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Digital all-computer simulation in managerial problems

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DIGITAL ALL-COMPUTER SIMULATION
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ABSTRACT

Management problems are becoming more and more complex. True scientific management demands the exact consideration of all the factors that are significant to the problem under study. In dealing with practical problems, one cannot tease out separate psychological, economic, or technological aspects as the problem mostly always involves working with a total integrated organizational unit. Due to the abundancy and iterrelation of these factors, it made it more difficult if not impossible in many cases to reach a solution by the analytical methods. Thus, many managerial problems if they are to be accurately identified and quantifiably solved, should be simulated.

Simulation has become a very important process or tool available to modern managers. But, to be a useful tool, it should be used effectively. Realizing:

1. The usefulness of the simulation process to managers to the point that in many cases it becomes imperative to simulate if a system is to be precisely understood or a problem is to be correctly solved.
2. The degree of controversy that has flared-up about the exact definition of simulation to the point that it has become difficult to state what is simulation and what is not.

The researcher has chosen to direct this research towards discovering a practical and systematic approach to simulating

managerial problems.

There are different types of simulations that are used for different purposes. All types are to be mentioned and defined in this research. But, due to the fact that *Digital all-computer simulation* is the type mostly used in solving managerial problems, it is to be the purpose of this research.

This research has been organized in the following manner:

1. Chapters 1, 2, and 3 explain in detail the concept, definition, and the different phases of the simulation process in order to set the stage for a detailed and practical discussion of a simulation application.
2. Chapter 4 presents an inventory control problem together with its solution via a digital all-computer simulation. The reason behind choosing such a problem is that inventory problems are the most common problems that managers encounter especially in the U.A.R. where inventory control has not received enough attention and where it is *the problem* to most if not all sectors (Government, public, and private).
3. Appendix I has been added to the research in order to provide the reader with the exact definitions of the terms that are mostly used in such operations. As uncertainty in business is more the rule than the exception, most of its models are probabilistic in nature. Appendix II has been added to introduce the elements of the probability theory and the most commonly used and encountered probability distributions to help the simulator in designing, manipulating, and reaching conclusions in such a context.

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INTRODUCTION

Simulation - as we are going to discuss it in this research - is a forecasting, planning, and problem-solving managerial technique that has emerged recently due to the change in the nature of managerial problems.

In the old days, organizations were relatively limited in size, in the number of products they handled, and in the size or number of markets they covered. Consequently, problems that faced managers were simple in the sense that it consisted of a very limited number of factors that were dealt with according to experience or what we call the *trial and error method.* The manager - from experience - chooses a solution and tries it, if it works, all the better, if it does not, he then - if his job or his organization still exists - tries another solution. That is why this method is also known as the *rule-of-thumb* method and its disadvantages are quite apparent. If the manager's predictions about how the system will react if a certain action was undertaken turn out to be true, then he has succeeded. But, what if his predictions were wrong? Or, what if there were more than one course of action that could be followed and he is not sure which one is best? Would he try them all? Wouldn't that be prohibitively expensive if not fatal to the system under study? What if there are human elements that could be affected, can he take the risk of experimenting on those live components to see what results he would get? Wouldn't that be inhuman?

Then, organizations grew, huge corporations that cover international markets came into existance, Taylor introduced the theory of "scientific management," the human element in organizations began to play a major role, labor unions were formed, and operations research techniques and the systems theory in management were introduced. Management came to be known as the conversion of the unrelated resources of men, machines, money, and information into an integrated system for objective accomplishment (6, 141). The *Analytical Method* for solving managerial problems replaced the old trial and error methods.

THE ANALYTICAL METHOD

Or as sometimes termed *Mathematical Analysis* whereby the manager or the problem-solver starts by analyzing the problem or system in order to discover all the pertinent factors (system's entities, attributes, parameters, and variables) relevant to the situation he is faced with and the relationships that exists between these factors. He then constructs a mathematical model in the form of one or a set of equations or inequalities that portrays the functional relationships that exists between these factors and solves it mathematically by calculus, linear programming, iteration,..., etc.

It was then found that in dealing with practical problems, one cannot tease out separate psychological, economic, or technological aspects as the problem mostly always involves working with a total integrated organizational unit. For example, when studying the functioning of a work group, one cannot isolate the interaction relationships between the members of the group from the nature of the task to be performed, and the types of relationships or interactions required for task performance (5, 353-354). That is to say that most of the problems contain a large number of variables and relationships to which the system is responsive.

Again, due to the uncertain nature of most business systems, problems usually contain variables that are subject to chance variations. For example, it is hard to predict human behavior such as customer behavior or demand in marketing problems, lead time in production problems,... etc.

These two characteristics of economic and business problems made it difficult if not impossible to construct mathematical models that portray these complex and random relationships and that at the same time can be solved analytically. Analytical solutions to system equations cannot be obtained for high-order systems or nonlinear systems or for the more realistic multiloop systems. Nonlinear behavior is beyond the reach of most mathematical processes.

Also, mathematical analysis has another drawback even in those cases when it might be possible to use direct mathematical attack. Because applications of typical mathematical analysis require explicit determination of all the relevant variables in the system, simplifying assumptions has to be made in order to carry out the analytical solution. These assumptions or simplifications cut down the usefulness of the technique as the obtained results may not be applicable in real life (6, 313). For example, in inventory control problems, it is sometimes necessary to assume that the rate of demand or the lead-time is constant - when in reality they are not and could not be - in order to simplify the relationships between the system variables to be able to build the equations (model) that can be mathematically solved.

That is not to say that mathematical analysis is useless; whenever analysis alone serves the purpose, it is best used as it is usually easier, quicker, and less expensive than other methods (8, 29). Also, mathematical analysis is an effective tool as it provides an understanding of what to look for in the system structure. The essential feature of the analytical method is that it is essentially deductive in character in the sense that through analysis, the problem-solver builds a mathematical model on assumptions that he assumes to be true or exsisting, generalizes, and thus reaches a solution to his specific problem (9, 178-179).

CHAPTER I

WHAT IS SIMULATION?

THE SIMULATION APPROACH

A method was needed that could predict the consequences of the complex and dynamic interrelations within business components, and experimentation was found to be the only way. The experimenter, instead of moving from general principles to specific ones as in analytical techniques, moves from the specific case to the general, i.e., his approach is inductive in character, or as J. Forrester calls it, an *Emperical Approach* (3, 585-592). Only, in this method, instead of experimenting on the real system, experiments are made on an *image* or *model* of the real system without tampering with the real system. The result is that we do not risk upsetting the real system without prior assurance that the changes contemplated will be beneficial. This technique is what is known as *Simulation* and is the only method known as far which makes possible experiments to validate theoretical predictions of systems behavior in cases where experimentation on the system under study would be impossible, prohibitively expensive, or complicated by the effects of interaction of the human observer with the system (7, 2).

More specifically then, simulation means:

1. There exists - either physically or still yet contemplated - an object system that has a problem that needs a solution.
2. This problem or situation is a dynamic and complex one. This means that, when analyzed, the problem is:
   A. Dynamic in the sense that time is an essential variable in it.
   B. Complex in the sense that the system is large in terms of the number of variables, parameters, relationships, and events to which the system is responsive.
   C. The system is subject to random fluctuations or random variables which refer to the uncertain nature of inputs of a system.
   D. Due to the complexity of the system and the existence of random variables, the relationships will be difficult if not impossible to be mathematically constructed, or as the mathematicians say, those relationships are not mathematically tractable (2, 26-27).
3. There are alternative solutions, courses of action, theories, or hypothesis that are in the mind of the problem-solver as to how this problem might be solved; only he is not sure which one will yield the best results. Or, he may not be able at the time to fully understand the dynamic and complex interactions within the system to be able to predict or suggest a solution. That is, he is still in an explanatory situation and wants to experiment on the system to understand it or to find out the best possible solution.

4. It is either impossible or too expensive to manipulate the object system (real world) to make the necessary experiments and take observations of the outcomes of these manipulation(s).
5. A model is thus constructed to duplicate the *essence* of the system or the relevant activity as it is sometimes impossible to duplicate the whole system into one single model that portrays all the different interactions that affect and are affected by its manipulation.
6. This model is then manipulated according to the problem-solver's theories, hypothesis, solutions, or courses of action and observations are taken of the behavior of the model due to this manipulation(s).

Diagrammatically, this procedure is represented in figure (1).

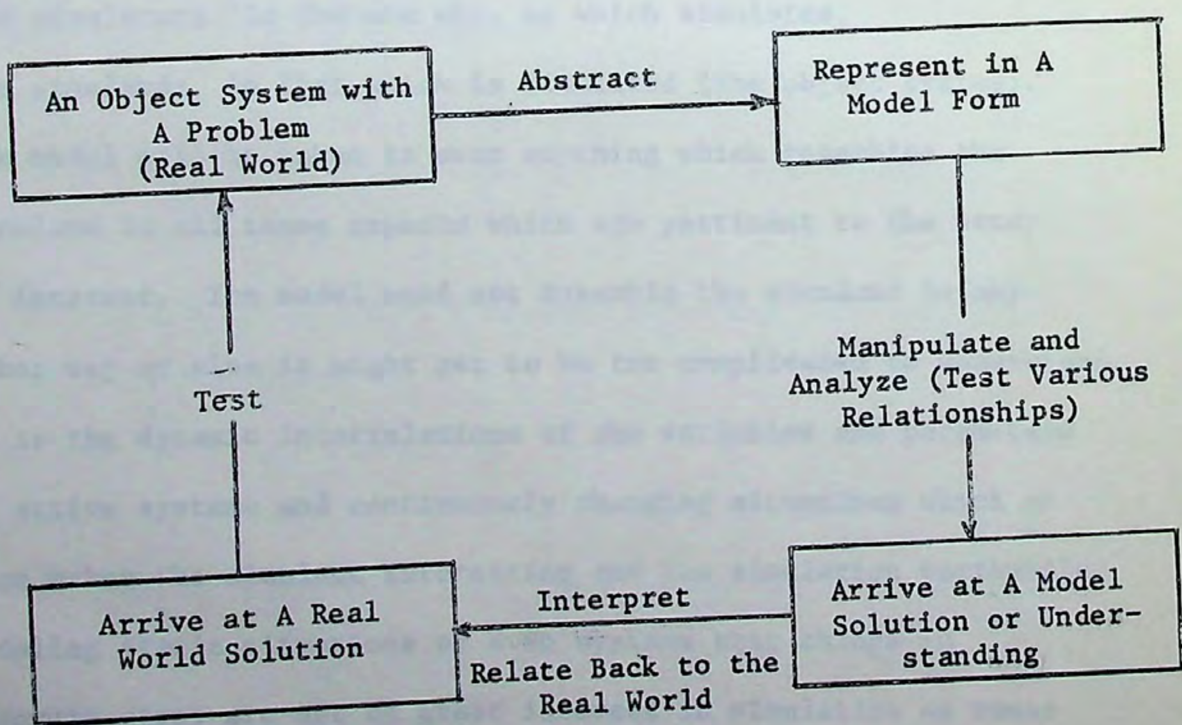

Figure (1): The General Form of the Simulation Process

7. The model solution is then interpreted to relate it back to the real world in order to reach a real world (object system) solution.

Diagramatically, this procedure is represented in figure (1).

SIMULATION DEFINED

Simulation can now be defined as: *The process whereby a model is constructed duplicating those aspects which are pertinent to the study of interest in the object system. This model is then manipulated (experimented upon) according to certain theories or hypotheses and observations are taken of the behavior of the model over time from which the dynamic behavior of the object system can be inferred.*

In other words, simulation provides a laboratory for analysis of problems. This implies that:

1. The simulator: is the one who, or which simulates.
2. The simuland: is that which is simulated (the object system).
3. The model will be taken to mean anything which resembles the simuland in all those aspects which are pertinent to the study of interest. The model need not resemble the simuland in any other way or else it might get to be too complicated to understand.
4. It is the dynamic interrelations of the variables and parameters of active systems and continuously changing situations which at once makes the simuland interesting and the simulation worthwhile. Modeling static situations or even systems that change in discrete steps are not of great interest in simulation as these can mostly be solved analytically.

Computers were not mentioned here as they are not necessary for simulation; it is just that they are the best tools as far available for the job (4, 3-4).

TYPES OF BUSINESS SIMULATION

There are three known types of business simulation:

1. Man-Model simulation:

   The distinguishing characteristic of man-model simulation is that some human counterparts of the object system are represented by live participants while any other human entities and the relevant nonhuman features of the object system are represented by a model. The essential property of man-model simulation is the interaction between live entities and the model. The whole simulation system is comprised of:

   A. The model.

   B. Live participants who assume the role of some counterparts in the object system.

   C. Administrative activities to coordinate and carry out the simulation process.

Hence the name man-model simulation.

2. Man-Computer simulation:

   This differs from man-model simulation by the use of computers to execute models. The purpose of those experiments are:

   A. To study the unknown behavior of subjects within a given and known simulated environment.

B. To evaluate the effect of alternative system configurations given known kinds of subjects employed.

C. To train students in simulated environments.

3. All-computer simulation:

These are simulations without a live component. They are sometimes called *Closed-loop* simulations as opposed to *Open-loop* simulations which contain a live component (8, 12-46). The practical power of computer simulation arises from:

A. The speed of the computer which ensures that simulation can be performed quickly. Several alternative plans can be tested in a relatively short period of time.

B. The computer can keep track of many different things going on at the same time. The complex interrelationships of a company may be taken into consideration concurrently. The simulator can combine enormous numbers of subsystems into a single experiment.

C. The accuracy of computers is equally important to the simulation. Computers can accurately handle the processing, thus ensuring the correctness of the output for evaluation (1, 3).

This last type of simulation will be the subject matter of this research. The next two chapters will be directed towards developing the theoretical framework in order to set the stage for a more detailed and practical discussion of an All-Computer simulation application in chapter IV.

Throughout this research, the words *simulator*, *experimenter*, and *designer* will be used interchangeably to indicate the person who is carrying out the simulation. This may not be the case in some problems especially complex ones as they could be different people comprising a group that is assigned such task, or different people assigned different tasks in a large and complicated simulation process. For the sake of simplification, the researcher will consider them all as one person that is assigned the carrying out of the whole simulation process.

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CHAPTER II

THE SIMULATION PROCESS

Following is a suggested methodology to be followed by the experimenter in order to carry out an all-computer simulation:

1. Problem formulation.
2. Constructing the mathematical model.
3. Validating the mathematical model.
4. Designing simulation experiments.
5. Developing the simulation model.
6. Executing the model.
7. Collecting and analyzing data.
8. Proposing new experiments if deemed necessary.
9. Presenting the final report.

Figure (2) illustrates diagramatically the simulation procedure according to the proposed sequence. Although this is a logical ordering of steps, there are cases when the experimenter will find it more convenient to tackle some phases before others according to the type of problem and the data available.

PROBLEM FORMULATION

There is an old saying that a problem well put is half solved. This phase is the starting point in any problem solving

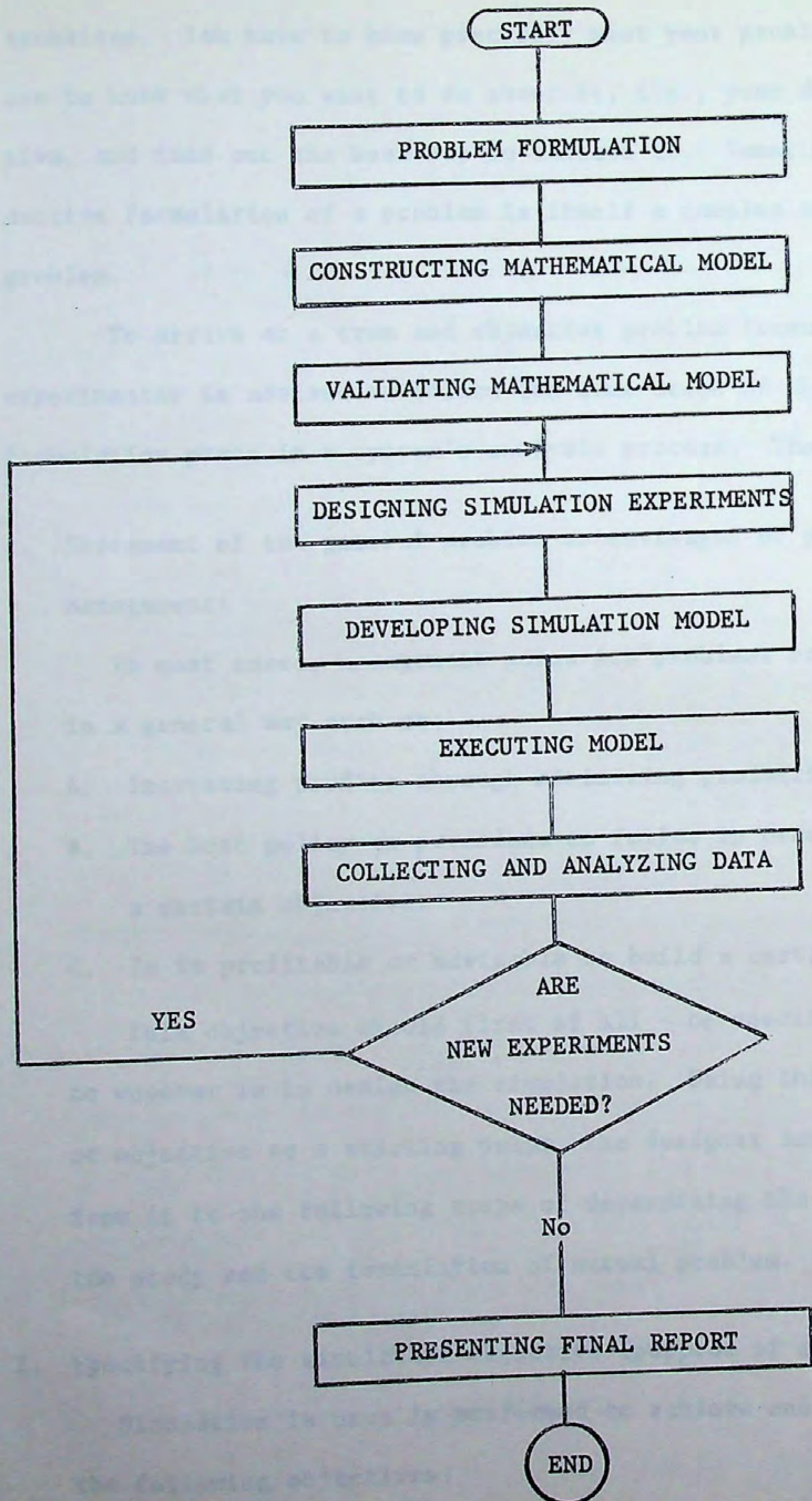


Figure (2): The Simulation Process

technique. You have to know precisely what your problem is if you are to know what you want to do about it, i.e., your definite objective, and find out the best way to achieve it. Sometimes, the productive formulation of a problem is itself a complex and technical problem.

To arrive at a true and objective problem formulation, the experimenter is advised to follow the same steps of the problem formulation phase in a system's analysis process. Those steps are:

1. Statement of the general problem as envisaged or posed by management:

   In most cases, management poses its problems or objectives in a general way such as:

   A. Increasing profits through minimizing production costs.

   B. The best policy or procedure to follow in order to reach a certain objective.

   C. Is it profitable or advisable to build a certain new system?

   This objective should first of all - be specified clearly to whoever is to design the simulation. Using this problem or objective as a starting point, the designer can proceed from it to the following steps of determining the purpose of the study and the formulation of actual problem.

2. Specifying the simulation objective (purpose of study):

   Simulation is usually performed to achieve one or more of the following objectives:

   A. For purposes of experimentation or evaluation; in other words, to try and predict the consequences of changes in policy, conditions, methods,... etc., without having to spend the money or taking the risk of actually making the changes in real life.

B. To learn more about the system in order to redesign or refine it. The very complexity of most of business and industrial systems makes necessary a means to provide understanding of both the system as a whole and of its parts.

C. To familiarize personnel with a system or situation, which may not exist as yet in real life.

D. To verify or demonstrate a new idea, a new system or approach; in other words, quantify the risks and benefits, and demonstrate the chances of success (16, 368-369).

The objective(s) of the simulation or the purpose of the study must be known and clear to the designer to enable him to formulate the actual problem and design his experiments productively. The objective may be stated or given to him clearly by management; or, he may have to deduce it from the problem that is given to him. For instance, if the problem posed to him is *to increase profits through minimizing production costs*, the designer may find during his analysis and investigation of the system that management needs to know more about the system in order to find the actual defect and modify it, or may be redesign a whole new system. The formulation of the problem by management and the data that is to be available to the designer to start from will specify the objective of the simulation. A common failure in determining simulation objectives is that the simulators become so engrossed in simulating that they try to extract more details from simulation than is needed or can be supported by the data available (6, 21).

3. Investigating and analyzing the system (system description):

Most business systems are so complex that it is not easy to find out what actually is wrong with them or the actual relationships that exists between the system components. When a problem involves a system of operations such as industrial or commercial ones, it can seldom be given a complete and accurate formulation by those who face it due to their interaction with the system itself (2, 100). To formulate the actual problem upon which the mathematical model that represents the interactions between the system components is to be constructed, the simulator should investigate and analyze the system and identify clearly:

A. System entities and their attributes.

B. What variables are of interest?

C. The exact interrelationships between the system parameters and variables, and how their values are determined?

D. Environmental factors and how they affect and are affected by the system behavior?

It is not always that the experimenter will find it easy or even be able to reach such conclusions. Systems might be so large and complex that a detailed and lengthy analysis by specialists is needed, or, they might not be clearly understood by the users or the analysts that several attempts (experiments through the use of models) has to be made prior to reaching a complete or even a satisfactory understanding of the system. As a matter of fact, this in itself might be the objective of the simulation.

In either case, system investigation and analysis is vital as it adds and provices insight and understanding to the system and pinpoints the areas where more investigation is needed.

4. Analysis of initial data:

While analyzing the system, the experimenter or the analyst if one is used, either depends upon historical data that shows the system behavior and that has been made available to him by management, or, if this data did not exist, or existed in insufficient or unrepresentative quantities, gathers the data he needs.

This data is usually in the form of tables (gathered statistics) showing the output of the whole system or all or some of its components when certain inputs take place. For this data to be of any meaning, it should:

A. Cover an extended period of time or trials so that the experimenter can get a clear picture about the trend of behavior of the system.

B. Cover all the relevant components of the system showing how each affects and is being affected by the other components.

In most situations, the experimenter will find that the latter is unavailable, thus, he will have to collect it during his analysis, or if not possible, make provisions when designing the simulation model, to gather such data during the first simulation runs. (See application in chapter IV).

C. Be expressed in a format that facilitates its understanding and analysis in order to reach conclusions about:

1) The pattern of behavior of different system components. For example, historical data may show that the demand for a certain facility follows a *poisson distribution*.

2) The different values that system parameters and variables did take in the past, from which the probable future values can be deduced according to the trend.

3) The relationships or correlation between the different parameters and variables.

4) The outstanding peculiarities in the system. For example, production costs may show to be decreasing at a very low rate whenever production quantities are increased when, according to the analysis and experience, it should have decreased at a much higher rate.

5) The type and amount of data that is lacking in order to render the analysis of the system complete. Consequently, provisions should be made, according to the situation, to collect this data.

6) What data is relevant to the situation, and which is to be used in the simulation experiments. This conclusion

will be very useful during the phase of designing the simulation experiments.

7) The type, shape, and frequency of data that is to be extracted from the actual simulation runs. That is, the quantitative and specific determination of the simulation objective(s).

8) Which data to use in model validation, and the exact or approximate results that he should get from the solution of the mathematical model when certain inputs are introduced into the model.

The methods for analyzing and inferring information from statistical data are explained in detail in Appendix II.

At this point, the following very important question should be raised:

When does initial data analysis take place? Should it be before or after constructing the mathematical model?

The researcher's idea is that there is a continuous interplay between:

A. Analyzing the system.

B. Analyzing initial data.

C. Mathematical model construction.

D. Model validation and reconstruction if necessary.

In the sense that any conclusions about one of them will bear on one or more of the others and that modifications or recourse to the other steps will be necessary.

5. Formulating the actual problem:

The simulator now knows what management objective(s) is and is aptly familiar with the system he is investigating. Based upon these two, he can now formulate exactly the actual problem with the system. For instance, if management's objective is to increase profits through minimizing production costs; the simulator, after investigating and analyzing the system, may discover that this could be achieved through either:

A. Better scheduling of job shops.

B. Increasing or decreasing quantities produced.

C. Increasing or decreasing quantities of raw-materials purchased.

Or, any other specific reason that he has identified during his analysis and which is the actual problem he is to solve.

It is not always that the simulator will have to go through all these steps, the problem may be clearly posed to him together with the simulation objective, and in such cases, all he has to do is only to investigate, analyze, and understand perfectly the system so as to be able to construct the mathematical model of the existing relationships between the system's components. Or, it may be that the simulator is himself the person in charge of the system, he understands it and knows his problem perfectly, hence, he can start directly on the following phase of constructing the methematical model. In simulation studies that are made to familiarize personnel with a system or situation, the simulation objective and the problem are both the same thing. Figure (3) illustrates diagramatically the subphases of the problem formulation phase.

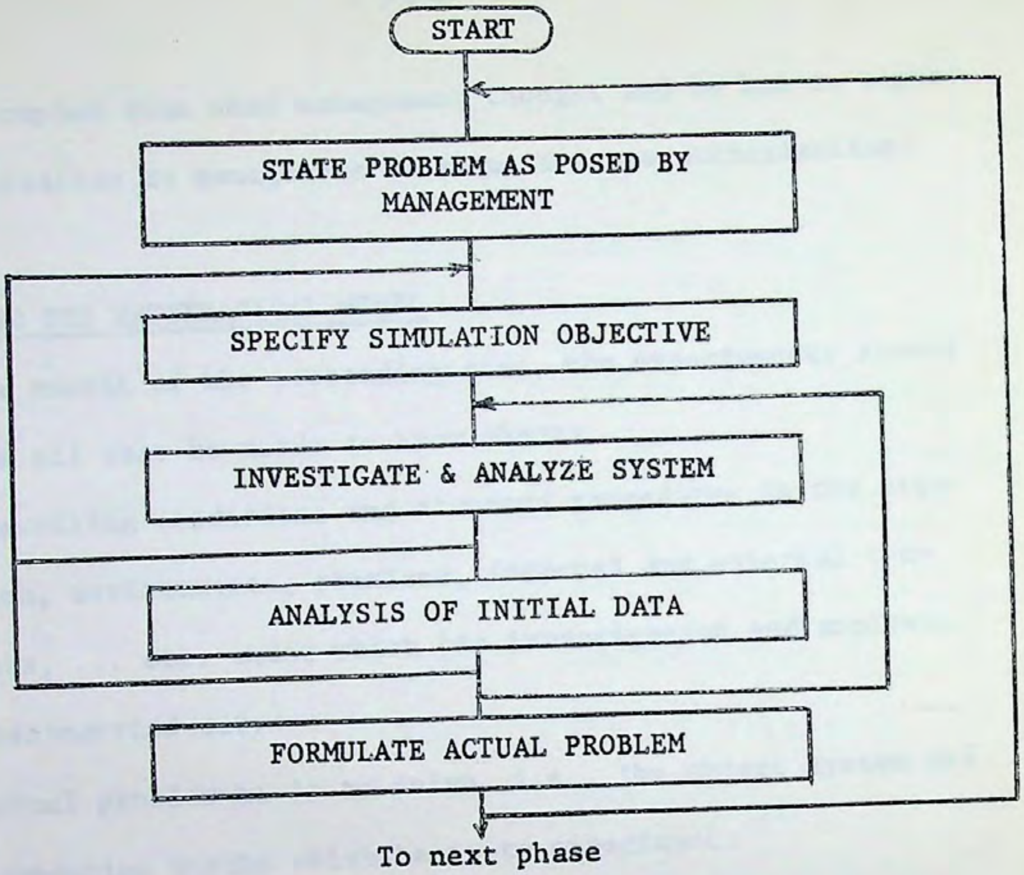


Figure (3): Subphases of the Problem Formulation Phase

In actuality, there is no demarcation lines between these subphases as they are all interrelated with each other, and are affecting and being affected by the result of each. For example, the output of the investigation and analysis step may show that the system is not well enough understood by the users and consequently, the simulation objective has to be changed to include that purpose. Also, the simulator may find when analyzing the initial data that the system has shown to be reacting in a certain unaccounted for fashion to certain inputs, and thus, he has to go back to the system analysis step to find the reason behind it. Or, the simulator may find after the final step of formulating the actual problem, that it is much bigger or complex than what management thought and he has to report this new situation to management to acquire a new authorization.

CONSTRUCTING THE MATHEMATICAL MODEL

As a result of the preceeding step, the experimenter should by now know all that he needs to know about:

A. The prevailing conditions and standard procedures in the organization, environmental premises, internal and external constraints, ... etc. under which his investigation and analysis has been carried out.

B. The actual problem he is to solve, i.e., the object system and the boundaries within which he is to experiment.

C. System antities, attributes, and variables of interest together with the existing interrelationships between them, and how their values are determined. Thus, he can construct the relevant mathematical relationships that exists between them.

D. The simulation objective which signifies to him the type and amount of data and experiments required from the simulation.

On the basis of this information, he then starts to construct the mathematical model for the object system as follows:

1. Recognizing and laying down of assumptions:

   These assumptions represent the prevailing conditions (premises) either internally or externally at the time of the analysis such as:

   A. The rate of demand or lead time whether it will be constant or fluctuating between certain limits according to a specified set of probabilities for the future planning period.

B. Working procedures that has to be followed such as actions when backordering occurs, whether servicing is on the basis of first arrive first served or according to least time, ...etc.

C. The method to be used in solving the model, such as assuming that generated random numbers are valid and representative.

These assumptions differ from those that are made in analytical solutions because they are not simplifying assumptions made to aid in carrying out the solution, but rather they are made:

A. Because they actually apply to the situation.

B. To be used as starting conditions for the simulation experiments. The experimenter can and usually does change them over time or every run in order to observe their effects and implications. That is, they are taken into consideration and not kept constant during the whole simulation process as in analytical solutions.

It is imperative that all assumptions must be stated clearly in the solution and in the final report for the following reasons:

A. Whenever a future study or modification of the system is attempted, recourse should be had to previous studies that were made on the system. Without knowing the assumptions upon which the previous solution was based, it would be difficult if not impossible to understand or appreciate the solution or the logic behind its choice.

B. Assumptions that are made at one time may not be valid at other times due to the change in time, environment, or technology. Thus, by referring to the assumptions of a previous solution, it may very well be found that those assumptions are the actual problem or the reason behind it.

2. Constructing the mathematical model:

Models cannot replace the real world; at best they reduce a complex system to manageable proportions or serve to crystalize our thinking and perception (3, 9).

Simulation models are basically mathematical models consisting of one or a set of equations or inequalities describing the properties of the system under study and whose solution or manipulation according to a certain theory and design explains or predicts changes in the state of the system.

Model building is a consequence of analytical efforts to abstract and describe the real world (object system). That is where mathematical analysis comes into the picture when dealing with dynamic and complex systems; it provides an understanding of what to look for in the system structure.

The following list generally describes the procedure to be followed in order to construct a mathematical model. These steps can be easily followed by the experimenter if the system is relatively simple and well understood, or they might be very complex and lengthy that they may need a system analysis by specialists if the system is too complex or not well enough understood:

A. Make an itemized list of all the variables and parameters of the system and the environment that contributes to its effectiveness. (From the detailed and complete analysis of the system that has been carried out in the problem formulation phase.)

B. Determine whether or not each of these components should be taken into consideration. Sometimes, although a component is affected by the decision in question, the effect may be too small relative to the sum of the effects on the other components and thus, it could be dropped or neglected. A word of caution here, in some cases, it may not be clear whether a component's effect is significant or not, in such cases it can be temporarily dropped, but it must be kept in mind and checked again for pertinence when testing and validating the model by comparing observations made taken from the model with observations taken from the real world (2, 162-165).

C. Combine all variables that are considered pertinent or representative of the system into a mathematical model according to your knowledge about the relationships that exists between them and about how their values are determined.

This approach to simulation is preferable to most experimenters. They first build a mathematical model of the simuland, then, formulate a theory or hypothesis according to which this mathematical model is to be manipulated in order to reach a solution to the problem, i.e., simulate the system behavior. This approach is convenient IF and only IF:

1) The mathematical model enriches the investigator's feel for the

problem.

2) The mathematical equations that are mechanized on the computer retain a recognizable correspondence with the real-life system they represent (11, 6-7).

But sometimes, the simuland is not well enough understood to be adequately described mathematically. Those are the cases where simulation comes into its own and really proves its value by accomplishing what would otherwise be impossible.

> *....That is where the power of simulation can amplify the intuition of the gifted empericist, crystallize his thinking, and capture his best ideas in a tangible form (11, 6).*

In such cases, the experimenter starts by developing a rudementary model in order to enable him to begin his primary investigation of the system, i.e., he constructs a starting point for his investigation. Experimenting on this rudementary model and comparing results obtained with observations on the actual system orients the experimenter's thinking and clearly indicate what refinements are required and what data are needed to implement them. When this idea is obtained, it is used to improve the model and the simulation. The process of constructing an acceptable model in such cases is illustrated in figure (4).

That is to say that the experimenter can either:

1) Construct a mathematical model and then simulate.

2) Simulate first on a rudementary model and observes until he is properly oriented to build an acceptable mathematical model.

Figure (5) illustrates the directions the experimenter can move in to complete his simulation loop.

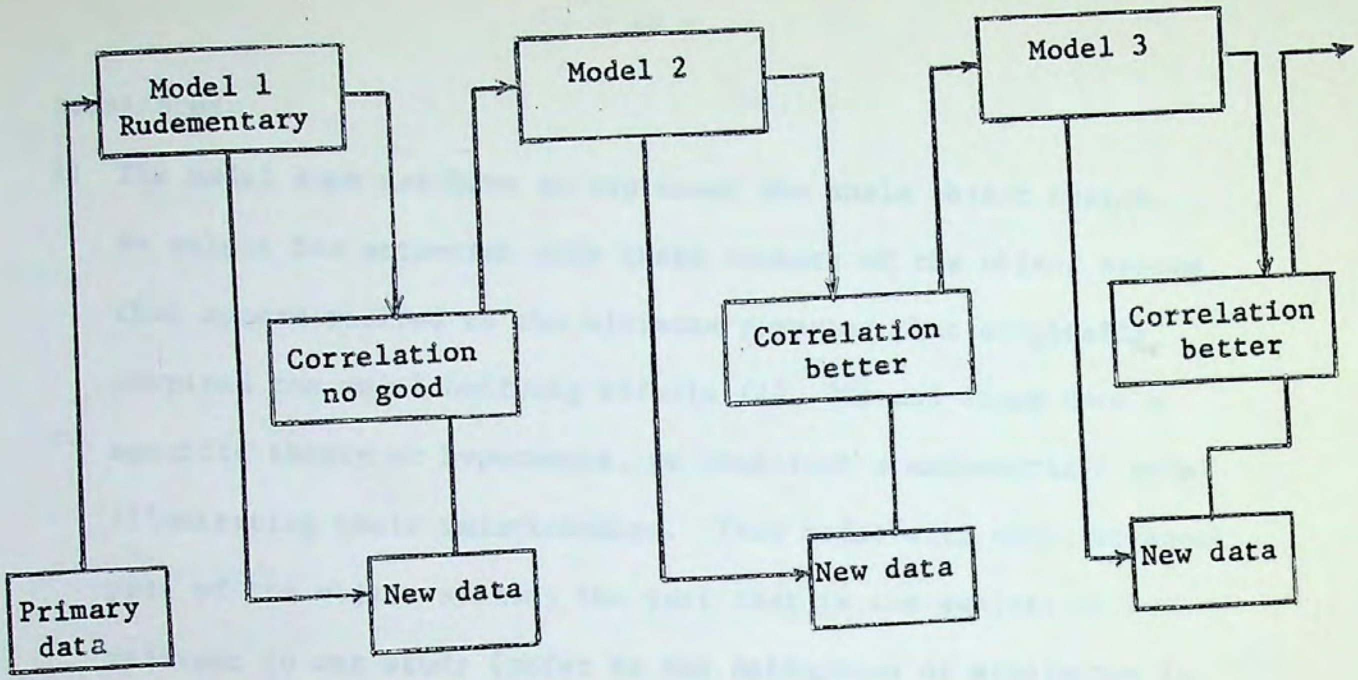


Figure (4): Steps towards the Development of an Acceptable Model
(Source: Elwood S. Buffa, "Operations Management: Problems and Models," 2nd ed., John Wiley & Sons, Inc., New York, 1968, p.12)

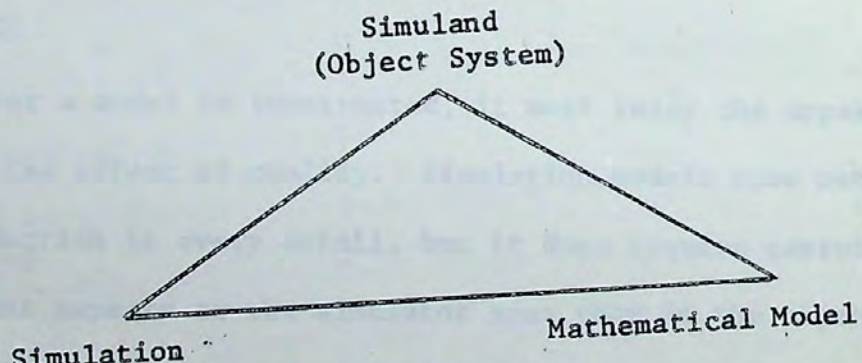


Figure (5): The Simulation Loop
(Source: John McLeod, "Simulation is Wha-a-at," From: John McLeod (ed.), "Simulation: The Dynamic Modeling of Ideas and Systems with Computers," McGraw-Hill Book Co., Inc., New York, 1968, p.6.)

Model building is by no means an easy job, it needs time, effort, and above all sound judgement in order to identify systems components and their properties, state what variables are of interest, their interrelationships, and how their values are determined. Two important factors must be balanced whenever model construction is considered:

1) The model does not have to represent the whole object system. We select for attention only those aspects of the object system that appear realted to the ultimate purposes that originally inspired the model building efforts (12, 30) and based upon a specific theory or hypothesis, we construct a mathematical model illustrating their relationships. This model will only represent part of the object system; the part that is the subject of or relevant to our study (refer to the definition of simulation in chapter I). Consequently, any system or object could in principle be represented by an unlimited number of models depending on the characteristics modelized and the problem that is to be solved (15, 3).

2) Whenever a model is constructed, it must relay the appearance of or the effect of reality. Simulation models does not require reproduction in every detail, but it does require capturing the relevant aspects as the simulator sees them in the object system. This effect is called *Verisimilitude* which means that, while the simulation is obviously an artificial representation, it has the quality of being true to life or to human experience by adding to the model all the characteristics that are relevant. The model must be complex enough to provide verisimilitude, yet be simple enough to be understood and processed in the available time (12, 58).

The amount of details to be included in the model depends upon:

1) The manpower and financial resources available to solve the problem.
2) The accessibility of information and data about the system and its environment.
3) The time frame within which the problem solution is required (5, 161).

Types of Models:

Mathematical models are classified into two types according to the characteristics of the system under study:

1) Deterministic models: are those models of systems which are devoid of uncertainty and thus, changes in the state of the system can be precisely predicted (5, 5). Those models are also called *nonstochastic models* and can - in most cases - be solved analytically or by iteration.
2) Stochastic models: are those models in which the functional relationships are subject to chance parameters; the output for a given set of inputs can be forecasted only in a probabilistic context (3, 14). Those chance parameters have a definite range of values, each one of which, depending on chance, can be attained with a definite probability.

The stochastic model is the feature that distinguishes a simulation from a mere sampling experiment in the classical sense (8, 307-319). This approach allows the generation of large amounts of data which otherwise might take years or months to accumulate.

As uncertainty in business is more the rule than the exception; the factor of chance plays a major role in determining inputs and outputs to and of business systems, thus, most of its models are stochastic.

Appendix II has been added to this research in order to furnish the experimenter with a clear idea about:

1) How to calculate probabilities in different situations.
2) How to find out, form, draw, or infer the probability distribution in any situation.
3) The situations or types of problems where the most common probability distributions apply.
4) How to assign or predict the values of a stochastic model's parameters and variables, and to infer the exact or the most approximate solutions or outcomes from the manipulation of such models. That is to say, how to design, manipulate, and reach conclusions in probabilistic situations.

VALIDATING THE MATHEMATICAL MODEL

The purpose of simulation models is to aid in understanding the object system we are interested in. The ultimate test of understanding is our ability to predict future behavior of the object system as then we may be able to control it in ways to meet our goals or we may only be able to understand why we can not control it the way we wish. To achieve this, we must make sure that our vehicle of research, i.e., our model represents the object system or, those aspects of the object system it is supposed to represent. That is to say, we must make sure that our model is VALID.

Though model validation is a step to be carried out after constructing the model, care must be exercised during the early steps of problem formulation and model construction as upon the accuracy of those two steps depends the accuracy and validity of the model.

Much has been written on validity; in its simplest form, validity involves answering the following question:

Does the model represent the phenomena (reality) it is supposed to represent?

IF the answer to this question is NO, then this second question must also be answered:

Where did the model fail to represent reality accurately?

To answer the first question, we start by testing the model we have constructed after FIRST making sure that the model is internally correct in a logical and programming sense (10, 284), i.e., it has been correctly flowcharted and programmed if, at this stage, the mathematical model is so complex that it needs to be processed by the computer. If not, then model validation at this stage is usually performed manually. The program is then processed in the computer and tested either by observation or experimentation as follows:

1. By observation: whereby the simulator acquires new data from the object system and checks it against data generated on the same premises from the model to determine the correspondence between the model and the object system. If these observations cannot be accounted for by the model, then the model needs

revision, i.e., an answer to the second question.

2. By experimentation: whereby the simulator experiments on the model itself using historical data gathered from the object system as starting conditions for the model, then processes the model and compares model output to see whether it coincides or not with the output acquired from the object system. It can also be performed by gathering new data from the object system, then running the model on the same initial conditions using the same values for parameters and variables and ascertaining that the results from the model coincide with the results acquired from the object system. That is to say, by following this method, we can test the accuracy of the model's prospective or retrospective prediction of the system's functioning (2, 591).

Choosing either way of testing the model depends upon:

1. The type of model and the nature of system it represents.
2. The type of data available at this stage for testing.
3. The deductions the experimenter has made during his previous analysis to the system and initial data.

For instance, if you have enough reliable data about the system you are to simulate and you know exactly the prevailing conditions (inputs) when this data (outputs) was acquired, then you can use this historical data on the model as initial conditions, run the model and compare its output with the output from the object system that you have already available. If they coincide, then your model is valid. But supposing that you have insufficient or unreliable historical data to start from, then you will have to depend on new data acquired from the object system and compare it with its counterpart generated from the model. In other cases, if you are simulating a new system that has not as yet actually existed or a system that is not well enough understood and you want to validate the model you have constructed, then the only way to test this model will have to be by experiments on the model to see whether its behavior and outputs coincide with the real world or not.

Now, supposing that after testing the model - using either way according to the situation - you found that your model did not represent the object system it is supposed to represent, i.e., your model is not valid. In this case you will have to answer the second question, namely; *where did the model fail to represent the object system?* in order to take the necessary measures to modify the model to make it valid. The reasons for model failure to represent reality are:

1. The model may fail by including variables which are not pertinent. For instance, the model may express the average number of sales calls required to convert a prospect into an account as a function of the size of the account when in actual cases, these two variables have no relationship.
2. The model may fail to include a variable which does have a significant effect on the system's functioning. For example, the model may not express the dependence of purchasing cost on the quantity purchased in a situation where there are price breaks depending on the quantity that is purchased, i.e., the purchasing cost is a function of the quantity purchased. The

model may not include the quantity purchased as one of the variables.

3. The model may not accurately express the actual relationship which exists between one or more of the pertinent independent variables. For example, the model may express a relationship as linear, when in fact it is not.

4. Finally, even if the model does not have any of the foregoing three deficiencies, it may still fail to represent the object system accurately if the parameters were not properly evaluated. For instance, we may consider that the lead time in an inventory control model is 10 days when in fact, it is more or less than that or, it might not be constant at all, but takes variable quantities (2, 577-578).

Figure (6) illustrates the process of constructing and validating the mathematical model.

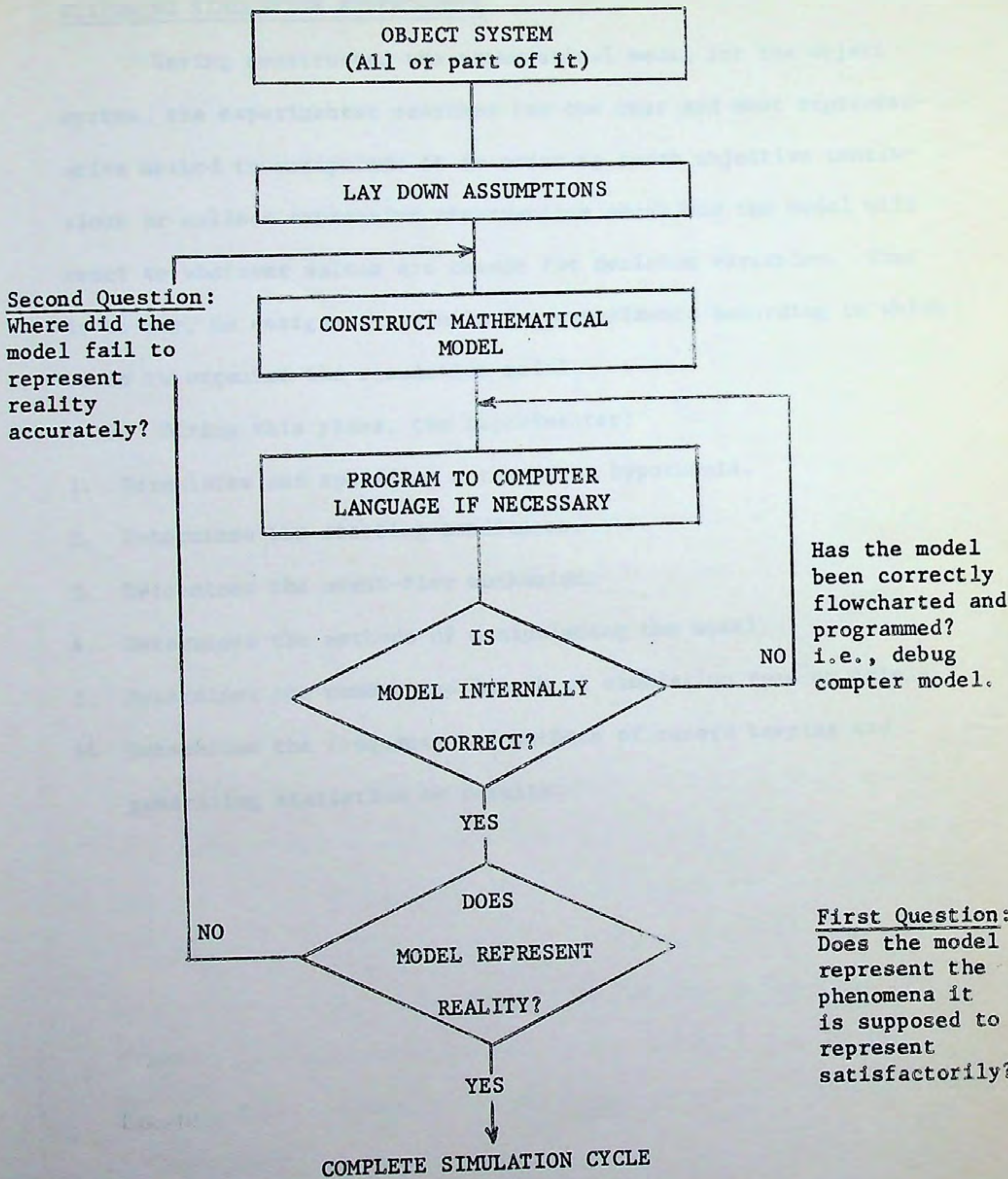


Figure (6): Model Construction and Validation Process

DESIGNING SIMULATION EXPERIMENTS

Having constructed the mathematical model for the object system, the experimenter searches for the best and most representative method to manipulate it in order to reach objective conclusions or collect expressive observations about how the model will react to whatever values are chosen for decision variables. That is to say, he designs the simulation experiments according to which he is to organize the simulation model.

During this phase, the experimenter:

1. Formulates and specifies a theory or hypothesis.
2. Determines the starting conditions.
3. Determines the event-flow mechanism.
4. Determines the methods of manipulating the model.
5. Determines the number and length of simulation runs or trials.
6. Determines the frequency and methods of record keeping and generating statistics or results.

1. Formulating and specifying a theory or hypothesis:

Before constructing a simulation model, and the moment we begin an inquiry or start solving a problem that we have already formulated, we should start with what we know from existing theory for that subject area. Sometimes, our knowledge about the existing system or the interactions within it is weak, or that we do not have a good theory at hand; hence we start by making tentative statements, .e., hypothesis from which to proceed. To start without a theory or hypothesis will cause our investigations to founder in a mass of confusion. That is not to say that our theories or hypothesis will always turn out to be right; in many cases, after testing them, our findings might very well change or modify them. Most theories are too general to be tested in any meaningful way. They have to be expressed more specifically to suit the problem at hand. Then a simulation model is designed on the basis of this theory to give operational opportunity to its implications with respect to the problem or the system (12, 24-27). To make this clear, let us consider the case of a factory that produces a certain item X. The problem is to find out the optimum quantity to produce in order to minimize total cost per unit of this item X.

There is a well known general economic theory that states: *The bigger the quantity produced per one machine(s)-layout; the less the cost per unit.*

There is also the theory of *Decreasing Utility* that states: *There is a limit to the amount produced after which the cost per unit will start to increase again.*

Now, in order to be able to test these theories, we have to make them more specific and applicable to the case under study. Assuming that the experimenter has found out during the phase of investigation and analysis that:

A. Any amount produced can be sold, i.e., there is no limit to demand.

B. The factory is producing 1000 units per layout and still is not achieving the minimum cost per unit.

C. Any amount produced that was less than 1,000 units entailed more cost per unit.

Now, the experimenter can formulate a hypothesis to start from. This hypothesis in this case will be:

Producing less than 1,000 units is not economical, there is a certain quantity more than 1,000 units that will minimize production costs before and after which, production costs per unit will increase.

In this example, the experimenter has:

A. Started with a general existing theory for the subject under study.

B. Laid down assumptions that he had deduced during the analysis of the system and the initial data.

C. Compared what he deduced from the analysis of the system and the initial data with what he knows from existing theory for the subject area and formed a hypothesis.

D. Expressed this hypothesis more specifically so as to apply to the problem.

E. On the basis of this hypothesis, he then should start to design the methods or ways according to which he is to manipulate the mathematical model in order to reach conclusions about whether his hypothesis is valid or not, and from it, he can also calculate the optimum amount to produce. That is to say, the experimenter transorms the mathematical model into a simulation model.

This simulation model is then run by the computer to study and test the hypothesis. The observations generated from the computer runs may confirm this hypothesis or, they might prove that certain modifications or refinements should be made in it. The experimenter, according to the generated data, modifies the hypothesis, design another simulation models or modifies the previous one, tests it again and so on until he is sure that he has reached a solution to his problem. The relationship between object systems, theories or hypothesis, and, simulation models is illustrated in figure (7).

2. Determining starting conditions:

According to the definition, a simulation run represents the operation of a system in a model form from a given starting point for a certain period of time. By starting point here, we mean the state of the system at the beginning of the simulation run. Starting conditions therefore are relevant to the results obtained from the model. For example, in an inventory management problem, if we start our run by assuming - as starting conditions - that

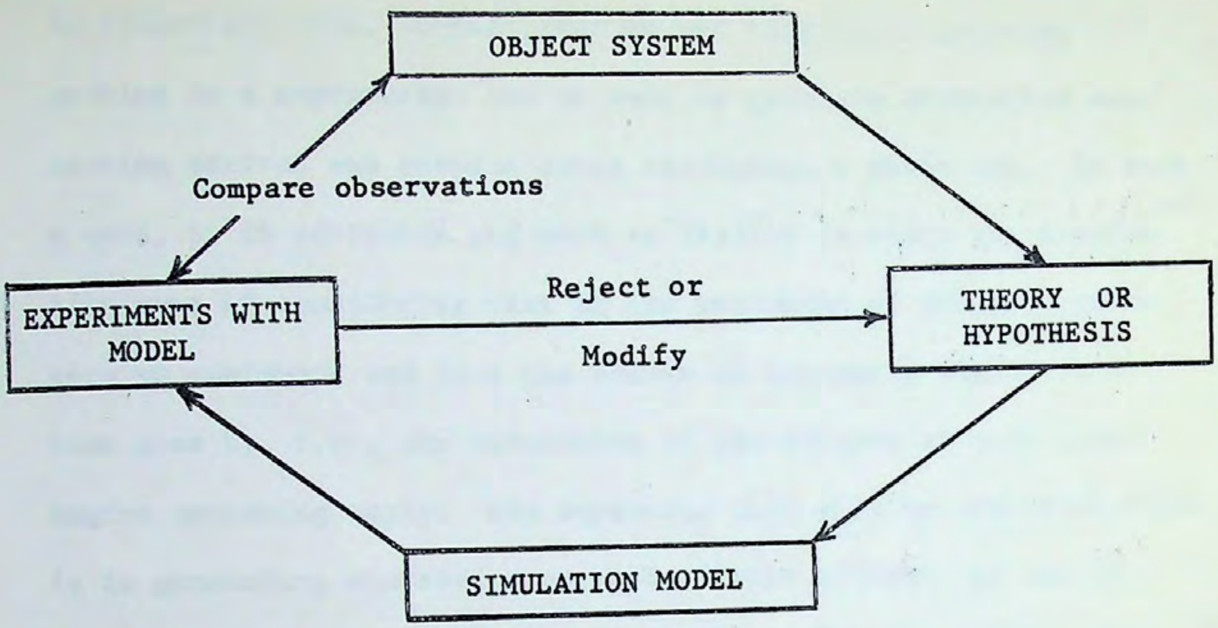


Figure (7): The Relationship between Object System, Theories, and, Simulation Models.
(Source: Richard F. Barton, A Primer on Simulation and Gaming, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1970, p.28.)

the inventory level is zero, then the quantity of goods in inventory at a certain production schedule and a certain consumption rate will be different from the quantity of goods in inventory if we assumed that the inventory level at the start was 100 units at the same production schedule and the same consumption rate.

Determining starting conditions is not an easy job, in fact, it is very tricky. As a general guide, determining starting conditions depends upon:

A. The system under study.

B. The type of the problem.

C. The results that are seeked; i.e., the type of output.

To illustrate this, suppose that we are tackling a queueing problem in a supermarket and we want to generate statistics concerning arrival and service rates throughout a whole day. In such a case, it is advisable and more to reality to start the simulation runs by considering that at the beginning of the day, there were no customers and that the number of customers builds up as time goes by, i.e., the simulation of the process in this case begins operating empty. Now supposing that what we are interested in is generating statistics associated with arrival and service rates representative of highly congested conditions at the peak of the rush hour; it would be quite misleading to start gathering statistics with empty queues as under congested conditions, queues are usually very long so that initial conditions close to this state would be truly representative. To achieve the congested state, you can start the model empty and let it run until you reach the forecasted congested state and at that moment in time, start gathering the needed statistics. This way is easy as it solves the problem of determining starting conditions, but it has the following disadvantages:

A. It needs a long computer time which is quite expensive.

B. Some experiments are usually needed to determine how long to let the model run before beginning to gather statistics.

Another approach is to assign certain values as starting conditions that represents the state of the system as near as possible to the congested state so that there is little or no startup period (10, 296-297).

The difficulty with this method is that it may not be easy to determine *a priori* what the values of those starting conditions should be, but it is rare that the problem-solver has no such information (13, 13). In fact, some experiment and/or observations similar to those needed to determine how long to let the model run before beginning to gather statistics in the first approach would furnish the problem-solver with what he needs, besides of course his basic knowledge about the system he is simulating that has been gathered during the analsis of the system and the initial data.

3. Determining event-flow mechanism:

The most important characteristic of simulation models is that TIME or a count of repetitions(trials) is an essential element of the model and the results of the simulation runs are values of the variables that describe the state of the system at the end of the time interval or trial. The computer program for the simulation must be so designed that it moves the model through the simulated time causing events to occur in the proper order and with the proper time interval between successive event occurrences (7, 158).

There is also the fact that while the components of a real system function simultaneously, the digital computer executes its instructions one at a time and thus can consider the system components only one at a time (13, 95-97). For this reason, it is important that when designing simulation models, the event-flow mechanism be constructed in such a way so that the simulated performance of the system components are synchronized in time.

There are two basic ways to structure simulation models in terms of proceeding through the occurrence of events.

A. The fixed increment method:

By this method, the model is moved through time in small and equal increments. The size of the increment is to be sufficiently small so that the discrete approximation to the continuous system is acceptable. A master clock is included in the model to control the time flow and is set at zero at the beginning of the simulation. During each interval, it is to be determined whether any events have occurred, and if so, proper bookkeeping is performed and appropriate logic tests are made. The master clock is then

$$\Delta t_1 = \Delta t_2 = \Delta t_3 \ldots$$

Figure (8): The fixed increment method.

advanced by the suitable increment Δt and the process is repeated until one of the following conditions occur:

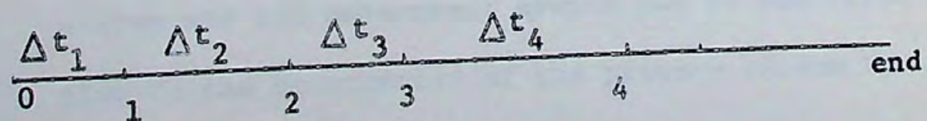
1) The clock reaches a preset ending value of simulated time, trials, or runs.
2) The event counter reaches a set maximum.
3) Nothing is scheduled to happen (12, 161-162).

This method is convenient for large simulations whenever one of the following conditions exists:

1) The relationships between random variables are not clearly specified, or,
2) There are no known relationships between the random variables (7, 106). The main disadvantage of this method is that it may be that no events occur during a time increment and thus the model cycles uselessly consuming expensive computer time.

B. The variable increment method:

Also known as the *next event* method. By this method, the occurrence times of the events resulting from the simulated performance of all system components are determined at the beginning of the simulation and the master clock is set to zero. The master clock is then updated to the time of the next event occurrence Δt, and proper bookkeeping together with appropriate logic tests are performed. The master clock is then updated again to the time of the second event



$$\Delta t_1 \neq \Delta t_2 \neq \Delta t_3 \neq \Delta t_4 \cdots$$

Figure (9): The variable increment methods.

occurrence Δt_2 and the process is repeated until one of the following conditions occur:

1) The clock reaches a perset ending value of simulated time, trials or runs.
2) The event counter reaches a set maximum.
3) Nothing further is scheduled to happen.

Sometimes, when determining the occurrence times of the events, the event times on the time continuum represent different kinds of events. In such cases, the model needs only to be cycled to the next *relevant* event and not to the directly next event.

These two features of variable increment method: a) moving directly to the next event, and; b) executing only the relevant events, result in a substantial reduction in computer time as compared to fixed time increment method.

This method is convenient in relatively small simulations whenever one of the following conditions exists:

1) The relationships between random variables are clearly specified.
2) The performance of the system is characterized by a primary random variable governing the entrance into the system and all subsequent events can be described relative to the occurrences of the primary random variable.

The choice between which of methods to be used depends upon:

1) The nature of the problem.
2) The type of available data.

There are also times when both methods are used in the same problem according to the nature of the variable that is being investigated.

4. Determining methods of manipulating the model:

The essential feature of experiments conducted under simulated conditions is the ability of answering the following question: *What would happen if such-and-so change were made?* To reach an answer to this question, the experimenter should be able to control the model and the changes in such a way that enables him to determine precisely the reaction or change in the state of the system due to any change in the inputs to the system decision variables. Experimentation with all-computer simulation models achieves the idealization of the perfectly controlled experiment (12, 37).

Before discussing the different methods of model manipulation, we should first get acquainted with:

A. The Monte Carlo technique.

B. The nature of inputs to stochastic and nonstochastic models.

The Monte Carlo technique:

Monte Carlo is a game of chance technique that is applied to solve probabilistic problems. The essence of this technique is to determine the outcome(s) at random at critical decision points in a stochastic process (5, 32).

According to definition, a *Stochastic Variable* is defined if:

a) the set of its possible values is given, and;

b) the probability of attaining each of these values is also given.

With this information, both the density and distribution curves can be plotted.

Simulation studies are mainly directed towards answering the question:

WHAT would happen (reactions or events) IF certain conditions or factors were predominant?

In *stochastic* simulation studies, the predominant factor is the *chance* factor. Thus, in fact, what we want to actually find out is the answer to this question:

WHAT events are likely to occur due to the chance factor alone; knowing in advance the probability of occurrence of each event?

That is to say, we are not trying to learn anything about the random variable as we already know its probability distribution and any other characteristic of relevance (from the analysis of the system and the initial data); rather, what we want to know is:

How it is going to occur in complex situations?

Our basic problem then is:

HOW to select the values of the random variable whose distribution is known so that we can simulate its occurrence in the real world?

Thus, while the main goal of the probability theory is to describe the distribution of the probability numbers over events, the goal of the Monte Carlo technique is to create events; to proceed from probabilities to events (11, 152).

Because we have no idea what the next event would be in the sequence of events we are considering, it is customary to assume that the events would occur randomly. For this purpose, we *use* a sequence of independent observations of a random variable whose distribution function is known, to *generate* a sequence of independent observations of another random variable whose distribution is also known. The original known sequence is often referred to as the *Basic Sequence* and is usually a sequence of random numbers from a uniform distribution. This process is known as the *Monte Carlo Sampling* (7, 61-62).

At this point, two vital questions are raised:

1) Why did we choose random numbers in particular?
2) How can we simulate a sequence of any type of distribution which in most cases is nonuniform, by a sequence of uniformly distributed random numbers? Wouldn't that affect the validity of the results?

Why did we choose random numbers?

Random numbers are merely a quantitative reflection of randomness (12, 158). The prominant factor that makes random numbers different from other numbers is the fact that future numbers cannot be predicted with a knowledge of past numbers than without this knowledge, hence, the probability of obtaining a particular number in any trial does not change trial after trial. This is also the assumption of independent events in the probability theory.

A uniform random number ranges between *ZERO* and *ONE* and is randomly selected from the uniform probability distribution function

$$F(X) = \int_0^1 dx$$

Whenever desirable, those numbers on the unit length can be translated to numbers on other intervals of any length by simple arithmetic.

Those characteristics of random numbers render them most suitable to reflect randomness of occurrence.

How can we simulate a sequence of nonuniform distribution by a sequence of uniform distribution?

To answer this, let x be a random variable that possesses a density function f(x) and a distribution function F(X). Also, let u be a uniformly or rectangularly distributed random variable - according to whether we are dealing with discrete or continuous events - on the interval (1, 0) that has a probability distribution function $\phi(u)$.

Now, since both u and F(X) are defined on the unit interval, then

$$u = F(X)$$

According to the law of the inverse function, we also have

$$x = F^{-1}(u)$$

Let us also consider other values x_1, u_1 such that

$$u_1 = F(X_1)$$

Then $$p(x \leqslant x_1) = F(x_1)$$

And $$p(u \leqslant u_1) = \phi(u_1) = u_1$$

Then $$p(u \leqslant u_1) = p(x \leqslant x_1)$$

That is to say that random sampling from a uniform or rectangular distribution within the interval (0, 1) can be made equivalent to random sampling of any known distribution. Consequently, we need not have a random number generator for each type of distribution, instead, we can sample randomly from any known distribution by sampling from a uniform distribution and transforming the results to the other known distribution (12, 73-74).

Figure (10) illustrates the process of transformation.

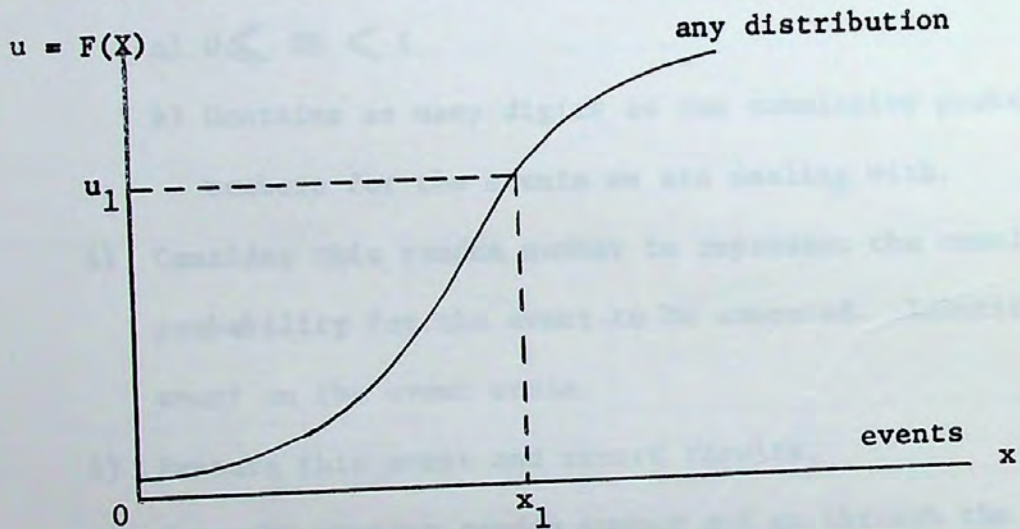


Figure (10): Monte Carlo Sampling of a Continuous Distribution

Thus, we can now say that the Monte Carlo simulation process involves the following steps:

1) Calculate the cumulative probabilities for the events on the unit length making sure that the last probability is exactly *ONE*.

2) Identify discrete events (or point events for continuous probabilities) corresponding to each interval on the unit length by associating each event with the lower bound of the

next interval. Since the list of events is independent and collectively exhaustive, then one of the events at least must occur (*no event* is considered an event). For this reason, we let the number zero if generated represent the first event, the last event is associated with the number one which is the final cumulative probability on the unit length.

3) Generate one uniform random number (RN) that has the following properties:

   a) $0 \leq RN < 1$

   b) Contains as many digits as the cumulative probability numbers for the events we are dealing with.

4) Consider this random number to represent the cumulative probability for the event to be executed. Identify this event on the event scale.

5) Execute this event and record results.

6) Generate another random number and go through the steps 3 to 5 again and again as many times as designed in the experiment.

Figure (11) illustrates diagramatically the Monte Carlo simulation process.

The nature of inputs to stochastic and nonstochastic models:

Inputs to an all-computer simulation model may be of two kinds:

1) Starting conditions, parameters, and input time path values.

2) Random numbers which are required - in stochastic models

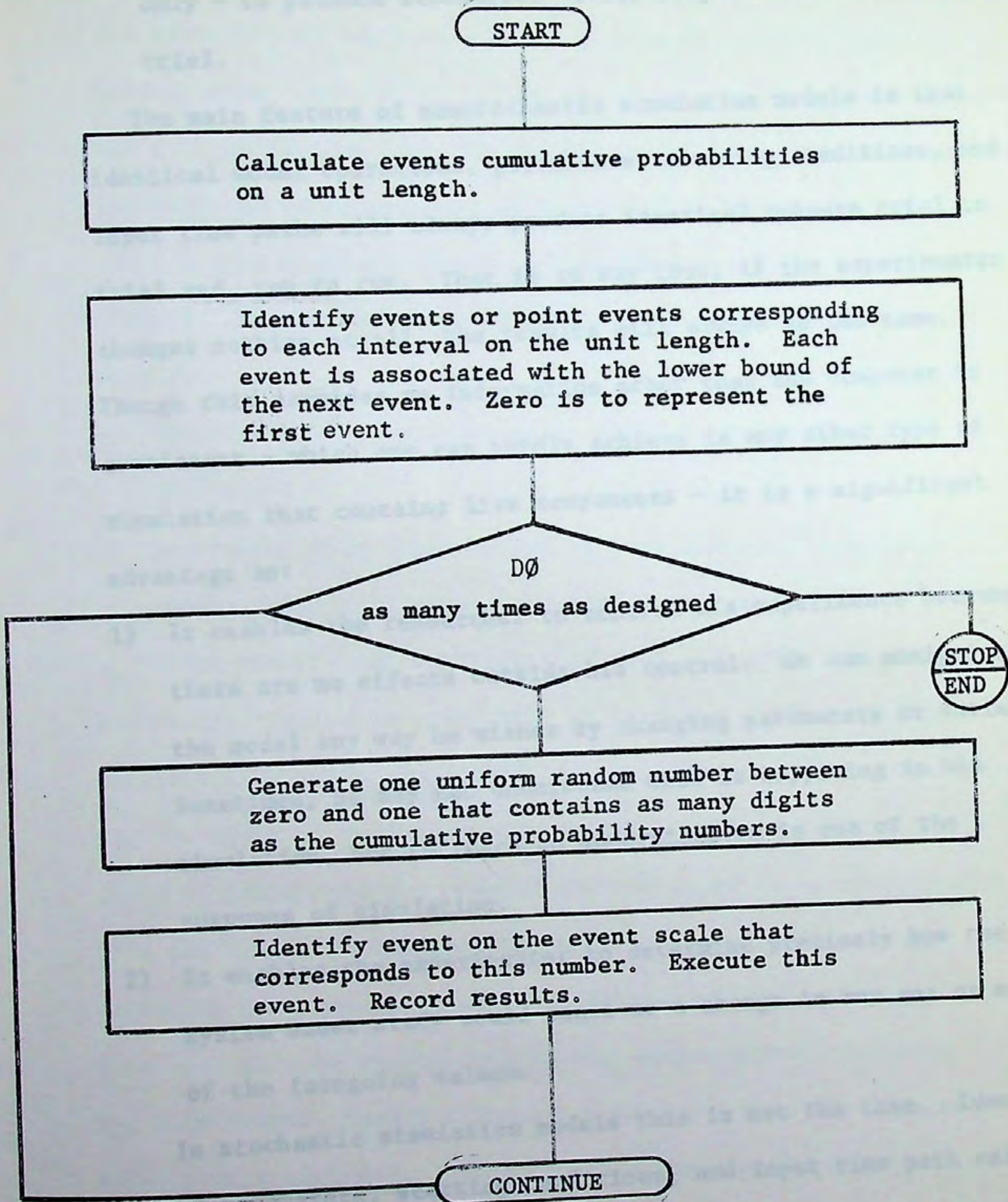


Figure (11): The Monte Carlo Simulation Process for a Single Event only - to produce stochastic variability (events) trial after trial.

The main feature of nonstochastic simulation models is that identical model operations, parameters, starting conditions, and input time paths will always produce identical outputs trial to trial and, run to run. That is to say that, if the experimenter changes nothing at all, the results will always be the same. Though this provides no information other than the computer is consistent - which one can hardly achieve in any other type of simulation that contains live components - it is a significant advantage as:

1) It enables the researcher to control his experiments because there are no effects outside his control. He can manipulate the model any way he wishes by changing parameters or variables. Sometimes, he may not understand what is happening in his simulation, but to reach an understanding is one of the purposes of simulation.
2) It enables the experimenter to determine precisely how the system under study would react to a change in any one or more of the foregoing values.

In stochastic simulation models this is not the case. Identical parameters, starting conditions, and input time path values produce varying outputs trial to trial and run to run due to the stochastic variability introduced by random numbers. This feature in stochastic simulation models is not a drawback, on the contrary, this is where simulation comes into its own and actually demonstrates its superiority on other techniques. In such cases, the experimenter controls all the inputs to the model either by holding them constant throughout the trials or runs, or changes one or more of them from run to run or trial to trial and observes the performance or response of the model to the random changes alone or the combined effects of random changes and other input changes. Such types of problems cannot be solved analytically because:

1) The mathematical relationships would be so complex and untractable.
2) Generating this large amount of data manually to predict random variations might take months or years.

Random numbers are distinguished from all other inputs to a stochastic simulation model such as starting conditions,input variables, ... etc. The reason for this distinction is that we want to think of random numbers as being inherent in the operations of the stochastic process and not an input or operation that the simulator can fashion to his purpose because those variables take random values that depend solely on chance and which can only be represented by the random number technique. For this reason, random numbers are excluded from the list of model inputs although, logically, they are and can only be classed as model inputs because; the model or part of it needs them to produce stochastic events.

Methods of model manipulation:

Now that the nature and effect of the different inputs to the model have been understood, we can state that the experimenter can manipulate the simulation model and design his experiments by either of the following methods according to the situation:

1) Changing the parameters from run to run.
2) Changing the input variables at different moments of time during a run or from run to run.
3) Changing both parameters and input variables from run to run. This method is known as *Input Variations*.

The changes mentioned above do not constitute a change in the model itself, they are just changes in the application of the model and it is this flexibility that enables the experimenter to perform many experiments with a single model.

4) Experimental changes can be made in the model itself, i.e., change its operations to represent intended future modifications and compare the new system's performance or effectiveness with the existing one.
5) Changing both input and model to study the combined effect of input and model variation by running changed data on a changed model (object system) (12, 32-38). In such cases, to avoid confounding effects, it is necessary to run either: a) changed data on a given model, or b) a changed model on a given data, in order to have a basis for comparison by knowing what each one of the changes would contribute, then correlate results statistically to get a clear picture of the

combined effects.

Thus, according to the objective(s) of the investigation, the experimenter prepares a list enumerating the type and values of the changes he wishes to investigate and design his experiments (runs) accordingly. This will also help him decide how many runs he will need. Sometimes, the types or values of changes cannot be predicted in advance as they may depend upon the results obtained from previous runs. In such cases, the experimenter designs his first runs or experiments only according to the available information, runs his model, then, when he gets the results of these first experiments, he can decide what type and how many more changes he should investigate and how many more runs he needs.

It is not always that the researcher will find that all the system's operations and inputs are controllable, that is to say, some elements of the object system might vary outside the control of management, i.e., vary uncontrollably. Still, due to experience and, as a result of the previous analysis of the system and the initial data, management can - to a certain degree - predict the values or the limits within which those uncontrollables might change. In such cases, the advisable way to sequence simulation runs is to:

1) Run the model on available and valid historical data and collect results.
2) Introduce predicted uncontrollables and run them with the controllables used in historical data.
3) Change controllables and run the model with the historical

values of the uncontrollables.

4) Make adaptive changes in the values of the controllables according to the results you have already obtained and run them with the predicted uncontrollables.

Following this procedure, managers can investigate:

a) The consequences of their estimates of future uncontrollables.

b) The consequences of alternative managerial responses to these changed conditions.

c) The effects these changed policies would have had in the past.

The computer programs that provide these services are sometimes called "What-If" programs. Repetition of these cycles is "Management By Simulation" (12, 113-114).

5. Determining number and length of runs or trials:

The more runs you make with different inputs in each, the better idea you get about the optimum operating conditions as you get to know how the system will react under various different conditions. Similarly, a longer run will portray a better picture of the system because the random factors have a better chance to *average out* (1, 58-60) as a longer run can be viewed in simulation studies as equivalent to taking a large sample, i.e., it resembles taking a larger sample from the system and gathering observations about each of its components.

Another advantage of longer runs is that it usually obliterates the effects of starting conditions (10, 296) specially in cases when you are not sure about their representativeness.

However, making numerous or long simulation runs consume expensive computer time. Thus, a balance must be struck between the two alternatives; a) reaching accurate results; and, b) minimizing computer costs. There seems to be no clear-cut rules for fixing the number or the proper length of runs (3, 497), however the following points can be used as a guide while planning runs:

A. For each run, we will want to vary one factor and measure the corresponding results in order to compare the various alternatives under consideration. Unless we expect the differences between the alternatives to be clear-cut, it would be desirable to enlist the aid of a statistician in planning the runs (1, 58-59). Thus, the number of runs can be determined by the number of parametric or variable changes that you want to experiment upon unless there is a certain ambiguity in the results obtained from some runs that makes it necessary to increase the number of trials to obtain more representative statistical data.

B. In many cases, the length of the run cannot be determined in advance because our purpose may be either to:

   1) Process the model until specific conditions or observations are generated, i.e., time itself may be one of the information outputs that we seek. For example, in the previous queueing problem, we might want to know how long it would take to reach the congested state. In such a case, we cannot determine the length of the run beforehand, we just keep the model running until we reach that

state and then compute the time needed.

2) Trace a system's response to a certain change, in which case the simulation run is continued until the variables have settled down (3, 497-498). For instance, suppose that in an inventory control model we want to find out the system's response to an increase or decrease in lead time and are using as starting conditions the contemporary state of the system when lead time is 5 days. When the lead time variable is changed from 5 days to 7 days, a *transient* period will occur at the beginning of the run during which the inventory level will start to fall rapidly, the amount and times of shortages will increase rapidly too, ... etc. These rapid reactions does not truly represent the system's response to the change in lead time. What they actually represent is the system's *instantaneous* response to a change in one of its major variables. After a certain period of time, the system sort of digests this new change and the state of the system assumes a new stable condition whereby the inventory level will decrease at a steady rate or fluctuate in an orderly or semi-orderly fashion and so with the other elements of the system. Another transient state will occur at the beginning of the simulation run when the lead time is changed to 2 days instead of seven, only this time there will be an increase in inventory level and a decrease in shortages. So, in order to

trace the system's true response, we should keep the model running until the transient state has passed and the variables settle down before collecting output data.

6. Determining frequency and methods of record keeping and generating statistics or results:

Because the model is installed in the computer, there is only one way to observe its behavior - through output. The simulation designer thus, must specify - when flowcharting the computer model - the types and shape of the data he needs the computer to generate, together with the points at which this data is to be collected.

The output of simulation experiments may consist of one or more of the following:

A. Records of parameters and starting conditions.

B. Time paths of input and generated variables.

C. Selected variable values from selected states of the model.

D. Additional created outputs that are combinations of or measures of the foregoing data.

E. Just the ending conditions.

Thus, according to the:

A. The simulation objective.

B. The type of the problem.

The designer should decide which of the above mentioned outputs he needs.

Also, he must determine the frequency of generating each of these outputs. Having determined the time-flow mechanism, the methods of model control, and the number and length of the runs, he should determine whether he will need to generate all or part of the foregoing data at:

- Each time increment.
- Each occurrence of an event.
- Certain points of simulated time.
- Specific states of the system.
- The end of each trial.
- The end of each run.
- The end of the whole simulation process.

Computers have the capacity to generate huge quantities of data, but the designer must differentiate between quantity and quality. Only relevant and meaningful data is to be generated, more data than that will only result in:

A. An uncalled for loss in computer time and consequently expenses.

B. Confounding whoever is analyzing this data.

C. More burden during the data reduction process.

Figure (12) illustrates the subphases of the experiment design phase. Again, it is of great importance to point out that, here also, no demarcation lines actually exist between the different subphases. The designer will always have to refer to previous subphases or make provisions for subsequent ones while designing each subphase. For example, while determining the methods of controlling the model, the results obtained from the determination of the event-flow step are the basis to be used in this

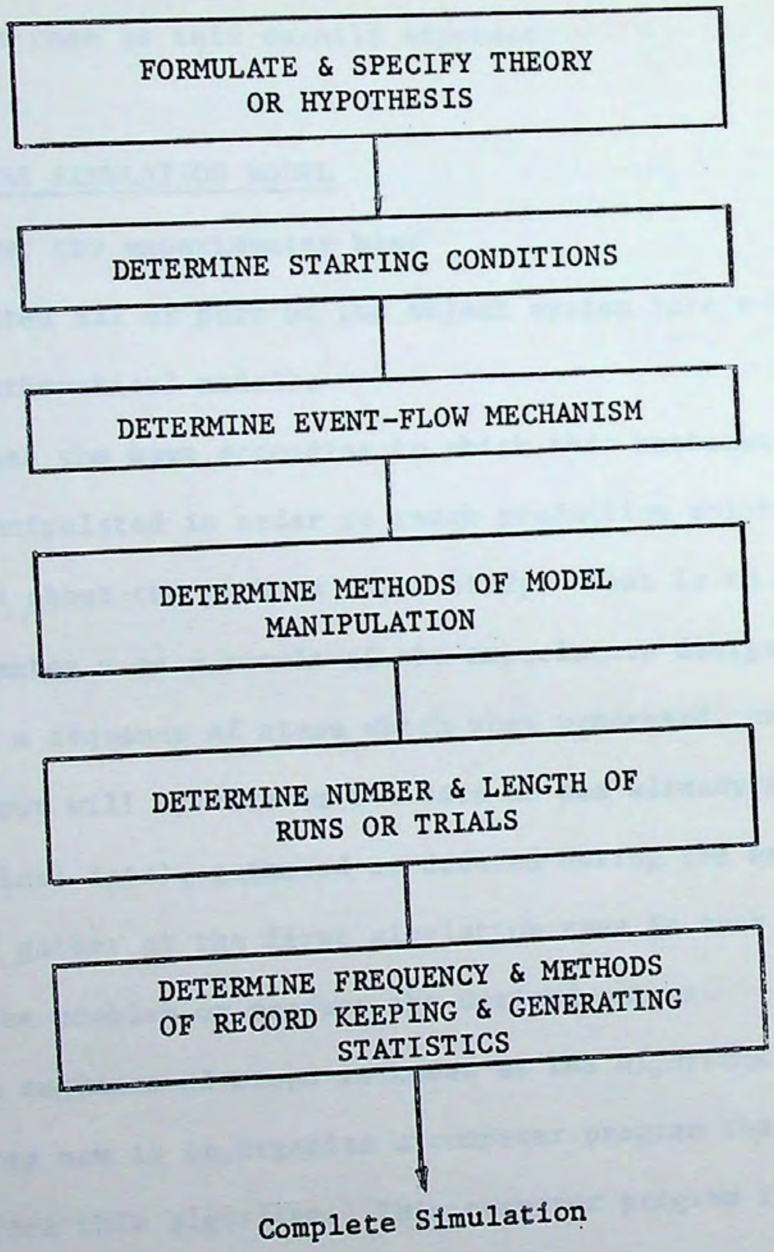


Figure (12): Subphases of the Experiments Design Phase determination. At the same time, the designer should keep in mind the number and length of runs that he will have to perform in such a case as this entails expenses.

DEVELOPING THE SIMULATION MODEL

By now, the experimenter has:

1. Represented all or part of the object system into a mathematical form (mathematical model).
2. Determined the ways according to which this mathematical model is to be manipulated in order to reach productive solutions or conclusions about the problem under study. That is to say, the experimenter - as a result of the experiments design phase - has devised a sequence of steps which when processed, evaluated, or worked out will operate on the data he has already available (historical data), gathered or deduced during the analysis phase, or will gather at the first simulation runs in such a way as to solve the problem or produce the desired output.

This sequence of steps is known as the *algorithm* (3, 18). The next step now is to organize a computer program that enables him to perform this algorithm. This computer program is the *simulation model*.

Simulation models should possess the following properties:

1. They represent all or a significant part of an object system.
2. They can be executed or manipulated.
3. Time or a count of repetitions is one of their variables.

4. Their purpose is to aid understanding of the object system by:
   A. Explaining past behavior of the object system.
   B. Predicting future behavior of the object system (12, 27-28).

During this phase, the experimenter:

1. Determines the method of generating random numbers (in stochastic models only).
2. Flow chart and program the simulation process.

Random number generation:

The characteristics of random numbers demands that the experimenter should not know in advance what the next number he is to use would be. That is why the original method for generating random numbers was by using labeled chips in an urn and each time, the experimenter draws a chip from the urn and uses the number labeled on it. This method proved to be impractical for the following reasons:

A. It is too time consuming especially when there is a need for a large amount of those numbers.
B. It is sometimes necessary, in complex sampling experiments, to repeat the calculations and/or the experiments by using the same random numbers as a method for checking the results of the simulation or trying alternative solutions. This cannot be realized by the original method as we have no way of repeating the same previously drawn numbers.

Thus, through the use of electronic devices, tables of random numbers were produced and published in many texts. Because those numbers can be regenerated and the experimenter knows in advance what the next numbers would be, it was found that this state is contradictory to the definition of random numbers, hence they are usually referred to as *Pseudorandom numbers*. That is not to say that those numbers are not actually random as we can in fact generate a set that is very much random and possesses all the properties of random numbers, but it is the state of mind of the experimenter, generator, or drawer that makes the difference. As long as he can know the next number or generate the same set again, they are not considered 100% random.

Some of the methods for generating random numbers in simulation experiments are:

1) Through the use of a table of random numbers and drawing as much numbers as needed from the table. Each number is then punched on a separate card and fed to the computer according to the program. This method has the following disadvantages:

   a) It takes a long time to punch those numbers on cards or tapes if the simulator is using a large amount of them,

   b) It demands a very large storage capacity that might not be convenient even for very large computers.

2) By using the power-residue concept to obtain random numbers. This is the most widely used method either for decimal or binary computers.

   A. For decimal computers:

   Using the relationship

$$u_{n+1} = x.u_n \pmod{10^d}$$

where:

u_n is the previous random number.

d represents the number of significant digits in a computer word.

x is a constant multiplier chosen so as to obtain the largest possible sequence of random numbers without repetition.

With a proper choice of x, this equation can produce a nonrepeating sequence, or period, of 0.5 billion numbers for d = 10.

Figure (13) portrays a subprogram named FUNCTION RANUM that is designed to generate random numbers according the power residue method with d = 10 & x = 100003.

```
*1010

      FUNCTION RANUM (IU, IN)
C     RANDOM NUMBER GENERATOR
C
      IF(IN) 30,40,30
   30 IN = 0
      IX = 100003
      SHIFT = 10.**(-10.)
   40 IU = IX*IU
      RU = IU
      RANUM = RU*SHIFT
      RETURN
      END
```

Figure (13): Subprogram for random number generation

Note that:

a) The first card in the program is a parameter card used to increase the floating point and the fixed point precision.

b) The calculations for each random number are made in fixed-point arithmatic, the 10 rightmost digits are retained after each operation, and the answer is converted to a 10-digit floating-point number between 0 and 1.

c) The variable IN, which must be initially defined in the main program as any integer other than zero, is used to avoid re-defining variables in the subprogram after the first number has been generated (14, 274).

B. For binary computers:

Using the relationship, or the generation formula

$$u_n = u_{n-1} (2^{18} + 3) (\text{mod } 2^{35})$$

where:

u is any odd number, was found to be statistically sound and to have a long period. The number generated u_n is then converted to a number between 0 and 1.

The above methods were presented for the purpose of illustrating how random numbers are generated in the computer. However, FORTRAN compilers almost always come equipped with several functions or subprograms which are brought into play whenever they are required and random number generators are one of those subprograms. Thus, whenever the computer encounters to the right of the equal sign in any one of the algebric statements the expression "RANDOMF(X)" or "FURNB(0)", ... etc., the computer will replace the expression with a new random number whose magnitude is less than one and equal to or greater than zero, i.e., a decimal fraction. If we want a two-digit random integer, we simply move the decimal point to the right two places, truncate via the statement:

M = RANDOM F(X) * 100.0

and the variable M has a two digit random integer stored in it.

In cases where the compiler is not equipped with a random number generator, the power residue method explained before is best used.

Flowcharting and programming the simulation model:

During this step, the experimenter:

A. Writes - according to sequence - the procedure he is to follow to solve the problem.

B. Designs the format of the input data.

C. Designs the format of the output data.

D. Draws in flow chart form all the arithmetic and logic steps to be performed by the computer, i.e., flowchart the simulation model.

E. Expresses this flow chart in computer language, i.e., program the simulation model (9, Introduction).

The steps from B to E can either be performed by the experimenter himself if he knows how, or he can seek the help of a specialized programmer.

The computer languages that are used in programming simulation models are:

A. Special purpose programs such as the GPSS (General Purpose

System Simulation), SIMSCRIPT, ... etc. These programs have been prepared to simulate specific problems or types of systems. The use of such programs provide the quickest and cheapest way to implement a simulation study on the computer. But, there are cases when those programs will not have the capabilities that the user desires to process his model as they do not apply to all cases or systems.

B. General purpose languages such as FORTRAN, COBOL, ALGOL, and PL/1. Virtually, any simulation model can be programmed in a general purpose computer language, only, programs written in those languages usually require a longer time to program and are more complicated than special purpose programs (10, 214-215).

Since it is not the purpose of this research to explain simulation languages, the only point that is to be stressed here is that the choice of the language to be used depends upon:

1) The availability of a ready made special purpose program that suits the problem under study and the purposes (design) of the experimenter.
2) If a special purpose program does not exist, a general purpose language is used to program the model.

EXECUTING (RUNNING) THE SIMULATION MODEL

During this phase, the experimenter:

1. Validates and tests the simulation model.
2. Runs the model on the computer, i.e., executes the simulation.

Validating and testing the model:

This step involves answering the following questions according to the following sequence:

A. Has the simulation model been correctly flowcharted and programmed? If not, then programming errors must be corrected first.

B. Does the simulation model represent the real situation? This question is answered by running the model on the computer in what is known as the *pilot run*. The experimenter then makes further observations and measurements of the system. As new data is obtained, the model is checked against it to determine the correspondence between the model and the real system. If there is no correspondence, then the model must be revised (3, 10).

This pilot run also helps the experimenter to estimate the model processing time so as to calculate costs and reserve computer time.

Running the model:

Once the model has been validated and tested, the simulation is then ready to be processed. The experimenter then executes the simulation according to the controls he has designed and gathers the data that is generated. Care must be taken to observe the model during the runs in order to discover any new phenomena that might show up and which the simulator has not considered during his analysis or, may be, never knew it existed.

COLLECTING AND ANALYZING DATA

Enough data must be gathered from the simulation trials and runs to enable the simulator to verify the validity of the model, and to compare the effects of the proposed changes with the original model. This data is usually in the shape of records of parameter values, starting conditions, time paths of input and generated variables, ... etc. For this large amount of data to yield information to management, it goes through the following steps:

1. Data collection:

   During this step, the simulator gathers the data generated from the simulation trials or runs according to the design of the experiment.

2. Data reduction:

   This step is defined as *the process whereby a mass of ungrouped, unorganized output data is transformed into a body of regrouped, reorganized data in a format that facilitates its analysis and from which data analysis can follow directly* (5, 92-94). It is an intermidiate process between the generation and collection of data, i.e., between the all-computer simulation model output, and the analysis of this output.

   During this step, the experimenter:

   A. Groups the generated data into tables that illustrates the values of the different parameters and the corresponding values of the other related variables at different points in time, or different system states.

B. Draws graphs that portrays the relationships between different parameters and variables.

C. Extracts the correlation between different variables and parameters

Or any other format to facilitate the interpretation of the output data (see Appendix II).

3. Data analysis:

The experimenter studies the relationships that by now should have been revealed between the different variables or components of the system and deduces the effects of the proposed changes or solutions on the original system.

At this point, and while making the final analysis; the experimenter might find out that there are points that are still ambiguous or that the response of the model is not clearly defined. The analysis will also point out the areas that still needs clarification or more stress, the kind of input data that should be added, the output data that is missing, and the experiments that should be carried to render the simulation productive. Thus, he proposes and designs new experiments and processes the model again.

The whole simulation procedure is illustrated sequentially by the flow diagram in figure (14).

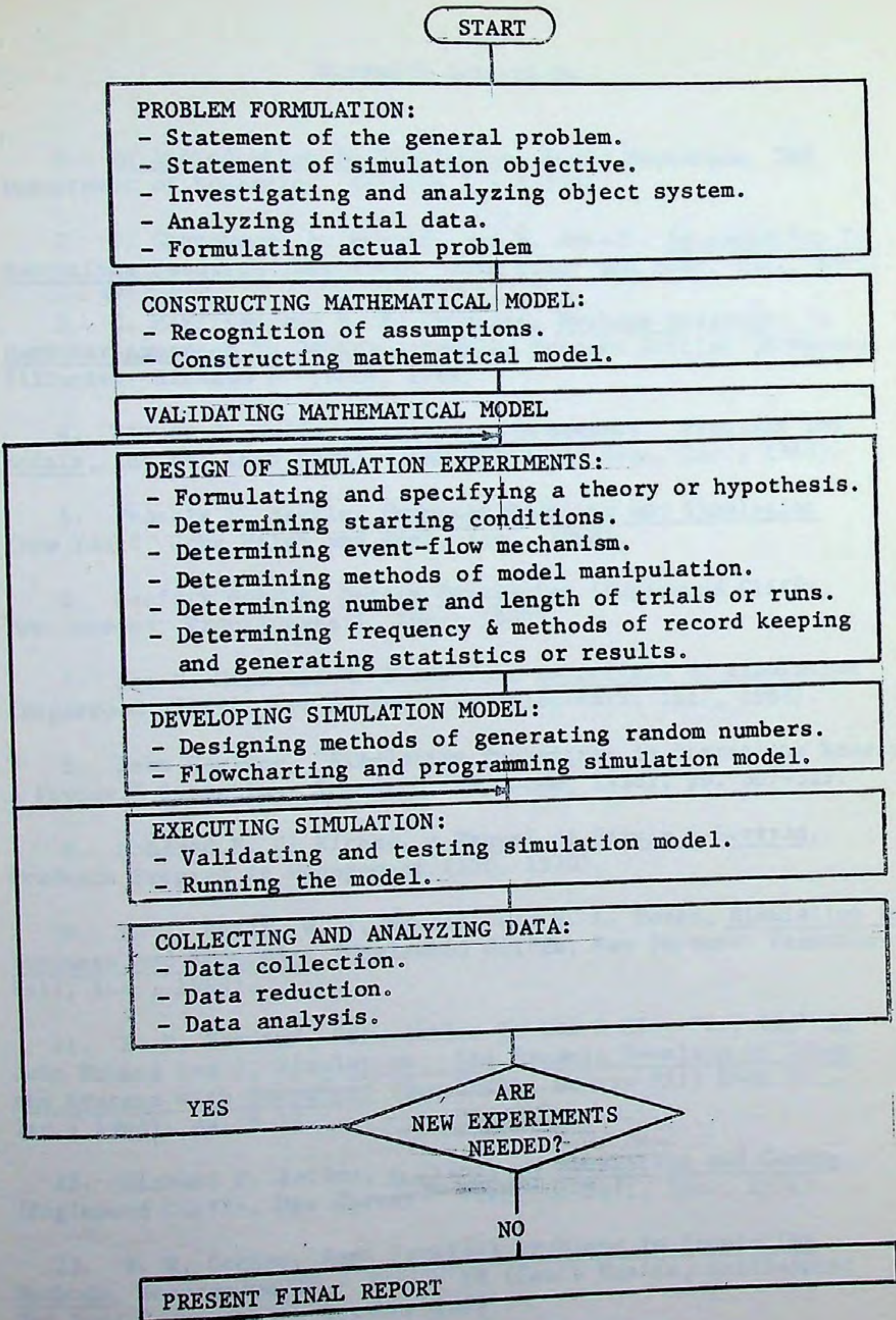


Figure (14): Flow Diagram of the whole Simulation Process

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CHAPTER III

CONCLUSION

Simulation is a very useful and effective tool that provides managers with the following advantages:

1. Decisions concerning systems that are still in a conceptual stage can be easily made without having to build the system to obtain answers. A model that forecasts system failure is less expensive than a system that is built and fails.
2. System performance can be simulated and observed under all conceivable conditions even those that cannot be controlled in the natural environment. Field testing for systems can be reduced or eliminated.
3. Models are extremely flexible and are adaptable to almost any reasonable application. There are no real-world constraints on the inputs to the model, thus, real-world as well as hypothetical situations can be studied and the best conditions or solutions can be predicted in advance.
4. The experimenter has complete control over the model and its parameters. In stochastic situations, chance elements associated with a system can be simulated by Monte Carlo technique and the system outcome(s) in such probabilistic context can be predicted (2, 10-11).

All-computer simulation differs from mathematical analysis in the way it treats problems as:

1. Analysis is usually a single attack methodology with models that represent either all possible instances of object system behavior or a single representative case. Simulation is a case-by-case technique that produces a specific number of individual representations of an object system. For this reason, simulation rather than analysis became the accepted method for studying object system dynamics.
2. Analysis produces conclusions directly from the model. Simulation assists in the solution rather than actually solves the problem. Simulation models - in most cases - have no capacity to optimize; they just illustrate what might happen if a system is set up to operate in a certain way with whatever values are chosen for the decision variables (3, 14), and then generalizes from repeated applications of the model.
3. Simulation may use analytical models, but not vice versa. One form of simulation is repeated application of an analytical model with different inputs for each application (4, 109-110).

Although simulation is a very handy technique and managers are almost usually tempted to use it; still, it is an expensive and relatively lengthy process. Whenever a problem is encountered, a manager must decide whether the problem is worth simulating or not?

Successful applications of simulation results from the effective balancing of the following factors:

1. Availability of data which will be used to supply the parameter values of the simulation. This will influence the objectives, expected results, and the type of model to be used.
2. Cost of data collection should be commensurate with expected results.
3. Cost of developing and manipulating the model must be weighted against the degree of detail and sophistication required.
4. Time constraints influence the choice of technique and consequently expected results. If there is not enough time to construct and simulate a sophisticated enough model, would you choose another technique if it is possible? Or, would you construct a model that is not so sophisticated but will yield approximate results?
5. Resulting from the previous step, the confidence level requirements affect the cost and the type of both the model and the data.
6. Again associated with the previous two factors, the degree of validity of the simulation data will influence the planned objectives and the credibility of results.
7. The value of the results which at the least consists of:
   A. A mere methodical approach to the definition of the real-life system.
   B. The model becomes a mean for establishing communication and providing visibility of the system.
   C. The numerical results provide insight into the behavior of the real system.

That is to day, the mere process of constructing a simulation of a system can in itself be a beneficial learning experience of the system that cannot be gained in any other way (2, 15-16).

8. Finally and above all, the objective of the simulation is the base from which all the other factors derive their relative weights.

   Do the objectives justify the simulation?

   Is the question that must be carefully answered and weighed (1, 43-56).

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CHAPTER IV

ILLUSTRATION

THE PROBLEM

1. A certain company deals in a principle item X. The company records show that the demand rate and lead times for replenishing stocks for the past two years were:

Weekly Demand	Frequency
11	10%
12	40%
13	25%
14	15%
15	7%
16	3%
	100%

Table (1): Demand rate Distribution

Lead Time in Weeks	Frequency
2	15%
3	45%
4	20%
5	15%
6	5%
	100%

Table (2): Lead Time Distribution

2. The company has reasons to believe that these rates will hold for the coming year.
3. Following are the costs and other data associated with this

item:

Unit holding cost (UHC)	20%
Cost per unit (CPU)	L.E. 380
COST PER ORDER PROCESSED (CPO)	L.E. 20
Demand per year (DPY)	680 units
Stock-out cost (SOC)	L.E. 10/unit

4. The company management wants to find out:

   A. The optimum quantity Q.

   B. The optimum reorder point R.

   That would minimize total annual costs (TAC).

5. Management decided to call an outside expert to solve this problem for them.

THE SOLUTION

PHASE I: PROBLEM FORMULATION

1. The problem as stated by management is:

   A. How many units of item X should the company order at one time?

   B. At what point as the stock level drops should the company place an order?

   In order to minimize total annual costs.

2. The simulation objective here is:

   To try and predict the consequences of changes in purchasing policy, compare these policies, and choose the optimum one without taking the risk or the time of actually making the change in real life.

3. System investigation and analysis:

We are not interested here in the operations of the whole company, but only with that part of the company that deals with purchasing, keeping, and selling. This subsystem of the company is to be our *object system*.

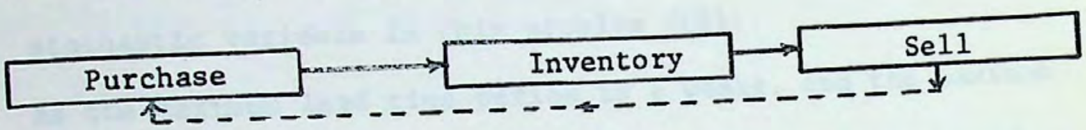

Figure (15): The object System

Analysing this object system, we can see that:

1) The company purchases annually but in installments, a certain number of units of item X that is equal to the anticipated demand per year (DPY = 680 units).

2) As the number of items purchased/year is fixed, and the price/item is also fixed, then the cost of goods purchased is also fixed.

3) Total annual cost of this item is given by:

   TAC = Cost of goods purchased
   + Total annual inventory cost

4) Thus, in the right-hand part of the TAC equation, the only variable is the annual inventory cost, and for this reason it is to be the subject of the analysis.

5) Although total demand per year is known, yet, demand per week is subject to random fluctuations. It fluctuates between 11 & 16 units/week according to the given frequences which we will consider as their probability of occurrence. Thus,

the weekly demand is a stochastic variable (WD).

6) Also, the lead time for delivery of purchased goods is subject to random fluctuations between 2 & 6 weeks according to another given set of frequences which we will also consider as its probability of occurrence. That is, lead time is another stochastic variable in this problem (LT).

7) As the maximum lead time period is 6 weeks, and the maximum demand per week is for 16 units, then, the *maximum* demand during lead time periods (DDLT) that the company can expect to occur is:

   6 weeks x 16 units/week = 96 unit/lead time

   That is to say that if the company does not wish to have any shortages, it should place an order the moment its inventory level reaches 96 units.

8) As the minimum lead time period is 2 weeks, and the minimum demand per week is for 11 units, then the *minimum* DDLT that the company should expect is for:

   2 weeks x 11 units/week = 22 units/lead time

   That is to say that the minimum reorder point could not in any case be less than 22 and even then, the company will always have stock-outs except in those cases when DDLT is at its minimum.

9) *Average* or *expected* DDLT is equal to:

   expected demand x expected lead time

   Expected demand is given by:

   11 x 0.10 = 1.10

12 x 0.40 = 4.80

13 x 0.25 = 3.25

14 x 0.15 = 2.10

15 x 0.07 = 1.05

16 x 0.03 = 0.48

Total 12.78 units

Expected LT is given by:

2 x 0.15 = 0.30

3 x 0.45 = 1.35

4 x 0.20 = 0.80

5 x 0.15 = 0.75

6 x 0.05 = 0.30

Total 3.50 weeks

Thus,

Expected DDLT = 12.78 x 3.5 = 44.73 units/LT period.

That is to say that the average number of units demanded per lead time, or the central tendency of demand during lead time periods is 45 units/LT period.

10) If the company does not wish to permit any stock-outs to occur, then it should place an order for a certain quantity Q every time the stock level reaches 96 units, i.e., whenever R = 96 units. That would mean that the inventory level, and consequently average inventory would be high. Thus, inventory holding costs and consequently TAC would increase.

11) For the company to decrease average inventory, it can do one

or both of the following:

a) decrease the ordering quantity Q.

b) decrease the reorder point R.

Figure (16) illustrates the consequences of decreasing Q and/or R in a simplified case where both demand and lead time are constant.

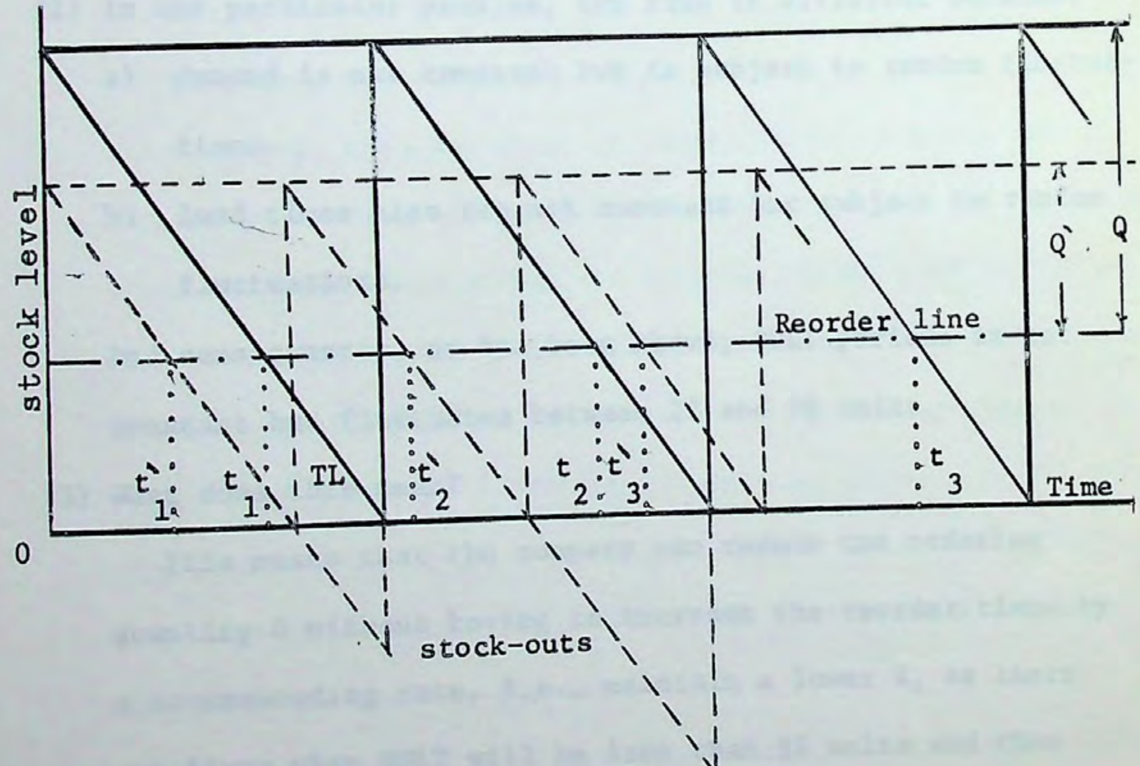

Figure (16): The Inventory Pattern when Q and/or R are Decreased

When the ordering quantity was Q and R was 96 units, the company placed an order at t_1, t_2, t_3, ... etc. When Q was decreased to Q\` without changing R, i.e., without permitting stock-outs, the company had to order more frequently at $\grave{t}_1$, $\grave{t}_2$, $\grave{t}_3$, ... etc. where $t_1 > \grave{t}_2$, and $t_2 > \grave{t}_2$ and so on.

As each order processed costs the company L.E. 20, then annual ordering costs increased. If the company maintains the same reorder points, i.e., decreased R as well, then a time lag would always occur between the exhaustion of inventory and the replenishing time. During this time lag TL, all demand will be backordered, i.e., stock-outs occur.

12) In our particular problem, the case is different because:

a) demand is not constant but is subject to random fluctuations.

b) lead times also are not constant but subject to random fluctuations.

And consequently, as has been shown, DDLT periods is not constant but fluctuates between 22 and 96 units.

13) What does this mean?

This means that the company can reduce the ordering quantity Q without having to increase the reorder times by a corresponding rate, i.e., maintain a lower R, as there are times when DDLT will be less than 96 units and thus stock-outs do not have to occur all the times. That is to say, the company can reduce its inventory holding costs by reducing both Q & R on the condition that it will permit stock-outs to occur from time to time.

14) Stock-outs costs the company L.E. 10/unit. The question now becomes:

Could the company strike a balance between:

a) stock-out costs, and

b) inventory holding costs.

in such a way that the result would be a substantial decrease in total annual costs?

To answer this question, we will try to get an idea about how many times stock-outs are expected to occur under different values of R, and how much they will cost.

Supposing the company decided to decrease R from 96 units to 95 units, then the company should expect a stock-out of one unit to occur during those periods when DDLT is 96 units. Now, *how many times would the company expect that to happen?* The probability that DDLT will be 96 units = the joint probability that LT will be 6 weeks *and* that a demand rate of 16 units will occur on 6 *consecutive* weeks. This is equal to: The probability of LT of 6 weeks *and* the joint probability of a demand of 16 units *and* a demand of 16 units *and* a demand of 16 units ..... up to *six* times.

$= (0.05) \times (0.03) \times (0.03) \times (0.03) \times (0.03) \times (0.03) \times (0.03)$

$= (0.05) \times (0.03)^{6} = 36 \times 10^{-12}$

That means that DDLT will be for 96 units 36 times every 10^{12} lead times, or that, the company will have 36 units short every 10^{12} lead times.

The *expected value* of stock-outs/LT period, or the *expected* stock-out/LT period in units short which we will label ESO is given:

$$(96 - 95) \times 36 \times 10^{-12} = 36 \times 10^{-12}$$

The *annual* stock-out cost is given by:

L.E. 10 x The expected value of stock-out/LT period when R is 95 units (ESO) x Number of orders/year

$$= \text{L.E. } 10 \times 36 \times 680/Q = 360 \times 10^{-12} \times 680/Q$$

which will turn out to be a very small quantity that can be neglected.

15) Again, suppose the company decided to decrease R to 94 units instead of 96 units, then there will be times when the company will undergo a shortage of 2 units when DDLT is for 96 units, and times when the company will have a shortage of 1 unit when DDLT is for 95 units.

We have already calculated the probability of DDLT to be for 96 units in the previous step, thus, we can say that the company will have a shortage of 2 units 36 times every 10^{-12} lead times, i.e., the probability of a stock-out of 2 units is 36×10^{-12}.

For the shortage to be 1 unit only, then DDLT must be for 95 units. That is, in the 6 weeks of lead time, 5 weeks will have a demand rate of 16 units, and any one week of the 6 weeks will have a demand rate of 15 units and which can happen in 6 different ways as follows:

Ways in which demand for 95 can occur	Probability that this event can happen
16 - 16 - 16 - 16 - 16 - 15	0.03 x 0.03 x 0.03 x 0.03 x 0.03 x 0.07

Or,

16 - 16 - 16 - 16 - 15 - 16 $\quad 0.03 \times 0.03 \times 0.03 \times 0.03 \times 0.07 \times 0.03$

Or,

16 - 16 - 16 - 15 - 16 - 16 $\quad 0.03 \times 0.03 \times 0.03 \times 0.07 \times 0.03 \times 0.03$

And so on in 6 different ways.

The combined probability of these events is the *sum* of their individual probabilities of occurrence which is equal to

$$(0.03)^5 \times (0.07) \times 6$$

The joint probability for DDLT to be for 95 units is given by: The probability that LT will be 6 weeks x The combined probability that the demand rates will follow any one of the above mentioned sequences

$$= (0.05) \times (0.03)^5 \times (0.07) \times 6 = 510 \times 10^{-12}$$

The *expected* value of stock-out/LT period (ESO) when R = 94 is calculated as follows:

$$(96 - 94) \times 36 \times 10^{-12} = 72 \times 10^{-12}$$

$$+ (95 - 94) \times 510 \times 10^{-13} = 510 \times 10^{-12}$$

$$= 582 \times 10^{-12}$$

The annual stock-out cost can be calculated by the same previous way and it is evident that it will be a very small amount too.

16) We can go on like this for the whole different combinations of lead time and demand rates, but as has already been shown, it will take a lot of calculations and time. Still, we need these ESO's to calculate the stock-out costs at different

reorder points, thus, we will program the computer to calculate it in a table form showing the different reorder points and the ESO's associated with it according to the following general formula:

$$ESO(JR) = \sum_{DDLT=96}^{JR+1} (DDLT - JR) \times P(DDLT)$$

where:

JR is the different values that R can assume from 35 units to 96 units.

ESO(JR) is the expected value of stock-out at R = JR

JR + 1 is the value of JR that exceeds DDLT by one unit because if R = DDLT, no stock-outs will occur.

P(DDLT) is the probability that demand during lead time will be equal to the amount under consideration (from JR = 96 units to JR + 1 units consecutively), and which has already been calculated.

17) The above calculations in paragraph (15) proves that it is possible to decrease R and consequently the inventory level and holding costs causing a very slight increase in stock-out costs,

18) We can then state that our problem is to balance stock-out costs against:

a) holding costs, and

b) ordering costs

In order to find the policy that minimizes TAC.

19) Figure (17) illustrates a sample of the company's inventory

pattern where:

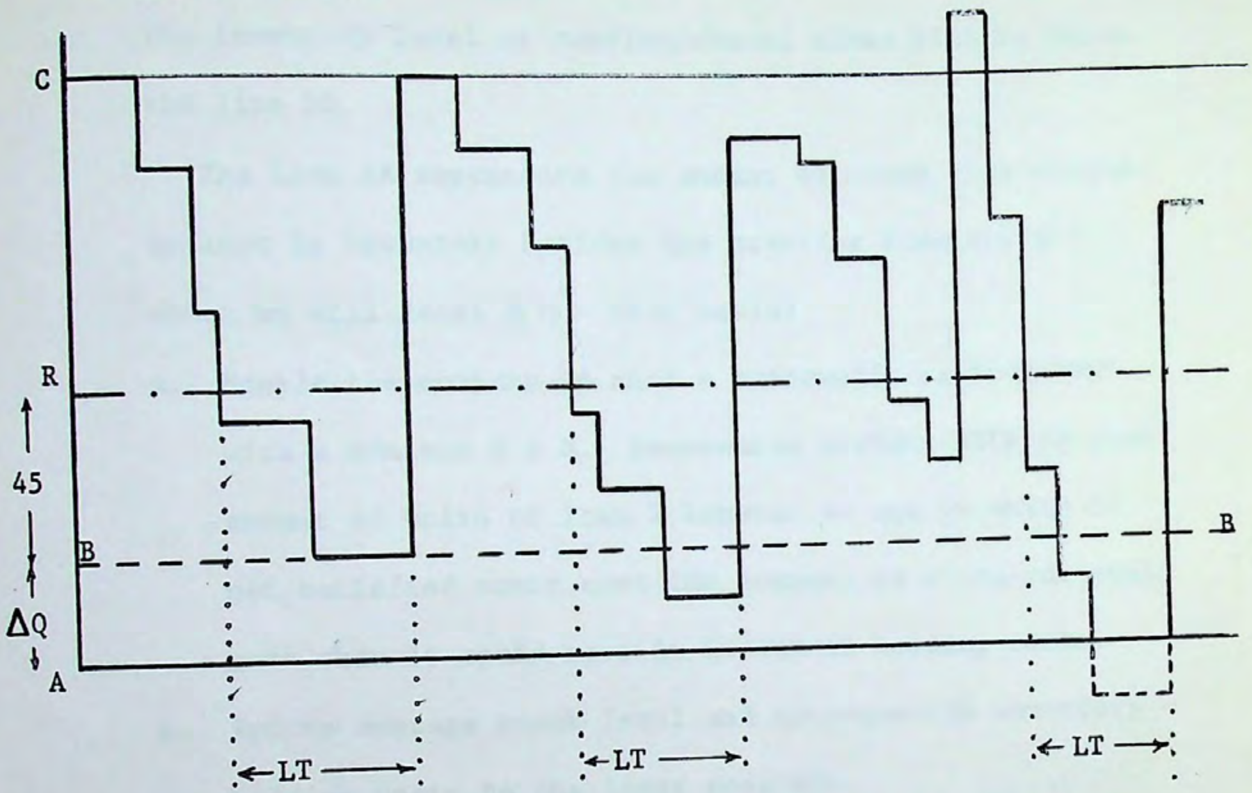

Figure (17): A Sample of the Company's Inventory Pattern

AR is the reorder point that will cause the amount and consequently the cost of shortages to be less than the decrease in the inventory holding costs, i.e., the optimum R. As the *average* DDLT is 45 units, then most probably R $\geqslant$ 45 or else stock-outs will occur most of the times.

The point B is 45 units below R, hence the line BB represents the inventory level at the times of replenishments in most of the cases, i.e., the central tendency of inventory level at replenishment time.

But there are times when DDLT $<$ 45 units and hence the inventory level at replenishment times will be above the line BB, and times when DDLT $>$ 45 units and consequently the inventory level at repelenishment times will be below the line BB.

The line AA represents the amount of items that should be kept in inventory besides the ordering quantity Q - which we will label Δ Q - that would:

A. Enable the company to meet a *reasonable* maximum DDLT with a minimum Q & R. Reasonable maximum DDLT is that amount of units of item X between 45 and 96 which if not satisfied would cost the company in stock-out costs more than it would save in inventory holding costs.

B. Reduce average stock level and consequently inventory holding costs to the least possible.

What we want to find out are the values of Q & R corresponding to the line AA.

Notice:

In this problem, it was more productive to the simulator to analyze the object system and the historical (initial) data concurrently. This is usually the case except when the object system is large and complex that it needs to be broken down first into sybsystems to uncover and understand the interactions that goes on within the subsystems. Then, all the subsystems are tied together according to their functional relationships and the mathematical model is built. After that, the experimenter studies the initial data to gather statistics that enables him to compare the real system with the results he is to obtain from the mathematical model.

This analysis also proved to the simulator that the initial data available is not enough and that, in order to solve the model, he has to calculate:

A. The probability of occurrence of each different DDLT.

B. The expected stock-out values at different reorder points.

Due to the fact that the experimenter has found out during the analysis that it will take long and tedious calculations to calculate the different values that these two variables can assume, he will make provisions while designing the simulation model, to calculate it at the beginning of the simulation trials.

4. Formulating the actual problem:

After this detailed analysis, the actual problem has been revealed to be:

The balancing of stock-out costs against:

a) holding costs; and,

b) ordering costs,

to find the policy that minimizes total annual costs.

PHASE II: CONSTRUCTING THE MATHEMATICAL MODEL

1. Assumptions:

We are to proceed in the solution of this problem on the following assumptions:

A. The frequency distributions that were given are going to hold for the next year to which we are planning.

B. There is no seasonal pattern for demand or lead time, both are random, and the given frequences will hold all year long.

C. Stock-out cost is linear, i.e., the cost of a stock-out of two units is twice that of one unit, and the cost of a stock-out of three units is three times that of one unit, and so on.

D. All units of item X demanded but not available are back ordered but are filled immediately when a new replenishment arrives.

2. Constructing the mathematical model:

As shown in the analysis of the problem, the only variable in the total costs equation is the total annual inventory cost, and it is to be the purpose of our analysis.

Total annual inventory cost which we will label TAIC is the sum of the following costs:

A. Annual holding costs (AHC).

B. Annual ordering costs (AOC).

C. Annual stock-out costs (ASOC).

Annual holding cost:

From the analysis of the problem in figure (17), we have seen that there is a certain reorder point R corresponding to a certain Q which if maintained would minimize annual inventory costs. Placing orders for quantities Q whenever inventory level reaches this R does not mean that there will not be stock-outs, but that the decrease in inventory holding cost will be greater than the costs due to stock-outs.

It has also been shown that this point should in any case exceed the average (expected) DDLT, i.e., 45 units and is naturally less than maximum DDLT, i.e., 96 units.

Assuming that this point is $>$ 45 units by the amount ΔQ.

Then, as:

Inventory holding cost = Average inventory x UHC

And, Average inventory = $Q/2 + \Delta Q$

From figure (17).

$\Delta Q = R - 35$

Then

Average inventory = $Q/2 + R - 45$

In cases when demand during a lead time period $<$45, then, following this period, the ordering quantity Q will increase the stock level than the average level by a quantity equal to 45 - this specific DDLT. Also, in cases when a stock-out occurs, and due to the fact that unsatisfied demand is satisfied the moment a new quantity is received, then following a lead time when a stock-out occurs, the stock level will be less than the average level by a quantity equal to the amount of that stock-out (see figure 17).

These two differences in the average stock level tend to cancel each other and consequently, average inventory level can be considered to be: $Q/2 + R - 45$.

Thus,

AHC = (Q/2 + R - 45) x L.E. 380 x 0.20 .... (1)

Annual ordering cost:

This cost is equal to:

The number of orders throughout the year x cost/order processed

Thus,

AOC = 680/Q x L.E. 20 .... (2)

Annual stock-out cost:

As was shown before, this cost is given by:

The cost of one stock-out x The *expected* value of stock-out per lead time period when R is at a certain value x Number of orders per year.

Thus,

ASOC = L.E. 10 x ESO x 680/Q .... (3)

Adding (1), (2), and (3), we can calculate TAIC as follows:

TAIC = (Q/2) + R - 45)(380)(0.20)
+ (680/Q)(20)
+ (10)(ESO)(680/Q) .... (4)

This equation is the mathematical model that is to be manipulated.

PHASE III: DESIGNING SIMULATION EXPERIMENTS

1. Formulating a theory or hypothesis:

From the previous analysis, we have seen that lowering R by two units would cause a stock-out per lead time period of 582×10^{-12} units. That is approximately 6 times every

(10 billion) times which is quite insignificant. At the same time, this decrease would cause a substantial decrease in inventory holding costs as the holding costs for two units is given by:

2 units x L.E. 380 x 20% = L.E. 142

Thus, we are to proceed in this problem on the following hypothesis:

In this specific situation, an inventory policy that makes a stock-out impossible, i.e., where R = 96 units, will cost more in inventory holding costs than it will save in stock-out costs.

2. Determing starting conditions:

   1) We know from analysis that:

      R can have any value from 1 to 96 units; and

      Q can have any value from 1 to 680 units (DPY).

      Thus, if we decide to let the simulation process begin operating empty, then we will need 96 trials for the different values of R each tried 680 times for the different values of Q.

      96 changes in R x 680 changes in Q = 65,280 trials which would consume a relatively long and expensive computer time.

   2) We also know that the *expected* DDLT is 45 units, thus, it is only logical that we should start our system at a minimum reorder point of 45 units or else there will be shortages most of the time and consequently TAIC would increase.

3) From figure (17) we can see that:

Stock level = Q + R - 45

As we have agreed that R as a starting condition = 45, Then, after the first time that a DDLT for 45 units occurs, the stock level will equal Q. As the minimum R is 45 units, then, Q should have a minimum value of 45 units and even then, the company will have to place an order the same day it receives a replenishment. Thus, we will assign Q the value of 45 as a starting condition and which is the minimum value it can have.

4) This way we have saved trying 45 values for R and 45 values for Q, i.e.,

$45 \times 45 = 2,025$ trials

Besides that, we will work out a way in the program so as to end each trial the moment TAIC starts to increase again, and this will save more computer time.

3. Determining event-flow mechanism:

1) The event we are interested in is the number of stock-outs that would occur under certain conditions. These conditions are the different values that the variables R & Q can assume.

2) As the relationship between Q & R is not clearly specified due to the random fluctuations in both; we cannot determine *a priori* the occurrence times of the events (stock-outs), hence, we will have to use the fixed increment method.

3) To determine the size of the increment; we must consider the following points:

   a) We want to make the size of the increment sufficiently small so that the discrete approximation to the continuous system is acceptable.

   b) We cannot state beforehand the exact magnitude of the increase that would make a significant effect on the amount of stock-outs.

   c) A change of one unit in either Q or R would change annual inventory holding cost by a significant amount.

   L.E. 380 x 20% = L.E. 76

   Thus, we will change both R & Q by increments of one unit each time and calculate the effects on TAIC.

4. Determining methods of model manipulation:

   1) The two variables of interest in our model are Q & R.
   2) We will calculate beforehand the ESO at different values of R.
   3) We want to know the effect on the number of stock-outs at different values of Q & R, i.e., the combined effect of changes in both R & Q.
   4) Thus, we will set R at a certain value (45 units), then assign to Q all its possible values (from 45 to 680 units) and calculate TAIC in each case.
   5) As we know that the TAIC curve is a usually U curve, then there is a certain Q that will render TAIC minimum, i.e., before and after which TAIC will increase. Thus, to shorten

the time of the trials, we will compare TAIC each time we change Q with the previous TAIC; as long as TAIC decreases, we will keep on increasing Q. The moment TAIC starts to increase again, we stop increasing Q and choose the previous TAIC as the minimum TAIC and record the corresponding R & Q.

To illustrate this, let us consider that we are experimenting at $R = R_j$

Let us assume that at the *ith* trial where $Q = Q_i$, TAIC was found to be equal to $TAIC_i$

Then we increased Q by one unit to become Q_{i+1}

and TAIC was found to be equal to $TAIC_{i+1}$

We then compare $TAIC_i$ with $TAIC_{i+1}$

If $TAIC_{i+1} < TAIC_i$ we let $TAIC_i = TAIC_{i+1}$

And continue our trials by increasing Q by one unit.

If $TAIC_{i+1} > TAIC_i$

Then $TAIC_i$ is the bottommost point of this TAIC curve, and we record Q_i & R_j.

6) We will then increase R by one unit, find the minimum TAIC by the same method and record its value and the values of the corresponding R & Q.

7) These values of TAIC will represent a family of U curves each one of which is in a different plane corresponding to its values of Q's and R's. The collection of these curves forms a bowel in a three dimensional space (see figure 18). Our objective is to find the bottommost point of this bowel and its Q & R coordinates as they will represent the optimum

Q & R values corresponding to the minimum TAIC.

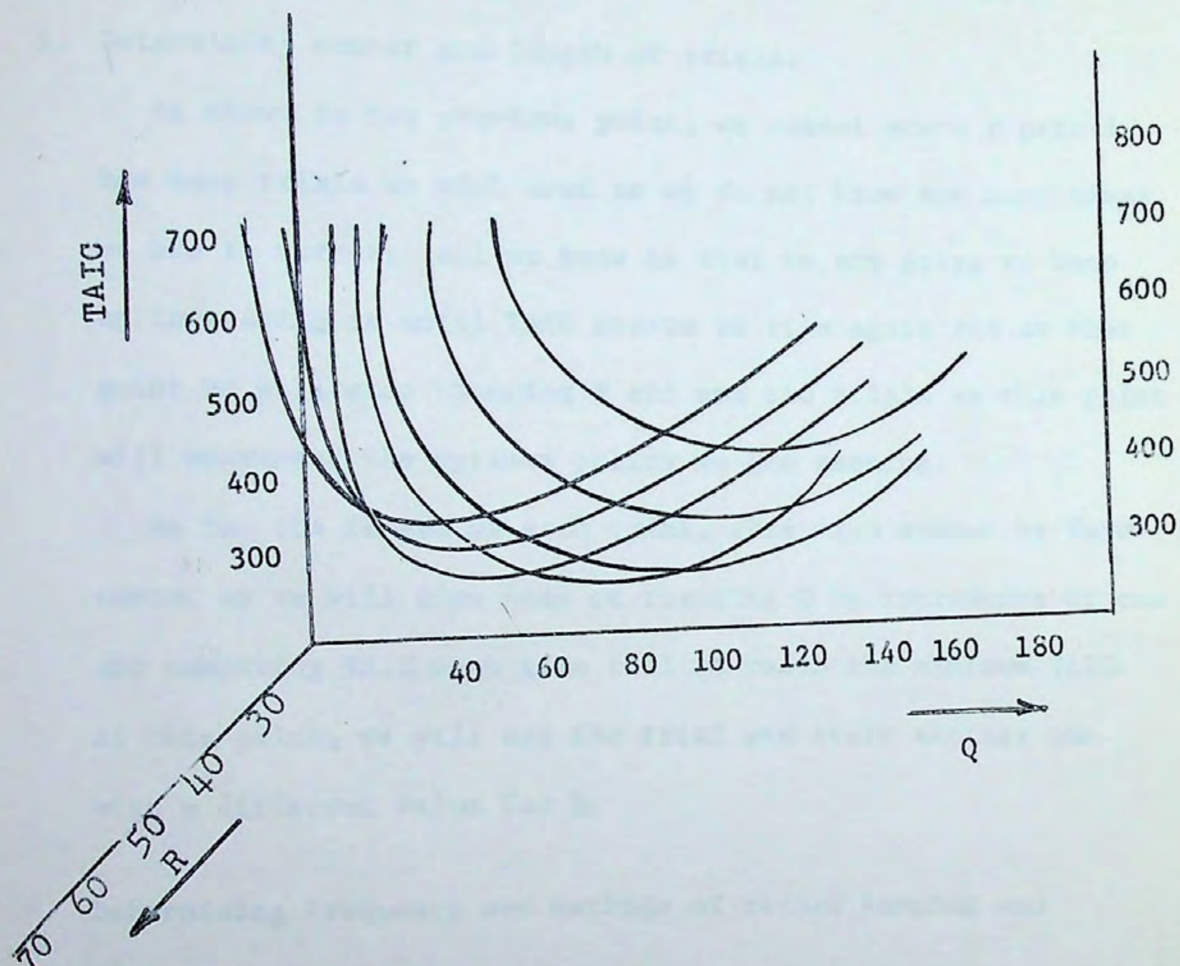

Figure (18): The Relationship between TAIC, Q, and R.

8) As we want to find the minimum TAIC at all the different combinations of R & Q; we will compare the value of TAIC we calculate at a certain R with the previous value TAIC at the preceeding R. As long as TAIC decreases, we will increase R by one unit and calculate the corresponding TAIC. If TAIC increases, then the previous TAIC is the minimum TAIC and we stop increasing R, record TAIC, R,

and Q. Their values will represent the optimum values for the combination of R & Q.

5. Determining number and length of trials:

As shown in the previous point, we cannot state a priori how many trials we will need as we do not know how many times we are to vary R. All we know is that we are going to keep on increasing it until TAIC starts to rise again and at that point we will stop changing R and end the trials as this point will represent the optimum policy we are seeking.

As for the length of each trial, this also cannot be forecasted as we will also keep on changing Q by increments of one and comparing TAIC each time till we reach the minimum TAIC. At this point, we will end the trial and start another one with a different value for R.

6. Determining frequency and methods of record keeping and generating statistics:

We will need two types of outputs in this simulation:

1) Outputs that are necessary for the experiments. That is, outputs in the form of statistics that are to be used in the calculations during the simulation trials. There are:

   a) A table containing the amounts of all the possible demands during any lead time period and the probability that this demand will occur in the following form:

Possible Demand during Lead Time in Units (DDLT)	Probability of this Demand
22	...
23	...
..	...
..	...
..	...
Up to 96	...

These values will be used to calculate the ESO table explained below.

b) A table containing the different possible reorder points (from R = 45 to R = 96) and the *expected* value of stock-outs at each value of R in the following form:

The Reorder Point (R)	Expected Value of Stock-out/LT (ESO)
45	.....
36	.....
..	.....
..	.....
..	.....
Up to 96	.....

The statistics in this table are to be used to calculate the stock-out costs under different values of R during the simulation trials.

In fact, both these tables are only necessary for the simulation runs and thus, we can program the computer only to store them (by a dimension statement) and use them without having to write them in the output sheet. But as these statistics might be useful to management in later studies that they may wish to carry, and also in the final report if management wants to discuss the reasons behind choosing this policy, we will instruct the computer to write them in the output sheet.

3) The optimum TAIC with the values of the corresponding R & Q at the end of the whole simulation process.

PHASE IV: DEVELOPING COMPUTER MODEL

Design of process generator:

We are going to use a deck of cards containing 1,000 random numbers each punched on a separate card as our source and every time we need to use one, we will instruct the computer to read a card and use the number on it.

Program simulation model:

The computer language we are to use is a general purpose language FORTRAN. The programming methodology consists of the following seven steps:

1. Problem statement
2. Input data list
3. Procedure
4. Input design
5. Output design
6. Flow chart
7. Program

Before going ahead with the programming procedure, we will present a list of the symbols that are to be used throughout the program.

LT Lead time.

KDDLT Demand during lead time. The reason for the "K" is that DDLT is an integer quantity.

KDDLT(I) Demand during lead time; the (I) classifies the different amounts of possible demand where I = 22, 23, ...96.

KQ Ordering quantity Q; also an integer.

JR Reorder point R; also an integer.

ESO Expected value of stock-out/LT period.

ESO(JR) Expected value of stock-out at different values of R where R = 45, 36, 37, ...96.

P(I) Probability that DDLT will equal I where I = 22, 23, ... 96.

TAIC Total annual inventory cost.

TAICQ The value of TAIC as Q is increased.

TAICR The value of TAIC as R is increased.

TAICOP Minimum or optimum TAIC.

KQOP The optimum ordering quantity Q.

JROP The optimum reordering point R.

Programming Methodology:

1. Problem statement:

A. To calculate, store, and print out:

1) Probability of occurrence of different DDLT's from DDLT = 22 to DDLT = 96 units/LT period.

2) Expected value of stock-out at different values of R from R = 45 to R = 96 units.

B. Simulate the company's inventory pattern and calculate TAIC at different values of R & Q in order to find out TAICOP and its corresponding R & Q and print them.

2. Input data list:

   A. A deck of cards containing 1,000 random numbers each punched on a separate card.

   B. The other needed data will be generated by the computer in the first phase of the program.

3. Procedure:

   A. Calculate by a Monte Carlo method the different DDLT periods that can occur in 200 simulated lead times according to tables (3) & (4) of cumulative probabilities and random number indexes as follows:

Lead Time in Weeks	Probability of Occurrence	Cumulative Probability	Random Number Index
2	0.15	0.15	00-14
3	0.45	0.60	15-59
4	0.20	0.80	60-79
5	0.15	0.95	80-94
6	0.05	1.00	95-99

Table (3): Cumulative Probabilities and Random Number Index for Lead Time Periods

Weekly Demand	Probability of Occurrence	Cumulative Probability	Random Number Index
11	0.10	0.10	00-09
12	0.40	0.50	10-49
13	0.25	0.75	50-74
14	0.15	0.90	75-89
15	0.07	0.97	90-96
16	0.03	1.00	97-99

Table (4): Cumulative Probabilities and Random Number Index for Weekly Demand

1) Read a random number from a card.
2) Identify the lead time period associated with this number.
3) Read as many more random numbers as there are weeks in the lead time period identified in the previous step and identify the weekly demand associated with each number.
4) Add up these weekly demands to get DDLT in this identified lead time.
5) Read another random number, identify new LT and calculate the DDLT associated with it by the same procedure.
6) Keep on calculating lead time periods for 200 times and each time calculate the DDLT associated with it.

B. Calculate the frequency of occurrence of each amount of DDLT from 22 to 96 units in these 200 simulated lead times.

C. Divide each frequency by 200 to get the probability of occurrence of each DDLT.

D. Store these probabilities.

E. Print out these probabilities in a table form.

F. Calculate the ESO's at different values of R from R = 45 units to R = 96 units as follows:

1) Suppose that we want to calculate ESO when R = 45 units.

2) Calculate first the expected value of stock-out when DDLT = 96 units according to following formula:

   (96 - 45) x Probability that DDLT = 96 units.

3) Then calculate the expected value of stock-out when DDLT = 95 units according to the formula:

   (95 - 45) x Probability that DDLT = 95 units.

4) Again calculate the expected value of stock-out when DDLT = 94 units according to the formula:

   (94 - 45) x Probability that DDLT = 94 units.

5) Keep on calculating the expected values of stock-outs by the same method until DDLT = 46 units as no stock-out will occur when DDLT = 45 units.

6) Sum up all the values of expected values of stock-outs that has been calculated. Their sum will equal the expected value of stock-out when R = 45 units (ESO(45)).

7) Change R to 46 and find ESO(46) by the same method. Only this time, keep on calculating the expected values of stock-outs until DDLT = 47 units as again no shortages will occur if DDLT = 46 units. That is to say that we always keep on calculating the ESO's until DDLT = R + 1.

8) The general formula for the calculation of ESO's at different values of R is

$$ESO(JR) = \sum_{DDLT = 96}^{JR + 1} (DDLT - JR) * P(DDLT)$$

Where:

JR is the different values that R can assume from 45 units to 96 units.

ESO(JR) is the expected value of stock-out at R = JR.

JR + 1 is that value of R that exceeds DDLT by one unit.

P(DDLT) is the probability that DDLT will be equal to the amount under consideration (from 96 units to JR + 1 units consecutively) and which has already been calculated in the first part of the program.

G. Store these ESO values.

H. Print out these ESO's in a table form.

I. Simulate the company's inventory pattern and calculate TAIC at different values of R & Q as follows:

1) Set R = 45 units.

2) Set Q = 45 units and calculate TAIC according to the formula:

$$TAIC = ((45.0/2.0) + R - 45.0) * (380.0 * 0.2) + (680.0/45.0) * 20.0 + (10.0 * ESO(45) * (680.0/45.0))$$

3) Reset Q = 46 units and calculate TAIC(46) by the same formula.

4) Compare TAIC(45) with TAIC(46); if TAIC(45)   TAIC(46), put TAIC(45) = TAIC(46) and increase Q to 47 and do the same again.

5) Keep on increasing Q by one unit at a time until TAIC starts to increase; record the minimum TAIC(Q) and the values of the corresponding R & Q.

6) Set R = 46 units, go through the whole procedure again and record the minimum TAIC and the corresponding R & Q.

7) Compare this TAIC with the preceeding TAIC, as long as TAIC decreases, keep on increasing R by one unit at a time and calculate TAIC until it starts to increase; record the minimum TAIC and its corresponding R & Q.

8) These values of R & Q represents the optimum policy that the company should follow in order to achieve the calculated minimum TAIC.

J. Print out the optimum values of TAIC, Q, and R.

4. Input design:

See figure (19).

5. Output design:

See figure (20).

6. Flow chart:

See figure (21).

7. Program:

See figure (22).

PROGRAM : INVENTORY SIMULATION (IS) Page 1 of IS
PROGRAMMER : AHMED ZAKI Date 5 / 5 / 71

CARD No.	Column Number									NR																																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49
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Figure (19) : Input Design Sheet

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55 x                       INVENTORY SIMULATION

56 x                      BY:  AHMED S. ZAKI

60 x                           5/5/71


45 x        POSSIBLE DEMAND          10 x    PROBABILITY OF
44 x       DURING LEAD TIME          11 x     THIS DEMAND
49 x           IN UNITS
51 x             DDLT                20 x        P(I)

52 x              22                 21 x        .....
                  23                             .....
                  ..                             .....
                  ..                             .....
        up to     96                             .....
```

Then on a new page:

```
45 x          REORDER POINT          10 x   EXPECTED VALUE OF
70 x                                           STOCK-OUT/LT
51 x                  R              23 x          ESO

50 x                  45             20 x       ........
                      36                        ........
                      ..                        ........
                      ..                        ........
         up to        96                        ........
```

Then, on a new page:

```
55 x                      OPTIMUM POLICY

 5 x   THE OPTIMUM ORDERING QUANTITY =         xxx UNITS

 5 x   THE OPTIMUM REORDER POINT     =          xx UNITS

 5 x   MINIMUM ANNUAL INVENTORY COST = L.E. xxxxxx.xx
```

Figure (20): Output Design Sheet

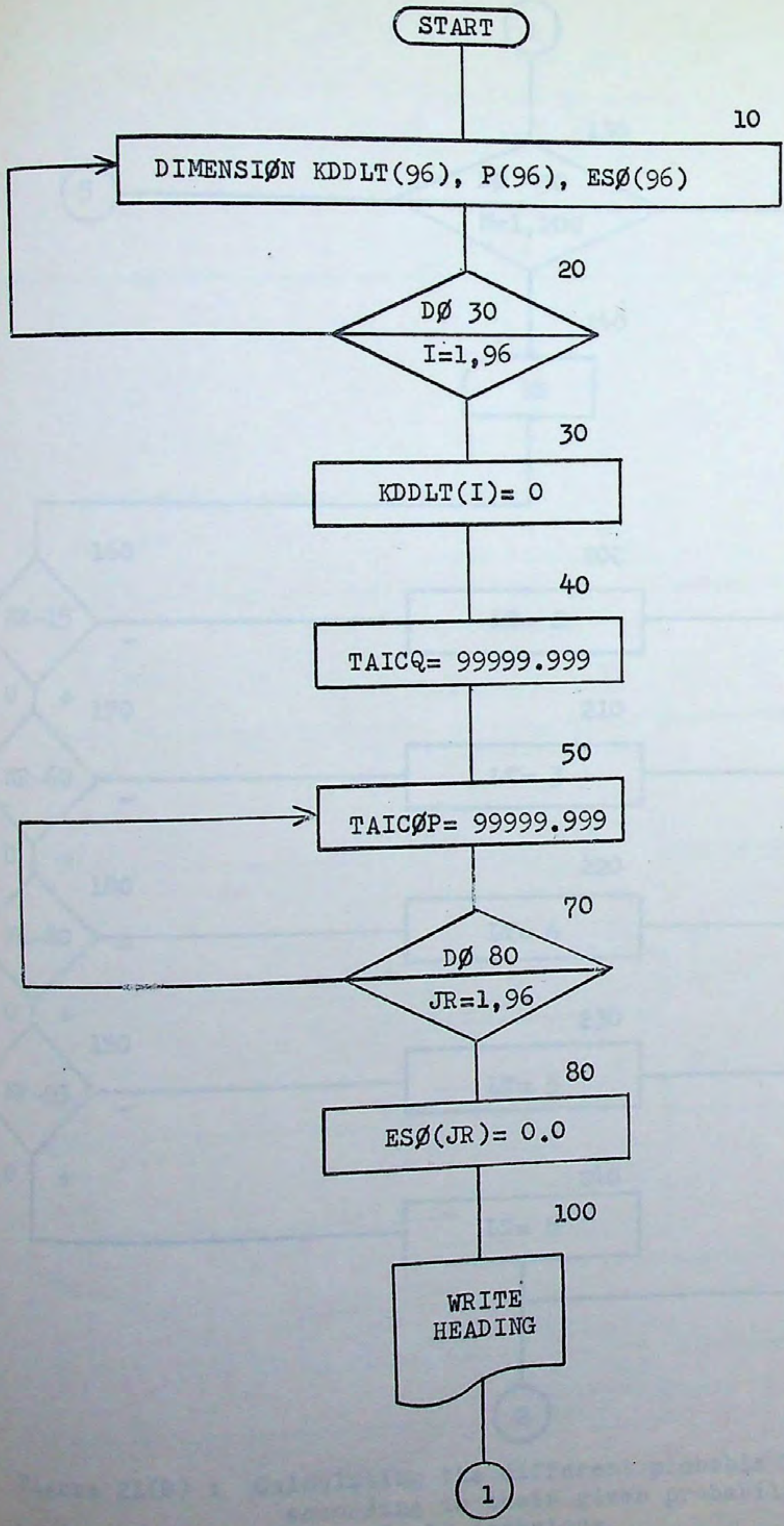

Figure 21(A) : Initializing The Program.

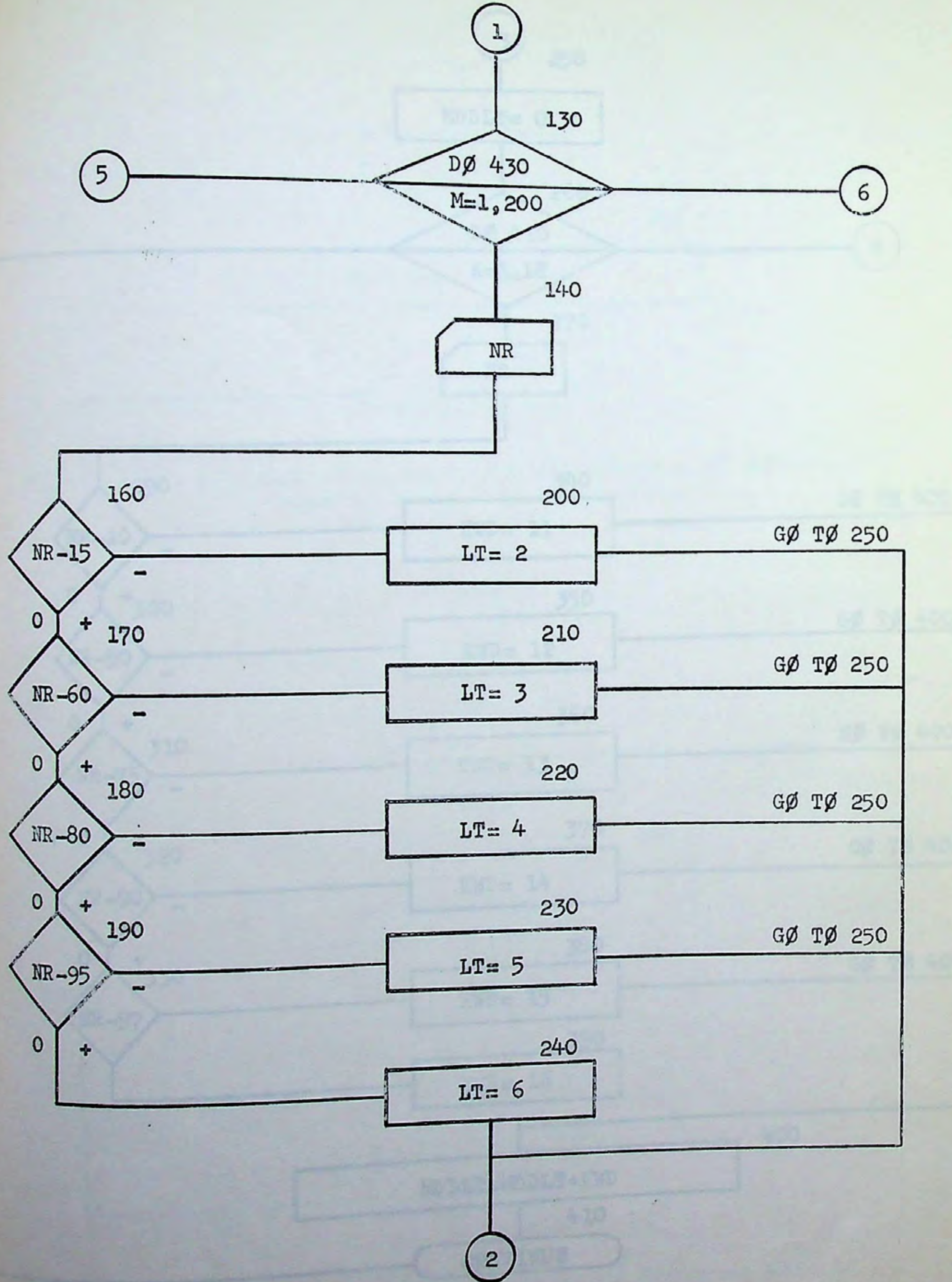

Figure 21(B) : Calculating the different probable lead time periods according to their given probabilities via a Monte Carlo technique.

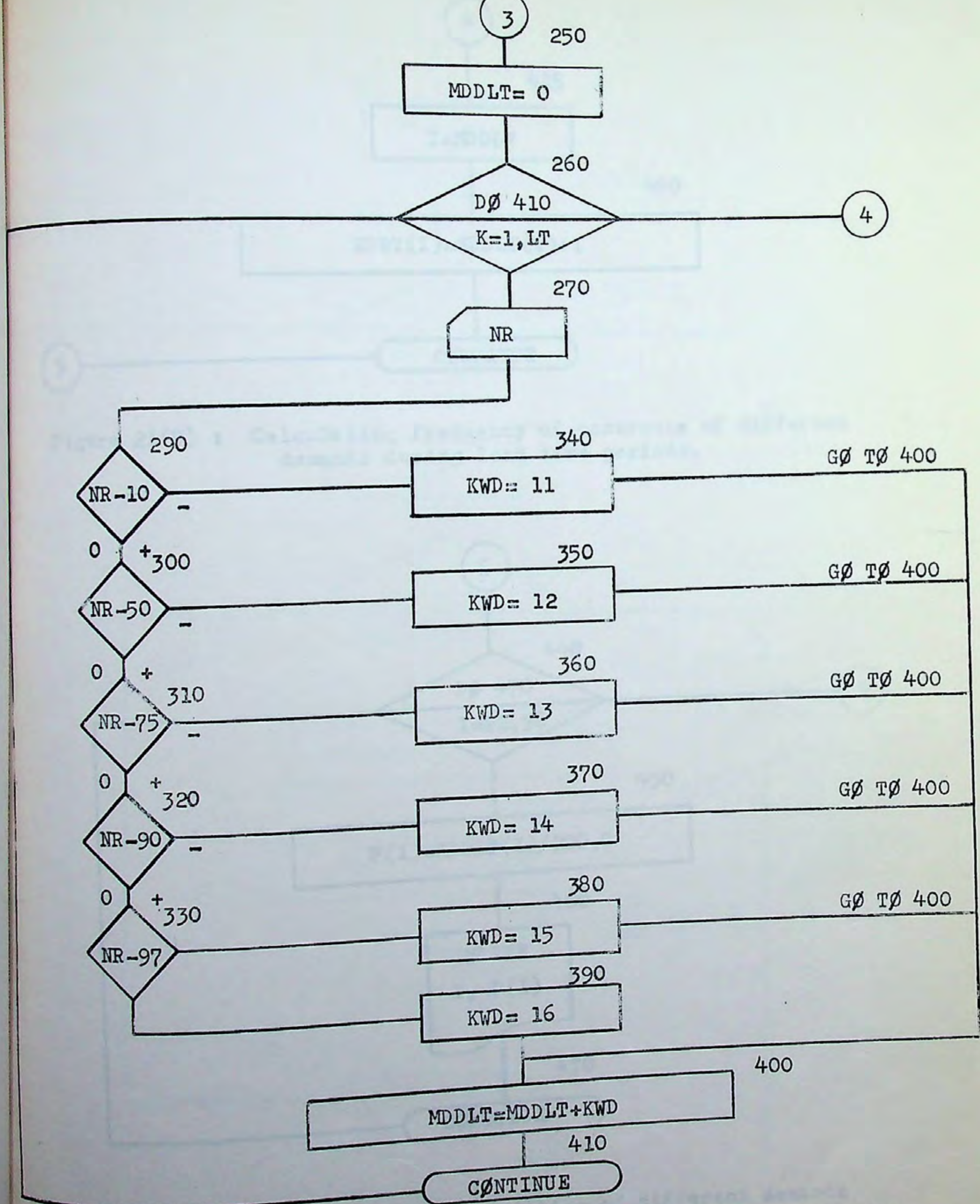

Figure 21(C) : Searching for the different probable demands during lead time periods according to their given probabilities via a Monte Carlo Technique.

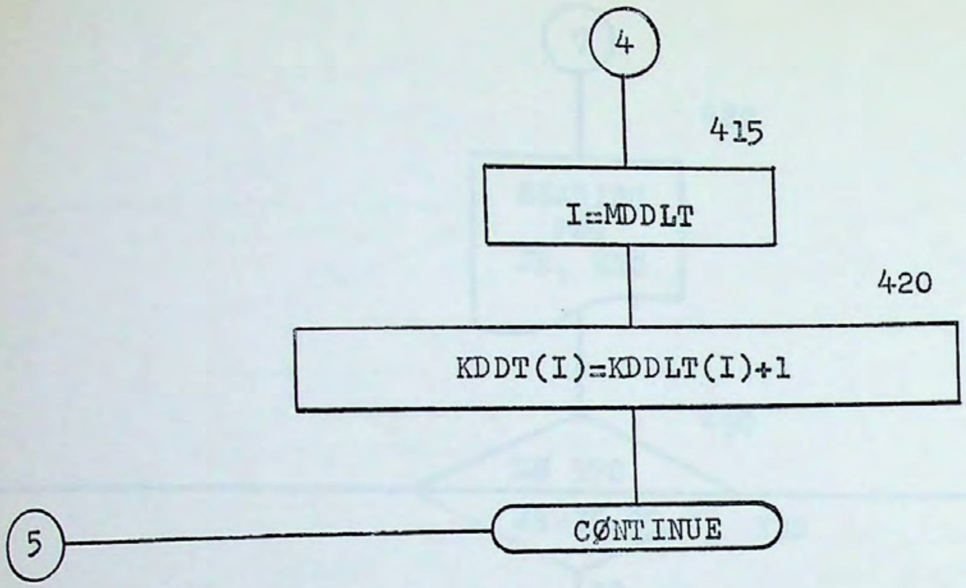

Figure 21(D) : Calculating frequency of occurence of different demands during lead time periods.

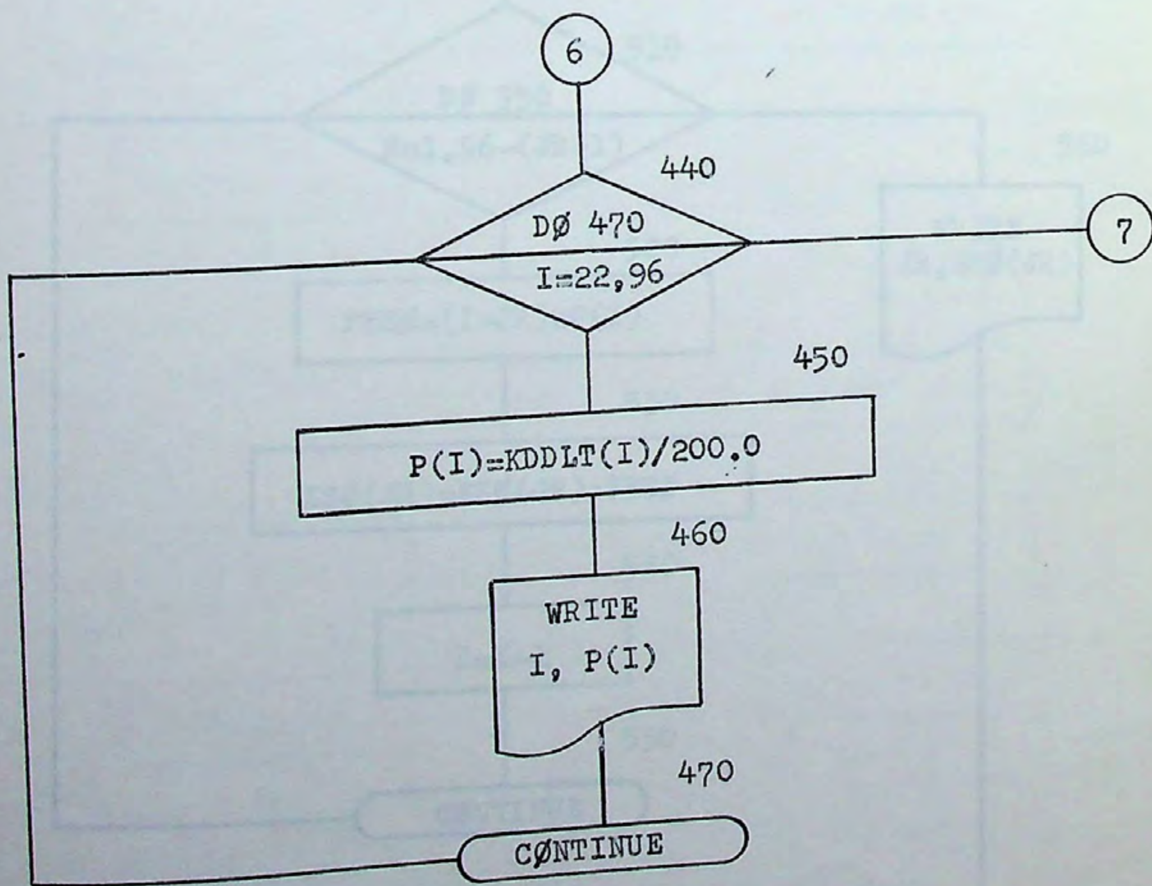

Figure 21(E) : Calculating probabilities of different demands during lead time periods.

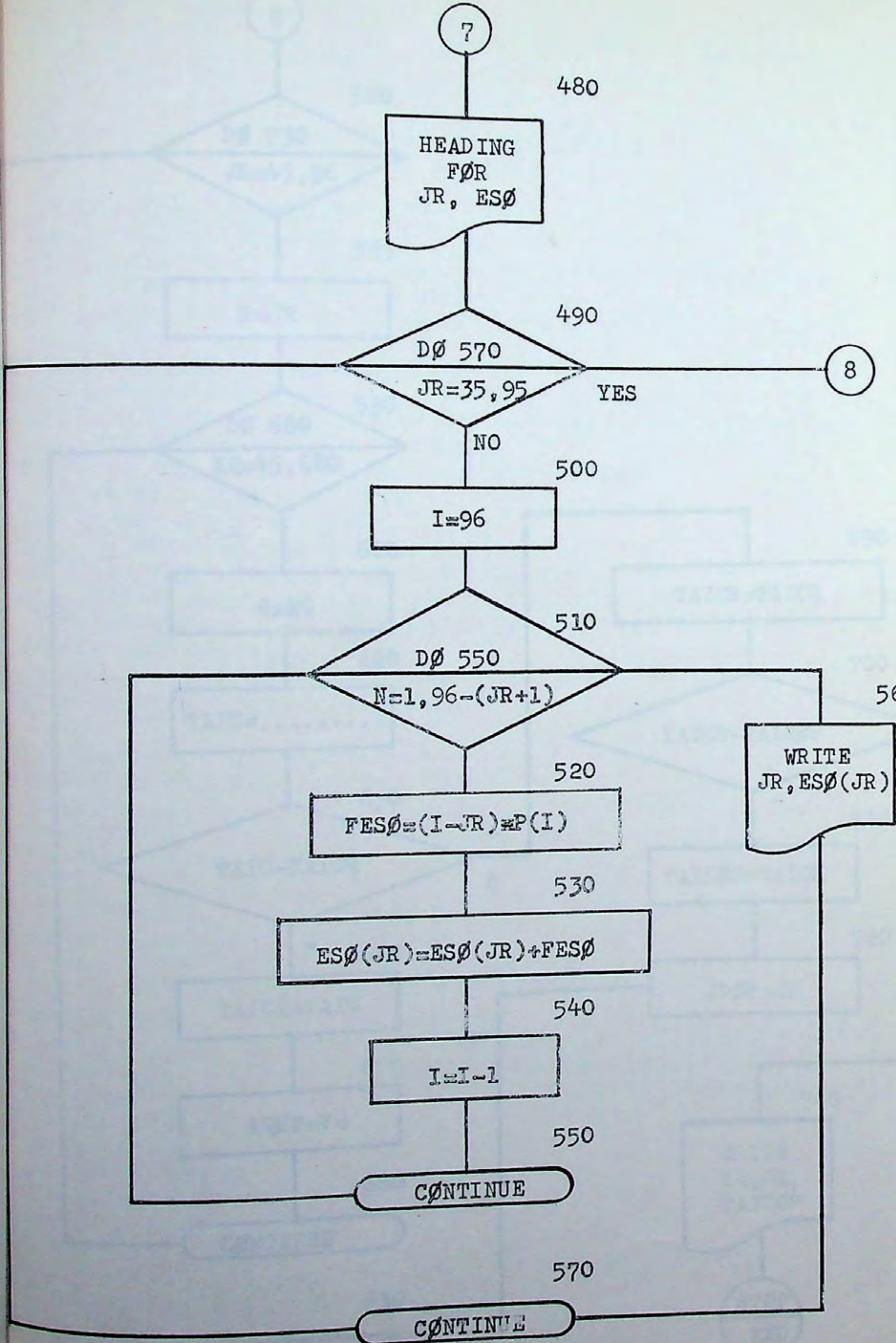

Figure 21(F) : Calculating values of expected stock-out values at different reorder points.

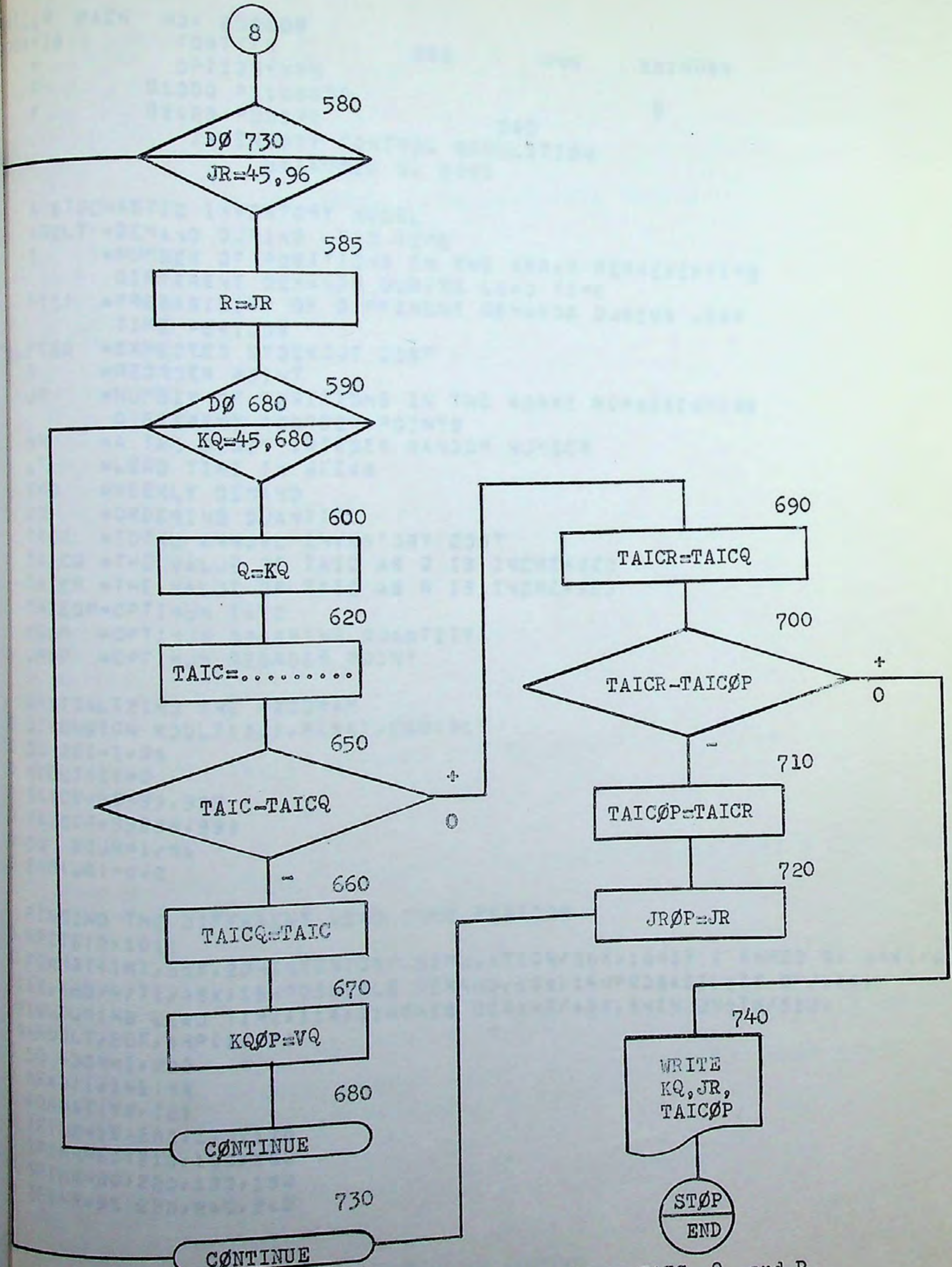

Figure 21 (G) : Calculating optimum TAIC, Q, and R.

```
PILER PACK  NO. 200005
000PIS       FORT/FN
 P           OPTIONYYYN          EN1        NNN     231N064
 F        01000 P1108002
 F        02623 P01136                                   S
                                      060
          INVENTORY CONTROL SIMULATION
               BY:AHMED S. ZAKI

 A STOCHASTIC INVENTORY MODEL
 KDDLT =DEMAND DURING LEAD TIME
 I     =NUMBER OF POSITIONS IN THE ARRAY REPRESENTING
        DIFFERENT DEMANDS DURING LEAD TIME
 P(I)  =PROBABILITY OF DIFFERENT DEMANDS DURING LEAD
        TIME PERIODS
 FESO  =EXPECTED STOCKOUT COST
 R     =REORDER POINT
 JR    =NUMBER OF POSITIONS IN THE ARRAY REPRESENTING
        DIFFERENT REORDER POINTS
 NR    =A TWO DIGIT INTEGER RANDOM NUMBER
 LT    =LEAD TIME IN WEEKS
 KWD   =WEEKLY DEMAND
 KQ    =ORDERING QUANTITY
 TAIC  =TOTAL ANNUAL INVENTORY COST
 TAICQ =THE VALUE OF TAIC AS Q IS INCREASED
 TAICR =THE VALUE OF TAIC AS R IS INCREASED
 TAICOP=OPTIMUM TAIC
 KQOP  =OPTIMUM ORDERING QUANTITY
 JROP  =OPTIMUM REORDER POINT

 INITIALIZING THE PROGRAM
10 DIMENSION KDDLT(96),P(96),ESO(96)
20 DO 30I=1,96
30 KDDLT(I)=0
40 TAICQ=99999.999
50 TAICOP=99999.999
70 DO  80JR=1,96
80 ESO(JR)=0.0

 FINDING THE DIFFERENT LEAD TIME PERIODS
00 WRITE(2,101)
01 FORMAT(1H1,55X,20HINVENTORY SIMULATION/56X,18HBY : AHMED S. ZAKI/6
 11X,6H3/4/71/45X,15HPOSSIBLE DEMAND,10X,14HPROBABILITY OF/44X,
 216HDURING LEAD TIME,11X,11HTHIS DEMAND/49X,8HIN UNITS/51X,
 34HDDLT,20X,4HP(I)//)
30 DO 430M=1,200
40 READ(1,141)NR
41 FORMAT(9X,I2)
60 IF(NR-15)200,170,170
70 IF(NR-60)210,180,180
80 IF(NR-80)220,190,190
90 IF(NR-95)230,240,240
```

Figure (22) : The Simulation Program

```
200 LT=2
205 GO TO 250
210 LT=3
215 GO TO 250
220 LT=4
225 GO TO 250
230 LT=5
235 GO TO 250
240 LT=6
C
C   CALCULATING DEMAND DURING EACH LEAD TIME PERIOD
250 MDDLT=0
260 DO 410K=1,LT
270 READ(1,271)NR
271 FORMAT(9X,I2)
290 IF(NR-10)340,300,300
300 IF(NR-50)350,310,310
310 IF(NR-75)360,320,320
320 IF(NR-90)370,330,330
330 IF(NR-97)380,390,390
340 KWD=11
345 GO TO 400
350 KWD=12
355 GO TO 400
360 KWD=13
365 GO TO 400
370 KWD=14
375 GO TO 400
380 KWD=15
385 GO TO 400
390 KWD=16
400 MDDLT=MDDLT+KWD
410 CONTINUE
C
C   SUMMING UP SIMILAR DEMANDS DURING LEAD TIME PERIODS
 415 I=MDDLT
 420 KDDLT(I)=KDDLT(I)+1
 430 CONTINUE
C
C   CALCULATING PROBABILITIES OF DIFFERENT DEMANDS DURING LEAD TIMES
 440 DO470I=22,96
 442 P(I)=KDDLT(I)/200.0
 460 WRITE(2,461)I,P(I)
 461 FORMAT(52X,I2,21X,F5.3)
 470 CONTINUE
C
C   CALCULATING EXPECTED STOCK-OUT COST AT DIFFERENT REORDER POINTS
 480 WRITE(2,481)
 481 FORMAT(1H1,45X,13HREORDER POINT,10X,17HEXPECTED VALUE OF/
   170X,12HSTOCK-OUT/LT/51X,1HR,23X,3HESO//)
 490 DO 570JR=45,95
 500 I=96
 510 DO 550N=1,96-(JR+1)
 520 FESO=(I-JR)*P(I)
 530 ESO(JR)=ESO(JR)+FESO
```

Figure (22) : Continued

```
540 I=I-1
550 CONTINUE
560 WRITE(2,561)JR,ESO(JR)
561 FORMAT(50X,I2,20X,F8.5)
570 CONTINUE
    COMPARING TOTAL ANNUAL INVENTORY COST AT DIFFERENT VALUES OF
    REORDER POINTS AND ORDERING QUANTITIES TO FIND MINIMUM TAIC
580 DO 730JR=45,96
585 R=JR
590 DO 680KQ=45,680
600 Q=KQ
620 TAIC=((Q/2.0)+R-45.0)*(380.0*0.2)+(680.0/Q)*20.0
   1+10.0*ESO(JR)*(680.0/Q)
650 IF(TAIC-TAICQ)660,690,690
660 TAICQ=TAIC
670 KQOP=KQ
680 CONTINUE
690 TAICR=TAICQ
700 IF(TAICR-TAICOP)710,740,740
710 TAICOP=TAICR
720 JROP=JR
730 CONTINUE
740 WRITE(2,741)KQOP,JROP,TAICOP
741 FORMAT(1H1,55X,14HOPTIMUM POLICY//5X,32HTHE OPTIMUM ORDERING QUANT
   1ITY   =,7X,I3,2X,5HUNITS//5X,32HTHE OPTIMUM REORDER POINT      =,8X
   2,I2,2X,5HUNITS//5X,32HMINIMUM ANNUAL INVENTORY COST   =,1X,4HL.E.,2
   3X,F9.2)
750 STOP
    END

LED  SUBROUTINE

RIT    QFINIO     QFREAD     QIOEI      ICNVIR     QIHER      ICHS
OP

LAR  ALLOCATION

A4 I          000A8 TAICQ      000AC TAICOP     000B0 JR         000B4
B8 NR         000BC LT         000C0 MODLT      000C4 K          000C8
CC N          000D0 FESO       000D4 R          000D8 KQ         000DC
E0 TAIC       000E4 KQOP       000E8 TAICR      000EC JROP

AY  ALLOCATION

F0 KODLT      00270 P          003F0 ESO
```

Figure (22) ; Continued

```
FULL FORTRAN IV  RF02          PROGRAM IS     00            MEMORY MAP                                      DATE 11/05
ALLOCATION            START ADDR  END ADDR   DEC LENGTH    START ADDR  END ADDR  DEC LENGTH       START ADDR

*** OBJECT PROGRAM ON PACK  200017
                                               25540
TOTAL MEMORY                                    3584
RESIDENT EXEC                                  19908
PROGRAM                00 0E00    00 53C4          0
TOTAL COMMON           00 F800    00 F800        512
EXEC OVERLAY AREA                               1536
UPPER EXEC

MAIN PROGRAM (RES)     00 0E00    00 53C4      19908
BUFFERS                00 0E00    00 0E88        136
FILE=BUFFER TABLES     00 0E88    00 0F74        236
CLASS 1 SUBRTNES       00 0F74    00 4E58      16100
OVERLAY DIRECTORY      00 4E58    00 4E58          0
CODING AND DATA        00 4E58    00 53C4       3436
CLASS 2 SUBRTNES       00 5BC4    00 53C4          0

          UNITABLA     00 0F74           :CHS        00   00 101C          APREJCT 02    00 1034
          CDDECTBA00   00 1044           CDVFY1      16   00 114C          GET     03    00 1188
          PCREAD  15   00 11E0           PRINTPUT11       00 11F4          PRVFY1  05    00 132C
          PZWRITE 07   00 1348           QFINIO      15   00 138C          QIANDO  67    00 1424
          QINOUT  12   00 1998           QSTP        10   00 3214          ZCONSERB01    00 3290
          ZCONSERR33   00 3630           ZDECODE     18   00 3A1C          ZFLARITH02    00 3A7C
          ZFLASCBN17   00 43D0           ZFLBNASC20       00 4780          ZFLOAT  10    00 435C
          ZFXASCBN12   00 4C30           ZFXBNASC05       00 4070

***IS                  00 4E58    00 53C4       3436

              TEXT     00 53C8    00 53C4
```

Figure (23): The Computer Memory Map

PHASE 6: EXECUTING THE SIMULATION MODEL

The simulation program is then punched on cards or any other suitable input media and tested for language errors on the computer and then compiled. This simulation model was run on an NCR Century 200 and it took:

2 minutes for testing

3 minutes for compilation

5 minutes for coumputations and print out.

—

12 minutes total computer time.

The memory map for this program is shown in figure (23).

PHASE 7: COLLECTING AND ANALYZING DATA

Tables (5), (6), and figure (24) represent the generated computer output. This output was analyzed in order to verify the results and write the final report about the observations and proposals. The analyses are included in the final report.

In this particular case, the generated results looked unrealistic and the researcher needed to carry out more experiments to see whether those results are actually sound or not. The experiments needed were simpler to carry manually than by the computer, and that is how they were done. In some cases, these experiments might need large and complicated calculations that another computer run(s) are necessary.

Possible demand during lead time DDLT	Probability of this demand P(I)	Cumulative probability Probability that demand is greater than DDLT
22	0.005	0.995
23	0.010	0.985
24	0.035	0.950
25	0.040	0.910
26	0.050	0.860
27	0.005	0.055
28	0.010	0.845
29	0.005	0.845
30	0.000	0.840
31	0.000	0.840
32	0.000	0.840
33	0.000	0.840
34	0.000	0.840
35	0.020	0.820
36	0.055	0.765
37	0.120	0.645
38	0.075	0.570
39	0.065	0.505
40	0.030	0.475
41	0.035	0.440

Table (5): Generated Probabilities and Cumulative Probabilities of Different Demands During Lead Time Periods

Table (5)--Continued

Possible demand during lead time DDLT	Probability of this demand P(I)	Cumulative probability Probability that demand is greater than DDLT
42	0.015	0.425
43	0.005	0.420
44	0.000	0.420
45	0.005	0.415
46	0.005	0.410
47	0.010	0.400
48	0.005	0.395
49	0.020	0.375
50	0.050	0.325
51	0.025	0.300
52	0.030	0.270
53	0.025	0.245
54	0.025	0.220
55	0.005	0.215
56	0.000	0.215
57	0.000	0.215
58	0.000	0.215
59	0.005	0.210
60	0.000	0.210
61	0.005	0.205
62	0.025	0.180

Table (5)--Continued

Possible demand during lead time DDLT	Probability of this demand P(I)	Cumulative probability Probability that demand is greater than DDLT
63	0.015	0.165
64	0.040	0.125
65	0.015	0.110
66	0.015	0.095
67	0.000	0.095
68	0.005	0.090
69	0.010	0.080
70	0.005	0.075
71	0.000	0.075
72	0.000	0.075
73	0.005	0.070
74	0.010	0.060
75	0.005	0.055
76	0.015	0.040
77	0.005	0.035
78	0.005	0.030
79	0.005	0.025
80	0.015	0.010
81	0.005	0.005
82	0.000	0.005
83	0.005	0.000

Table (5)--Continued

Possible demand during lead time DDLT	Probability of this demand P(I)	Cumulative probability Probability that demand is greater than DDLT
84	0.000	0.000
85	0.000	0.000
86	0.000	0.000
87	0.000	0.000
88	0.000	0.000
89	0.000	0.000
90	0.000	0.000
91	0.000	0.000
92	0.000	0.000
93	0.000	0.000
94	0.000	0.000
95	0.000	0.000
96	0.000	0.000

REORDER POINT R	EXPECTED VALUE OF STOCK-OUT/LT ESO
45	6.33499
46	5.91499
47	[illegible]
48	5.09499
49	4.66999
50	4.31999
51	3.98999
52	3.69499
53	3.42499
54	3.19999
55	2.98499
56	2.76999
57	2.55499
58	2.33499
59	2.12499
60	1.90999
61	1.68000
62	1.48500
63	1.28000
64	1.14000
65	1.01500
66	0.92000
67	0.82000
68	0.72000
69	0.63500
70	0.56000
71	0.48500
72	0.40500
73	0.32500
74	0.26000
75	0.19000
76	0.14500
77	0.10500
78	0.07000
79	0.03000
80	0.01500
81	0.01000
82	0.00000
83	0.00000
84	0.00000
85	0.00000
86	0.00000
87	0.00000
88	0.00000
89	0.00000
90	0.00000
91	0.00000
92	0.00000
93	0.00000
94	0.00000
95	0.00000

Table (6) : Generated expected stock-out values at different reorder point

OPTIMUM POLICY

THE OPTIMUM ORDERING QUANTITY	45 UNITS
THE OPTIMUM REORDER POINT	45 UNITS
MINIMUM ANNUAL INVENTORY COST	LE.2969.51

Figure (24): Generated Optimum Policy

FINAL REPORT

THE SIMULATION MODEL SOLUTION

The company can achieve minimum total annual inventory cost, and consequently, minimum annual costs if it places an order for 45 units each time its stock level reaches 45 units.

Following this policy,

The number of orders/year = 680/45 = 15.1   i.e., 16 orders/year.

The total annual inventory cost = L.E. 2962.45

THE CONSEQUENCES OF THIS POLICY

Table (7) which is extracted from table (5) of generated probabilities and cumulative probabilities shows that:

DDLT	Probability that DDLT =	Probability that DDLT <	Probability that DDLT >
45	0.005	0.590	0.415
50	0.005	0.680	0.325
55	0.050	0.790	0.215
60	0.000	0.790	0.210

Table (7): Probabilities and Cumulative Probabilities of Selected Demands during Lead Time Periods.

1. In 59% of the lead times (from 9 to 10 lead times), DDLT is expected to be less than 45 units. This means that at the momemt

a replenishment arrives, the stock level in inventory is equal to the amount of the order (45 units) plus the difference between the safety stock (45 units) and DDLT, i.e., 45 + (45-DDLT). Thus, the company withdraws from inventory until its stock level reaches 45 units and the places an order for a new quantity at the point OP_1 in figure (25).

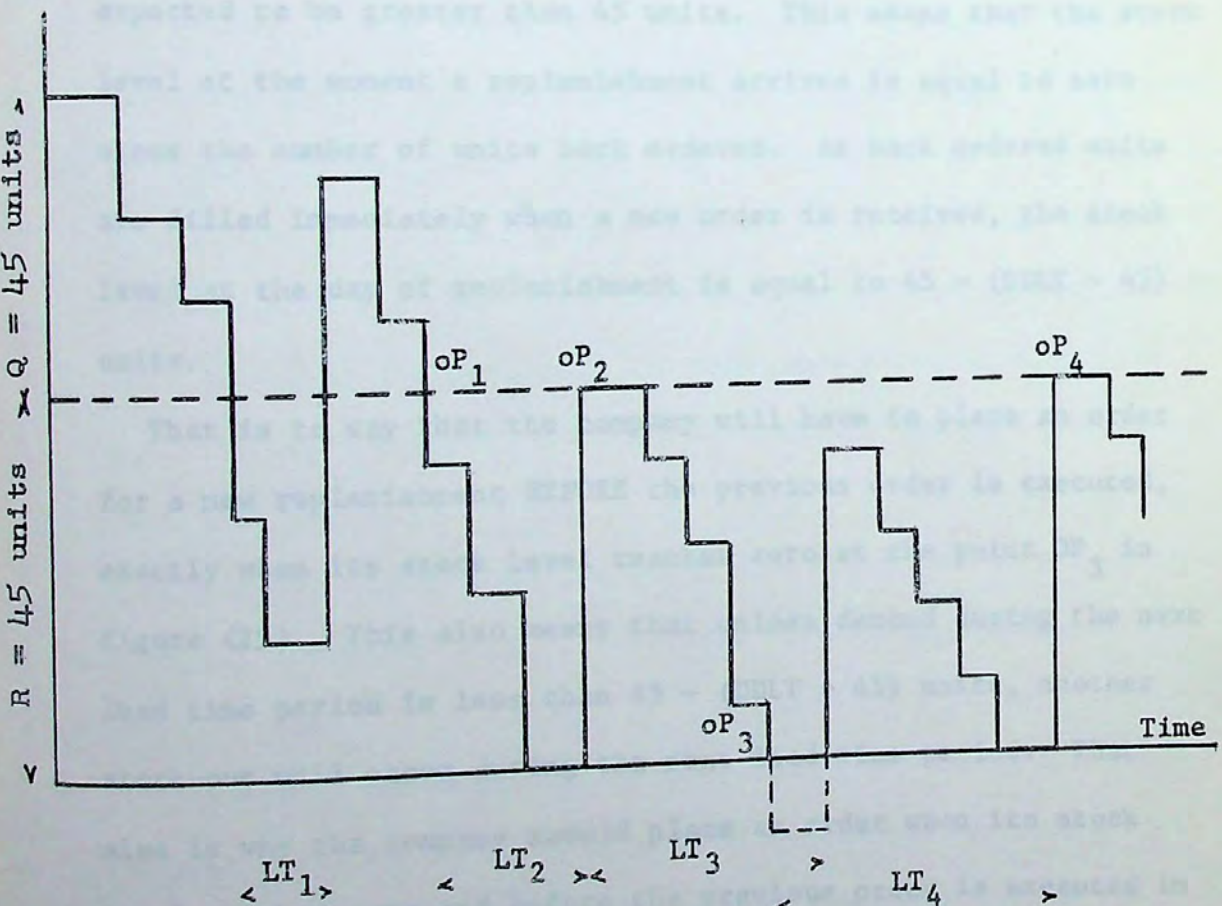

Figure (25): The Company's Expected Inventory Pattern

2. In 0.5% of the lead times, DDLT is expected to be equal to 45 units. This means that at the moment a replenishment arrives,

the stock level is equal to the amount of the order (45 units) plus the difference between the safety stock and DDLT, i.e., 45 + (45 - 45). Thus, the inventory level at this moment is equal to 45 units. This means that the company should place an order for a new quantity the same day it receives the previous order at the point OP_2 in figure (25).

3. In 41.5% of the lead times (from 6 to 7 lead times), DDLT is expected to be greater than 45 units. This means that the stock level at the moment a replenishment arrives is equal to zero minus the number of units back ordered. As back ordered units are filled immediately when a new order is received, the stock level at the day of replenishment is equal to 45 - (DDLT - 45) units.

That is to say that the company will have to place an order for a new replenishment BEFORE the previous order is executed, exactly when its stock level reaches zero at the point OP_3 in figure (25). This also means that unless demand during the next lead time period is less than 45 - (DDLT - 45) units, another stock-out will occur during the next lead time period. That also is why the company should place an order when its stock level reaches zero and before the previous order is executed in order to decrease the length of the next lead time period and thus, protects itself, to a certain extent, against having another stock-out during the following lead time.

ALTERNATIVE POLICIES

If the company wishes to decrease the frequency of stock-out occurrence, it can either:

1. Increase the order point (R) only. This decreases the frequency of stock-out occurrence and its magnitude.
2. Increase the ordering quantity (Q) to a quantity $\grave{Q} > Q$. This: a) decreases the number of orders/year and consequently the number of lead times, and, b) increases the stock level in inventory at the time of replenishment by the amount $\grave{Q} - Q$ which protects, to some extent according to the value of Q, the company from having a stock-out during the following lead time period.
3. Increase both the order point (R) and the reordering quantity (Q). This decreases the frequency and magnitude of stock-outs plus protects the company from having a stock-out during the following lead time period.

Table (8) illustrates the effects on stock-out occurrence and total annual inventory cost at different inventory policies. This table shows that:

1. By increasing the amount of safety stock (R) from 45 units to 50 units, the frequency of stock-out occurrence decreases from 41.5% to 32.5%, i.e., about 25% of the stock-out times. In other words, stock-outs occurs 4 to 5 times instead of 6 to 7 times/year while the total annual inventory cost increases by 2.9% only.
2. Increasing (R) to 55 units decreases the percentage of stock-out occurrence from 41.5% to 21.5%, i.e., about 50% of the

Q	R	Number of stock-outs per year	Total annual inventory cost L.E.	Increase in cost from minimum cost L.E.	Percentage of increase
45	45	6 to 7	2962.45	000000	000
45	50	5	3040.20	77.75	2.9%
45	55	3 to 4	3209.95	247.50	8 %
45	60	3 to 4	3438.90	467.45	16 %
50	45	5 to 6	3033.56	71.11	2.5%
55	45	5 to 6	3097.50	135.05	4.5%
60	45	5	3224.70	262.25	9 %
50	50	4 to 5	3139.52	177.07	6 %
55	50	4	3252.68	290.23	9 %
60	50	3 to 4	4135.16	1172.71	39 %

Table (8): Comparative Effects of Alternative Policies

stock-out times. This causes stock-outs to occur 3 to 4 times/year instead of 6 to 7 times/year while the total annual inventory cost increases by 8% only.

3. Increasing (R) to 60 units will in fact have no effect on the number of stock-outs per year as the difference between the probability that DDLT will equal to 55 units and that DDLT will equal 60 units is only 0.005, thus, the percentage of stock-out occurrence will decrease by 0.5% only (see table 7) while the total annual inventory cost increases by 16% - 8%, i.e., 8%.

4. By increasing Q only, the company decreases the number of orders/year and consequently, the number of stock-outs/year. Also, the company protects itself, to a certain extent according to the value of Q, from hav ing two consecutive stock-outs. For example, if Q is increased to 50 units instead of 45, then the number of stock-outs/year drops to

   14 x 41.5% = 6 times

   instead of from 6 to 7 times/year while the total annual inventory cost increases by 2.5%.

   Again, if the company decides to increase Q to 60 units, the number of orders/year decreases to 12 instead of 16 and consequently, the number of stock-outs/year drops to 5 times instead of 6 to 7 times/year while the total annual inventory cost increases by 9%. Comparing this policy with that of increasing (R) only to 50 units we can see that the company can achieve the same results (decreasing number of stock-outs to 4 instead of 6 to 7 times/year as in the number 1 solution) by an increase of 2.9% instead of this 9%. That is to say that increasing Q alone will not benefit the company as better or at least similar results with less expenses can be achieved by increasing (R) only.

5. Increasing both (Q) and (R) achieves no great improvement over increasing (R) alone as the number of orders/year is not affected unless Q is increased substantially (to 60 units at least). This causes the annual inventory cost to increase by a large amount (39%) that is not commensurate with the achieved results.

THE OPTIMUM POLICY

From table (8) and the above analysis, it is the researcher's opinion that the company should place an order for 45 units every time its stock level reaches 55 units or zero if a stock-out occurs. This policy will result in:

1. Decreasing the number of stock-outs/year to 3 instead of 6 times/ year.
2. Increasing total annual inventory cost by 8% only which is minor compared to the benefits the company will gain by following that policy.

MANUAL SOLUTION

The manual solution to this problem is to be carried out in order to:

1. Validate the results obtained from the computer runs.
2. Illustrate the amount of computations needed to solve this problem and the time it consumes in order to verify the fact that computers are practically a necessity whenever simulation is considered in the sense that without them; computations and calculations would consume so long a time that may render the simulation objective obsolete by the time they are finished. It will also illustrate the possibility of making calculation errors which can be avoided through the use of computers.

During the manual solution, we will follow the following steps:

1. Calculate possible demands during possible lead time periods by a Monte Carlo technique (see table 9).
2. Add up similar demands during lead time periods to calculate frequency of occurrence of each demand (see table 10).
3. Divide frequency of occurrence by number of trials to calculate relative frequency (approximate probability) as in table (10).

At this moment, we will stop and check those results against:

1. Computer results:

Table (11) portrays the results obtained by the computer with 100 & 200 trials i.e., different sample sizes. It also

Trial No.	RN	Corresponding Lead Time	RN	Corresponding Weekly Demand	Demand during Lead Time Period
1	03	2	38 17	12 12	24
2	32	3	69 24 30	13 12 12	37
3	03	2	48 88	12 14	26
4	71	4	27 80 33 90	12 14 12 15	53
5	78	4	55 87 16 34	13 14 12 12	51
6	45	3	59 16 68	13 12 13	38
7	79	4	33 59 20 59	12 13 12 13	50
8	42	3	34 99 66	12 16 13	41
9	48	3	15 20 73	12 12 13	37
10	60	4	44 18 58 61	12 12 13 13	50

Table (9): Manually Calculated Probable Demands During Probable Lead Time Periods

Table (9)--Continued

Trial No.	RN	Corresponding Lead Time	RN	Corresponding Weekly Demand	Demand during Lead Time Period
11	18	3	00 32 65	11 12 13	36
12	20	3	59 99 10	13 16 12	41
13	95	6	73 52 68 66 30 21	13 13 13 13 12 12	76
14	97	6	63 19 23 58 00 60	13 12 12 13 11 13	74
15	75	4	70 66 08 60	13 13 11 13	50
16	58	3	07 67 69	11 13 13	37
17	27	3	95 26 49	15 12 12	39
18	91	5	72 29 41 41 68	13 12 12 12 13	62
19	23	3	48 98 06	12 16 11	39

Table (9)--Continued

Trial No.	RN	Corresponding Lead Time	RN	Corresponding Weekly Demand	Demand during Lead Time Period
20	45	3	15	12	
			19	12	
			15	12	36
21	67	4	90	15	
			58	13	
			68	13	
			73	13	54
22	34	3	95	15	
			93	15	
			95	15	45
23	32	3	92	15	
			95	15	
			39	12	42
24	27	3	92	15	
			10	12	
			75	14	41
25	85	5	41	12	
			05	11	
			82	14	
			20	12	
			48	12	61
26	60	4	43	12	
			88	14	
			94	15	
			74	13	54
27	74	4	61	13	
			76	14	
			11	12	
			92	15	54
28	01	2	55	13	
			66	13	26

Table (9)--Continued

Trial No.	RN	Corresponding Lead Time	RN	Corresponding Weekly Demand	Demand during Lead Time Period
29	96	6	43	12	
			01	11	
			70	13	
			11	12	
			78	14	
			27	12	74
30	88	5	74	13	
			28	12	
			74	13	
			65	13	
			66	13	64

Possible DDLT	Frequency of Occurrence	Probability P(I)	Cumulative Probability DDLT Than
22	-		1.000
23	-		
24	1	0.033	0.967
25	-		0.967
26	2	0.066	0.901
27	-		
28	-		
29	-		
30	-		
31	-		
32	-		
33	-		
34	-		
35	-		0.901
36	2	0.066	0.835
37	3	0.099	0.736
38	1	0.033	0.693
39	2	0.066	0.627
40	-	-	0.627
41	3	0.099	0.528
42	1	0.033	0.495
43	-		
44	-		0.495
45	1	0.033	0.462
46	-		
47	-		
48	-		
49	-		0.462
50	3	0.099	0.363
51	1	0.033	0.330
52	-	-	0.330
53	1	0.033	0.297
54	3	0.099	0.198
55	-	-	
56	-		
57	-		
58	-		
59	-		
60	-		0.198
61	1	0.033	0.165

Table (10): Frequency, Relative Frequencies (Approximate Probabilities), and Cumulative Probabilities of Different Probable Demands during Lead Time Periods

Table (10)--Continued

Possible DDLT	Frequency of Occurrence	Probability P(I)	Cumulative Probability DDLT Than
62	1	0.033	0.132
63	-		0.132
64	1	0.033	0.099
65	-		
66	-		
67	-		
68	-		
69	-		
70	-		
71	-		
72	-		
73	-		0.099
74	2	0.066	0.033
75	-		0.033
76	1	0.033	0.000
77	-	0.000	0.000
78	-	0.000	0.000
79	-	0.000	0.000
80	-	0.000	0.000
81	-	0.000	0.000
82	-	0.000	0.000
83	-	0.000	0.000
84	-	0.000	0.000
85	-	0.000	0.000
86	-	0.000	0.000
87	-	0.000	0.000
88	-	0.000	0.000
89	-	0.000	0.000
90	-	0.000	0.000
91	-	0.000	0.000
92	-	0.000	0.000
93	-	0.000	0.000
94	-	0.000	0.000
95	-	0.000	0.000
96	-	0.000	0.000

DDLT	Calculated Probability		
	Computer		Manual 30 trials
	200 trials	100 trials	
22	0.005	0.010	0.000
23	0.010	0.010	0.000
24	0.035	0.030	0.033
25	0.040	0.050	0.000
26	0.050	0.060	0.066
27	0.005	0.000	0.000
28	0.010	0.010	0.000
29	0.005	0.000	0.000
30	0.000	0.000	0.000
31	0.000	0.000	0.000
32	0.000	0.000	0.000
33	0.000	0.000	0.000
34	0.000	0.000	0.000
35	0.020	0.010	0.000
36	0.055	0.090	0.066
37	0.120	0.120	0.099
38	0.075	0.060	0.033
39	0.065	0.060	0.066
40	0.030	0.050	0.000
41	0.035	0.050	0.099
42	0.015	0.020	0.033
43	0.005	0.010	0.000
44	0.000	0.000	0.000
45	0.005	0.010	0.033
46	0.005	0.010	0.000
47	0.010	0.020	0.000
48	0.005	0.000	0.000
49	0.020	0.010	0.000
50	0.050	0.060	0.099
51	0.025	0.030	0.033
52	0.030	0.000	0.000
53	0.025	0.010	0.033
54	0.025	0.040	0.099
55	0.005	0.000	0.000
56	0.000	0.000	0.000
57	0.000	0.000	0.000
58	0.000	0.000	0.000
59	0.005	0.000	0.000
60	0.000	0.000	0.000
61	0.005	0.010	0.033

Table (11): Computer Results Versus Manual Solution Results

Table (11)--Continued

DDLT	Calculated Probability		
	Computer		Manual
	200 trials	100 trials	30 trials
62	0.025	0.030	0.033
63	0.015	0.010	0.000
64	0.040	0.030	0.033
65	0.015	0.030	0.000
66	0.015	0.010	0.000
67	0.000	0.000	0.000
68	0.005	0.000	0.000
69	0.010	0.010	0.000
70	0.005	0.000	0.000
71	0.000	0.000	0.000
72	0.000	0.000	0.000
73	0.005	0.000	0.000
74	0.010	0.020	0.066
75	0.005	0.000	0.000
76	0.015	0.020	0.033
77	0.005	0.000	0.000
78	0.005	0.000	0.000
79	0.005	0.000	0.000
80	0.015	0.000	0.000
81	0.005	0.000	0.000
82	0.000	0.000	0.000
83	0.005	0.000	0.000
84	0.000	0.000	0.000
85	0.000	0.000	0.000
86	0.000	0.000	0.000
87	0.000	0.000	0.000
88	0.000	0.000	0.000
89	0.000	0.000	0.000
90	0.000	0.000	0.000
91	0.000	0.000	0.000
92	0.000	0.000	0.000
93	0.000	0.000	0.000
94	0.000	0.000	0.000
95	0.000	0.000	0.000
96	0.000	0.000	0.000

shows the corresponding results obtained by the manual solution. Comparing these results, we can detect the following discrepencies:

A. There are certain amounts of DDLT that has a zero probability in all cases. This is due to the fact that these amounts (events) possess a very small probability of occurrence. For example, as we have seen in the analysis of the problem and data, the probability that DDLT will be equal to 96 units is 36×10^{-12}; also for DDLT to be equal to 95 units is 582×10^{-12}. In the computer solution, we were interested only in calculating probabilities up to 3 decimal digits as those that exceed that are in fact very RARE events that can be neglected without affecting the results, thus their probabilities are considered to be equal to zero.

B. There are certain DDLT's that in the manual solution has probabilities of zero, while when calculated by the computer, they possessed a certain probability. This is due to the fact that by increasing the number of trials, i.e., the sample size, we provided a chance for other possible events to occur. For example, DDLT(23) in the manual solution has a zero probability, i.e., it is an impossible event. When we used the computer and increased the number of trials to 100, this event occurred with a probability of 0.010. Also, DDLT(27) in both the manual solution and the computer solution with 100 trials has a probability of zero, i.e., an impossible event, while when we increased the number of trials to 200 it did have a probability of 0.005, i.e., it could occur.

Thus, we can state that, by increasing the number of trials or the sample size, we actually give chance to all possible events to occur, that is to say, we make the list really *collectively exhaustive* as it should be. This is another example to show the value of the computer in simulation studies with probabilistic characteristics because in manual solutions, the sample size should be limited IF the calculations are to be carried out in suitable time.

2. Analytical results:

Let us take as an example DDLT(37). In the manual solution, this event has a probability of 0.099. When we increased the number of trials in the computer solution, its probability increased to 0.120.

Now, let us calculate the actual probability of this event by the analytical method:

This amount of demand can occur IF and only IF lead time is 3 weeks AND the demand rates were:

11 & 11 & 15 OR

12 & 12 & 13 OR

11 & 12 & 14 OR

11 & 13 & 13.

Each of the above mentioned possibilities can happen in 3 different ways. For example, the first possibility can take any one of the following forms:

11 & 11 & 15 OR

11 & 15 & 11 OR

15 & 11 & 11.

Thus, the probability of occurrence of this event is:

p(3 weeks) x p(11) x p(11) x p(15)

+ p(3 weeks) x p(11) x p(15) x p(11)

+ p(3 weeks) x p(15) x p(11) x p(11)

That is;

0.45 x 0.10 x 0.10 x 0.07

+ 0.45 x 0.10 x 0.07 x 0.10

+ 0.45 x 0.07 x 0.10 x 0.10

That is;

0.45 x 3 (0.00070) = 0.45 x 0.0021

And so on for the other possibilities.

Thus, the probability of this event to occur is:

0.45 x 3 (0.10 x 0.10 x 0.07) = 0.45 x 0.00210

+ 0.45 x 3 (0.40 x 0.40 x 0.25) = 0.45 x 0.12000

+ 0.45 x 3 (0.10 x 0.40 x 0.15) = 0.45 x 0.01800

+ 0.45 x 3 (0.10 x 0.25 x 0.25) = 0.45 x 0.01875

= 0.45 x (0.15885) = 0.07135

This probability is smaller than 0.120. What can we deduce from this?

A. When the number of trials was increased, we gave chance to this event to happen more often, thus, its probability increased from 0.099 to 0.120.

B. When we applied the analytical solution, the probability decreased because in such a solution, the probabilities calculated are the realistic collectively exhaustive probabilities where each event from DDLT = 22 to DDLT = 96 has a chance to occur. Thus, the list of events contains all the possible events, i.e., all types of events. That is to say, the number *one*, which is the sum of all possible events will be distributed among all the possible events, thus, the probability of each event will decrease relatively.

3. Conclusion:

   A. In probalistic studies, the number of trials (sample space) must be large enough if the obtained results are to be representative.

   B. Large samples need a large amount of calculations that consumes a lot of time and gives chance to calculation errors.

Thus, computers, if available, must be used in simulation studies if the results are to be realistic and obtained in due time.

APPENDIX I

GLOSSARY OF TERMS

Activity	An activity is any process that causes changes in the system (4, 2).
Analog	A relation of likeness between two things or of one thing to another, consisting in the resemblance not of the things themselves but of two or more attributes, circumstances, or effects (9).
Analog Models	Employ one set of properties to represent some other set of properties which the system being studied possesses, (e.g., for certain purposes, the flow of water through pipes may be taken as an analog of the *flow* of electricity in wires (1, 158).
Computer Models	A mathematical model expressed or written according to a particular set of rules so that the model may be processed by the computer (2, 12).
Continuous Systems	Are those systems where the predominant activities of the system cause smooth changes in the attributes of the system entities. When such a system is modeled mathematically, the variables of the model representing the attributes are controlled by continuous functions. In such systems, the relationships describe the *rates* at which attributes change, so

that the model consists of differential equations (4,29).

Collectively Exhaustive Events

When the list of outcomes of a given action includes every possible outcome of that action so that no outcome other than the ones listed is possible, the list is said to be collectively exhaustive. A list of outcomes *(events)* can be both mutually exclusive and collectively exhaustive (5, 34-45).

Digital Computer

A computer that operates with numbers expressed directly as digits in a decimal, binary, or other system (10).

Discrete Systems

Are those systems in which the changes are predominantly discontinuous and the term *event* is used to describe the occurrence of a change in a point in time. An event may cause a change in the value of some attribute of an entity, it may create or destroy an entity, or it may start or stop an activity (4, 123). Few systems are wholly discrete or continuous, however, in most systems one type of change predominates so that systems can usually be classified as being continuous or discrete (4, 4-5).

A Dynamic Mathematical Model	Is a model that displays the relationships between the system attributes when the system is moving through time. It allows the changes of system attributes to be derived as a function of time (4, 10).
Event	An event is a subset of the sample space of an experiment. For example, if a pair of dice is being tossed, an event is that "at least one head is obtained" (2, 91-92).
Event Space	Is the set that contains all the mutually exclusive, collectively exhaustive outcomes or points of the event (2, 91-92).
Experiment	Is any process that results in observing the performance of the system or its model under a certain set of conditions (5, 1-11).
Generated Variables	Are those arising as a consequence of the operations of the model. Sometimes called *Endogenous Variables* (6, 31).
Input Variables	Are those variables arising external to the model. Sometimes called *Exogenous Variables* (6, 31).
Mathematical Models	Use mathematics to designate properties of the system under study by means of

	a mathematical equation or a set of such equations (1, 158). Sometimes, they are called *symbolic Models* because they use symbols to represent variables.
Model	A description or analog used to help visualize something that cannot be directly observed (10).
Mutually Exclusive Events	Events are said to be mutually exclusive if one and only one outcome can take place at a time (7, 52).
Object System	Is the system we want to study; it is the *object* or subject matter of an investigation or learning (6, 4-5).
Outputs	Are the data we wish to obtain from a run of the simulation (6, 4-5). This data may be records of parameters and starting conditions, time paths of input and generated variables, selected variable values from selected states of the model, or just the ending conditions We may also create outputs that are combinations of or measures of the foregoing data.
Parameters	Are those attributes of the system that do not change during the simulation

because of anything that occurred during the simulation (5, 144). They can be changed from one simulation run to another only at the command of the experimenter. They are also called *Variable Constants*, or *Input Contants*.

Population Is the total set of elements about which knowledge is desired. It may refer to:

a) physical items such as population of all cars of a certain type that have been produced, or

b) the totality of numbers representing the measurements of a certain characteristic such as the life span of the motors of above mentioned cars. Here, the population consists of a set of numbers representing time or kilometric measures and not the physical items themselves (5, 34-35).

The population is sometimes referred to as the *Universal set* and the important thing is that it must be definable.

Random A sequence of trials in which the results follow no recognizable pattern (3, 256).

Random Sample A sample such that each element in the population from which it was drawn has an equal and independent chance to be included in the sample.

Random Variable — A value or magnitude that changes, occurrence after occurrence or event after event, in no predictable sequence (7, 80). For example, tomorrow's sales of a certain item by a store that has no way of knowing exactly what tomorrow's sales will be is a random variable.

Real Time — Is the time we live in; the passing of real hours, days, months, or years (6, 29). Object systems behave in real time.

Relationships — Are those connections between components, variables, and parameters that control the changes of the state in the system. They specify how the values of different variables in the system are related to each other and to the parameters of the system (5, 145).

Run of a Simulation — Is cycling through the operation of the model for a measurable amount of simulated time (6, 31).

Sample — A sample is a subset of a population. The purpose of a sample is to yield inferences about the population from which it was drawn. To achieve this, a sample must truly represent the population from which it was drawn. The two important features of a sample are: a) its size, and b) the manner in which it was selected (5, 36).

Sample Space

Is the set of all mutually exclusive, collectively exhaustive outcomes pertinent to an experiment (2, 89). An experiment may have more than one sample space according to the phenomena being observed. For example, if traffic in a sqaure is being observed, and the sample space is the set of all cars that cross the square at a certain time, the purpose of observation may be:

a- the number of cars of a certain color,

b- the number of cars of a certain type, or

c- the number of cars of a certain type and color.

For each of these phenomena there is a specific sample space, but the one to use is the one that is relevant to the experiment.

Simulated Time	Is a variable in simulation models meant to represent real time. It is not necessary that simulated time bear a constant ratio to real time (6-29).
Starting Conditions	Simulation models are simply sequences of operations. When a simulation model is used, there must be some input external to the model that determines the starting conditions for the operation. Thus, starting conditions are initial values given to input and generated variables (6-31). The first value in any complete time path is always a starting condition.
State of the System	Is a description of all the entities, attributes, and activities as they exist at one point in time (4, 2).
A Static Mathematical Model	Is a model that displays the relationships between the system attributes when the system is in equilibrium (4, 10).
System	A system is a collection of entities or things (animate or inanimate) which receives certain outputs, with the objective of maximizing some functions of the inputs and outputs (8, 28).

System Attributes

Attributes are the properties of entities (2, 1). Entities are described by listing their attributes. For example, in an inventory control situation, an item in inventory is an entity that may have the following attributes:

a) Quantity at a particular time,

b) Monthly cost of storing one unit of it,

c) Number of units consumed per year,

and a host of other attributes. An entity, specially in business systems, usually has many attributes, but we need only to choose those attributes that are relevant to the case under study and ignore the rest.

System Entities

An entity is something that has separate and distinct existence and objective or conceptual reality (10). For example, the entities in a firm are those physical objects such as machines, raw materials, clerks, machine operators... etc. Also, they are those abstract objects such as profit goals, sales quotas, production standards, and goals.

System Environment

A system is often affected by changes occurring outside it. Some system activities may also produce changes that do not react on the system. Such changes occurring outside the system are said to occur in the system environment (4, 3). An important step in modeling systems is to decide upon the boundary between the system and its environment. The decision may depend upon the purpose of the study.

Time Path — Is a record of a single variable, either input or generated, showing its values at each moment of time during a simulation run (6, 31).

Variable — Is an entity that can take on different values or be represented by different symbols during a single simulation run (6, 31).

FOOTNOTE REFERENCES

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APPENDIX II

ELEMENTS OF THE PROBABILITY THEORY
AND
THE MOST COMMONLY USED
PROBABILITY DISTRIBUTIONS

THEORY OF PROBABILITY

Probability is a numerical quantity that represents the chance or the likelihood that a certain event will occur, it is always denoted by the letter p.

Probability (p) of an event = Number of times the event occurs/ Total number of possible events.

The two main features of probability are:

1. p is always a positive number.
2. p of an event is always neither less than zero nor bigger than one.

$$0 \leqslant p \leqslant 1$$

That is to say that the probability of an impossible event is zero, and the probability of an event that is certain is one. The convention that probabilities are numbers between zero and one is a mean for scaling uncertainty between impossibility and complete certainty. Sometimes, probabilities are described as a percentage where the probability number or fraction is multiplied by 100. In this case,

$$0\% \leqslant p\% \leqslant 100\%$$

According to the type of the system and the nature of the events that are described, probabilities fall into two categories:

1. Discrete probabilities: When the events in question are distinct, such as the number of machine breakdowns per period or the amount of inventory at a certain time, separate probability numbers may be associated with each event. Those numbers are called discrete probabilities.

2. Continuous probabilities: When the events cannot be identified distinctly, for instance, events may be stated in terms of length, time, size...etc. which are numbers from a continuum of numbers where at any interval on the continuum there is an infinity of numbers, hence an infinity of possible events (4, 148-150).

Probability Rules:

Considering that an experiment is being carried with only two possible outcomes or events A and B, the following table states the rules that apply to those events and the conditions relevant to the application of each.

The same rules apply if we had more than two events. Supposing for example that a certain action can produce *n* mutually exclusive events $A_1, A_2, \ldots, A_n$. Then according to the theorem of addition, the probability that one of these *n* events occurs is equal to the sum of their separate probabilities and is expressed as:

$$p(A_1 \text{ or } A_2 \text{ or } \ldots A_n) = p(A_1 + A_2 + A_3 \ldots + A_n) = p(A_1) + p(A_2) \ldots + p(A_n) = \sum_{i=1}^{n} p(A_i)$$

Further, if one of the events must occur and $A_1, A_2, \ldots, A_n$ represent all possible events, that is they are collectively exhaustive, then $p(A_1 \text{ or } A_2 \text{ or } \ldots A_n) = \sum_{i=1}^{n} p(A_i) = 1$

Also, the probabilities of combinations of those events are calculated by the multiplication formula which states that the probability that stochastically independent events occur together equals the product of the probabilities of occurrence of each event and is expressed by (2, 36-37):

$$p(A_1 \text{ and } A_2 \text{ and } \ldots\ldots A_n) = p(A_1 A_2 A_3 \ldots A_n) = \prod_{i=1}^{n} p(A_i)$$

Events A & B	Addition Rules Probability that either A or B occur. p(A, B) = p(A or B)	Multiplication Rules Probability that both A & B occur. p(A, B) = p(A & B)
Mutually exclusive: events cannot both occur together.	p(A) + p(B)	0
Conditionally dependent: the occurrence of one event changes the probability of the occurrence of another event.	p(A) + p(B) - p(A) p(B/A) = p(A) + p(B) - p(B) p(A/B)	p(A) p(B/A) = p(B) p(A/B)
Independent: occurrence of the events are not influenced by one another.	p(A) + p(B) - p(A) p(B)	p(A) p(B)

Table (12): Probability Rules
(Source: Clifford H. Springer and Others, Probabilistic Models: Volume Four of the Mathematics for Management Series, Richard D. Irwin, Inc., Homewood Illinois, 1968, p. 14.)

Combinations and Permutations:

Many business problems involve finite sample sapces with equally likely outcomes, and in such cases the frequences of interest must be counted. In some problems, attention may be given to the order of arrangement, i.e., AB and BA must both be counted. This is known as *permutation* and is computed according to the following formula:

The number of permutations of n things taken k at a time is

$$nP_k = P_{n,k} = \frac{n!}{(n-k)!}$$

In other cases, the frequences may have to be chosen with no attention given to the order of arrangement, i.e. AB and BA are considered redundant. These are known as *combinations* and are computed according to the following formula:

The number of combinations of n-things taken k at a time is

$$nC = C_{n,k} = \binom{n}{k} = \frac{n!}{K!(n-k)!}$$

PROBABILITY DISTRIBUTIONS

A probability distribution is a systematic arrangement of numerical data usually illustrated by a graph of frequency of measurement along the ordinate (vertical axis) and the measure-ment along the abscissa (horizontal axis) as shown in figure (26).

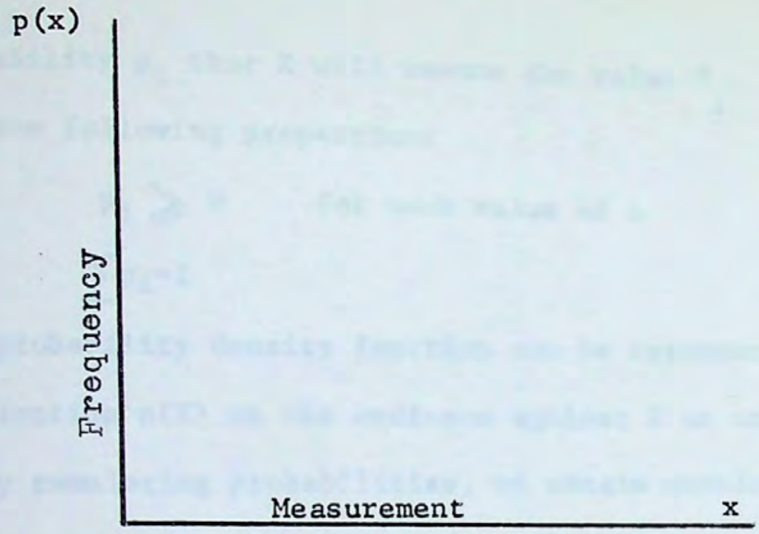


Figure (26): The Graphic Representation of Probability Distribution

Depending on the type of probability considered which can either be discrete or continuous, probability distributions are also categorized into:

1. Discrete probability distributions:

If a variable can assume a discrete set of values X_1, X_2, ..., X_n with respective probabilities p_1, p_2, ..., p_n where $p_1+p_2+...+p_n= 1$, then a discrete probability for X has been defined. The function p(X) which has the respective values p_1, p_2, ..., p_n for $X=X_1$, X_2, ..., X_n is called the *probability frequency function* of X. Because X can assume certain values with given probabilities, it is usually callled a *discrete random variable*. A random variable is also called a *chance variable* or *stochastic variable*.

More specifically then, a discrete probability distribution is a function p(X) of the discrete variable X yielding the probability p_i that X will assume the value X_i. This function has the following properties:

$$p_i \geq 0 \qquad \text{for each value of } i$$

$$\sum p_i = 1$$

The probability density function can be represented graphically by plotting p(X) on the ordinate against X on the abscissa.

By cumulating probabilities, we obtain *cumulative probability distributions* or *probability distribution functions* defined as:

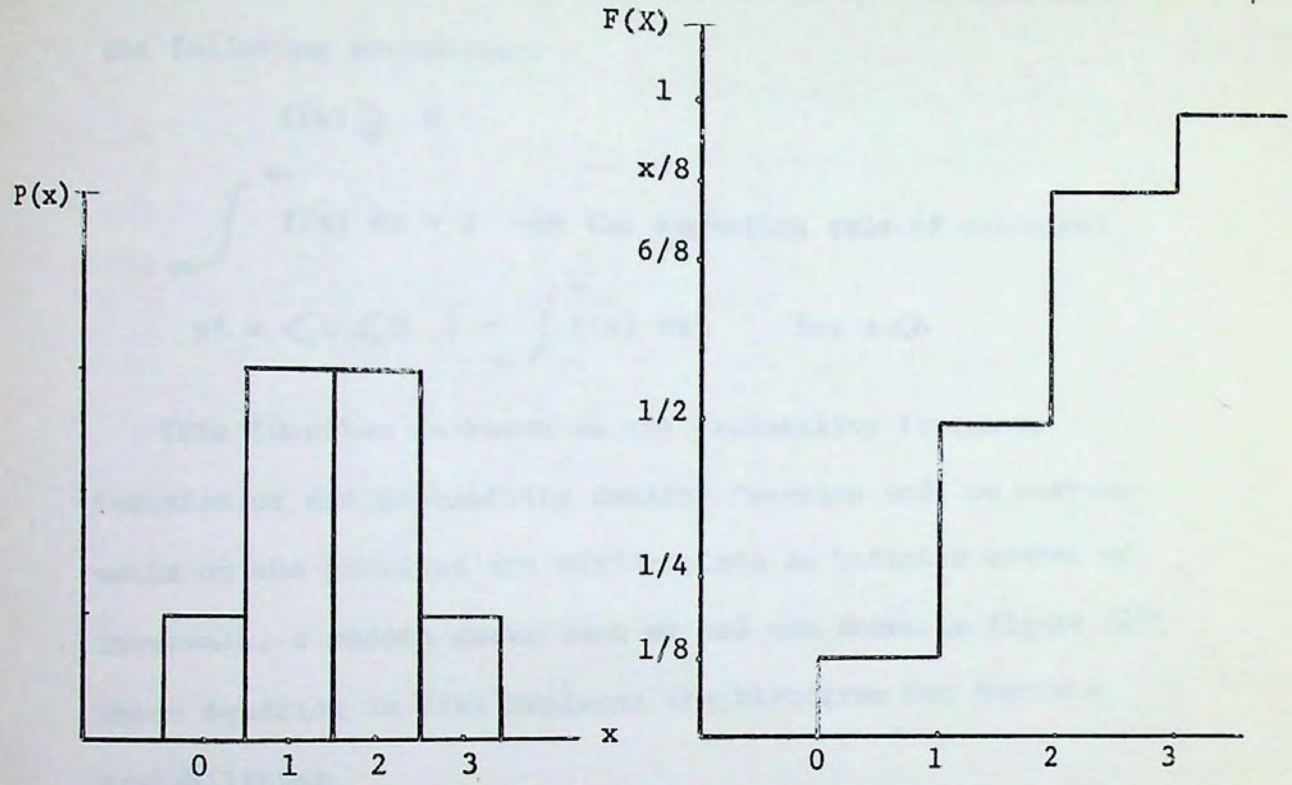
$$F(a) = p(x \leqslant a) = \sum_{x_i \leqslant a} P_i$$

F(a) is the probability that X will have any value less than or equal to *a*. The distribution function can be represented graphically by plotting the cumulative probability F(X) on the ordinate against x on the abscissa.

As an example, let us consider that we have the following discrete data about the number of breakdowns in a certain machine in a factory.

Number of breakdowns x	0	1	2	3
Probability p(x)	1/8	3/8	3/8	1/8
Cumulative probability F(X)	1/8	4/8	7/8	1

The probability density and distribution functions for this data is represented graphically as in figure (27).



a) Probability Density Function

b) Probability Distribution Function

Figure (27): Graphic representation of Discrete Probability Distributions

Note that in this example, a histogram was drawn for x, that is we have considered x as a continuous variable even though it is actually discrete as it is more representative in most cases. Also, the sum of the areas of the rectangles equals one.

2. Continuous probability distributions:

The ideas of the discrete distributions can be extended to the case where the variable x may assume a continuous set of values. Thus, a continuous probability distribution is a function f(x) of the continuous random variable x that has the following properties:

$$f(x) \geqslant 0$$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \quad \text{(by the summation rule of calculus)}$$

$$p(\,a < x \leqslant b\,) = \int_{a}^{b} f(x)\,dx \qquad \text{for } a < b$$

This function is known as the probability frequency function or the probability density function and, as measurements on the abscissa are divided into an infinite number of intervals, a smooth curve such as the one shown in figure (28) whose equation is f(x) replaces the histogram for discrete probabilities.

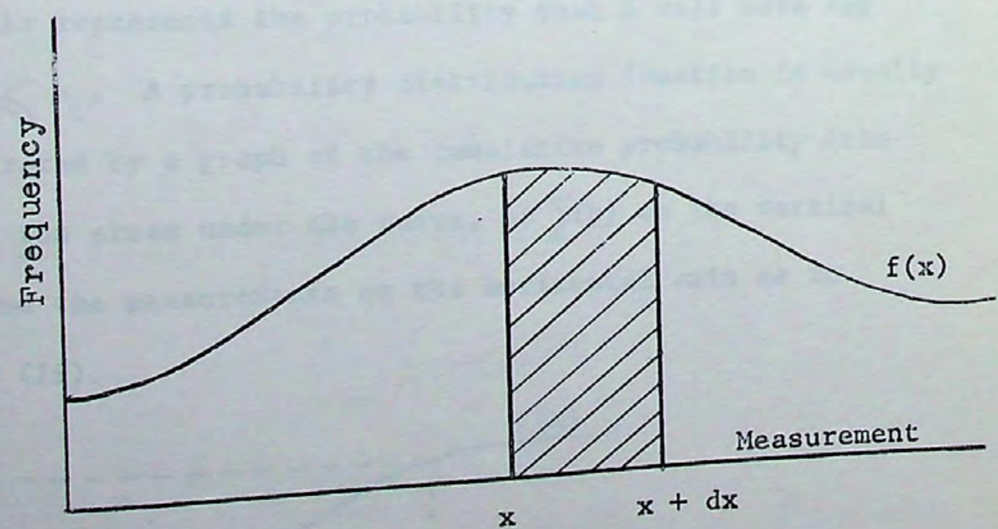


Figure (28): Continuous Probability Density Function Curve

The total area under the density function curve is always represented by unity. The shaded area under a portion of this curve is the probability p of a random measurement x_i lying between the two corresponding horizontal values x and x+dx usually denoted by

$$p(x < x_i < x+dx)$$

The probability distribution function F(X) is the cumulative probability (or the integral) of the frequency function f(x). Thus, the probability density graph is in fact a graph of the first derivative of the probability distribution function F(X), i.e., f(x) is the first derivative of F(X) and conversely, F(X) is the integral of f(x).

$$\frac{d\,F(X)}{dx} = f(x)$$

and $$F(X) = \int_{x_o}^{x_i} f(x)\,dx$$

This represents the probability that X will have any value $\leqslant x_i$. A probability distribution function is usually illustrated by a graph of the cumulative probability (the sum of the areas under the curve, or $\sum(p)$ on the vertical axis and the measurements on the horizontal axis as in figure (29).

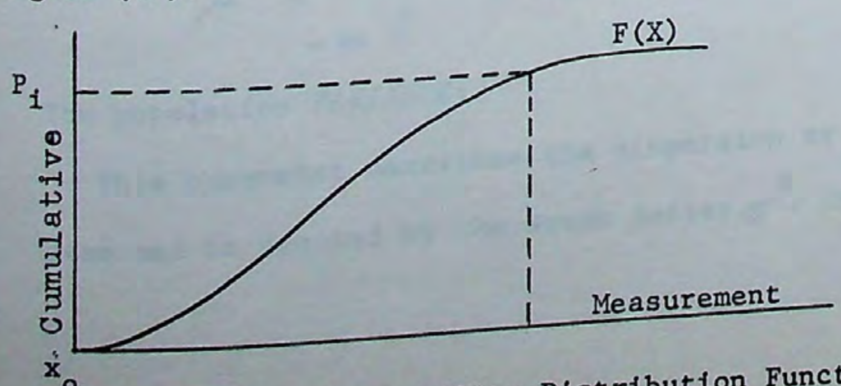


Figure (29): Probability Distribution Function Curve

Measures of Probability Frequency Characteristics:

In simulation studies, a population refers to the total set of numerical elements - measurements - about which knowledge is desired. The important thing about a population is that it should be definable, and once defined, it is considered fixed and is described by a frequency function. A frequency function implies the existence of certain parameters known as *population parameters* which are constant values that never vary within a specified population (3, 36).

The most popular and practical parameters (characteristic features) of a population probability frequency function f(x) are:

1. The population *MEAN*:

   The mean of a random variable x is defined as the first moment or (center of gravity) about the origin (zero measurement) on the frequency function curve f(x). To calculate the mean:

   A. For a discrete distribution

   $$\mu = \sum_{-\infty}^{\infty} x\, f(x)$$

   B. For a continuous distribution

   $$\mu = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

2. The population *VARIANCE*:

   This parameter describes the dispersion or spread about the mean and is denoted by the Greek letter σ^2. It is also sometimes called the *spread* or the *mean square deviation*. The square root of the variance is called the *standard deviation* or *root mean square*.

The variance is given by

A. For a discrete distribution

$$\sigma^2 = \sum_{-\infty}^{\infty}(x - \mu)^2 \; f(x) = \sum_{-\infty}^{\infty} x^2 f(x) - \mu^2$$

B. For a continuous distribution

$$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 \; f(x) \; dx = \int_{-\infty}^{\infty} x^2 f(x) \; dx - \mu^2$$

3. The Expected Value:

The *expected value*, the *weighted average*, or the *mean* of a random variable is the sum of the products of each value that this random variable can assume multiplied by its probability of occurrence. For example, suppose that a certain store deals in a certain item. The daily demand (DD) for this item is a stochastic variable that can assume different values according to different probabilities as shown in the table below.

Daily Demand in units DD	Probability f(DD)
3	0.2
4	0.3
6	0.4
9	0.1
	1.0

Daily Demand and Probability of Occurrence

The expected value EDD is given by:

$(3 \times 0.2) + (4 \times 0.3) + (6 \times 0.4) + (9 \times 0.1)$

$= 0.6 + 1.2 + 2.4 + 0.9 = 5.1$ units

The value 5.1 is not thought of as the value that could occur in any one day, but, if DD was observed for a large number of days, the *average* number of units demanded per day will be equal to 5.1 units. EDD is a function of the random variable DD, and is a parameter characterizing the central tendency of the function.

In symbolic form, the expected value of a random variable x is given by

$$EX = \sum x.f(x)$$

Sample Statistics:

In many cases, it will not be convenient to collect observations from a population and we have to rely on samples from this population. In such cases, we calculate *sample statistics* defined as *values calculated from a sample that may be used to estimate a population parameter* (3, 36).

Since samples from a population are not identical, then sample statistics are not fixed but are distributed. That is to say that a sample statistic is a random variable that has its frequency distribution. The two important sample statistics are:

1. The sample *MEAN*:

Usually denoted by $\bar{x}$ and is given by

$$\bar{x} = 1/n \sum x$$

Where n is the number of measurements x in the sample.

2. The sample *VARIANCE*:

Usually denoted by s^2 and is given by

$$s^2 = 1/n-1 \sum x^2 - \bar{x}^2$$

$\bar{x}$ and s^2 are both estimates of the population mean μ and population variance σ^2 respectively. If n is large, then both $\bar{x}$ and s^2 are unbiased estimates of μ and σ^2. That is to say, if n is large, then, regardless what the original population is (any population), the sampling distribution of $\bar{x}$ is normal with mean= μ (the population mean) and a standard deviation= $\sigma \sqrt{N}$. The standard deviation in this case is referred to as *standard error*. In cases when σ is not known, s may be used instead and the error would be very small. The following table illustrates the relationships between population parameters and sample statistics.

	Population	Sample	Sampling Distribution of $\bar{x}$
Mean	μ	$\bar{x}$	μ
Standard Dev.	σ	s	$\frac{\sigma}{\sqrt{N}}$ $\frac{s}{\sqrt{N}}$

Table (13): The Relationship between Population and Sample Characteristics

Multivariate Probability Distributions:

Thus far, we have been discussing the characteristics of univariate probability distributions. In cases when there are more than one variable that are stochastically independent, e.g., X and Y, we simply apply the multiplication formula. Thus, the probability density function f(x,y) is given by

$$f(x,y) = f(x) \cdot f(y) \quad \text{(which represents a surface)}$$

The probability distribution function is also given by

$$F(X,Y) = \int^{x}\int^{y} f(x,y)\, dx\, dy$$

In distributions with more than two variables x , x , ...x ,

we have

$$F(X_1, X_2, \ldots\ldots, X_n) = \int X_1, X_2, \ldots X_n \int f(x_1, x_2, \ldots., x_n)$$

Where $f(x_1, x_2, \ldots, x_n) = f(x_1) \cdot f(x_2)\ldots\ldots f(x_n) = \prod_{i=1}^{n} f(x_i)$

Table (14) portrays the most popular characteristics of probability distributions.

TYPES OF DISTRIBUTIONS

There are many types of probability distributions for both univariate and multivariate variables. In practice, it has been found that there are only a few types of them that closely approximate the most naturally occurring distributions and thus are mostly used. They are to be classified here according to the type of probability distribution whether it is discrete or continuous.

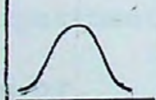
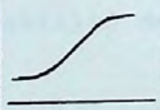
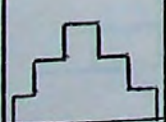
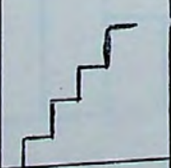
Probability Frequency		Probability Distribution		First Moment	Second Moment
Function	Graph	Function	Graph	(mean)	(variance)
Continuous					
$P(x < x_i < x+dx)$ $= f(x)$ f(x) is the first derivative of F(X)	Frequency versus measurement 	$P(x_i \leqslant x) = F(X)$ $F(X) = \int_a^b f(x)dx$ F(x) is the integral of f(x)	Cumulative percentage 	$\sigma = \int_{-\infty}^{\infty} xf\ (x)dx$ $\bar{x} = \frac{1}{N}\sum x$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)d_2$ $S^2 = \frac{1}{N-1}\sum x^2 - \bar{x}^2$
Discrete					
$P(x < x < x+ x)$ f(x) f(x) is the term of F(X)	Frequency versus measurement 	$P(x \leqslant x) = F(X)$ $F(X) = \sum_a^b f(x)$ F(X) is the Simulation of f(x)	Cumulative percentage 	$\mu = \sum_{-\infty}^{\infty} xf(x)$ $\bar{x} = \frac{1}{N}\sum x$	$\sigma^2 = \sum_{-\infty}^{\infty}(x-\mu)^2 f(x)$ $S^2 = \frac{1}{N-1}\sum x^2 - \bar{x}^2$

Table (14): Characteristics of Probability Distributions
(Source: Francis F Martin, "Computer Modeling and Simulation," John Wiley & Sons, Inc., New York, 1968, pp. 44-45.)

1. Discrete Distributions:

A. The Binomial Distribution:

This type of distribution known sometimes as *Bernoulli distribution* is characterized by the fact that the random variable can assume only *TWO* distinct values, a *success* or a *failure*, occurrence or nonoccurrence, effective or noneffective...etc. The probability of a success - which always refers to the event we are seeking - is always denoted by p and that of a failure is denoted by q. Thus, $p + q = 1$

The probability of getting a success in *n* trials of an experiment, i.e., the probability density function is given by

$$f(s) = \frac{n!}{s!(n-S)!} \quad p^{s} \quad q^{n-s}$$

The graphical representation of a binomial distribution is shown in figure (30).

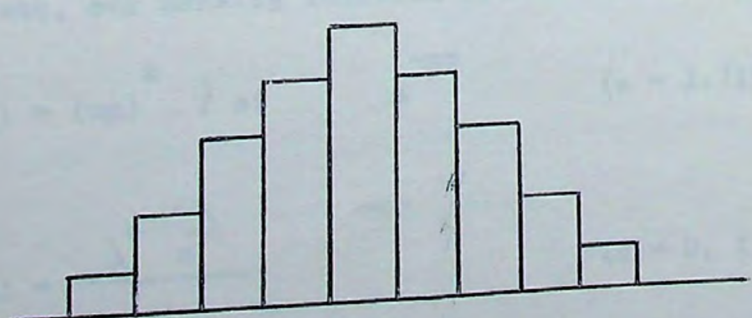


Figure (30): A Binomial Frequency Distribution (No. of Successes in Independent Trials)

The probability distribution function is given by

$$F(k) = \sum_{o}^{s=k} \frac{n!}{s!(n-S)!} p^s q^{n-s} \quad (k = 0, 1, 2, \ldots\ldots, n)$$

The mean and variance are given by the formulas

$$\mu = np$$

$$\sigma^2 = npq$$

$$\sigma = \sqrt{npq}$$

B. The Poisson Distribution:

This distribution is applicable in situations where some kind of event such as a *defect* or *change* which will also be called a *success* occurs randomly in time or over distances, areas, or volumes. The average rate of occurrence of the event is considered constant in a poisson process and is usually denoted by λ. Thus, if we let *s* in the binomial probability distribution remain finite for a large number of trials as *n* approaches infinity and *p* approaches zero in such a manner that the product *np* which is also λ remain constant, our density function becomes

$$f(s) = (np)^s / s! \quad e^{-np} \qquad (e = 2.71828)$$

or

$$f(s) = \frac{\lambda^s e^{-\lambda}}{s!} \qquad (s = 0, 1, 2, \ldots.)$$

That is to say that in the binomial distribution, if *n* is large while *p* a success is close to zero so that q = 1 - p is close to one, then by placing $np = \lambda =$ constant, $q \approx 1$, and $p \approx 0$, we get a poisson distribution. The event in such cases is called a *rare event*. In practice, an event is considered rate if $n \geq 20$ while $np < 5$. In such cases, a binomial distribution is very closely approximated by a poisson distribution. Thus, the poisson distribution can be considered as the limit of the binomial distribution.

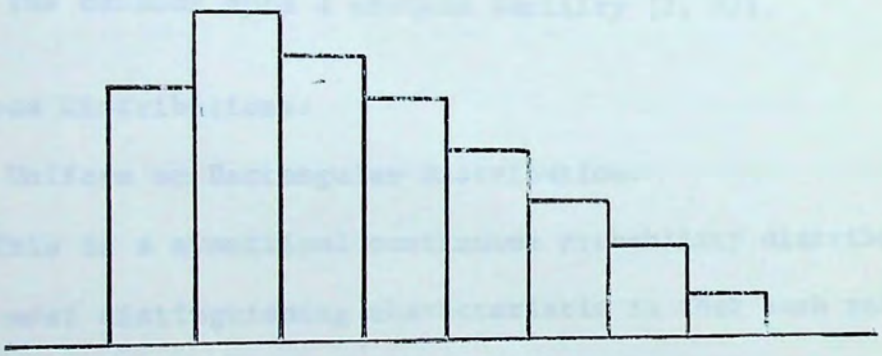


Figure (31): A Poisson Distribution
(Occurrence in an Interval of Time)

The probability distribution function is given by

$$F(m) = \sum_{s=0}^{m} f(s) = \sum_{s=0}^{m} \frac{(np)^{s}}{s!} e^{-np} = \sum_{s=0}^{m} \frac{\lambda^{s} e^{-\lambda}}{s!}$$

Where f(s) is the probability that *m* or fewer successes will occur when the average number of successes is np or .

The mean and variance are given by

$$\mu = np = \lambda$$
$$\sigma^2 = np = \lambda$$

That is, the mean and variance are identical.

Situations which are known to follow the poisson distribution are:

1) The number of defects of a manufactured article.
2) The number of accidents in some unit of time.
3) The number of insurance claims in some unit of time.
4) Arrivals or departures of travelers or customers at a specified point.
5) The demands upon a certain facility (2, 52).

2. Continuous Distributions:

A. The Uniform or Rectangular Distribution:

This is a symetrical continuous probability distribution. Its most distinguishing characteristic is that each value of the random variable has the same probability of occurring. Thus, it is a flat distribution in which the probability density function is given by

$$f(x) = 1 \qquad 0 \leqslant x \leqslant 1$$

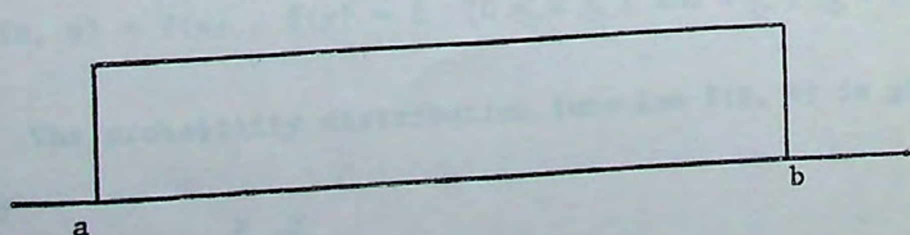


Figure (32): Graphical Representation of the Uniform Distribution (Uniform between a & b which are the Equiprobable point events)

The probability distribution function is given by

$$F(X) = \int_0^x dx$$

As variability in nature is usually nonuniformly distributed, there is probably no physical phenomena that can be described by this distribution. Still, whenever there are purely random choices to be made among several alternatives, the uniform distribution is frequently applied (2, 49), i.e., we assume uniformity when little is known about the distribution. For the same reason, this distribution is mostly used in generating random numbers where each digit has the same probability of occurring between the limits zero and one. The mean and variance between zero and one are given by

$$\mu = \int_0^1 x\ dx = 0.5$$

$$\sigma = \int_0^1 (x - 0.5)^2\ dx = 0.0833$$

$$\sigma = \sqrt{0.0833} = 0.2887$$

Approximately 57.74% of all measurements fall within the limits of $\pm 1\ \sigma$. By the same token, the bivariate uniform probability density function f(x, y) is - according to the multiplication formula - given by

$$f(x, y) = f(x) \cdot f(y) = 1 \quad (0 \leqslant x \leqslant 1 \text{ and } 0 \leqslant y \leqslant 1)$$

The probability distribution function F(X, Y) is given by

$$F(X, Y) = \int_0^x \int_0^y dx . dy = 1$$

In such a case, there is an equal likelihood for any point x, y occurring within the unit square bounded by the points 0.0; 1.0; 1.1; 0.1. Also

$$\mu = \int_0^1 \int_0^1 x\, y\, dx\, dy \qquad \text{and } \mu_x = 0.5 \quad \& \; \mu_y = 0.5$$

$$\sigma_x^2 = 0.0833 \qquad \& \; \sigma_y^2 = 0.0833$$

$$\sigma_x = 0.2887 \qquad \& \; \sigma_y = 0.2887$$

Approximately one third of all measurements fall within the $\pm 1\, \sigma_x$ & $\pm 1\, \sigma_y$ limits.

B. The Normal Distribution:

Also known as the *Gaussian Distribution*. This is one of the most useful distributions that is bell-shaped and is symmetrical about the mean. The two inflection points of the density graph occur at $\mu \pm \sigma$.

The probability frequency function is given by

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}}\, e^{-(x-\mu)^2/2\sigma^2} \qquad (\text{for } -\infty < x < \infty)$$

The graphical representation of this curve is shown in figure (33).

The probability distribution function is given by

$$F(X) = \int_{-\infty}^{x} f(x)dx = \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{x} e^{-(x-\mu)^2/2\sigma^2}$$

In practice, the *Standardized Normal Distribution* or curve is used. For this curve, μ is at the origin and $\sigma^2 = 1$. The variable z replaces x and is given by

$$z = x - \mu / \sigma$$

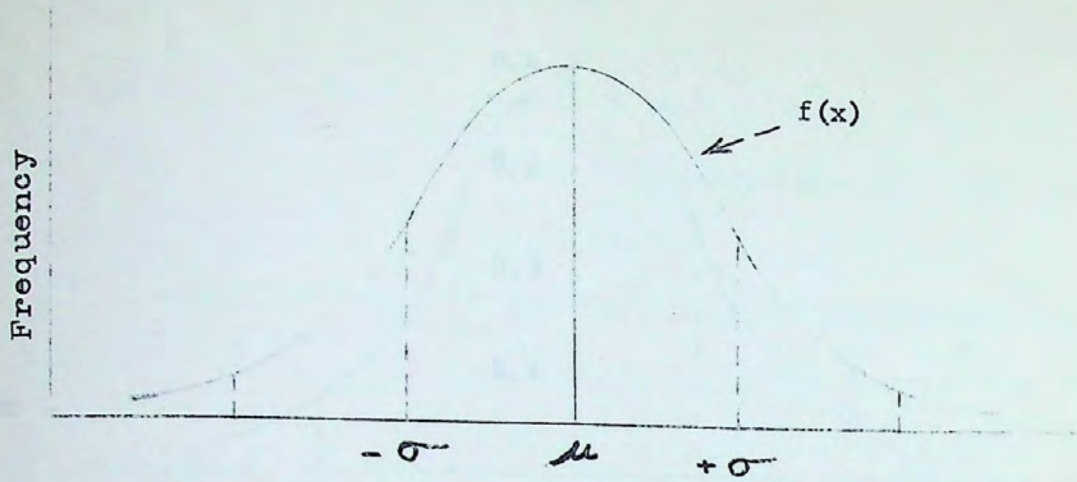


Figure (33): Normal Probability Density (Frequency) Function

And is said that z is normally distributed with mean zero and variance one, thus,

$$f(z) = (1/\sqrt{2\pi})\ e^{-z^2/2}$$

The approximate spread about the mean in units is:

$\pm\ 1\sigma$ includes about 68.3% of all the cases.

$\pm\ 2\sigma$ includes about 95.5% of all the cases.

$\pm\ 3\sigma$ includes about 99.7% of all the cases.

Figure (34) illustrates the shape and characteristics of the standard normal curve.

If n is large, and if neither p nor q is too close to zero, the binomial distribution is closely approximated by a normal distribution with standardized variable z given by

$$z = x - \mu/\sigma = x - np/\sqrt{npq}$$

This approximation becomes better with increasing n and in the limiting case is exact. That is the binomial curve is

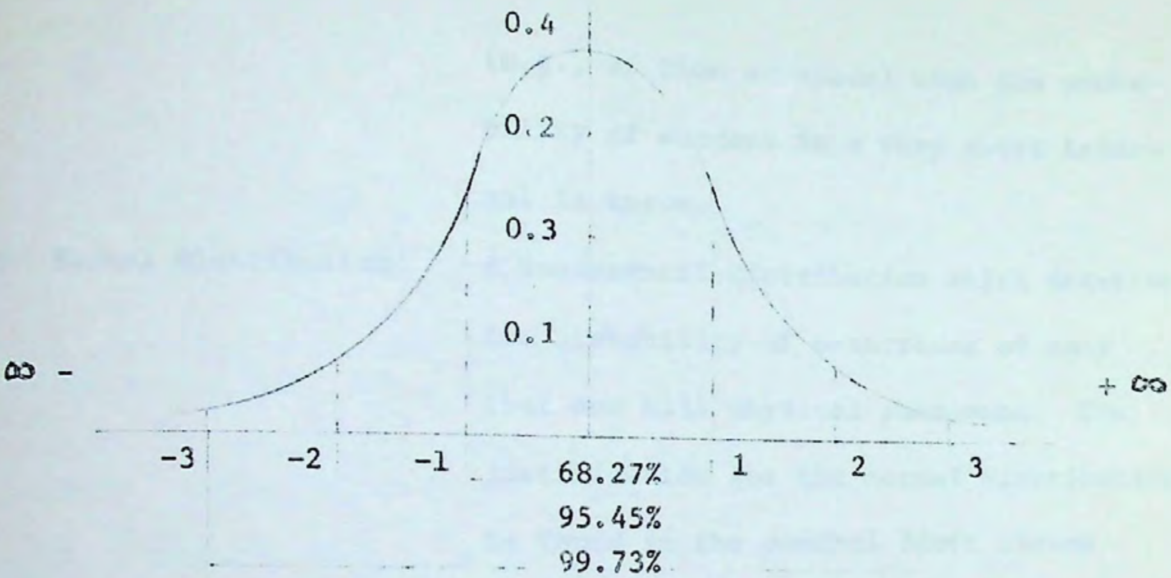


Figure (34): Standard Normal Curve
(Total area under the curve = unity)

the same as the normal when the measurements on the abscissa (z) tends to $\pm\infty$. In practice, the normal is good approximation to the binomial regardless of n, if $np > 5$. (1,153).

Interim Summary:

Having discussed the most popular distributions, we can now summarize their use in:

1. Binomial Distribution: Gives the probability of obtaining exactly x successes in a sample of n trials, where the probability of success on a single trial is known.

2. Poisson Distribution: Gives the probability of the occurrence of exactly x successes in a given interval

(e.g., of time or space) when the probability of success in a very short interval is known.

3. Normal Distribution: A fundamental distribution which describes the probability of occurrence of many (but not all) physical phenomena. The justification for the normal distribution is found in the *central limit theorm* which states that sample means tend to be normally distributed regardless of the nature of the population from whence the samples were drawn.

4. Uniform Distribution: gives the probability of occurrence of any one of several equally likely events.

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