
	$\|V\|$(pu)	$\delta(rad)$	$\|V\|$(pu)	$\delta(rad)$	$\|V\|$(pu)	$\delta(rad)$	$\|V\|$(pu)	$\delta(rad)$
1	1.0175	0	1.0825	0	1.0175	0	1.0825	0
2	1.0175	-0.0912	1.0825	-0.0388	1.0175	-0.0905	1.0825	-0.0380
3	1.0375	-0.1082	1.1025	-0.0431	1.0375	-0.1082	1.1025	-0.0431
4	0.9539	-0.0888	1.0190	-0.0571	0.9539	-0.0890	1.0190	-0.0572
5	0.9471	-0.1169	1.0122	-0.0654	0.9471	-0.1179	1.0122	-0.0663
6	0.9689	-0.1367	1.0340	-0.0717	0.9689	-0.1367	1.0340	-0.0717

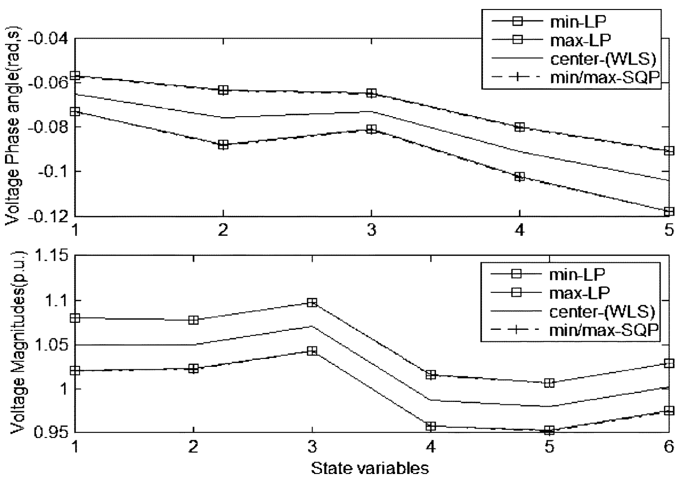

Fig. 1. Estimated states for the six-bus system redundancy ≈ 2.

TABLE II
EXECUTION TIME OF WLS-LP AND SQP (CPU: PEN 4, 1.7 GHZ)

# of measurements	CPU time	
	Linear (WLS-LP)	Non-Linear(SQP)
23(redundancy $\approx$ 2)	0.201 sec	10.725 sec
67 (Full)	0.270 sec	15.743 sec

Table I presents results obtained by both methods, when applied to the six-bus network. For the nonlinear method, the upper and lower uncertainty bounds of the state variables are found using (1)–(3) with $\tau \equiv 3\%$. The same tolerance was also used for the linear formulation. A WLS estimator was used to compute the (center point) estimated states. Then, (4)–(8) are used to find the upper and lower bounds. It is apparent that both formulations provide almost identical estimates. The sesults of Table I are illustrated in Fig. 1. We also notice that the solution obtained by WLS is strictly bounded by the solution of sequential quadratic programming (SQP) and WLS-LP.

Table II shows the execution time for both methods with different redundancy levels. A redundancy ≈ 2 and full set of measurements are used. Clearly, the linear (WLS-LP) outperforms the nonlinear method in these tests. The WLS, however, is known to give deceptive results in the case where contaminated measurements are used. In this situation, a robust estimator, such as least median squares (LMS) and least trimmed squares (LTS), may be used for accurate estimation of the center point [5]. It is important to stress that proposed formulations assume that the transducer tolerances $\underline{\tau}$ must be known and fixed. Practically, this is not necessarily guaranteed, because such tolerances will become unknown as instruments age under the action of various unknown processes, and systematic recalibration procedures are rarely done in the field. That is due to massive amounts of meters, which, in turn, lead to maintenance being impractical and extremely expensive [6].

IV. CONCLUSIONS

Two formulations of uncertainty analysis in power system state estimation are presented in this study. The uncertainty is modeled via deterministic upper and lower bounds on measurement errors, which take into account known meter accuracies. Both methods provided almost identical estimates when applied to the six-bus test system. It is concluded from execution time analysis that WLS-LP is faster than SQP and more appropriate for uncertainty interval estimation in larger power networks.

REFERENCES

[1] A. Abur and M. K. Celik, "Least absolute value state estimation with equality and inequality constraints," *IEEE Trans. Power Syst.*, vol. 8, no. 2, pp. 680–686, May 1993.
[2] H. Singh, F. L. Alvarado, and W.-H. E. Liu, "Constrained LAV state estimation using penalty functions," *IEEE Trans. Power Syst.*, vol. 12, no. 1, pp. 383–388, Feb. 1997.
[3] K. A. Clements, P. W. Davis, and K. D. Frey, "Treatment of inequality constraints in power system state estimation," *IEEE Trans. Power Syst.*, vol. 10, no. 2, pp. 567–574, May 1995.
[4] F. C. Schweppe, *Uncertain Dynamic Systems*. Englewood Cliffs, N.J.: Prentice-Hall, 1973.
[5] P. J. Rousseeuw and A. M. Leroy, *Robust Regression and Outlier Detection*. New York: Wiley, 1987.
[6] M. M. Adibi and D. K. Thorne, "Remote measurement calibration," *IEEE Trans. Power Syst.*, vol. PWRS-1, no. 2, pp. 194–202, May 1986.