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The American University in Cairo School of Sciences and Engineering

Nonlinear Control for Two Links Flexible Manipulator

By

Mohamed Ibrahim Shalaby

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Robotics Control and Smart Systems (RCSS)

> Under supervision of: Dr. Maki Khalil Habib Professor, Mechanical Engineering Department

> > August, 2017

DEDICATION

Dedicated to my parents, wife and daughters.

ACKNOWLEDGMENTS

I would like to acknowledge my thesis supervisor, Prof. Maki K. Habib for providing me continuous guidance, advice and knowledge in both academic and non-academic sides to help me finish my thesis which without I wouldn't have started or finished.

ABSTRACT

Recently the use of robot manipulators has been increasing in many applications such as medical applications, automobile, construction, manufacturing, military, space, etc. However, current rigid manipulators have high inertia and use actuators with large energy consumption. Moreover, rigid manipulators are slow and have low payload-to arm-mass ratios because link deformation is not allowed. The main advantages of flexible manipulators over rigid manipulators are light in weight, higher speed of operation, larger workspace, smaller actuator, lower energy consumption and lower cost.

However, there is no adequate closed-form solutions exist for flexible manipulators. This is mainly because flexible dynamics are modeled with partial differential equations, which give rise to infinite dimensional dynamical systems that are, in general, not possible to represent exactly or efficiently on a computer which makes modeling a challenging task. In addition, if flexibility nature wasn't considered, there will be calculation errors in the calculated torque requirement for the motors and in the calculated position of the end-effecter. As for the control task, it is considered as a complex task since flexible manipulators are non-minimum phase system, under-actuated system and Multi-Input/Multi-Output (MIMO) nonlinear system.

This thesis focuses on the development of dynamic formulation model and three control techniques aiming to achieve accurate position control and improving dynamic stability for Two-Link Flexible Manipulators (TLFMs). LQR controller is designed based on the linearized model of the TLFM; however, it is applied on both linearized and nonlinear models. In addition to LQR, Backstepping and Sliding mode controllers are designed as nonlinear control approaches and applied on both the nonlinear model of the TLFM and the physical system.

The three developed control techniques are tested through simulation based on the developed dynamic formulation model using MATLAB/SIMULINK. Stability and performance analysis were conducted and tuned to obtain the best results. Then, the performance and stability results obtained through simulation are compared. Finally, the developed control techniques were implemented and analyzed on the 2-DOF Serial Flexible Link Robot experimental system from Quanser and the results are illustrated and compared with that obtained through simulation.

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NOMENCLATURE

- ρ_1 Link-1 Mass Density, Kg/m
- ρ_2 Link-1 Mass Density, Kg/m
- l_1 Link-1 Length, m
- l_2 Link-2 Length, m
- m_1 Link-1 Mass, Kg
- m_2 Link-2 Mass, Kg
- *m*_{h2} Second Hub Mass, Kg
- *m_p* Payload Mass, Kg
- J_{o1} Link-1 inertia, Kg.m²
- J_{o2} Link-2 inertia, Kg.m²
- J_{h1} Link-1 Hub inertia, Kg.m²
- J_{h2} Link-2 Hub inertia, Kg.m²
- J_p Payload inertia, Kg.m²
- (EI)₁ Link-1 Flexural Rigidity, N.m²
- $(EI)_2$ Link-2 Flexural Rigidity, N.m²
- ε_1 Deflection in Link-1, m
- ε_2 Deflection in Link-2, m
- T Total kinetic energy, J
- T_{h1} The kinetic energy at Link-1 Hub, J
- T_{h2} The kinetic energy at Link-2 Hub, J
- T_{l1} The kinetic energy of Link-1, J
- T_{l2} The kinetic energy of Link-2, J
- T_p The kinetic energy of payload
- U Total elastic energy, J
- U_1 The elastic energy stored in Link-1, J
- U_2 The elastic energy stored in Link-2, J

Acronyms

FM Flexible Manipulator

TLFM Two-Link Flexible Manipulator

AMM Assumed Mode Method

- FEM Finite Element Method
- LPM Lumped Parameter Model
- LQR Linear Quadratic Regulator
- SMC Sliding Mode Controller

MIMO Multiple-Input/Multiple-Output

HIL Hardware In the Loop

Chapter 1

Introduction

1.1 Overview

In recent years, the use of industrial robots and service robots in different applications from simple to dangerous tasks in hazardous areas has increased, such as medical applications, automobile, construction, manufacturing, military, space, etc. Robotic manipulators are used to perform accurate tasks such as assembly/disassembly, sorting, packaging, palletizing, depalletizing, welding, etc. In addition, they are used to perform dangerous tasks in hazardous areas such as space and underwater, and in the presence of radiation [1], [2].

Most of the existing robotic manipulators are designed and built to maximize stiffness in an attempt to minimize the vibration of the end-effector and to achieve good position accuracy. This high stiffness is achieved by using heavy material and a bulky design. Hence, the existing heavy rigid manipulators are shown to be inefficient in terms of power consumption or speed with respect to the operating payload. Also, for a high speed of the task the operation of high precision robots is severely limited by their dynamic deflection, which persists for a period of time after a move is completed this is due to the vibration occurs in the manipulators' links, Figure 1.1 shows an example rigid link manipulator. Accordingly, the time needed for the vibration to vanish called settling time. The settling time required for this residual vibration delays subsequent operations, thus conflicting with the demand of increased productivity, this delay depends on the size of the robot and the speed of its motion. These conflicting requirements between high speed and high accuracy have rendered the robotic assembly task a challenging research problem. Also, many industrial manipulators face the problem of arm vibrations during high speed motion [3]-[5].

In order to improve industrial productivity, it is required to reduce the weight of the arms and/or to increase their speed of operation. For these purposes it is very desirable to build flexible robotic manipulators. Compared to the conventional heavy and bulky robots, flexible link manipulators have the potential advantage of lower cost, larger work volume, higher operational speed, greater payload-to-manipulator-weight ratio, smaller actuators, lower energy consumption, better maneuverability, better transportability and safer operation due to reduced inertia. But the greatest disadvantage of these manipulators is the vibration problem due to low stiffness, Figures 1.2 and 1.3 show an example for flexible link manipulators. Due to the importance and usefulness of this field, researchers worldwide are nowadays engaged in the investigation of dynamics and control of flexible manipulator [3], [6]-[8].



Figure 1.1. An example for rigid link manipulator [9]



Figure 1.2. An example for single link flexible manipulators [10]



Figure 1.3. An example for two-link flexible manipulators [11]

The properties and capabilities provided by flexible manipulators stand for a clear challenge in opening new applications for robots. Situations, where the workspace is constrained, or when it is required to perform operations such as assembly in space, prevents the use of classical, rigid-link, industrial robot configurations. For these applications structural mass and stiffness must be reduced, to allow entering very confined workspaces and/or to permit costeffective launching, and to enlarge manipulator reach out and dexterity. This could be of interest not only in space applications, but also in the industrial sector [12].

Some known examples are the application of fast, flexible manipulators in the food industry (robotic packing and palletizing) and in assembly tasks. Flexibility is also becoming an important issue for other fields such as machine tools and civil engineering machinery, for example, tunnel boring machines, excavators, and so on, where requirements for extending tools life, increasing accuracy and speeding up overall. Other potential areas of application are manipulation in nuclear and other hazardous environments, car/vehicle painting, manufacturing of electronic hardware and food industry [13].

Research on flexible link manipulators has a range from a single-link manipulator rotating about a fixed axis to three-dimensional multi-link arms. However, experimental work, in general, is almost exclusively limited to single-link manipulators. This is because of the complexity of multi-link manipulator systems, resulting from more degrees of freedom and the increased interactions between gross and deformed motions. It is important for control purposes to recognize the flexible nature of the manipulator system and to build a suitable mathematical framework for modeling it. The use of dynamic models for flexible link manipulators is threefold: forward dynamics, inverse dynamics and controller design. Flexible manipulators are distributed parameter systems that can be represented as rigid body with flexible movements [12], [13]. There are two physical limitations associated with such systems:

- a. The control torque can only be applied at the joints, and
- b. Only a finite number of sensors of bounded bandwidth (maximum frequency it can operate) can be used and at restricted locations along the links of the flexible manipulators.

The control of flexible manipulators to maintain accurate positioning is extremely challenging. Due to the flexible nature of the system, the dynamics are highly non-linear and complex. Problems arise due to lack of sensing as there should be finite numbers of sensors available with its infinite degree of freedom, vibration due to system flexibility, imprecise positional accuracy and the difficulty in obtaining accurate model for the system. Therefore, flexible manipulators have not been favored in production industries, due to un-attained endpoint positional accuracy requirements in response to input commands. In this respect, a control mechanism that accounts for both the rigid body and flexural motions of the system is required.

Moreover, the complexity of this problem increases when a flexible manipulator carries a payload. Practically, robot manipulators are required to perform a single or sequence of tasks along a pre-planned trajectory. The complexity of the problem increases dramatically for multi-link flexible manipulators such as two-link flexible manipulator [9], [13], [14].

The behavior of single flexible link manipulators leads to limited performance for the manipulator. Thus, the performance of the multi-link flexible manipulators becomes reliable through the utilization of efficient controllers. These manipulators can be a promising substitute for the multilink rigid manipulators. Nonetheless, the controller design for multilink flexible manipulators is a challenging task due to their non-linear dynamic formulation. This non-linear dynamics is perhaps among the main reasons that few studies on the controller design of multi-link flexible manipulators have been reported compared to those for single flexible link manipulators with a linear dynamic model [12], [15].

Two link Flexible Manipulators (TLFMs) are more suitable in industry, aerospace, nuclear plant, military, defense, agriculture, home care, etc., in comparison with single link and multi-link FMs. Thus, it is interesting and important to present the extensive and exclusive review on different aspects of dynamical complexities, modeling, control problems and control techniques reported on TLFMs [13].

1.2 Thesis Structure

This thesis is organized into four main parts. Literature review on both modeling and control of flexible manipulators is discussed in Chapter 2. Chapter 3 shows the research methodology. Dynamic modeling of the system is explored in Chapter 4. Control techniques are considered in Chapter 3. Chapter 6 presents results and discussion. Finally, Chapter 7 shows the conclusion and future work.

Chapter 2

Literature Review

2.1 Modeling

There are two kinds of errors introduced when the flexibility effect is not considered during the formulation of mathematical model. The first kind of error is introduced in the torque requirement for the motors and the second kind results in the positioning inaccuracy of the end-effector. The positioning of the end-effectors for precision jobs should involve very small amplitudes of vibration, ideally no vibration at all. Therefore, to achieve greater accuracy one has to start with very accurate mathematical models for the system [3], [16].

The original dynamics of flexible link manipulators, being described by partial differential equations and thus possessing an infinite dimension(Flexible manipulators are distributed parameter systems, hence infinite degrees of freedom are required to characterize the dynamic behavior of the system) is not available to be used directly in both system analysis and control design. Most commonly the dynamic equations are truncated to some finite dimensional models with either the assumed modes method (AMM) or the finite element method (FEM) [3], [17].

In modeling flexible link manipulators, the most widely used methods to develop manipulators dynamics are based on energy principles and Lagrange's equations. Energy principle-based models of flexible manipulators are spatially continuous of infinite order. To generate spatially discrete models the assumed-mode method (AMM), the Finite-Element Method (FEM) or Lumped Parameter Model (LPM) are generally used [13], [18].

The AMM and the FEM methods use either the Lagrangian formulation or the Newton–Euler recursive formulation. According to [12], modeling can be classified as follow:

- a. Lagrange's equation and AMM,
- b. Lagrange's equation and FEM,
- c. Euler–Newton equation and AMM, and
- d. Euler-Newton equation and FEM,

The Lagrangian Mechanics and the assumed mode method have been used to drive a proposed dynamic model of a single link flexible manipulator having a revolute joint. The link has been considered as an Euler–Bernoulli beam subjected to large angular displacement [17].

LPM models the system as a lump of masses and massless spring. This method is the simplest among the three modeling methods, but generally it does not give accurate result. It includes two approaches that define the characteristic of constrained mass and spring. First, an experimental approach that uses Holzer method and an experimental approach that uses a train of experiments to define the parameters of a flexible link, as a second approach [13]

A common way to describe a flexible link is by means of the Euler–Bernoulli equation. This model is valid under the assumptions that: (i) the link is slender with uniform geometric and inertial characteristics, (ii) the link is flexible in the lateral directions and stiff with respect to axial forces and to torsion and bending forces due to gravity, and (iii) nonlinear deformation and friction can be neglected. The Euler–Bernoulli beam with linear kinematics is a linear infinite-dimensional model which takes into account only perpendicular deformation with respect to an unstressed reference configuration [18].



Figure 2.1. Euler-Bernoulli beam [4]

Then, the Euler–Bernoulli equation that is shown in Figure 2.1 for the link is given as follows:

$$(EI)\frac{\partial^4 p(x,t)}{\partial x^4} + \rho \frac{\partial^2 p(x,t)}{\partial t^2} = 0.2$$
(2.1)

P uniform mass/unit length

y(x,t) is the the deflection

P(x, t) represent the position of a point on the flexible arm

EI is flexural rigidity

 m_t tip payload

 I_h is the hub moment of inertial

T is the torque applied to the link

In AMM formulation, the elastic deflection of the beam is represented by an infinite number of separable frequency modes. Since the low frequency modes are dominant in the system's dynamics, and controllers/actuators generally act as low-pass filters, the modes are truncated to a finite number to obtain a system with a finite dimensionality.

The main drawback of AMM method is the difficulty in finding modes for links with non-regular cross sections and multi-link manipulators [13].

Many authors used the finite element method where the elastic deformations are analyzed by assuming a known rigid body motion and later superposing the elastic deformation with the rigid body motion. In order to solve a large set of differential equations derived by the finite element method, a lot of boundary conditions have to be considered, which are, in most situations, uncertain for flexible manipulators. Using the assumed mode method to derive the equations of motion of the flexible manipulators, only the first several frequency modes are usually retained by truncation and the higher modes are neglected [3], [19].

In the Lagrange's equation and AMM the deflection of the flexible link manipulator is represented as a summation of modes. Each mode is assumed as a product of two functions; one dependent on the distance along the length of the manipulator, and the other, a generalized coordinate, dependent on time. In principle, the summation amounts to an infinite number of modes should be taken into consideration. However, for practical purposes, a small number of modes are used. The Lagrange's equation and FEM is conceptually similar to the above AMM method. Here the generalized coordinates are the displacement and/or slope at specific points (nodes) along the flexible link manipulator [12], [16].

In FEM method, the behavior of the elasticity of a flexible manipulator is first observed on a rigid body motion in which the elastic deformation is then superimposed. The main advantage of FEM over the other approximation methods in the modeling of flexible links is that the connections in FEM are supposed to be clamped-free with minimum two modes shape per link. Another important advantage of FEM, especially over analytical solution methods is its easiness of handling with nonlinear conditions. The FEM can also handle irregularities in the structure and mixed boundary conditions, i.e., it is suitable in handling applications involving irregular structures. Though there are many advantages, including the generalized coordinates where each coordinate has its physical meanings, the concept of natural frequency is lost. Moreover, analyzing approximate flexibility with FEM gives rise to an overestimated stiffness matrix especially in complex problems. However, due to the large state space equations involved, the numerical simulation time may be exhausting for FEM models [13].

The Euler–Newton's method is a more direct means of calculating system dynamics. The rate of change of linear and angular momentum is derived explicitly in this method, rather than via Lagrange's equation. Newton's second law is used to balance these terms with the applied forces. In simulation, the linear and angular momentums of the manipulator are unknown while the actuator forces are known [20].

Expressing the former in terms of a set of assumed modes or finite elements leads to a dynamic model relating the time dependency of the modes/elements to the external forces.

The basic approach in the Euler–Newton and AMM is to divide the manipulator into a number of elements and carry out a dynamic balance on each element. For a large number of elements this is a very tedious process. On the other hand, it is far easier to include non-linear effects without complicating the basic model [12].

2.1.1 Single Link

Although the AMM has been widely used, there are several ways to choose link boundary conditions (such as geometric contrarians and maximum motor torques) and number of selected modes. Many researchers studied single-link flexible manipulators using Lagrange's equation and the AMM. However, fewer researches used a Newton–Euler formulation to model a single-link flexible manipulator. Though many researchers studied flexible link manipulators with revolute joints, only few works are reported on prismatic joints [3], [16]. Similar to AMM, FEM was applied to study single-link flexible manipulators using Lagrange's equation and the FEM [21].

2.1.2 Two Links

Modeling of two link flexible systems started without taking bending-torsion of the flexible link into account, and then it was taken into consideration for system with two rotary joints. Afterwards, AMM was used widely with the Lagrange's equation after showing that the conventional Lagrange's equation for rigid manipulators is not accurate in modeling the system [3], [22].

FEM was used for two links systems and for multi-link systems using both Lagrange's and Newton-Euler equations. A decoupling method was proposed studying both links independently without transferring vibration from the first link to the second link. Most of this work was made for planer two flexible link manipulators [3], [21]

2.2 Control

According to [2], [13], the available literature addresses a wide range of topics, related to the control of flexible link manipulators and this involves:

- a. Tip Position Control,
- b. Tip Tracking Control,
- c. Contact Force Control, and
- d. Observer Design

Control techniques was first applied to control the end-effector of a single flexible manipulator by measuring the tip position and using that measurement as a basis for applying torque to joint at the base of the beam. However, they only considered a linearized model and also the arm can sweep only in the XY plane, so that it is not affected by the gravity [3], [18].

Vibration control techniques for flexible link manipulators are generally classified into two categories: **passive** and **active** control. **Passive control** utilizes the absorption property of matter and thus is realized by a fixed change in the physical parameters of the structure, for example, adding a layer of viscoelastic material on the flexible link in order to increase the damping properties of the flexible manipulator . **Active control** of flexible link manipulators can in general be divided into two categories; open-loop and closed-loop control.

Open-loop control involves altering the shape of actuator commands by considering the physical and vibration properties of the flexible link manipulators. The approach may account for changes in the system after the control input is developed. Closed-loop control differs from open-loop control in that it uses measurements of the system state and change the actuator input accordingly to reduce the system response oscillation [12], [23].

Open-loop control methods involve the development of suitable forcing functions in order to reduce the vibration at resonance modes. The methods developed include shape command methods, the computed torque technique and bang-bang control [12], [23].

- **a.** Shaped command methods attempt to develop forcing functions that minimize vibrations and the effect of parameters that affect the resonance modes. Common problems of concern encountered in these methods include long move (response) time, instability owing to un-reduced modes and controller robustness in the case of a large change of the manipulator dynamics.
- **b.** Computed torque approach, depending on the detailed model of the system and desired output trajectory, the joint torque input is calculated using a model inversion process. The technique suffers from several problems, owing to, for instance, model inaccuracy, uncertainty over implement ability of the desired trajectory, sensitivity to system parameter variations and response time penalties for a causal input.
- **c. Bang-bang control** involves the utilization of single and multiple switched bangbang control functions. Bang-bang control functions require accurate selection of witching time, depending on the representative dynamic model of the system. A minor modeling error could cause switching error and thus result in a substantial increase in the residual vibrations.

As per [3], [13], [17], the **Closed-loop** control methods include

- a. PID Control,
- b. Feedback Linearization
- c. Observer Based Control
- d. Adaptive Control,
- e. Fuzzy Logic Control
- f. Neural Network Based Control,
- g. Backstepping Control,
- h. Sliding Mode Control,
- i. LQR/LQG Based Control
- j. H_inf control.

2.2.1 Single Link

In [24] an augmented Sliding Mode Control (SMC) technique was investigated for slewing flexible manipulators shown in Figures 2.2 & 2.3.



Figure 2.2. Flexible single link with harmonic drive geometry [24]



Figure 2.3. Flexible single link with harmonic drive test setup [24]

A conventional sliding surface uses a first order system including a combination of error and error rate terms. The augmented sliding surface includes an enhanced term that helps to reject flexible degrees-of-freedom; the novel aspect of this method is that the flexible body generalized accelerations are neglected in the control law development. The controller was still shown to be stable in the presence of flexible body generalized accelerations, unmodeled dynamics, disturbances and model uncertainties. The algorithms are theoretically developed and experimentally tested on a revolute single flexible link robot. Results are shown in Figures 2.4 & 2.5 showing that augmented SMC has less oscillation and better settling time



Figure 2.4. Simulation results [24]



Figure 2.5. Experimental results [24]

In [25], two robust non-linear controllers have been developed in this study to control the rigid and flexible motions of a single-link robotic manipulator. The controllers consist of a conventional sliding mode controller (CSMC) and a fuzzy sliding mode controller (FSMC). The parameters of FSMC are determined by fuzzy inference systems, and it has been designed herein based on two Lyapunov functions. Results are shown in Figure 2.6.



Figure 2.6. Simulation Results of Angular Displacement for FSMC and CSMC [25]

In [26] describes an adaptive neuro-fuzzy control system for controlling a flexible manipulator with variable payload. The system schematic is shown in Figure 2.7.



Figure 2.7. Schematic representation of the flexible manipulator system [26]

The controller proposed by [26]constitutes a fuzzy logic controller (FLC) in the feedback configuration and two dynamic recurrent neural networks in the forward path. A dynamic recurrent identification network (RIN) is used to identify the output of the manipulator system, and a dynamic recurrent learning network (RLN) is employed to learn the weighting factor of the fuzzy logic. It is envisaged that the integration of fuzzy logic and neural network based-controller will encompass the merits of both technologies, and thus provide a robust controller for the flexible manipulator system. Results are shown in Figure 2.8.



Figure 2.8. Step response, (a) response of the system (b) identification output of the system [26]

In [10] proposed a control strategy with friction compensation using neural networks to control single link flexible manipulator using a conventional nonlinear harmonic drive actuator. Experimental system layout is shown in Figure 2.9.



Figure 2.9. Experimental system layout [10]

The proposal consists of the utilization of a control law in parallel to a nonlinear friction compensation mechanism based on NNs. Results are shown in Figure 2.10.



Figure 2.10 Closed- and open-loop experimental results [10]

In [4] a modified PID control (MPID) is proposed which depends only on vibration feedback to improve the response of the flexible arm by measuring the joint angle, joint velocity and the tip deflection. Experimental setup is shown in Figure 2.11.



Figure 2.11. Experimental setup [4]

The arm moves horizontally by a DC motor on its base while a tip payload is attached to the other end. A simulation for the system with both PD controller and the proposed MPID controller is performed.

The robustness of the proposed controller is examined by changing the loading condition at the tip of the flexible arm. The MPID control which is a modification of the classical PID controller by replacing the classical integral term with a vibration feedback term to include the effect flexible modes of the beam in the generated control signal. Results are shown in Figures 2.12 & 2.13.



Figure 2.12 Tip deflection for step response of angle 15[4]



Figure 2.13 Joint angle for step response of angle 15 [4]

In [27] A two-stage generalized proportional integral GPI-controller design scheme is proposed in connection with an online closed-loop continuous-time estimator of the natural frequency of a flexible robot. This methodology only requires the measurement of the angular position of the motor and the coupling torque. The experimental setup is shown in Figure 2.14.



Figure 2.14. Flexible manipulator experimental setup [27]

The proposed controller is for the control of an uncertain flexible robotic arm with unknown mass at the tip, including a Coulomb friction term in the motor dynamics. A fast nonasymptotic algebraic identification method developed in continuous time is used to identify the unknown system parameter and update the designed certainty equivalence GPI controller. Results are shown in Figure 2.15 & 2.16.



Figure 2.15. Online estimation of ω [27]



Figure 2.16. Trajectory tracking [27]

In [28] presented design and development of a robust control based on linear quadratic regulator (LQR) for a flexible link manipulator, the system structure is shown in Figure 2.17.



Figure 2.17. Flexible link structure [28]

System performances were evaluated in terms of input tracking capability of hub angular position response, end-point displacement, end-point residual and joint velocity of the single link.

For the controller of the system, LQR was developed to solve flexible link robustness by handling the vibration and perform input tracking capability of angular position of the link. Results are shown in Figure 2.18.



Figure 2.18. Hub position for step input [28]

In [29] an adaptive fuzzy output feedback approach is proposed for a single-link robotic manipulator coupled to a brushed direct current (DC) motor with a non-rigid joint. The controller is designed to compensate for the nonlinear dynamics associated with the mechanical subsystem and the electrical subsystems while only requiring the measurements of link position. Using fuzzy logic systems to approximate the unknown nonlinearities, an adaptive fuzzy filter observer is designed to estimate the immeasurable states. The result of the output trajectory is shown in Figure 2.19.



Figure 2.19. Trajectories of y (solid line) and yr (dash-dotted) [29]

In [8] developed adaptive position control with low-pass and band-stop filtered input preshaping vibration controllers have and realized in a closed loop joint based configuration for a single-link flexible manipulator which is shown in Figure 2.20.



Figure 2.20. Single-Link flexible system [8]

The filter based vibration controllers have been developed on the basis of the resonance modes of the system which represents the dominant vibration modes of the system. The filter is used to filter out the input energy at the dominant vibration modes of the system so that the manipulator is not excited at those frequencies. Two alternative approaches can be adopted to filter out the input energy at natural frequencies of the system. The first method is to pass the input torque through a low-pass filter.

This will attenuate energy input at all frequencies above the filter cut-off frequency. The second method to remove input energy at system natural frequencies is to use (narrow-band) band-stop filters with center frequencies. Results are shown in Figure 2.21.



Figure 2.21. The hub-angle (without payload): (a) without filter (b) Low pass filter (c) Band-pass filter [8]

In [17] the Lagrange mechanics and the assumed mode method have been used to derive a proposed dynamic model of a single link flexible manipulator having a revolute joint. Photo and schematic of the experimental setup is shown in Figure 2.22.



Figure 2.22. Photo and schematic diagram of the experimental setup of the flexible system [17]

The proposed model has been used to investigate the effect of two main design parameters, the payload, and the open loop control torque profile. The results of the investigation show that as long as the rest-to-rest rotational maneuver is considered, the payload has a dominant effect on the elastic deflection of the manipulator. Simulation and experimental results are shown in Figure 2.23.



Figure 2.23 Simulated (a) and experimental (b) results for SDRE controller [17]

2.2.2 Two Links

In [30] proposed a nonlinear control law and adaptive control law has been presented for the motion control of flexible manipulators. The system experimental setup is shown in Figure 2.24.



Figure 2.24. Two-Link flexible manipulator [30]

Asymptotical stability of the closed-loop system has been guaranteed by using the wellknown Lyapunov theory. Experiments for a two-link flexible arm have realistically demonstrated the effectiveness of the proposed schemes.

In addition, some highly oscillatory behavior of the vibration modes can be observed when the motor rotates, which is due to the difficulty of obtaining perfect state measurements and/or the existence of high frequency external noise from the air-injection device, this device injects air towards the link to represent external noise source. Results are shown in Figure 2.25.

However,[31] showed that the applied technique[30] can produce unexpected stability results.


Figure 2.25. (a) Trajectory response of joint 1. (b) Trajectory response of joint 2- (c) Tracking error of joint 1. (d) Tracking error of joint 2[30]

In [5] adaptive energy-based robust control was presented for both closed loop stability and automatic tuning of the gains of additional control terms to the conventional PD controller for multi-link flexible manipulator shown in Figure 2.26.



Figure 2.26. Geometry of multi-link flexible manipulator [5]

The control objective is to rotate each link of the robot to the desired angular position and simultaneously suppress the residual vibration. Simulation was carried out on two-link Flexible manipulator. Results are shown in Figure 2.27.



Figure 2.27. Results for adaptive-energy based robust control (ABRC) and for energy based robust control (EBRC) with three different gains [5]

In [32] a two-link flexible manipulator was controlled by three methods and the results are compared. The system layout is shown in Figure 2.28.



Figure 2.28. Planar two-link flexible arm [32]

The three applied methods are PD control, PD control augmented by a nonlinear correction term feedback, where the correction term is a function of the deflection of each link, and an adaptive fuzzy controller with the nonlinear correction term feedback. Results are shown in Figures 2.29, 2.30 and 2.31.



Figure 2.29. Deflection of the shoulder link and the elbow link for the pure PD [32]



Figure 2.30. Deflection of the shoulder link and the elbow link for the PD plus nonlinear deflection feedback controller [32]



Figure 2.31. Deflection of the shoulder link and the elbow link for the hybrid control method [32]

In [6] a sliding mode controller is proposed for a two-link flexible manipulator to address its non-minimum phase characteristics using the output redefinition method. The manipulator is decomposed into two parts by input-output linearization, namely, an input-output subsystem and the zero dynamics of the overall system respectively. A sliding mode control strategy is designed to make the input-output subsystem converge to their equilibrium points in finite time. Results are shown in Figures 2.32 and 2.33.



Figure 2.32. Tip position of first arm [6]



Figure 2.33. Tip position of second arm [6]

In [33] a robust control method of a two-link flexible manipulator with neural networks based quasi-static distortion compensation is proposed and experimentally investigated. The experimental setup is shown in Figure 2.34.



Figure 2.34. Experimental setup two-link flexible manipulator [33]

The dynamics equation of the flexible manipulator is divided into a slow subsystem and a fast subsystem based on the assumed mode method and singular perturbation theory. A decomposition based robust controller is proposed with respect to the slow subsystem, and H1 control is applied to the fast subsystem. The overall closed-loop control is determined by the composite algorithm that combines the two control laws. Furthermore, a neural network compensation scheme is also integrated into the control system to compensate for quasi-static deflection. Tracking error in X and Y direction are shown in Figures 2.35 and 2.36.



Figure 2.35. Tracking error in X direction [33]



Figure 2.36. Tracking error in Y direction [33]

In [7] proposed combined control strategy based on neural network (NN) and the concept of sliding mode control (SMC) systematically. The experimental two-link flexible system is shown in Figure 2.37.



Figure 2.37. Experimental two-link flexible manipulator [7]

The chattering phenomenon in conventional SMC is eliminated by incorporated a saturation function in the proposed controller, and the computation burden caused by model dynamics is reduced by applying a two-layer NN with an analytical approximated upper bound, which is used to implement a certain functional estimate. In addition, the Lyapunov analysis can guarantee the signals of closed-loop system bounded (will not go to infinity) and the online NN adaptive laws can make the system states converge to the sliding surface. Tip tracking errors for first and second link are shown in Figures 2.38 and 2.39.



Figure 2.38 Tip Tracking error of the first link [7]



Figure 2.39. Tip tracking error of the second link [7]

In [34] a neuro-sliding-mode control (NSMC) strategy was developed to handle the complex nonlinear dynamics and model uncertainties of flexible-link manipulators.



Figure 2.40. Two-link flexible manipulator structure [34]

The designed composite controller was based on a singularly perturbed model of flexible-link manipulators when the rigid motion and flexible motion are decoupled. The NSMC is employed to control the slow subsystem (rigid motion) to track a desired trajectory with a traditional sliding mode controller to stabilize the fast subsystem (flexible motion) which represents the link vibrations. Results for the error in control for joints 1 and 2 are shown in Figure 2.41 and 2.42.



Figure 2.41. Error of joint angle 1 for both Sliding Mode Control (SMC) and Neuro-Sliding Mode control (NSMC) [34]



Figure 2.42. Error of joint angle 2 for both Sliding Mode control (SMC) and Neuro-Sliding Mode control (NSMC) [34]

In [35] assessed the dynamic model and proposed control for two-flexible system shown in Figure 2.43.



Figure 2.43 Structure of two-link flexible manipulator [35]

The proposed controller is radial basis function neural network (RBFNN) controller for solving flexible link vibration, achieve high-precision position tracking, and payload effect robustness are shown in Figures 2.44 & 2.45.



(b) Link-2.

Figure 2.44. Angular position of the system without load [35]



(b) Link-2.

Figure 2.45. Angular position of the system with incorporating payload 0.1 kg [35]

In [36] a hybrid control scheme consisting of a fuzzy nonsingular terminal sliding mode (NTSM) controller and a genetic algorithm, was proposed for the tip-position control of an uncertain two-link flexible manipulator. By the designed fuzzy NTSM controller, the inputoutput subsystem is guaranteed of fast convergence, strong robustness and perfect capability of eliminating chattering as shown in Figure 2.46.



Figure 2.46. Tip position control for first link (y1) and second link (y2) [36]

In [22] presented the dynamic modeling and active vibration control of planar multilink manipulators having flexible links. An LQG controller with a KBF has been proposed for two-link flexible manipulator. The Multi-link flexible manipulator layout is shown in Figure 2.47.



Figure 2.47. Multilink flexible manipulator [22]

Optimal control theory has been employed to develop a vibration suppression strategy based on the use of collocated sensor/actuator pairs, which can represent ideal piezoelectric pairs and LQG controller with a KBF has been proposed. Results are shown in Figure 2.48.



Figure 2.48. End-effector displacement for two link flexible manipulator, thin-line is open loop and bold line is closed loop [22]

Chapter 3

Research Methodology

3.1 Research Challenges

Flexible manipulators have been an active field of research for some time now. However, despite nearly two decades of research, adequate closed-form solutions do not exist. This is mainly because flexible dynamics are modeled with partial differential equations, which give rise to infinite dimensional dynamical systems that are, in general, not possible to represent exactly or efficiently on a computer. In addition, in practice, the sensors are always installed at the boundaries of flexible links so that the spatially distributed positions and velocities are not measurable via these sensors [2].

The major need for the flexible manipulators arises for improving the industrial productivity and for space application by achieving the following [3], [6]:

- a. Reduce the weight of the arms,
- b. Increase their speed of operation,
- c. Advantage of lower cost,
- d. larger work volume,
- e. Greater payload-to-manipulator-weight ratio,
- f. Smaller actuators needed,
- g. Lower energy consumption,
- h. better maneuverability and better transportability, and
- i. Safer operation due to reduced inertia

Some of the limitations of associated with flexible manipulator [13]:

- a. Control complexity mainly due to:
 - i. Non Minimum Phase system, and
 - ii. Underactuation Problem
- b. Uncertainties due to Truncation of flexible modes, and
- c. MIMO and Nonlinear System.

Testing on a physical system is crucial since flexible robotics is normally referred to single link systems. Two link systems are still in the experimental stage, and research remains to be done with experimental systems in order to gain a better understanding of the dynamics and control issues facing multiple link flexible robots [3], [13].

3.2 Objectives

This thesis focused on the development of applying nonlinear control techniques for position control of a flexible manipulator system that hasn't been widely used in the literature. Controllers developed through simulations using MATLAB/SIMULINK, and once stability and good performance had been achieved, the controllers were transferred onto an experimental system.

The objectives of this thesis are:

- a. Investigate the available model of two links flexible manipulators and develop their mathematical models,
- b. Investigate and compare different control techniques such as LQR, Backstepping and Sliding Mode control
- c. Apply control technique on simulation environment.
- d. Hardware testing using the available 2DOF Flexible Manipulator System, and
- e. Analyze and compare results for simulation and experimental results

3.3 Methodology

This thesis focuses on the development of dynamic formulation model and three control techniques aiming to achieve accurate position control and improving dynamic stability for Two-Link Flexible Manipulators (TLFMs). LQR controller is designed based on the linearized model of the TLFM; however, it is applied on both linearized and nonlinear models. In addition to LQR, Backstepping and Sliding mode controllers are designed as nonlinear control approaches and applied on both the nonlinear model of the TLFM and the physical system.

The three developed control techniques are tested through simulation based on the developed dynamic formulation model using MATLAB/SIMULINK. Stability and performance analysis were conducted and tuned to obtain the best results. Then, the performance and stability results obtained through simulation are compared. Finally, the developed control techniques were implemented and analyzed on the 2-DOF Serial Flexible Link Robot experimental system from Quanser and the results are illustrated and compared with that obtained through simulation. Figure 3.1 shows the research methodology followed in this thesis.



Figure 3.1. Research methodology

Chapter 4

Modeling of Two-Link Flexible Manipulators

4.1 Introduction

As mentioned in Chapter 2, the main two methods for modeling the Two-Link Flexible Manipulators (TLFMs) are the Assumed Mode Methods (AMM) and the Finite Element method (FEM). AMM method has been used in the research over the FEM due to [3]:

- a. Less computational load for simulation and control, and
- b. Vibration Modes are clearer in AMM in contrary to FEM.

Considering the above, this thesis adopted AMM approach for modeling TLFM.

4.2 AMM Method

AMM is a method where the infinite dimensional model of a system is truncated to finite dimensional model series in terms of combination of mode eigenfunctions which are also called as mode shapes and time dependent generalized coordinate. The following equation describes the deflection of a flexible link using AMM approach and illustration is shown in Figure 4.1:

$$\varepsilon_i(x_i, t) = \sum_{k=0}^N \varphi_{ik}(x_i) \,\delta_{ik}(t) \tag{4.1}$$

where:

i: 1.....M, where M is the number of flexible links

k: 1N, Where N is number of AMM modes used to describe ith flexible link

t: Time

 x_i : Is the x-axis position for i^{th} flexible link $(0 \le x_i < l_i); \quad l_i$: length of i^{th} link

 $\varepsilon_i(x_i, t)$: Deflection of i^{th} link at x_i

 $\varphi_{ik}(x_i)$: Is the k^{th} mode shape function of link *i*; When k = 0, for i^{th} link, it is called as the zeroth mode, which gives the characteristic similar to rigid manipulator without any flexible deflections. $\varphi_{ik}(x_i)$ is calculated from the boundary condition described by equations 4.2 to 4.5

 $\delta_{ik}(t)$: Time dependent generalized coordinate that is associated with each assumed frequency mode and referred as flexible coordinates



Figure 4.1. Flexible links manipulator system

There are various ways of choosing the boundary conditions for this method. Selection of suitable boundary conditions of the AMM is important for FMs to fit into a proper application. Usually, it is selected based on the set nearest to the natural modes of the system. According to the general beam vibration, there are four applicable theories for boundary conditions: Pinned-pinned, Clamped-pinned, Clamped-free and Clamped-clamped. For instance, clamped-pinned boundary condition gives simpler coefficient of joint torques while Pinned-pinned simplifies calculation of the tip position [19], [37], [38].

The four boundary conditions are described as:

a. Pinned-pinned

$$\varphi_k(x_k) = \psi_k \sin(\beta_k x_k) \qquad (4.2)$$

$$\psi_k = \frac{\cosh(\beta_k l_k)}{\cos(\beta_k l_k)}$$

 ψ_k and β_k are mathematical parameters and don't represent real angles.

 β_k for all of the four boundary condition, it is the solution of the following equation [37]:

$$1 + \cosh(\beta_k l_i) \cos(\beta_{ik} l_i) - \frac{M_L \beta_k}{\rho_i} (\sin(\beta_{ik} l_i) \cosh(\beta_i l_i) - \cos(\beta_{ik} l_i) \sinh(\beta_i l_i)) \\ - \frac{J_L \beta_k^3}{\rho_i} (\sin(\beta_i l_i) \cosh(\beta_i l_i) + \cos(\beta_i l_i) \sinh(\beta_i l_i) + \frac{M_L J_L \beta_i^4}{\rho_i^2} (1 - \cos(\beta_i l_i) \cosh(\beta_i l_i)) = 0$$

All the parameters of β_k for all modes are illustrated in section 4.3.

b. Clamped-pinned

$$\varphi_k(x_k) = \sin(\beta_k x_k) - \sinh(\beta_k x_k) + \psi_k \cos(\beta_k x_k) - \cosh(\beta_k x_k)$$

$$\psi_k = -\frac{\sin(\beta_k l_k) + \sinh(\beta_k l_k)}{\cos(\beta_k l_k) + \cosh(\beta_k l_k)}$$
(4.3)

c. Clamped-free

$$\varphi_k(x_k) = \sin(\beta_k x_k) - \sinh(\beta_k x_k) + \psi_k \cos(\beta_k x_k) - \cosh(\beta_k x_k)$$

$$\psi_k = -\frac{\cos(\beta_k l_k) + \cosh(\beta_k l_k)}{\sin(\beta_k l_k) - \sinh(\beta_k l_k)}$$
(4.4)

d. Clamped-clamped

$$\varphi_k(x_k) = \sin(\beta_k x_k) - \sinh(\beta_k x_k) + \psi_k \cos(\beta_k x_k) - \cosh(\beta_k x_k)$$

$$\psi_k = -\frac{\cos(\beta_k l_k) - \cosh(\beta_k l_k)}{\sin(\beta_k l_k) + \sinh(\beta_k l_k)}$$
(4.5)

As per [19], [37]-[39] Clamped-pinned boundary condition is the best condition suites the modeling for multi-link flexible manipulators in general and TLFMs in specific.

4.3 Mathematical Modeling of TLFM

This section focuses on the development of a combined Euler-Lagrange and AMM algorithm characterizing the dynamic behavior of the two-link flexible manipulator system as per [19], [37]. The following assumptions are made for the development of a dynamic model of the flexible manipulator [13], [19], [37]:

- a. Each link is assumed to be long and slender. Therefore, transverse shear and the rotary inertia effects are negligible,
- b. The motion of each link is assumed to be in the horizontal plane,
- c. Links are considered to have constant cross-sectional area and uniform material properties, i.e. with constant mass density and Young's modulus, etc,
- d. Each link has a very small deflection,
- e. Motion of the links can have deformations in the horizontal direction only,
- f. The kinetic energy of the rotor is mainly due to its rotation only, and the rotor inertia is symmetric about its axis of rotation, and
- g. The backlash in the reduction gear and coulomb friction effects are neglected.

The following coordinate frames are then established for Two-Link Flexible Manipulator shown in Figure 4.2:

- a. Inertial frame (X_0, Y_0)
- b. The rigid body moving frame associated to link (X_1, Y_1) and link (X_2, Y_2) ,
- c. The flexible body moving frame associated to link 1 (\hat{X}_1 , \hat{Y}_1) and link 2 (\hat{X}_2 , \hat{Y}_2),
- d. The rigid motion is described by the joint angles θ_i , and
- e. $\varepsilon_i(x_i)$ stand for the transversal deflection of link *i* at x_i , $0 \le x_i \le l_i$, l_i being the *i*th link length.



Figure 4.2. Two-link flexible manipulator system with frames

The joint (rigid) rotation matrix A_i and the rotation matrix E_i of the (flexible) link at the endpoint are, respectively

 A_i is the rigid joint rotation matrix:

$$A_{i} = \begin{bmatrix} \cos(\theta_{i}) & -\sin(\theta_{i}) \\ \sin(\theta_{i}) & \cos(\theta_{i}) \end{bmatrix}$$
(4.6)

 E_i is the rotation matrix of the flexible link at the end point of the link

$$E_{i} = \begin{bmatrix} 1 & -\arctan\left(\check{e}_{ie}\right) \\ \arctan\left(\check{e}_{ie}\right) & 1 \end{bmatrix}$$

Since the deflection of the link is very small $\check{\varepsilon}_{ie}$, hence $\arctan(\check{\varepsilon}_{ie}) \cong \check{\varepsilon}_{ie}$

$$E_{i} = \begin{bmatrix} 1 & -\check{\varepsilon}_{ie} \\ \check{\varepsilon}_{ie} & 1 \end{bmatrix}$$
where $\check{\varepsilon}_{ie} = \frac{\partial \varepsilon_{i}}{\partial x_{i}}|_{x_{i}=l_{i}}$

$$(4.7)$$

Let ${}_{i}^{i}p$ be the position of a point along the deflected link i with respect to frame (X_i,Y_i) and p_{i} is the absolute position of the same point in frame (X₀,Y₀). In addition, Let ${}_{i+1}{}_{i}^{i}r$ be the position of the origin of frame (X_i,Y_i) with respect to frame (X_{i+1},Y_{i+1}) and r_{i} is the absolute position of the frame *i*.

$$p_{i} = r_{i} + W_{i} {}^{i}_{i} p$$

$$r_{i+1} = r_{i} + W_{i} {}^{i}_{i+1} r$$
(4.8)

Where *W* is the global transformation matrix from (X_0, Y_0) to (X_i, Y_i) , which follows the following equation

$$W_i = W_{i-1}E_{i-1}A_i$$

The following identities as used in the dynamic modeling

$$A_i^T A_i = E_i^T E_i = S^T S = I$$

where

$$S = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

The dynamic equation of motion for two links flexible manipulator is derived using Lagrangian approach by computing the kinetic energy T and potential energy U of the system, the lagrangian equation is formed which is given by, L = T - U.

The total kinetic energy of the system is given by the sum of the following components [37]:

$$T = \sum_{i=1}^{2} T_{hi} + \sum_{i=1}^{2} T_{li} + T_{p}$$
(4.9)

where:

T: Total kinetic energy

 T_{hi} : is the kinetic energy at hub *i* of mass m_{hi} and momnent of inertia J_{hi}

 T_{li} : is the kinetic energy of link *i*

 T_p : is the kinetic energy of payoad of mass m_p and momnent of inertia J_p located at the end of second link.

 ρ_i is the linear density of link *i*

and

$$T_{hi} = \frac{1}{2} m_{hi} \, \dot{r}_i^T \, \dot{r}_i + \frac{1}{2} J_{hi} \, \dot{\alpha}_i^2 \tag{4.10}$$

$$T_{li} = \frac{1}{2} \int_0^{l_i} \rho_i(x_i) \, \dot{p}_i^T(x_i) \, \dot{p}_i(x_i) dx_i$$
(4.11)

$$T_p = \frac{1}{2} m_p \, \dot{r}_{n+1}^T \, \dot{r}_{n+1} + \frac{1}{2} J_p (\dot{\alpha}_n + \dot{\check{\epsilon}}_{ne})^2 \tag{4.12}$$

The potential energy (without considering gravity i.e. horizontal plane motion) is given by [37]:

$$U = \sum_{i}^{2} U_{i} = \sum_{i}^{2} \frac{1}{2} \int_{0}^{l_{i}} EI_{i} x_{i} \left[\frac{d^{2} \varepsilon_{i}(x_{i},t)}{dx_{i}^{2}} \right]^{2} dx_{i}$$
(4.10)

where U_i : is the elastic energy stored in link I and EI_i its flexural rigidity

Finit-dimensional approximation of link deflection is investigated next using AMM method. Links are modeled as Euler-Bernoulli beams of uniform linear density ρ_i , and constant flexural rigidity EI_i with deformation $\varepsilon_i(x_i, t)$ satisfying the following partial differential equation,

$$EI_{i}\frac{\partial^{4}\varepsilon_{i}(x_{i},t)}{\partial x_{i}^{4}} + \rho_{i}\frac{\partial^{2}\varepsilon_{i}(x_{i},t)}{\partial t^{2}} = 0 \qquad i=1,2 \qquad (4.11)$$

As mentioned in section 4.2, AMM method, $\varepsilon_i(x_i, t)$ can be expressed as a linear combination of the product of boundary condition $\varphi_{ik}(x_i)$ and time-dependent generalized coordinates $\delta_{ik}(t)$ as per equation 4.1:

$$\varepsilon_i(x_i,t) = \sum_{k=0}^N \varphi_{ik}(x_i) \,\delta_{ik}(t)$$

The solution of equation 4.1 is in form of [37]:

$$\varphi_{ik}(x_i) = m_i [\cos(\beta_{ik} x_i) - \cosh(\beta_{ik} x_i + \gamma_{ik} (\sin(\beta_{ik} x_i) - \sinh(\beta_{ik} x_i))]$$
(4.12)

where m_i is the mass of link *i* and γ_{ik} is given by:

$$\gamma_{ik} = \frac{\sin\beta_{ik} - \sinh\beta_{ik} + \frac{M_L\beta_{ik}}{\rho_i}(\cos\beta_{ik} - \cosh\beta_{ik})}{\cos\beta_{ik} + \cosh\beta_{ik} - \frac{M_L\beta_{ik}}{\rho_i}(\sin\beta_{ik} - \sinh\beta_{ik})}$$
(4.13)

and β_{ik} is constant value that represent the solution of the following equation:

$$1 + \cosh(\beta_{ik}L_i)\cos(\beta_{ik}L_i) - \frac{M_L\beta_{ik}}{\rho_i}(\sin(\beta_{ik}L_i)\cosh(\beta_{ik}L_i) - \cos(\beta_{ik}L_i)\sinh(\beta_{ik}L_i)) - \frac{J_L\beta_{ik}^3}{\rho_i}(\sin(\beta_{ik}L_i)\cosh(\beta_{ik}L_i) + \cos(\beta_{ik}L_i)\sinh(\beta_{ik}L_i) + \frac{M_LJ_L\beta_{ik}^4}{\rho_i^2}(1 - \cos(\beta_{ik}L_i)\cosh(\beta_{ik}L_i)) = 0$$

$$(4.14)$$

where:

 M_{Li} is the mass at the tip end of link *i* and J_{Li} is the inertia at the tip end of link *i*

$$M_{L1} = m_2 + m_{h2} + m_P$$
$$M_{L2} = m_P$$
$$J_{L1} = J_{o2} + J_{h2} + J_p + L_2^2 m_P$$
$$J_{L2} = J_p$$

and

 m_2 : mass of the second link

 m_{h2} : mass of the motor hub for second link

 m_p : mass of the payload

 J_{o2} : inertia of second link around joint-2 axis

 J_{h2} : inertia of motor hub of link around joint-2 axis

 J_p : inertia of the payload

 ρ_i : density of link *i*

Once β_{ik} is available the natural frequency of the *kth* frequency mode of link *i* can be calculated from

$$\omega_{ik} = \beta_{ik}^2 \sqrt{\frac{(EI)_i}{\rho_i}} \tag{4.15}$$

The dynamic model of two-link flexible manipulator is obtained using the Language-Euler equations:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = f_i \qquad \qquad i=1,2,\dots,6$$
(4.16)

where $\{f_i\}$ are the generalized forces performing work on $\{q_i\}$

As a result of this procedure and by solving equations 4.14 to 4.16, the equations of motion for a planar 2-link flexible arm can be written in the familiar closed form

$$B(q)\ddot{q} + H(q,\dot{q}) + Kq = u$$
(4.17)
where $q = [\theta_1 \quad \theta_2 \quad \delta_{11} \quad \delta_{12} \quad \delta_{21} \quad \delta_{22}]^T$ and

u is the 2-vector of joint (actuator) torques.

 $u = [\tau_1 \quad \tau_2 \quad 0 \quad 0 \quad 0 \quad 0]^T$

B is the positive- definite symmetric inertia matrix,

$$B = \begin{bmatrix} B_{11} & B_{12} & B_{13} & B_{14} & B_{15} & B_{16} \\ B_{21} & B_{22} & B_{23} & B_{24} & B_{25} & B_{26} \\ B_{31} & B_{32} & B_{33} & B_{34} & B_{35} & B_{36} \\ B_{41} & B_{42} & B_{43} & B_{44} & B_{45} & B_{46} \\ B_{51} & B_{52} & B_{53} & B_{54} & B_{55} & B_{56} \\ B_{61} & B_{62} & B_{63} & B_{64} & B_{65} & B_{66} \end{bmatrix}$$

H is the vector of coriolis and centrifugal forces,

$$H = [h_1 \quad h_2 \quad h_3 \quad h_4 \quad h_5 \quad h_6]^T$$

K is the stiffness matrix,

<i>K</i> =	Г0	0	0	0	0	ך 0	
	0	0	0	0	0	0	
	0	0	$m_1\omega_{11}$	0	0	0	
	0	0	0	$m_1\omega_{12}$	0	0	
	0	0	0	0	$m_1\omega_{21}$	0	
	Lo	0	0	0	0	$m_1\omega_{22}$	

Where elements of the stiffness matrix appear associated with the flexible coordinates $\delta_{ik}(t)$ and it represents the mass multiplied by the natural frequency of the mode.

All coefficients used in the B and H Matrices are shown in Appendix A

4.4 Physical Parameters and Model Coefficients

Table 4.1 below show the physical parameters used for model to calculate all model coefficients. The below data are of 2-DOF Serial Flexible Link Robot from Quanser company [11]. Further details about the hardware system are shown in Appendix B.

Parameter	Unit	Symbol (i for	Link 1	Link2
		mik number)		
Mass Density	Kg/m	$ ho_i$	2829	2487
Length	m	l_i	0.202	0.2018
Width	m	wd _i	0.076	0.038
Thickness	m	Th _i	0.00127	0.00089
Link Mass	Kg	m_i	0.46	0.157
Second Hub Mass	Kg	m_{h2}		1.01
Payload Mass	Kg	m_p		0.23
Link Inertia	Kg.m ²	J _{oi}	0.0109	0.0075
Hub Inertia	Kg.m ²	J _{hi}	0.0073	0.0065
Payload Inertia	Kg.m ²	J_p		0.0087
Flexural Rigidity	N.m ²	$(EI)_i$	1868	433

Table 4.1: Physical parameters

Using the mathematical formulation in Appendix A and Table 4.1, the elements of B and H matrices are calculated as follow:

 $B11 = 0.945 + 10.34c_2 + (12.2t_1 - 9.5t_2)s_2$ $B12 = 43.67 + 0.5804c_2 + (-405.01t_1 + 73.45t_2)s_2$ $B13 = -210 + 33.45c_2 + (27.48t_2 + 55.43\delta_{12})s_2$ $B14 = 19.54 + 60.54c_2 + (-22.45t_1 + 12.65\delta_{11})s_2$ $B15 = 35.61 + 55.65c_2 - 98.54t_1s_2$ $B16 = 68.13 - 120.03c_2 + 58.61t_1s_2$ B21 = 0B22 = 34.64

 $B23 = 74.23 + 5.54c_2 + (16.98t_2 - 8.11t_3)s_2$ $B24 = 0.331 - 11.32c_2 + (19.8t_2 + 13.45t_3)s_2$ B25 = 104.56B26 = -95.35B31 = 0B32 = 0 $B33 = 0.64 + 11.34c_2 + 5.5t_2s_2$ $B34 = 3.56 + 14.65c_2 + 52.4t_3s_2$ $B35 = 17.37 + 11.94c_2 + 74.21t_3s_2$ $B36 = 64.67 + 4.63c_2 + 22.4t_3s_2$ B41 = 0B42 = 0B43 = 0 $B44 = 48.6 + 12.4c_2 - 15.83t_2s_2$ $B45 = -53.4 + 30.64c_2 + 7.87t_3s_2$ $B46 = 18.96 - 42.45c_2 + 1.13t_3s_2$ B51 = 0B52 = 0B53 = 0B54 = 0B55 = -33.95B56 = 264.34B61 = 0B62 = 0B63 = 0B64 = 0B65 = 0B66 = -49.56 $t_1 = 54.67\delta_{11} + 0.85\delta_{12}$ $t_2 = -12.45\delta_{21} + 11.45\delta_{22}$

$$\begin{split} t_3 &= -64.43\delta_{11} + 49.33\delta_{12} \\ c_2 &= \cos\theta_2 \\ s_2 &= \sin\theta_2 \\ h1 &= \left[(12.4\theta_2 - 23.4\delta_{11}16.8\delta_{12} + 11.3\delta_{21} + 2.6\delta_{22})\theta_1 \\ &+ (23.4\theta_2 - 12.4\delta_{11} - 1.45\delta_{12} + 9.43\delta_{21} + 11.61\delta_{22})\theta_2 \\ &+ (5.73\delta_{21} + 8.65\delta_{22})\delta_{11} + (18.54\delta_{21} - 29.54\delta_{22})\delta_{12} \right]s_2 \\ &+ \left[(4.41\theta_1 + \theta_2 - 4.32\delta_{21} - 16.74\delta_{22})t_1 \\ &+ (13.5\theta_1 - 4.24\theta_2 - 6.75\delta_{21} - 11.45\delta_{22})t_2 + 2.65\delta_{12}\delta_{11} \\ &- 9.76\delta_{11}\delta_{12} \right]\theta_2c_2 \\ h2 &= (38.54\theta_1 - 24.5\delta_{11} - 2.23\delta_{12})\theta_1s_2 \\ &+ \left\{ \left[(15.64\theta_1 - 3.52\delta_{21} + 8.65\delta_{22})t_1 + (2.67\theta_1 - 19.54\delta_{11} + 21.1\delta_{12})t_2 \\ &- 9.12\delta_{12}\delta_{11} + 15.63\delta_{11}\delta_{12} \right]\theta_1 \\ &+ \left[(14.6\delta_{11} - 9.05\delta_{12})t_2 + (16.83\delta_{21} - 6.54\delta_{22})t_3 \right]\delta_{11} \\ &+ \left[5.65\delta_{12}t_2 + (15.96\delta_{21} + 16.4\delta_{22})t_3 \right]\delta_{12} \right]c_2 \\ h3 &= \left[(12.45\theta_2 + 3.45\theta_1 + 17.53\delta_{12} - 32.45\delta_{21} + 18.56\delta_{22})\theta_1 \\ &+ (-10.45\theta_2 + 4.64\delta_{11} - 17.48\delta_{12} - 16.75\delta_{21} + 20.43\delta_{22})\theta_2 \\ &+ (-11.45\delta_{21} + 15.54\delta_{22})\delta_{11} + (14.52\delta_{21} - 22.41\delta_{22})\delta_{12} \right]s_2 \\ &+ \left[(14.42\theta_2 - 31.34\theta_1 + 2.42\delta_{21} - 17.45\delta_{22})t_2 \\ &+ (34.54\theta_2 - 17.54\delta_{21} - 18.94\delta_{22})t_3 + 22.53\delta_{12}\theta_1 \right]\theta_2s_2 \\ h4 &= \left[(-15.64\theta_2 + 25.78\theta_1 + 19.76\delta_{12} - 13.53\delta_{21} + 24.65\delta_{22})\theta_1 \\ &+ (11.43\theta_2 - 34.67\delta_{11} + 25.76\delta_{12} + 17.65\delta_{21} - 11.23\delta_{22})\theta_2 \\ &+ (23.33\delta_{21} - 16.76\delta_{22})\delta_{11} + (28.65\delta_{21} - 4.21\delta_{22})\delta_{12} \right]s_2 \\ &+ \left[(5.63\theta_2 + 27.43\theta_1 - 6.78\delta_{21} - 8.65\delta_{22})t_2 \right] \right]s_2 \\ &+ \left[(5.63\theta_2 + 27.43\theta_1 - 6.78\delta_{21} - 8.65\delta_{22})t_2 \right] \right]s_2 \\ &+ \left[(5.63\theta_2 + 27.43\theta_1 - 6.78\delta_{21} - 8.65\delta_{22})t_2 \right] \\ &+ \left[(5.63\theta_2 + 27.43\theta_1 - 6.78\delta_{21} - 8.65\delta_{22})t_2 \right] \\ &+ \left[(5.63\theta_2 + 27.43\theta_1 - 6.78\delta_{21} - 8.65\delta_{22})t_2 \right] \\ &+ \left[(5.63\theta_2 + 27.43\theta_1 - 6.78\delta_{21} - 8.65\delta_{22})t_2 \right] \\ &+ \left[(5.63\theta_2 + 27.43\theta_1 - 6.78\delta_{21} - 8.65\delta_{22})t_2 \right] \\ &+ \left[(5.63\theta_2 + 27.43\theta_1 - 6.78\delta_{21} - 8.65\delta_{22})t_2 \right] \\ &+ \left[(5.63\theta_2 + 27.43\theta_1 - 6.78\delta_{21} - 8.65\delta_{22})t_2 \right] \\ &+ \left[(5.63\theta_2 + 27.43\theta_1 - 6.78\delta_{21} - 8.65\delta_{22} \right] \\ &+ \left[(5.63\theta_2 + 27.43\theta_1 - 6.78\delta_{21}$$

$$+ (19.53\dot{\theta}_2 + 25.6\dot{\delta}_{21} - 16.44\dot{\delta}_{22})t_3 - 11.56\delta_{12}\dot{\theta}_1]\dot{\theta}_2s_2$$

$$h5 = (11.34\dot{\theta}_1 - 24.54\dot{\delta}_{12} - 2.34\dot{\delta}_{21})\dot{\theta}_1s_2 + [-5.7t_1\dot{\theta}_1 + (-6.41\dot{\delta}_{21} + 17.43\dot{\delta}_{22})t_3]\dot{\theta}_2c_2$$

$$h6 = (-34.5\dot{\theta}_1 + 7.35\dot{\delta}_{12} + 12.43\dot{\delta}_{21})\dot{\theta}_1s_2$$

$$+ [14.54t_1\dot{\theta}_1 + (23.64\dot{\delta}_{21} + 24.09\dot{\delta}_{22})t_3]\dot{\theta}_2c_2$$

Chapter 5

Control Techniques for Two-Link Flexible Manipulators

This chapter focuses on the development of three nonlinear control techniques that include LQR, backstepping and sliding mode. The aim of these control techniques is to improve position control of TLFMs in terms of dynamic performance and steady state error. These control techniques are used with both linearized and nonlinear models of the system as relevant to the control technique.

5.1 LQR Controller

5.1.1 LQR technique [40],[13], [41]

LQR technique is a linear optimal control technique that aims to achieve good system performance while minimizing the amount of actuation used in achieving the desired performance.

A measure of the quality of a controller is formulated in terms of a performance index (J). This index is used to design the controller and depends on the control signal and the state vector. In this way the 'optimal' control signal is found resulting in the minimum value of the index J. The job of the designer is not to determine control parameters directly, but to define the appropriate measure for the performance index and to minimize it.

(5.1)

The performance index is defined by the following equation:

$$J = \int_0^\infty z^T Q z dt + \int_0^\infty u^T R u dt$$

where:

J: Performance Index or cost function

z: is the state vector

u: is the control (input) vector

The matrices R and Q are "weight matrices" that determine the relative importance of the error.

For a linear state space system with *z* state vector can represented as:

$$\dot{z} = Az + Bu$$

$$y = Cz + Du$$
(5.2)

The LQR control input is u = -Kz with its general illustration shown in Figure 5.1.



Figure 5.1: General LQR controller with z state vector

where K is the controller gain vector and it is calculated using the following equation

$$K = R^{-1}B^T P \tag{5.3}$$

P is the solution of Riccati equation

$$A^{T}P + PA - PBR^{-1}B^{T}P + Q = 0 (5.4)$$

By selecting proper positive elements of weight matrices R & Q, the P matrix can be calculated from equation 5.4. The selection of R & Q matrices are done through trial and error tuning through simulation starting with identify matrix for both till reaching the desired performance.

5.1.2 LQR controller design

In order to design the LQR controller, the linear state representation need to be found from the dynamic equation of TLFM represented in Chapter 4. The linear state space is generated from the linearization of the nonlinear state space representation of the dynamic equation of TLFM.

Equation (4.20) represents the dynamic equation for the TLFM system under study;

$$B(q)\ddot{q} + h(q,\dot{q}) + Kq = u$$

Which can be re-written in the following form to simplify model implementation using MATLAB/Simulink:

$$\ddot{q} = B(q)^{-1}(u - h(q, \dot{q}) - Kq)$$

$$\ddot{q} = B(q)^{-1}u - B(q)^{-1}h(q, \dot{q}) - B(q)^{-1}Kq$$
(5.5)

The general state space representation of nonlinear systems is,

$$\dot{z} = f(z) + g(z)u$$

$$y = j(z) + p(z)u$$
(5.6)

where

f(z), g(z), j(z) and p(z) are nonlinear functions of state vector z

and

$$y = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

The state vector of the TLFM shown below

$$z = \begin{bmatrix} q \\ \dot{q} \end{bmatrix} = [z_1 \, z_2 \, z_3 \, z_4 \, z_5 \, z_6 \, z_7 \, z_8 \, z_9 \, z_{10} \, z_{11} \, z_{12}]^T$$

Where z is the state vector and the state variables are

$$z_{1} = \theta_{1}$$

$$z_{2} = \theta_{2}$$

$$z_{3} = \delta_{11}$$

$$z_{4} = \delta_{12}$$

$$z_{5} = \delta_{21}$$

$$z_{6} = \delta_{22}$$

$$z_{7} = \dot{\theta}_{1}$$

$$z_{8} = \dot{\theta}_{1}$$

$$z_{9} = \dot{\delta}_{11}$$

$$z_{10} = \dot{\delta}_{12}$$

$$z_{11} = \dot{\delta}_{21}$$

$$z_{12} = \dot{\delta}_{22}$$

Finding the \dot{z} vector

$$\dot{z} = \begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \begin{bmatrix} \dot{q} \\ B(q)^{-1}(u - h(q, \dot{q}) - Kq) \end{bmatrix}$$
(5.7)

In order to formulate the state space representation of the TLFM in equation 5.7 in the form of general state space representation of a nonlinear system shown in equation 5.6, the following matrices F & G are introduced,

$$F = B(q)^{-1}(-h(q,\dot{q}) - Kq) = \begin{bmatrix} F_1 & F_2 & F_3 & F_4 & F_5 & F_6 \end{bmatrix}$$
(5.8)

$$G = B(q)^{-1} = \begin{bmatrix} G_{11} & G_{12} & G_{13} & G_{14} & G_{15} & G_{16} \\ G_{21} & G_{22} & G_{23} & G_{24} & G_{25} & G_{26} \\ G_{31} & G_{32} & G_{33} & G_{34} & G_{35} & G_{36} \\ G_{41} & G_{42} & G_{43} & G_{44} & G_{45} & G_{46} \\ G_{51} & G_{52} & G_{53} & G_{54} & G_{55} & G_{56} \\ G_{61} & G_{62} & G_{63} & G_{64} & G_{65} & G_{66} \end{bmatrix}$$
(5.9)

The elements of the matrices F & G depend on the actual system under representation and the general state space representation in equation 5.6

Using equations 5.8 and equation 5.9 to formulate the detail presentation equation 5.7 in the form of equation 5.6

where

The mathematical formulation for F and G matrices are show in Appendix C.

p(z) = [0]

Next step is to linearize of the nonlinear model in equation 5.10 to put the system on the form of linear state space representation as shown below:

 $\dot{z} = Az + Bu$ y = Cz + Du where

A, B, C and D is state space matrices for the linearized model that can be found from [42]

$$A = \frac{df(z)}{dz}\Big|_{z=z_e}$$

$$B = \frac{dg(z)}{dz}\Big|_{z=z_e}$$

$$C = \frac{dj(z)}{dz}\Big|_{z=z_e}$$

$$D = \frac{dp(z)}{dz}\Big|_{z=z_e}$$
(5.11)

where z_e is the equilibrium point for the system defined as

From equation 5.11 and substitute with the parameters from Table 4.1 calculate

А	=										
F0	0	0	0	0	0	1	0	0	0	0	ך 0
0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	1
0	0	-980.8	85.78	64.42	565	0	0	840.58	-84.4	-44.55	886.98
0	0	18.7	-354.45	234.46	-475.5	0	0	-484.4	564.4	174.13	-97.5
0	0	238.7	-335.11	-854.5	-745.24	40	0	-937.55	-66.7	-135.98	-77.03
0	0	-85.6	464.68	3573.5	56.76	0	0	759	-664.4	624.23	-2476.02
0	0	9475.8	-474.3	967.6	-9.07	0	0	-424.3	-3464.76	-986.08	345.98
L0	0	-89	-4.5	-245.07	659.7	0	0	-3.3	759.09	235.87	-602.07 J
		B =	$\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 943.4 \\ 0.479 \\ 55.08 \\ -575.5 \\ -575.5 \\ 268.87 \\ 400.95 \\ \end{array}$	0 0 0 0 0 864.2 -489.23 134.9 $\cdot9473.85$ 18.94 -99.45	$\begin{array}{c} 0\\ 0\\ 0\\ 0\\ 0\\ 325.45\\ 440.5\\ 0.84\\ 94.46\\ 6328.57\\ -874.5 \end{array}$		0 0 0 57.8 235.5 698.4 978.3 57.67	$\begin{array}{c} 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 6\\ -2757\\ 0.75\\ 0.75\\ 8\\ 458.9\\ 8\\ -454\\ -2435\\ 8\\ -2435\\ 8\\ 3584\end{array}$	$\begin{array}{c} 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 7.68 \\ 7.68 \\ 7.64\\ 5\\ -0.2\\ 98 \\ 523\\ .65 \\ -744\\ 5.05 \\ -296\\ 4.4 \\ -698 \end{array}$	4 24 .5 .64 .09 3.4	
	<i>C</i> =	$=\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}$	$\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}$	$\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}$	0 0]				

_ ת	٥٦	0	0	0	0	01
D =	lo	0	0	0	0	0]

The selection of R & Q matrices are done through trial and error tuning through simulation starting with identify matrix for both till reaching the desired performance. The final content for R&Q matrices are listed below

	г140	0	0	0	0	0	0	0	0	0	0	ך0
	0	345	0	0	0	0	0	0	0	0	0	0
	0	0	440	0	0	0	0	0	0	0	0	0
	0	0	0	440	0	0	0	0	0	0	0	0
	0	0	0	0	45	0	0	0	0	0	0	0
0 –	0	0	0	0	0	80	0	0	0	0	0	0
ų –	0	0	0	0	0	0	77	0	0	0	0	0
	0	0	0	0	0	0	0	1000	0	0	0	0
	0	0	0	0	0	0	0	0	5000	0	0	0
	0	0	0	0	0	0	0	0	0	12	0	0
	0	0	0	0	0	0	0	0	0	0	33	0
	Γ0	0	0	0	0	0	0	0	0	0	0	1
	100	0	0	0	0	0	1					
	0	100	0	0	0	0						
D _	0	0	100	0	0	0						
К-	0	0	0	100	0	0						
	0	0	0	0	100	0						
	- 0	0	0	0	0	100	Lر					

5.2 Backstepping Controller

5.2.1 Backstepping technique [42],[13], [41],[43]

Backstepping is a recursive procedure that interlaces the choice of a Lyapunov function with the design of feedback control. It breaks a design problem for the full system into a sequence of design problems for lower-order (even scalar) systems. By exploiting the extra flexibility that exists with lower order and scalar systems, backstepping can often solve stabilization, tracking, and robust control problems under conditions less restrictive than those encountered in other methods.

Because of this recursive structure, the designer can start the design process at the knownstable system and "back out" new controllers that progressively stabilize each outer subsystem. The process terminates when the final external control is reached. Hence, this process is known as Backstepping.

Backstepping is applied to systems of lower triangular form, where each state is a function of itself and a function of the next state as shown below

$$\begin{aligned} \dot{\varepsilon}_1 &= f(\varepsilon_1, \varepsilon_2) \\ \dot{\varepsilon}_2 &= f(\varepsilon_1, \varepsilon_2, \varepsilon_3) \\ \vdots \\ \dot{\varepsilon}_n &= f(\varepsilon_1, \varepsilon_2, \varepsilon_3, \dots, \varepsilon_n, u) \end{aligned}$$

(5.12)

where

 ε is the state vector and u is the input for the system.

Figure 5.2 below shows a general illustrative diagram for backstepping technique.



Figure 5.2. Illustrates diagram the backstepping technique

The first step in backstepping design is to start with the first subsystem

$$\dot{\varepsilon}_1 = f(\varepsilon_1, \varepsilon_2)$$

$$\dot{\varepsilon}_2 = f(\varepsilon_1, \varepsilon_2, \varepsilon_3)$$
(5.13)

Defining the error e_1 and e_2 as

$$e_1 = \theta_d - \varepsilon_1$$

$$e_2 = \alpha_1 - \varepsilon_2 \tag{5.14}$$

where α_1 is the first virtual input appeared in the representation shown in equation 5.14. Differentiating both variables e_1 and e_2 in equation 5.14,

$$\dot{e}_1 = \theta_d - \varepsilon_2$$

$$\dot{e}_2 = \dot{\alpha}_1 - \dot{\varepsilon}_2$$
(5.15)

Then, define the first positive definite Lyapunov function V_1 as,

$$V_1 = \frac{1}{2}e_1^2 \tag{5.16}$$

Finding the derivative of Lyapunov function \dot{V}_1 from equation 5.16

$$\dot{V}_1 = e_1 \dot{e}_1 \tag{5.17}$$

 α_1 is virtual input that makes \dot{V}_1 negative definite.

Afterwards we define the second positive definite Lyapunov function V_2 is defined as,

$$V_2 = V_1 + \frac{1}{2}e_2^2 \tag{5.18}$$

and then define,

$$\varepsilon_3 = \alpha_2 \tag{5.19}$$

where α_2 is the second virtual input.

Finding the derivative of Lyapunov function \dot{V}_2 from equation 5.18

$$\dot{V}_2 = \dot{V}_1 + e_2 \dot{e}_2 \tag{5.20}$$

 α_2 is virtual that makes \dot{V}_2 negative definite

The second step is to back out to the second subsystem

$$\dot{\varepsilon}_{1} = f(\varepsilon_{1}, \varepsilon_{2})$$

$$\dot{\varepsilon}_{2} = f(\varepsilon_{1}, \varepsilon_{2}, \varepsilon_{3})$$

$$\dot{\varepsilon}_{3} = f(\varepsilon_{1}, \varepsilon_{2}, \varepsilon_{3}, \varepsilon_{4})$$
(5.21)

The second subsystems in equation 5.21 can be redefined with new state variable as below

$$\dot{\mu}_1 = f(\mu_1, \varepsilon_2)$$

$$\dot{\varepsilon}_3 = f(\mu_1, \varepsilon_2, \varepsilon_3)$$
(5.22)
where

where

$$\mu_1 = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix}$$

Apply the same steps that were applied to the first subsystem from equation 5.13 to the redefined subsystem in equation 5.22. The steps include defining the error states with as third and fourth virtual inputs as in equation 5.14. Then, define Lyapunov functions to find the virtual inputs accordingly. Repeating the aforementioned steps till reaching last subsystem with state ε_n where *u* is going to appear with the last subsystem and is going calculated based on last Lyapunov function. The total number of virtual inputs (α_1 , α_2 , ...) needed throughout the subsystems for system of order *n* is (*n*-1).

5.2.2 Backstepping controller design for TLFM

Starting from the system dynamic equation (4.20)

$$B(q)\ddot{q} + H(q,\dot{q}) + Kq = u$$

Rewrite equation 4.20 as follow

$$\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\delta} \end{bmatrix} + \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} + \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} \theta \\ \delta \end{bmatrix} = \begin{bmatrix} \tau \\ 0 \end{bmatrix}$$
(5.23)

where

$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$
$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$
$$\delta = \begin{bmatrix} \delta_{11} \\ \delta_{12} \\ \delta_{22} \end{bmatrix}$$
$$B_{11} \& K_{11} \in R^{2x2}$$
$$B_{12} \& K_{12} \in R^{2x4}$$
$$B_{21} \& K_{21} \in R^{4x2}$$
$$B_{22} \& K_{22} \in R^{4x4}$$
$$H_1 \in R^{2x1}$$
$$H_2 \in R^{4x1}$$

Breaking equation 5.23 into two equations as below

$$B_{11}\ddot{\theta} + B_{12}\ddot{\delta} + H_1 + K_{11}\theta + K_{12}\delta = \tau$$
(5.24)

$$B_{21}\ddot{\theta} + B_{22}\ddot{\delta} + H_2 + K_{21}\theta + K_{22}\delta = 0$$
(5.25)

Rewriting equation 5.25 as below

$$\ddot{\delta} = -B_{22}^{-1}(B_{21}\ddot{\theta} + H_2 + K_{21}\theta + K_{22}\delta)$$
(5.26)

Substituting equation 5.26 into equation 5.24

$$B_{11}\ddot{\theta} + B_{12}(-B_{22}^{-1}(B_{21}\ddot{\theta} + H_2 + K_{21}\theta + K_{22}\delta)) + H_1 + K_{11}\theta + K_{12}\delta = \tau$$

$$B_{11}\ddot{\theta} - B_{12}B_{22}^{-1}B_{21}\ddot{\theta} - B_{12}B_{22}^{-1}H_2 - B_{12}B_{22}^{-1}K_{21}\theta - B_{12}B_{22}^{-1}K_{22}\delta + H_1 + K_{11}\theta + K_{12}\delta = \tau$$

Rearranging,

$$(B_{11} - B_{12}B_{22}^{-1}B_{21})\ddot{\theta} + (H_1 + K_{11}\theta + K_{12}\delta - B_{12}B_{22}^{-1}H_2$$

$$-B_{12}B_{22}^{-1}K_{21}\theta - B_{12}B_{22}^{-1}K_{22}\delta) = \tau$$
(5.27)

In order to formulate the state space representation of the TLFM in equation 5.27 on the form of general representation of lower triangle systems, matrices M & N are defined

$$M = B_{11} - B_{12} B_{22}^{-1} B_{21} \tag{5.28}$$

$$N = H_1 + K_{11}\theta + K_{12}\delta - B_{12}B_{22}^{-1}H_2 - B_{12}B_{22}^{-1}K_{21}\theta - B_{12}B_{22}^{-1}K_{22}\delta$$
(5.29)

Substituting with equations 5.28 and 5.29 in equation 5.27 we got

$$M\ddot{\theta} + N = \tau \tag{5.30}$$

Equation 5.30 can be used as a base to apply backstepping control technique for it. Rewriting equation 5.30 as follow:

$$\ddot{\theta} = M^{-1}\tau - M^{-1}N \tag{5.31}$$

Defining the states for equation 5.31 as,

$$\varepsilon_1 = \theta$$

$$\varepsilon_2 = \dot{\theta} \tag{5.32}$$

State representation is then presented as below:

$$\dot{\varepsilon}_1 = \varepsilon_2$$

$$\dot{\varepsilon}_2 = M^{-1}\tau - M^{-1}N \tag{5.33}$$

Now equation 5.33 is on a similar form of equation 5.12, for purpose of position control we define θ_d as desired position for both links.

Applying design steps illustrated in section 5.2.1, we define error variable as e_1 and e_2

$$e_1 = \theta_d - \varepsilon_1 \tag{5.34}$$

$$e_2 = \alpha - \varepsilon_2 \tag{5.35}$$

where α is virtual input defined for the representation in equation 5.35.

Differentiating both variables e_1 and e_2

$$\dot{e}_1 = \dot{\theta}_d - \varepsilon_2 \tag{5.36}$$

$$\dot{e_2} = \dot{\alpha} - \dot{\varepsilon}_2 \tag{5.37}$$

Reference to equation 5.16, the first positive definite Lyapunov function V_1 is

$$V_1 = \frac{1}{2}e_1^2 \tag{5.38}$$

Finding the derivative of Lyapunov function \dot{V}_1 from equation 5.38

$$\dot{V}_1 = e_1 \dot{e}_1$$
 (5.39)

Substituting for \dot{e}_1 from equation 5.36 into equation 5.39

$$\dot{V}_1 = e_1(\dot{\theta}_d - \varepsilon_2) \tag{5.40}$$

Substituting for ε_2 from equation 5.35

$$\dot{V}_{1} = e_{1}(\dot{\theta}_{d} + e_{2} - \alpha)$$

$$\dot{V}_{1} = e_{1}(\dot{\theta}_{d} + e_{2} - \alpha)$$

$$\dot{V}_{1} = e_{1}\dot{\theta}_{d} + e_{1}e_{2} - e_{1}\alpha$$
(5.41)

Selecting the virtual input α that makes \dot{V}_1 a negative definite function by trying cancel all the positive terms,

$$\alpha = \dot{\theta}_d + c_1 e_1 \tag{5.42}$$

Substituting with equation 5.42 in equation 5.41

$$\dot{V}_{1} = e_{1}\dot{\theta}_{d} + e_{1}e_{2} - e_{1}(\dot{\theta}_{d} + c_{1}e_{1})$$

$$\dot{V}_{1} = e_{1}e_{2} - c_{1}e_{1}^{2}$$
(5.43)

 \dot{V}_1 is a negative definite $e_2 = 0$ for all values e_1

Reference to equation 5.18 a positive definite Lyapunov function V_2 is

$$V_{2} = V_{1} + \frac{1}{2}e_{2}^{2}$$

$$V_{2} = \frac{1}{2}e_{1}^{2} + \frac{1}{2}e_{2}^{2}$$
(5.44)

Finding the derivative of Lyapunov function \dot{V}_2 from equation 5.44

$$\dot{V}_2 = \dot{V}_1 + e_2 \dot{e}_2$$

Substituting for \dot{V}_1 from equation 5.43 and for \dot{e}_2 from equation 5.37

$$\dot{V}_2 = e_1 e_2 - c_1 e_1^2 + e_2 (\dot{\alpha} - \dot{\varepsilon}_2)$$
(5.45)

Substituting for $\dot{\epsilon}_2$ from equation 5.33 into equation 5.45

$$\dot{V}_2 = e_1 e_2 - c_1 e_1^2 + e_2 (\dot{\alpha} - (M^{-1}\tau - M^{-1}N))$$

$$\dot{V}_2 = e_1 e_2 - c_1 e_1^2 + e_2 \dot{\alpha} - e_2 M^{-1} \tau + e_2 M^{-1} N$$
(5.46)

Selecting the virtual input τ that makes \dot{V}_2 a negative definite function

$$\tau = M(-M^{-1}N + \dot{\alpha} + e_1 + c_2 e_2) \tag{5.47}$$

Substituting with τ from equation 5.47 in equation 5.46

$$\dot{V}_{2} = e_{1}e_{2} - c_{1}e_{1}^{2} + e_{2}\dot{\alpha} - e_{2}M^{-1}(M(-M^{-1}N + \dot{\alpha} + e_{1} + c_{2}e_{2})) + e_{2}M^{-1}N$$

$$\dot{V}_{2} = e_{1}e_{2} - c_{1}e_{1}^{2} + e_{2}\dot{\alpha} - e_{2}M^{-1}(N + M\dot{\alpha} + Me_{1} + Mc_{2}e_{2}) + e_{2}M^{-1}N$$

$$\dot{V}_{2} = e_{1}e_{2} - c_{1}e_{1}^{2} + e_{2}\dot{\alpha} - e_{2}M^{-1}N - e_{2}\dot{\alpha} - e_{2}e_{1} - c_{2}e_{2}^{2} + e_{2}M^{-1}N$$

$$\dot{V}_{2} = -c_{1}e_{1}^{2} - c_{2}e_{2}^{2}$$
(5.48)

 \dot{V}_2 is negative definite for all values e_1 and e_2

Differentiating virtual input α in equation 5.42 with

$$\dot{\alpha} = \ddot{\theta}_d + c_1 \dot{e}_1 \tag{5.49}$$

Substituting in τ equation 5.47 with $\dot{\alpha}$ from equation 5.49

$$\tau = M(-M^{-1}N + \ddot{\theta}_d + c_1\dot{e}_1 + e_1 + c_2e_2)$$
(5.50)

Selection of $c_1 \& c_2$ matrices is tuned during the simulation of the system to get the suitable performance for the system.

The selection of matrices c_1 and c_2 matrices are done through trial and error tuning through simulation starting with identify matrix for both till reaching the desired performance. The final content for matrices c_1 and c_2 matrices are listed below

$$c_1 = \begin{bmatrix} 100 & 0\\ 0 & 56 \end{bmatrix}$$
$$c_2 = \begin{bmatrix} 312 & 0\\ 0 & 64 \end{bmatrix}$$

As for stability investigation, backstepping controller gives a stable system since it designed based on Lyapunov function and its derivative, this in case that there is not possible condition in Lyapunov function derivative to be negative definite.

5.3 Sliding Mode Controller

5.3.1 Sliding mode technique [44],[13], [41], [45]

Sliding controller design provides a systematic approach to the problem of maintaining stability and consistent performance in the face of modeling imprecisions. Furthermore, by allowing the trade-offs between modeling and performance to be quantified in a simple fashion, it can illuminate the whole design process. Sliding control has been successfully applied to robot manipulators, underwater vehicles, automotive transmissions and engines, high-performance electric motors, and power systems.

Consider a single-input dynamic system:

$$x^{(n)} = f(X) + b(X)u$$
(5.51)

where,

n: the system order

 $x^{(n)}$ is the nth derivative of state x

u is the scalar control input,

x is the scalar output of interest (such as position)

X is the state vector. $X = \begin{bmatrix} x_1 & x_2 & \dots & x_n \end{bmatrix}^T = \begin{bmatrix} \dot{x} & \ddot{x} & \dots & x^{(n-1)} \end{bmatrix}^T$

f(X) and b(X) are nonlinear functions of the states.

The desired states vector is formulated in the following form

 $X_d = [x_{1d} \ x_{2d} \ \dots \ x_{nd}]^T = [\dot{x_d} \ \dot{x_d} \ \dots \ x^{(n-1)}_d]^T$
The tracking error \tilde{x} is expressed as,

$$\widetilde{x} = x - x_d = \begin{bmatrix} x \\ \dot{x} \\ . \\ . \\ x^{(n-1)} \end{bmatrix} - \begin{bmatrix} x_d \\ \dot{x}_d \\ . \\ . \\ x_d^{(n-1)} \end{bmatrix} = \begin{bmatrix} \widetilde{x} \\ \dot{\widetilde{x}} \\ . \\ \widetilde{x} \\ . \\ \widetilde{x}^{(n-1)} \end{bmatrix}$$
(5.52)

Defining a time-varying sliding mode surface s(x,t) = 0 in the state-space $\mathbb{R}^{(n)}$ by the following scalar equation,

$$s(x,t) = \left(\frac{d}{dt} + \lambda\right)^{n-1} \tilde{x}$$
(5.53)

where λ is a strictly positive constant related to the transient time.

The simplified, lst-order problem of keeping the scalar s at zero can now be achieved by choosing the control law u such that outside the sliding surface of s(x,t) = 0 that satisfy the following sliding condition:

$$\frac{1}{2}\frac{d}{dt}s^2 < -\eta|s| \tag{5.54}$$

where η is a strictly positive constant.

Figure 5.3 shows graphical presentation for sliding surface and effect of λ and η parameters.



Figure 5.3: Sliding surface and parameters effect [44]

The control law of single input is defined as

$$u = \hat{b}^{-1}(x_d^{(n-1)} - \hat{f} - k \, sgn(s)) \tag{5.55}$$

The control law stated equation 5.55 has two parts,

Continuous part

$$\hat{b}^{-1}(x_d^{(n-1)} - \hat{f})$$

and discontinuous part

$$\hat{b}^{-1}(-k \, sgn(s))$$

where \hat{b} and \hat{f} are the estimated values for b and f respectively due to uncertainties. where sgn function is shown in Figure 5.4.



Figure 55.4. Sgn function

The k gain affects both both λ , η and its value is found by tuning the response of the control behavior according to equation 5.55. The gain k needs be large enough to drive the back to zero again that associated with η as illustrated in Figure 5.3. Also, "k" can't be extremely large because this will cause a very high switching control activity that is associated with λ as shown in Figure 5.4. The gain "k" is multiplied by the function "-sgn(s)" to become "-k*sign(s)".

Since the implementation of the associated control switching is imperfect (for instance, in practice switching is not instantaneous, and the value of s is not known with infinite precision), this leads to chattering as illustrated in the Figure 5.5.



Figure 5.5. Chattering Effect

Chattering is undesirable in practice, since it involves high control activity and further may excite high frequency dynamics neglected in the course of modeling (such as unmodeled structural modes, neglected time-delays, and so on).

The discontinuous control law u is suitably smoothed by replacing the previous discontinues function "-k*sgn(s)" by " $-k*sat(s/\phi)$ " to achieve an optimal trade-off between control bandwidth and tracking precision where ϕ is the bandwidth of the saturation function.



Figure 5.6. Using saturation function "sat"

The sat function is as shown in Figure 5.7.



Figure 5.7. The Sat function

Hence the control law in equation (5-40) is modified as follow:

$$u = \hat{b}^{-1}(x_d^{(n-1)} - \hat{f} - k \operatorname{sat}(s/\phi))$$
(5.56)

Consider the multi-input dynamic system:

$$x_{i}^{(n_{i})} = f_{i}(X) + \sum_{j=1}^{m} b_{ij}(X)u_{j}$$

$$i = 1, \dots m \text{ and } j = 1, \dots m$$
where,
(5.57)

n: the system order

m: number of inputs

 x_i^n is the nth derivative of state x_i

 $u_{\rm j}$ is the jth scalar control input,

 x_i is the ith scalar output of interest

 $f_i(X)$ and $b_i(X)$ are nonlinear functions of the states.

Similar to the control law associated with single input case, the vector of control law u can be represented in the following form:

$$u = \hat{B}^{-1}(x_d^{(n-1)} - \hat{F} - \hat{K} \, sgn(S))$$
(5.58)

where

u is vector of u_i

 \hat{B} is matrix with b_{ij}

 \hat{F} is vector with f_i

 \widehat{K} sgn(S) is vector of k_i sgn(s_i)

In addition to avoid the chattering effect, control law is defined as

$$u = \hat{B}^{-1}(x_d^{(n-1)} - \hat{F} - \hat{K} sat(S/\phi))$$
(5.59)

where

 \widehat{K} sat (S/ϕ) is vector of k_i sat (s_i/ϕ_i)

5.3.2 Sliding mode controller design

Starting from equation 5.21,

$$\ddot{\theta} = M^{-1}\tau - M^{-1}N$$

Define state vectors ε_1 and ε_2

$$\varepsilon_1 = \begin{bmatrix} \theta_1 \\ \dot{\theta}_1 \end{bmatrix}$$
$$\varepsilon_2 = \begin{bmatrix} \theta_2 \\ \dot{\theta}_2 \end{bmatrix}$$

Then define the desired states as below

$$\varepsilon_{1d} = \begin{bmatrix} \theta_{1d} \\ \dot{\theta}_{1d} \end{bmatrix}$$
$$\varepsilon_{2d} = \begin{bmatrix} \theta_{2d} \\ \dot{\theta}_{2d} \end{bmatrix}$$

The error vectors can be calculated as

$$\widetilde{\varepsilon_1} = \varepsilon_1 - \varepsilon_{1d}$$
$$\widetilde{\varepsilon_2} = \varepsilon_2 - \varepsilon_{2d}$$

Defining both sliding surfaces

$$s_1(x,t) = \tilde{\varepsilon}_1 - \lambda_1 \tilde{\varepsilon}_1$$
$$s_2(x,t) = \tilde{\varepsilon}_2 - \lambda_2 \tilde{\varepsilon}_2$$

By applying control law in equation 5.58, we got

$$\tau = \begin{bmatrix} \widehat{M}(\varepsilon_{1d} + \widehat{M}^{-1}\widehat{N} - k_1 sgn(s_1))\\ \widehat{M}(\varepsilon_{2d} + \widehat{M}^{-1}\widehat{N} - k_2 sgn(s_2)) \end{bmatrix}$$
(5.60)

 k_1 and k_2 gains are tuned and selected as 300 and 260 respectively through trial and error and monitoring the system performance to get the suitable response.

To reduce the chattering, sat function is applied as per equation 5.59

$$\tau = \begin{bmatrix} \widehat{M}(\varepsilon_{1d} + \widehat{M}^{-1}\widehat{N} - k_1 sat(s_1/\phi) \\ \widehat{M}(\varepsilon_{2d} + \widehat{M}^{-1}\widehat{N} - k_2 sat(s_2/\phi)) \end{bmatrix}$$
(5.61)

where \widehat{M} and \widehat{N} are estimates of M and N matrices that were defined in equations 5.28 and 5.29 respectively. The estimation \widehat{M} and \widehat{N} is taken as the original M and N with 2% positive change of their value.

As for stability investigation, once satisfying the sliding condition that makes the sliding surface as invariant set, which means the system will tracking surface but it won't diverge out of it.

υ Chapter 6

Results and Discussion

This chapter presents the simulation and experimental testing using all of the developed control techniques and illustrated their results.

The simulation testing is implemented using MATLAB/Simulink and it is based on the nonlinear AMM model of TLFM derived in Chapter 4 and the linearized model of the system derived in Chapter 5 while applying the physical parameters showed in table 4-1. The experimental testing uses the 2-DOF Serial Flexible Link Robot from Quanser along with the nonlinear model. The dynamic performance is time domain describing the simulation and experimental results of all developed control technique are compared.

As illustrated in Chapter 5, the aim of the developed controllers is to achieve position control for both links of the TLFM. The input signal used as a desired position control is a square wave of 0.1 Hz and amplitude of ± 25 and ± 10 degrees for link1 and link2 respectively, input representation is shown in Figure 6.1.



(a) Input representation for link 1



Figure 6.1. (a) Input representation for link 1, (b) Input representation for link 2

6.1 Simulation Testing and Results

6.1.1 Open loop model

Figure 6.2 shows the implementation TLFM dynamic model in equation 4.20

$$B(q)\ddot{q} + H(q,\dot{q}) + Kq = u$$



Figure 6.2. TLFM open loop nonlinear model implementation on Simulink

Figure 6.3 below shows the internal structure of TLFM dynamics block



Figure 6.3. TLFM dynamics block subsystem in TLFM nonlinear model on Simulink

As mentioned the inputs for both links are square waves, selecting the following values for each input,

Theta 1 input: Amplitude: ± 25 Frequency: 0.1 Hz

Theta 2 input: Amplitude: ± 10 Frequency: 0.1 Hz

Frequencies for input signal used in the literature vary from 0.1-1 Hz.

Figures 6.4 and 6.5 show the open loop responses of Theta1 and Theta2 respectively associated with model of the system implementation using MATLAB/Simulink shown in Figure 6.2.

For both links, the link positions (Theta1 and Theta2) moved far away from the desired value when the input of each link is in the negative half of the square wave. While during the positive half of the desired input, the link positions moved towards the negative desired value of the input.



Figure 6.4. Link1open loop response of nonlinear model for square input 0.1Hz, Input signal in purple and Theta1 output in yellow

Theta2 (rad)



Figure 6.5. Link2 open loop response of nonlinear model for square input 0.1Hz, Input signal in purple and Theta2 output in yellow

Figure 6.6 shows the MATLAB/Simulink implementation of linearized model in equation in 5.11 on Simulink



Figure 6.6. TLFM open loop linearized model

Apply the same desired inputs used with the nonlinear model to the linearized model shown in Figure 6.6.

Figures 6.7 and 6.8 show the responses of Theta1 and Theta2 respectively for the mentioned desired input signals of the linear model implemented in Figure 6.6. The responses are very similar to the nonlinear models results.

```
Theta1 ▲
(rad)
```



Figure 6.7. Link1open loop response of nonlinear model for square input 0.1Hz, Input signal in purple and Theta1 output in yellow



Figure 6.8. Link2 open loop response of nonlinear model for square input 0.1Hz, Input signal in purple and Theta2 output in yellow

6.1.2 LQR controller

The general representation of LQR control input is u = -Kx and its block diagram representation shown in Figure 6.9.



Figure 6.9. General LQR controller with *x* state vector

Hence it is necessary to have the state vector in order to find the control input u = -Kx. However, the State-Space block in Simulink shown in Figure 6.10 can only generate the output from the block without generating the state vector.



Figure 6.10. State-space block of Simulink

Therefore, a new state space block was designed and implemented as shown in Figure 6.11 to help to generate the state vector beside the output. Figure 6.12 shows the integration of the new state space block with the TLFM linearized model.



Figure 6.11. State space block implementation with state vector generated from the block



Figure 6.12. TLFM Open loop linearized model with state space block implemented

The LQR controller designed in section 5.1.2 has been implemented as shown in Figure 6.13.



Figure 6.13. LQR controller implementation for linearized model

For the design laws mentioned in Chapter 5,

where

$$K = R^{-1}B^T P$$

and P is the solution of the following equation

$$A^T P + PA - PBR^{-1}B^T P + Q = 0$$

For the linearized TLFM system under study $Q \in R^{12x12}$ and $\in R^{6x6}$, for the tuning process depended on changing the values of Q and R matrices and monitor the dynamic performance parameter to get the best response. Below are the best concluded values for both matrices

	۲140 _آ	0	0	0	0	0	0	0	0	0	0	0
	0	345	0	0	0	0	0	0	0	0	0	0
	0	0	440	0	0	0	0	0	0	0	0	0
	0	0	0	440	0	0	0	0	0	0	0	0
0 =	0	0	0	0	45	0	0	0	0	0	0	0
	0	0	0	0	0	80	0	0	0	0	0	0
Q =	0	0	0	0	0	0	77	0	0	0	0	0
	0	0	0	0	0	0	0	1000	0	0	0	0
	0	0	0	0	0	0	0	0	5000	0	0	0
	0	0	0	0	0	0	0	0	0	12	0	0
	0	0	0	0	0	0	0	0	0	0	33	0
	Γ0	0	0	0	0	0	0	0	0	0	0	1-
	100	0	0	0	0	0	1					
	0	100	0	0	0	0						
D_	0	0	100	0	0	0						
N-	0	0	0	100	0	0						
	0	0	0	0	100	0						
	- 0	0	0	0	0	100	L(

Using A, B, C, D, R and Q matrices the LQR gain was found using MATLAB LQR function.

For stability check, the eigen values for A matrix was found all negative which implies the linearized system is stable a shown below:

$\lambda_1 = -22.43$	$\lambda_4 = -112.34$	$\lambda_7 = -209.5$	$\lambda_{10} = -84.79$
$\lambda_2 = -45.45$	$\lambda_5 = -990.12$	$\lambda_8 = -738.4$	$\lambda_{11} = -230.55$
$\lambda_3 = -962.44$	$\lambda_6 = -578.07$	$\lambda_9 = -328.93$	$\lambda_{12} = -65.8$

Figures 6.14 and 6.15 show the responses for Theta1 and Theta2 respectively for the input reference signals for the LQR controller on the linearized model.



Figure 6.14. Link1 LQR controller response of simulated linearized model for square input 0.1Hz, Input signal in purple and Theta1 output in yellow



Figure 6.15. Link2 LQR controller response of simulated linearized model for square input 0.1Hz, Input signal in purple and Theta2 output in yellow

The control signal of link1 and link2 are shown in Figures 6.16 and 6.17 respectively.



Figure 6.16. Control signal of Link1 for LQR controller for linearized model



Figure 6.17. Control signal of link2 for LQR controller for linearized model

Parameters describing the response of the system using LQR controller with linearized model are be shown in Table 6.1 below

Table 6.1. Response parameters of LQR controller linearized model simulation

	Rise Time (s)	Settling Time (s)	Overshoot %	St St Error%
Link 1	0.7	0.8	0.45	0
Link 2	0.8	0.8	0	0

The designed LQR controller for the linearized model was applied to the nonlinear model shown in Figure 6.18 to compare the effect of the linearization and its response on the nonlinear model.



Figure 6.18. Applying LQR controller to the nonlinear model

Figures 6.19 and 6.20 show the responses of Theta1 and Theta2 respectively for the desired input signals illustrating LQR controller performance with the nonlinear model.





Figure 6.20. Link2 LQR controller response of simulated nonlinear model for square input 0.1Hz, Input signal in purple and Theta2 output in yellow

The control signal of link1 and link2 are shown in Figures 6.21 and 6.22 respectively



Figure 6.21. Control signal for Link1LQR controller applied to nonlinear model



Figure 6.22: Control signal for Link2LQR controller applied to nonlinear model

Parameters describing the response of the system using LQR controller with the nonlinear model are shown in Table 6.2 below

	Rise Time (s)	Settling Time (s)	Overshoot %	St St Error%
Link 1	0.6	1.5	13.3%	22%
Link 2	0.5	1	12%	18%

Table 6.2. Response parameters of LQR controller nonlinear model simulation

It is clear that from the results of applying the LQR controller to the linearized model, that has a good dynamic response with zero steady state error and overshoot. However, when the LQR controller was applied to the nonlinear model, a steady state error appears in the position for both links. In addition an overshoot appeared in links' response as well. This is due to the fact that the LQR is a linear controller that was designed to work with linearized model and it can't handle the nonlinear dynamics in the nonlinear model which was not taken into consideration when designing the LQR controller.

6.1.3 Backstepping controller

The Backstepping controller designed in section 5.2 has been implemented as per equation 5.50

$$\tau = M(-M^{-1}N + \ddot{\theta}_d + c_1\dot{e}_1 + e_1 + c_2e_2)$$

where all parameters are known except c_1 and c_2 that are subjected to tuning.

Figure 6.23 shows the implementation of backstepping controller for the nonlinear model



Figure 6.23. Backstepping controller implementation for nonlinear model

For the tuning process depended on changing the values of c_1 and c_2 matrices and monitor the dynamic performance parameter to get the best response. Below are the suitable selected values for both matrices c_1 and c_2

$$c_2 = \begin{bmatrix} 100 & 0 \\ 0 & 56 \end{bmatrix} \qquad c_2 = \begin{bmatrix} 312 & 0 \\ 0 & 64 \end{bmatrix}$$

Figures 6.24 and 6.25 show the responses for Theta1 and Theta2 respectively for the desired input signals for the Backstepping controller with the nonlinear model.



Figure 6.24. Link1 backstepping controller response of simulated nonlinear model for square input 0.1Hz, Input signal in purple and Theta1 output in yellow

Theta2 (rad)



Figure 6.25. Link2 backstepping controller response of simulated nonlinear model for square input 0.1Hz, Input signal in purple and Theta2 output in yellow

Figures 6.26 and 6.27 show the control signals for link1 and link2 respectively



Figure 6.26. Control signal for link1 with backstepping control



Figure 6.27. Control signal for link1 with backstepping control

Parameters describing the response of the system using Backstepping controller are be shown in Table 6.3 below

Table 6.3: Response parameters of backstepping controller nonlinear model Simulation

	Rise Time(s)	Settling Time(s)	Overshoot%	St St Error%
Link 1	0.6	0.9	0	0
Link 2	0.6	0.7	0	0

As it can be seen from Figures 6.20 and 6.21, the backstepping controller tuning was easier than the LQR controller and it gave better performance with nonlinear model over the LQR with linear model and this is due to that the back stepping technique is a combination of control signals that each of them was accurately deigned for subsystem.

6.1.4 Sliding mode controller

The sliding mode controller designed in section 5.3 is implemented as per equation 5.61

$$\tau = \begin{bmatrix} \widehat{M}(\varepsilon_{1d} + \widehat{M}^{-1}\widehat{N} - \mathbf{k}_1 sgn(s_1) \\ \widehat{M}(\varepsilon_{2d} + \widehat{M}^{-1}\widehat{N} - \mathbf{k}_2 sgn(s_2)) \end{bmatrix}$$

Figure 6.28shows the implementation of sliding mode controller for the nonlinear model



Figure 6.28. Sliding mode controller implementation for nonlinear model

 k_1 and k_2 were tuned by monitoring the dynamic response of system and select the best tuning values which were concluded as $k_1 = 300$ and $k_2 = 260$.

Figures 6.29 and 6.30 show the responses for Theta1 and Theta2 respectively for the input reference signals for the backstepping controller on the nonlinear model.



Figure 6.29. Link1 sliding mode controller response of simulated nonlinear model with chattering for square input 0.1Hz, Input signal in purple and Theta1 output in yellow

Theta2 (rad)



Figure 6.30. Link2 sliding mode controller response of simulated nonlinear model with chattering for square input 0.1Hz, Input signal in purple and Theta2 output in yellow





Figure 6.31. Control signal for Link1 with SMC controller chattering case



Figure 6.32. Control signal for Link2 with SMC controller chattering case

Parameters describing the response of the system using sliding mode controller with nonlinear model with chattering can be shown in Table 6.4 below

Table 6.4. Response parameters of sliding mode controller nonlinear model with chattering simulation

	Rise Time(s)	Settling Time(s)	Overshoot%	St St Error %
Link 1	0.5		0	2.8
Link 2	0.5		0	3.3

As it can be seen from Figures 6.23 and 6.24, sliding Mode controller issue was the chattering problem which is generated due to the discontinuous part in the control law however it could reached a less rise time using sliding mode controller than the backstepping controller.

Applying control equation (5.42) to reduce the chattering

$$\tau = \begin{bmatrix} \widehat{M}(\varepsilon_{1d} + \widehat{M}^{-1}\widehat{N} - k_1 sat(s_1/\phi) \\ \widehat{M}(\varepsilon_{2d} + \widehat{M}^{-1}\widehat{N} - k_2 sat(s_2/\phi)) \end{bmatrix}$$

Figures 6.33 and 6.34 show the responses for Theta1 and Theta2 respectively for the input reference signals for the backstepping controller with the nonlinear model.



Figure 6.33. Link1 sliding mode controller response of simulated nonlinear model without chattering for square input 0.1Hz, Input signal in purple and Theta1 output in yellow



Figure 6.34. Link2 sliding mode controller response of simulated nonlinear model without chattering for square input 0.1Hz, Input signal in purple and Theta2 output in yellow

Figures 6.35 and 6.36 show the control signal for the link1 and link2 of SMC controller without chattering



Figure 6.35. Control signal for Link1 with SMC controller without chattering case



Figure 6.36. Control signal for Link2 with SMC controller without chattering case

Parameters describing the response of the system using sliding mode controller with nonlinear model without chattering can be shown in Table 6.5

	Rise Time (s)	Settling Time (s)	Overshoot%	St St Error%
Link 1	0.5	0.7	0	0
Link 2	0.5	0.7	0	0

Table 6.5. Response parameters of sliding mode controller nonlinear model without chattering simulation

As it can be seen from Figures 6.25 and 6.26, sliding Mode controller issue was the chattering problem was solved and the performance was improved.

6.2 Experimental Testing and Results

The used hardware throughout this thesis is 2-DOF Serial Flexible Link Robot from Quanser Company that is shown in Figure 6.37. This robot system consists of two DC motors driving a two-bar serial linkage via harmonic gearboxes. The primary link (link1) is rigidly clamped to the first drive (a.k.a. elbow) and carries at its end the second harmonic drive (a.k.a. shoulder) to which second flexible link (link2) is attached. Both motors are instrumented with quadrature optical encoders [11].



Figure 6.37. The 2DOF Serial Link Robot [11]

The implementation of each of the developed controllers depends on the states and the dynamics of the model. In relation to the hardware, it is possible only to measure two states represents the position of both links. Therefore, in order to be able to test the developed controllers on experimental system there is a need to find the remaining states (states related

to AMM modes). Hence, the dynamic model in both linearized and nonlinear was used as relevant to generate the missing states and integrate them with the physical model as shown in Figure 6.38.



Figure 6.38. Controller implementation to hardware system

The real time control of hardware system is done using Hardware In the Loop (HIL) technique that is supported by physical system 2-DOF Serial Flexible Link Robot from Quanser. In the HIL technique, the controller is implemented using Simulink where it receives the links' position (Theta1 and Theta2) feedback from the physical system through data acquisition input card and send the control action (torques for both motors) to the physical system through data acquisition input card. Figure 6.39 shows HIL implementation on 2-DOF Serial Flexible Link Robot from Quanser.



Figure 6.39: HIL Implementation on TLFM from Quanser

6.2.1 LQR controller

Figure 6.40 shows the implementation of LQR controller on hardware system.



Figure 6.40. LQR implementation on hardware system

Figures 6.41 and 6.42 show the responses for Theta1 and Theta2 respectively for the input reference signals for the LQR controller on hardware system.



Figure 6.41. Link1 LQR controller response of hardware system for square input 0.1Hz, Input signal in purple and Theta1 output in yellow



Figure 6.42. Link2 LQR controller response of hardware system for square input 0.1Hz, Input signal in purple and Theta2 output in yellow

Parameters describing the response of the system using for LQR controller on the hardware system is shown in Table 6.6 below

	Rise Time (s)	Settling Time (s)	Overshoot (%)	St St Error (%)
Link 1	1.6	1.6	0	24
Link 2	1.3	1.3	0	27

Table 6.6. Response parameters of LQR controller for hardware system

It is noted that from applying the LQR controller to hardware system and compared to applying the LQR controller to nonlinear model that, the rise time has increased and the overshoot has increased. In addition there is no overshoot when LQR was applied to the hardware system and the settling time has decreased. The difference between applying the LQR controller to the nonlinear model and to the hardware system is due to the modeling assumptions mentioned in 4.3 and due to number of AMM modes, was represented as two modes in implemented model.

6.2.2 Backstepping controller

Figure 6.43 shows the implementation of backstepping controller on hardware system.



Figure 6.43. Backstepping controller implementation on hardware system

Figures 6.44 and 6.45 show the responses for Theta1 and Theta2 respectively for the input reference signals for the backstepping controller on hardware system.



Figure 6.44: Link1 backstepping controller response of hardware system for square input 0.1Hz, Input signal in purple and Theta1 output in yellow



Figure 6.45. Link2 backstepping controller response of hardware system for square input 0.1Hz, Input signal in purple and Theta2 output in yellow

Parameters describing the response of the system using backstepping controller on the hardware system is shown in Table 6.7 below

Table 6.7. Response paramete	rs of backstepping controller	for hardware system
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	Rise Time (s)	Settling Time (s)	Overshoot %	St St Error
Link 1	1.2	1.2	0	0.95 %
Link 2	1	1.1	0	1.12 %

It is noted that from applying the backstepping controller to hardware system and compared to applying the backstepping controller to nonlinear model that, the rise time and the settling time have increased slightly while the overshoot remained zero and there is a slight steady state error around 1%. The difference between applying the backstepping controller to the nonlinear model and to the hardware system is due to the modeling assumptions mentioned in 4.3 and due to number of AMM modes, which are two in implemented model.

6.2.3 Sliding mode controller

Figure 6.46 shows the implementation of SMC controller on hardware system.



Figure 6.46. SMC controller implementation on hardware system

Figures 6.47 and 6.48 show the responses for Theta1 and Theta2 respectively for the input reference signals for the SMC controller with chattering on hardware system.



Figure 6.47. Link1 SMC controller response of hardware system with chattering for square input 0.1Hz, Input signal in purple and Theta1 output in yellow



Figure 6.48. Link2 SMC controller response of hardware system with chattering for square input 0.1Hz, Input signal in purple and Theta2 output in yellow

Parameters describing the response of the system using SMC controller on the hardware system with chattering are shown in Table 6.8 below

Table 6.8. Response parameters of SMC	controller with chattering for hardware system
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	Rise Time (s)	Settling Time (s)	Overshoot %	St St Error%
Link 1	0.7		2.3	1.8
Link 2	0.7		3.1	0

Figures 6.49 and 6.50 show the responses for Theta1 and Theta2 respectively for the input reference signals for the SMC controller without chattering on hardware system.






Figure 6.50. Link2 SMC controller response of hardware system w/o chattering for square input 0.1Hz, Input signal in purple and Theta2 output in yellow

Parameters describing the response of the system using SMC controller on the hardware system without chattering are shown in Table 6.9 below

	Rise Time (s)	Settling Time (s)	Overshoot %	St St Error%
Link 1	1.1	1.1	0	2
Link 2	0.9	1.2	0.5	0.1

Table 6.9. Response parameters of SMC controller w/o chattering for hardware system

It is noted that from applying the SMC controller to hardware system and compared to applying the SMC controller to nonlinear model that, the rise time and the settling time and overshoot have increased slightly and there is a steady state error around 2% for link 1 and 0.1% for link 2. The difference between applying the SMC controller to the nonlinear model and to the hardware system is due to the modeling assumptions mentioned in 4.3 and due to number of AMM modes, which are two in implemented model.

6.3 Results Summery Table

The summery of all parameters describing the response of the system using all developed controllers and their implementation on simulation and experimental testing is shown in Table 6.10.

No.	Control	Response Parameter	Results	
	Technique		Link 1	Link 2
1	LQR Controller	Overshoot %	0.45	0
	for Linearized	Settling Time	0.8	0.8
	Model	Rise Time	0.7	0.8
		St St Error%	0	0
2	LQR Controller	Overshoot %	22	18
	for Nonlinear	Settling Time	1.5	1
	Model	Rise Time	0.6	0.5
		St St Error%	22	18
3	LQR Controller	Overshoot %	0	0
	for Hardware	Settling Time	1.6	1.3
		Rise Time	1.6	1.3
		St St Error %	24	27
4	LQR Controller	Overshoot %	6	16.5
	of Quanser	Settling Time	1.8	2.3
	Results[11]	Rise Time	1	0.8
		St St Error%	0.5	15.3
5	Backstepping	Overshoot %	0	0
	Controller for	Settling Time	0.9	0.7
	Nonlinear Model	Rise Time	0.6	0.6
		St St Error%	0	0
6	Backstepping	Overshoot %	0	0
	Controller for	Settling Time	1.2	1
	Hardware	Rise Time	1.2	1.1
		St St Error%	0.95	1.12
7	Sliding Mode	Overshoot %	0	0
	Controller with	Settling Time		
	chattering for	Rise Time	0.5	0.5
	Nonlinear Model	St St Error%	2.8	3.3
7	Sliding Mode	Overshoot %	0	0
	Controller w/o	Settling Time	0.7	0.7
	chattering for	Rise Time	0.5	0.5
	Nonlinear Model	St St Error%	0	0
8	Sliding Mode	Overshoot %	2.3	3.1
	Controller with	Settling Time		
	chattering for	Rise Time	0.7	0.5
	Hardware	St St Error%	1.8	0
9	Sliding Mode	Overshoot %	0	0.5
	Controller w/o	Settling Time	1.1	0.9
	chattering for	Rise Time	1.1	1.2
	Hardware	St St Error%	2	0.1

Table 6.10. Summery table for dynamic performance parameters

Chapter 7

Conclusion and Future Work

7.1 Conclusion

This thesis presented a nonlinear dynamic model of a TLFM and the AMM with Lagrangian method was selected for modeling the system. In addition, the development of three control techniques: LQR, Backstepping and Sliding Mode were introduced. For the LQR, the dynamic model was linearized been formulated using the form of linear state space system. For the backstepping technique, the controller was designed based on Lyapunov function which assures the stability of the system. Finally, sliding mode controller was designed but with the chattering problem exists due to the discontinuity part in the control law. The chattering problem was resolved by changing the function of the discontinuity part to represent a boundary not a line which smoothed the control action accordingly.

The implementation and testing of the developed control techniques were performed through simulation and by using an experimental 2-DOF Serial Flexible Link Robot from Quanser Company. The results were summarized, discussed and compared against dynamic the performance of position control and the steady state error of both links under similar control inputs.

From the results and the comparison between the developed control techniques, the following were concluded:

- a. The LQR controller gave a better performance with the linearized model of the system than when it was used with the nonlinear model of the system in both simulation and experimental testing. This is due to the fact that the LQR controller was designed in association with the linearized model of the system and hence it was not able to handle the nonlinear dynamics of the TLFMs, since the model in highly nonlinear.
- b. Backstepping controller presented better performance than and sliding mode controller in terms of dynamic performance and steady stated error.
- c. Sliding Mode controller is associated with chattering problem which is generated due to the discontinuity part in the control law. However, it could reach the target with less rise time than that obtained using backstepping control techniques. The saturation function was used to improve the performance and reduce chattering.
- d. There slight differences between the results obtained through simulation and experimental testing. Such differences were due to modeling assumptions mentioned in 4.3 and due to number of AMM modes used to represent each link. In this thesis only two AMM modes were used.

7.2 Future Work

The recommendation for future work can be divided into two parts, the first part is to improve modeling and the second part is to improve control and performance.

As for the modeling part:

- a. Need to try different boundary conditions for the AMM modeling and compare the results accordingly,
- b. Increase the number of modes, since this thesis used two modes for each link, to investigate the effect on computational load, model accuracy and control performance. A proper balance should be reached between the number of AMM modes, the computational load and the required performance and
- c. Use FEM and AMM modeling for the same system. Then, evaluate the performance.

As for the control part:

- a. Using computational intelligence techniques such fuzzy logic, neural networks and/or genetic algorithms to optimize the parameters that relies on conventional manual tuning. This will save the tuning time for all controllers' type and contribute to improve the performance, and
- b. Apply computational intelligence techniques to replace the developed control techniques for the purpose to control the dynamic of the two-link flexible manipulator system and control its position with aim to check and compare its performance and compare it with other control techniques.

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Appendix A

Model Coefficients for TLFM Model [37]

Appendix A introduces the mathematical formulation of the parameters in B and H Matrices used for modeling of the TLFM stated in Chapter 4.

 $B_{11} = b_{111} + b_{112}c_2 + (b_{113}t_1 + b_{114}t_2)s_2$ $B_{12} = b_{121} + b_{122}c_2 + (b_{123}t_1 + b_{124}t_2)s_2$ $B_{13} = b_{131} + b_{132}c_2 + (b_{133}t_2 + b_{134}\delta_{12})s_2$ $B_{14} = b_{141} + b_{142}c_2 + (b_{143}t_2 + b_{144}\delta_{11})s_2$ $B_{15} = b_{151} + b_{152}c_2 + b_{153}t_1s_2$ $B_{16} = b_{161} + b_{162}c_2 + b_{163}t_1s_2$ $B_{22} = b_{221}$ $B_{23} = b_{231} + b_{232}c_2 + (b_{233}t_2 + b_{234}t_3)s_2$ $B_{24} = b_{241} + b_{242}c_2 + (b_{243}t_2 + b_{244}t_3)s_2$ $B_{25} = b_{251}$ $B_{26} = b_{261}$ $B_{33} = b_{331} + b_{332}c_2 + b_{333}t_2s_2$ $B_{34} = b_{341} + b_{342}c_2 + b_{343}t_2s_2$ $B_{35} = b_{351} + b_{352}c_2 + b_{353}t_3s_2$ $B_{36} = b_{361} + b_{362}c_2 + b_{363}t_3s_2$ $B_{44} = b_{441} + b_{442}c_2 + b_{443}t_2s_2$ $B_{45} = b_{451} + b_{452}c_2 + b_{453}t_3s_2$ $B_{46} = b_{461} + b_{462}c_2 + b_{463}t_3s_2$ $B_{55} = b_{551}$ $B_{56} = b_{561}$ $B_{66} = b_{661}$

where $s_2 = \sin \theta_2$, $c_2 = \cos \theta_2$, and

$$t_1 = t_{11}\delta_{11} + t_{12}\delta_{12}$$

$$t_2 = t_{21}\delta_{21} + t_{22}\delta_{22}$$

$$t_3 = t_{31}\delta_{11} + t_{32}\delta_{12}$$

$$h_{1} = \left[(h_{101}\dot{\theta}_{2} + h_{102}\dot{\delta}_{11} + h_{103}\dot{\delta}_{12} + h_{104}\dot{\delta}_{21} + h_{105}\dot{\delta}_{22})\dot{\theta}_{1} + (h_{106}\dot{\theta}_{2} + h_{107}\dot{\delta}_{11} + h_{108}\dot{\delta}_{12} + h_{109}\dot{\delta}_{21} + h_{110}\dot{\delta}_{22})\dot{\theta}_{2} + (h_{111}\dot{\delta}_{21} + h_{112}\dot{\delta}_{22})\dot{\delta}_{11} + (h_{113}\dot{\delta}_{21} + h_{114}\dot{\delta}_{22})\dot{\delta}_{12} \right]s_{2} + \left[(h_{115}\dot{\theta}_{1} + h_{116}\dot{\theta}_{2} + h_{117}\dot{\delta}_{21} + h_{118}\dot{\delta}_{22})t_{1} + (h_{119}\dot{\theta}_{1} + h_{120}\dot{\theta}_{2} + h_{121}\dot{\delta}_{11} + h_{122}\dot{\delta}_{12})t_{2} \right]s_{2}$$

$$+ h_{123}\delta_{12}\dot{\delta}_{11} + h_{124}\delta_{11}\dot{\delta}_{12} \dot{\theta}_2 c_2$$

$$\begin{aligned} h_2 &= (h_{201}\dot{\theta}_1 + h_{202}\dot{\delta}_{11} + h_{203}\dot{\delta}_{12})\dot{\theta}_1 s_2 \\ &+ \Big\{ \big[(h_{204}\dot{\theta}_1 + h_{205}\dot{\delta}_{21} + h_{206}\dot{\delta}_{22})t_1 \\ &+ (h_{207}\dot{\theta}_1 + h_{208}\dot{\delta}_{11} + h_{209}\dot{\delta}_{12})t_2 \\ &+ h_{210}\delta_{12}\dot{\delta}_{11} + h_{211}\delta_{11}\dot{\delta}_{12} \big]\dot{\theta}_1 \\ &+ \big[(h_{212}\dot{\delta}_{11} + h_{213}\dot{\delta}_{12})t_2 + (h_{214}\dot{\delta}_{21} + h_{215}\dot{\delta}_{22})t_3 \big]\dot{\delta}_{11} \\ &+ \big[h_{216}\dot{\delta}_{12}t_2 + (h_{217}\dot{\delta}_{21} + h_{218}\dot{\delta}_{22})t_3 \big]\dot{\delta}_{12} \Big\} c_2 \end{aligned}$$

$$h_{3} = \left[(h_{301}\dot{\theta}_{1} + h_{302}\dot{\theta}_{2} + h_{303}\dot{\delta}_{12} + h_{304}\dot{\delta}_{21} + h_{305}\dot{\delta}_{22})\dot{\theta}_{1} \right. \\ \left. + (h_{306}\dot{\theta}_{2} + h_{307}\dot{\delta}_{11} + h_{308}\dot{\delta}_{12} + h_{309}\dot{\delta}_{21} + h_{310}\dot{\delta}_{22})\dot{\theta}_{2} \right. \\ \left. + (h_{311}\dot{\delta}_{21} + h_{312}\dot{\delta}_{22})\dot{\delta}_{11} + (h_{313}\dot{\delta}_{21} + h_{314}\dot{\delta}_{22})\dot{\delta}_{12} \right]s_{2} \\ \left. + \left[(h_{315}\dot{\theta}_{1} + h_{316}\dot{\theta}_{2} + h_{317}\dot{\delta}_{11} + h_{318}\dot{\delta}_{12})t_{2} \right. \\ \left. + (h_{319}\dot{\theta}_{2} + h_{320}\dot{\delta}_{21} + h_{321}\dot{\delta}_{22})t_{3} \right] \right]s_{2}$$

$$\begin{split} h_4 &= \left[(h_{401}\dot{\theta}_1 + h_{402}\dot{\theta}_2 + h_{403}\dot{\delta}_{11} + h_{404}\dot{\delta}_{21} + h_{405}\dot{\delta}_{22})\dot{\theta}_1 \\ &+ (h_{406}\dot{\theta}_2 + h_{407}\dot{\delta}_{11} + h_{408}\dot{\delta}_{12} + h_{409}\dot{\delta}_{21} + h_{410}\dot{\delta}_{22})\dot{\theta}_2 \\ &+ (h_{411}\dot{\delta}_{21} + h_{412}\dot{\delta}_{22})\dot{\delta}_{11} + (h_{413}\dot{\delta}_{21} + h_{414}\dot{\delta}_{22})\dot{\delta}_{12} \right]s_2 \\ &+ \left[(h_{415}\dot{\theta}_1 + h_{416}\dot{\theta}_2 + h_{417}\dot{\delta}_{11} + h_{418}\dot{\delta}_{12})t_2 \\ &+ (h_{419}\dot{\theta}_2 + h_{420}\dot{\delta}_{21} + h_{421}\dot{\delta}_{22})t_3 \\ &+ h_{422}\delta_{11}\dot{\theta}_1 \right]\dot{\theta}_2c_2 \end{split}$$

$$h_{5} = (h_{501}\dot{\theta}_{1} + h_{502}\dot{\delta}_{11} + h_{503}\dot{\delta}_{12})\dot{\theta}_{1}s_{2} + [h_{504}t_{1}\dot{\theta}_{1} + (h_{505}\dot{\delta}_{11} + h_{506}\dot{\delta}_{12})t_{3}]\dot{\theta}_{2}c_{2}$$

$$h_{6} = (h_{601}\dot{\theta}_{1} + h_{602}\dot{\delta}_{11} + h_{603}\dot{\delta}_{12})\dot{\theta}_{1}s_{2} + [h_{604}t_{1}\dot{\theta}_{1} + (h_{605}\dot{\delta}_{11} + h_{606}\dot{\delta}_{12})t_{3}]\dot{\theta}_{2}c_{2}$$

$$\begin{split} b_{111} &= J_{h1} + J_{o1} + J_{h2} + m_{h2}\ell_1^2 \\ &+ J_{o2} + m_2\ell_1^2 + J_p + m_p(\ell_1^2 + \ell_2^2) \\ b_{112} &= 2(m_2d_2 + m_p\ell_2)\ell_1 \\ b_{113} &= 2(m_2d_2 + m_p\ell_2) \\ b_{114} &= -2\ell_1 \\ b_{121} &= J_{h2} + J_{o2} + J_p + m_p\ell_2^2 \\ b_{122} &= (m_2d_2 + m_p\ell_2)\ell_1 \\ b_{123} &= m_2d_2 + m_p\ell_2 \\ b_{124} &= -\ell_1 \\ b_{131} &= w_{11} + (J_{h2} + J_{o2} + J_p + m_p\ell_2^2)\phi_{11,e} \\ &+ (m_{h2} + m_2 + m_p)\ell_1\phi_{11,e} \\ b_{132} &= (m_2d_2 + m_p\ell_2)(\phi_{11,e} + \ell_1\phi_{11,e}') \\ b_{133} &= -(\phi_{11,e} + \ell_1\phi_{11,e}') \\ b_{134} &= -(m_2d_2 + m_p\ell_2)(\phi_{11,e}\phi_{12,e}' - \phi_{12,e}\phi_{11,e}') \\ b_{141} &= w_{12} + (J_{h2} + J_{o2} + J_p + m_p\ell_2^2)\phi_{12,e} \\ &+ (m_{h2} + m_2 + m_p)\ell_1\phi_{12,e} \\ b_{142} &= (m_2d_2 + m_p\ell_2)(\phi_{12,e} + \ell_1\phi_{12,e}') \\ b_{143} &= -(\phi_{12,e} + \ell_1\phi_{12,e}') \\ b_{144} &= -(m_2d_2 + m_p\ell_2)(\phi_{12,e}\phi_{11,e}' - \phi_{11,e}\phi_{12,e}') \\ b_{151} &= w_{21} + J_p\phi_{21,e}' + m_p\ell_2\phi_{21,e} \\ b_{152} &= (v_{21} + m_p\phi_{21,e})\ell_1 \\ b_{153} &= v_{21} + m_p\phi_{21,e} \\ b_{161} &= w_{22} + J_p\phi_{22,e}' + m_p\ell_2\phi_{22,e} \\ \end{split}$$

$$\begin{split} b_{221} &= J_{h2} + J_{o2} + J_p + m_p \ell_2^2 \\ b_{231} &= (J_{h2} + J_{o2} + J_p + m_p \ell_2^2) \phi_{11,e}' \\ b_{232} &= (m_2 d_2 + m_p \ell_2) \phi_{11,e} \\ b_{233} &= -\phi_{11,e} \\ b_{234} &= -(m_2 d_2 + m_p \ell_2) \phi_{12,e} \\ b_{241} &= (J_{h2} + J_{o2} + J_p + m_p \ell_2^2) \phi_{12,e}' \\ b_{242} &= (m_2 d_2 + m_p \ell_2) \phi_{12,e} \\ b_{243} &= -\phi_{12,e} \\ b_{244} &= -(m_2 d_2 + m_p \ell_2) \phi_{12,e} \\ b_{251} &= w_{21} + J_p \phi_{21,e}' + m_p \ell_2 \phi_{22,e} \\ b_{331} &= m_1 \\ b_{332} &= 2(m_2 d_2 + m_p \ell_2) \phi_{11,e} \phi_{11,e}' \\ b_{333} &= -2\phi_{11,e} \phi_{11,e}' \\ b_{341} &= 0 \\ b_{342} &= (m_2 d_2 + m_p \ell_2) (\phi_{11,e} \phi_{12,e}' + \phi_{12,e} \phi_{11,e}') \\ b_{351} &= (w_{21} + J_p \phi_{21,e}' + \phi_{12,e} \phi_{11,e}') \\ b_{352} &= (v_{21} + m_p \phi_{21,e}) \phi_{11,e} \\ b_{353} &= -(v_{21} + m_p \phi_{21,e}) \phi_{11,e} \\ b_{362} &= (v_{22} + m_p \phi_{22,e}) \phi_{11,e} \\ b_{363} &= -(v_{22} + m_p \phi_{22,e}) \phi_{11,e} \\ b_{363} &= -(v_{22} + m_p \phi_{22,e}) \phi_{11,e} \\ b_{441} &= m_1 \\ b_{442} &= 2(m_2 d_2 + m_p \ell_2) (\phi_{12,e} \phi_{12,e}' + \phi_{12,e} \phi_{12,e}') \\ b_{453} &= -(v_{21} + m_p \phi_{21,e}) \phi_{11,e} \\ b_{451} &= (w_{21} + J_p \phi_{21,e}' + m_p \ell_2 \phi_{21,e}) \phi_{12,e}' \\ b_{453} &= -(v_{21} + m_p \phi_{21,e}) \phi_{12,e} \\ b_{453} &= -(v_{22} + m_p \phi_{22,e}) \phi_{12,e} \\ b_{463} &= -(v_{22} + m_p \phi_{22,e}) \phi_{12,e} \\ b_{463} &= -(v_{22} + m_p \phi_{22,e}) \phi_{12,e} \\ b_{463} &= -(v_{22} + m_p \phi_{22,e}) \phi_{12,e} \\ b_{551} &= m_2 \\ b_{561} &= 0 \\ b_{661} &= m_2. \\ \end{cases}$$

```
t_{11} = \phi_{11,e} - \ell_1 \phi_{11,e}'
t_{12} = \phi_{12,e} - \ell_1 \phi_{12,e}'
t_{21} = v_{21} + m_p \phi_{21,e}
t_{22} = v_{22} + m_p \phi_{22,e}
t_{31} = \phi'_{11,e}
t_{32} = \phi'_{12,e}.
h_{101} = -2(m_2d_2 + m_p\ell_2)\ell_1
h_{102} = 2(m_2 d_2 + m_p \ell_2)(\phi_{11,e} - \ell_1 \phi'_{11,e})
h_{103} = 2(m_2 d_2 + m_p \ell_2)(\phi_{12,e} - \ell_1 \phi_{12,e}')
h_{104} = -2(v_{21} + m_p \phi_{21,e})\ell_1
h_{105} = -2(v_{22} + m_p \phi_{22,e})\ell_1
h_{106} = -(m_2 d_2 + m_p \ell_2)\ell_1
h_{107} = -(m_2 d_2 + m_p \ell_2) \ell_1 \phi'_{11,e})
h_{108} = -2(m_2d_2 + m_p\ell_2)\ell_1\phi'_{12,e})
h_{109} = -2(v_{21} + m_p \phi_{21,e})\ell_1
h_{110} = -2(v_{22} + m_p \phi_{22,e})\ell_1
h_{111} = -2(v_{21} + m_p \phi_{21,e})\ell_1 \phi'_{11,e}
h_{112} = -2(v_{22} + m_p \phi_{22,e})\ell_1 \phi_{11,e}'
h_{113} = -2(v_{21} + m_p \phi_{21,e})\ell_1 \phi'_{12,e}
h_{114} = -2(v_{22} + m_p \phi_{22,e})\ell_1 \phi'_{12,e}
h_{115} = 2(m_2d_2 + m_p\ell_2)
h_{116} = m_2 d_2 + m_p \ell_2
h_{117} = -(v_{21} + m_p \phi_{21,e})
h_{118} = -(v_{22} + m_p \phi_{22,e})
h_{119} = -2\ell_1
h_{120} = -\ell_1
h_{121} = -(\phi_{11,e} + \ell_1 \phi'_{11,e})
h_{122} = -(\phi_{12,e} + \ell_1 \phi'_{12,e})
h_{123} = -(m_2 d_2 + m_p \ell_2)(\phi_{11,e} \phi'_{12,e} - \phi_{12,e} \phi'_{11,e})
h_{124} = -(m_2 d_2 + m_p \ell_2)(\phi_{12,e} \phi'_{11,e} - \phi_{11,e} \phi'_{12,e})
 h_{201} = (m_2 d_2 + m_p \ell_2) \ell_1
 h_{202} = 2(m_2 d_2 + m_p \ell_2)\phi_{11,e}
 h_{203} = 2(m_2 d_2 + m_p \ell_2)\phi_{12,e}
 h_{204} = -(m_2 d_2 + m_p \ell_2)
 h_{205} = -(v_{21} + m_p \phi_{21,e})
```

```
h_{206} = -(v_{22} + m_p \phi_{22,e})
```

$$\begin{split} h_{206} &= -(v_{22} + m_p \phi_{22,e}) \\ h_{207} &= \ell_1 \\ h_{208} &= \phi_{11,e} + \ell_1 \phi_{11,e}' \\ h_{209} &= \phi_{12,e} + \ell_1 \phi_{12,e}' \\ h_{210} &= (m_2 d_2 + m_p \ell_2) (\phi_{11,e} \phi_{12,e}' - \phi_{12,e} \phi_{11,e}') \\ h_{211} &= (m_2 d_2 + m_p \ell_2) (\phi_{12,e} \phi_{11,e}' - \phi_{11,e} \phi_{12,e}') \\ h_{212} &= \phi_{11,e} \phi_{11,e}' \\ h_{213} &= \phi_{11,e} \phi_{12,e}' + \phi_{12,e} \phi_{11,e}' \\ h_{215} &= (v_{22} + m_p \phi_{22,e}) \phi_{11,e} \\ h_{216} &= \phi_{12,e} \phi_{12,e}' \\ h_{217} &= (v_{21} + m_p \phi_{21,e}) \phi_{12,e} \\ h_{218} &= (v_{22} + m_p \phi_{22,e}) \phi_{12,e} \\ h_{301} &= -(m_2 d_2 + m_p \ell_2) (\phi_{11,e} - \ell_1 \phi_{11,e}') \\ h_{302} &= -2(m_2 d_2 + m_p \ell_2) (\phi_{11,e} - \phi_{11,e} \phi_{12,e}') \\ h_{304} &= -2(v_{21} + m_p \phi_{21,e}) \phi_{11,e} \\ h_{305} &= -2(v_{22} + m_p \phi_{22,e}) \phi_{11,e} \\ h_{306} &= -(m_2 d_2 + m_p \ell_2) (\phi_{11,e} + \phi_{12,e}') \\ h_{306} &= -(m_2 d_2 + m_p \ell_2) \phi_{11,e} \\ h_{307} &= -2(m_2 d_2 + m_p \ell_2) \phi_{11,e} \\ h_{308} &= -2(m_2 d_2 + m_p \ell_2) \phi_{11,e} \phi_{11,e}' \\ h_{309} &= -2(v_{21} + m_p \phi_{21,e}) \phi_{11,e} \\ h_{310} &= -2(v_{22} + m_p \phi_{22,e}) \phi_{11,e} \\ h_{311} &= -2(v_{22} + m_p \phi_{22,e}) \phi_{11,e} \\ h_{313} &= -2(v_{22} + m_p \phi_{22,e}) \phi_{11,e} \phi_{11,e}' \\ h_{314} &= -2(v_{22} + m_p \phi_{22,e}) \phi_{11,e} \phi_{12,e}' \\ h_{315} &= -(\phi_{11,e} + \ell_1 \phi_{11,e}') \\ h_{316} &= -\phi_{11,e} \\ h_{317} &= -2\phi_{11,e} \phi_{11,e}' \\ h_{319} &= -(w_{22} + m_p \theta_{21,e}) \phi_{11,e} \\ h_{312} &= -(v_{22} + m_p \phi_{21,e}) \phi_{11,e} \\ h_{312} &= -(v_{22} + m_p \phi_{21,e}) \phi_{11,e} \\ h_{313} &= -(v_{21} + m_p \phi_{21,e}) \phi_{11,e} \\ h_{314} &= -2(v_{22} + m_p \phi_{22,e}) \phi_{11,e} \\ h_{315} &= -(\phi_{11,e} + \ell_1 \phi_{11,e}') \\ h_{316} &= -\phi_{11,e} \\ h_{312} &= -(w_{22} + m_p \phi_{21,e}) \phi_{11,e} \\ h_{321} &= -(v_{22} + m_p \phi_{21,e}) \phi_{11,e} \\ h_{322} &= -(m_2 d_2 + m_p \ell_2) (\phi_{11,e} \phi_{12,e}' - \phi_{12,e} \phi_{11,e}') \\ h_{322} &= -(m_2 d_2 + m_p \ell_2) (\phi_{11,e} \phi_{12,e}' - \phi_{12,e} \phi_{11,e}') \\ h_{322} &= -(m_2 d_2 + m_p \ell_2) (\phi_{11,e} \phi_{12,e}' - \phi_{12,e} \phi_{11,e}') \\ h_{322} &= -(m_2 d_2 + m_p \ell_2) (\phi_{11,e} \phi_{12,e}' - \phi_{12,e} \phi_{11,e}') \\ h_{322} &= -(m_2 d_2 + m_p \ell$$

$$\begin{split} h_{401} &= -(m_2d_2 + m_p\ell_2)(\phi_{12,e} - \ell_1\phi_{12,e}') \\ h_{402} &= -2(m_2d_2 + m_p\ell_2)(\phi_{11,e}\phi_{12,e}' - \phi_{12,e}\phi_{11,e}') \\ h_{403} &= 2(m_2d_2 + m_p\ell_2)(\phi_{12,e} - \phi_{12,e}\phi_{11,e}') \\ h_{404} &= -2(v_{21} + m_p\phi_{22,e})\phi_{12,e} \\ h_{405} &= -2(v_{22} + m_p\ell_2)\phi_{12,e}\phi_{11,e}' \\ h_{406} &= -(m_2d_2 + m_p\ell_2)\phi_{12,e}\phi_{11,e}' \\ h_{407} &= -2(m_2d_2 + m_p\ell_2)\phi_{12,e}\phi_{12,e}' \\ h_{409} &= -2(v_{21} + m_p\phi_{21,e})\phi_{12,e}\phi_{11,e}' \\ h_{410} &= -2(v_{22} + m_p\phi_{22,e})\phi_{12,e}\phi_{11,e}' \\ h_{410} &= -2(v_{22} + m_p\phi_{22,e})\phi_{12,e}\phi_{12,e}' \\ h_{411} &= -2(v_{22} + m_p\phi_{22,e})\phi_{12,e}\phi_{12,e}' \\ h_{413} &= -2(v_{21} + m_p\phi_{21,e})\phi_{12,e}\phi_{11,e}' \\ h_{414} &= -2(v_{22} + m_p\phi_{22,e})\phi_{12,e}\phi_{12,e}' \\ h_{415} &= -(\phi_{12,e} + \ell_1\phi_{12,e}') \\ h_{416} &= -\phi_{12,e}' \\ h_{417} &= -(\phi_{11,e}\phi_{12,e}' + \phi_{12,e}\phi_{11,e}') \\ h_{418} &= -2\phi_{12,e}\phi_{12,e}' \\ h_{419} &= -(m_2d_2 + m_p\ell_2)\phi_{12,e} \\ h_{420} &= -(v_{21} + m_p\phi_{21,e})\phi_{12,e} \\ h_{422} &= -(m_2d_2 + m_p\ell_2)(\phi_{12,e}\phi_{11,e}' - \phi_{11,e}\phi_{12,e}') \\ h_{501} &= (v_{21} + m_p\phi_{21,e})\phi_{11,e} \\ h_{502} &= 2(v_{21} + m_p\phi_{21,e})\phi_{11,e} \\ h_{503} &= 2(v_{21} + m_p\phi_{21,e})\phi_{11,e} \\ h_{504} &= v_{21} + m_p\phi_{21,e})\phi_{11,e} \\ h_{505} &= -(v_{21} + m_p\phi_{21,e})\phi_{11,e} \\ h_{601} &= (v_{22} + m_p\phi_{22,e})\phi_{11,e} \\ h_{601} &= (v_{22} + m_p\phi_{22,e})\phi_{11,e} \\ h_{603} &= 2(v_{22} + m_p\phi_{22,e})\phi_{11,e} \\ h_{604} &= v_{22} + m_p\phi_{22,e})\phi_{11,e} \\ h_{606} &= -(v_{22} + m_p\phi_{22,e})\phi_{12,e} \\ h_{605} &= -(v_{22} + m_p\phi_{22,e})\phi_{12,e} \\ h_{605} &= -(v_{22} + m_p\phi_{22,e})\phi_$$

Appendix B

2-DOF Serial Flexible Link Robot from Quanser [11]

Appendix B includes technical details of the 2-DOF Serial Flexible Link Robot from Quanser used for experimental testing in this thesis.

The 2-Degree-Of-Freedom (DOF) Serial Flexible Link (2DSFL) Robot is depicted in below Figure. This robot system consists of two DC motors driving via harmonic gearboxes (zero backlash) a two-bar serial linkage. Both links are flexible and instrumented with strain gauges. The primary link is rigidly clamped to the first drive (a.k.a. elbow) and carries at its end the second harmonic drive (a.k.a. shoulder) to which another flexible link is attached. Both motors are instrumented with quadrature optical encoders.

A nomenclature of the Two-Degree-Of-Freedom Serial Flexible Link mechanical assembly is shown below:



2-DOF Serial Flexible Link Robot

ID #	Description	ID #	Description
1	Harmonic Drive #1 (Shoulder)	2	Harmonic Drive #2 (Elbow)
3	DC Motor #1 (Shoulder)	4	DC Motor #2 (Elbow)
5	Motor #1 Encoder	6	Motor #2 Encoder
7	Flexible Link #1	8	Flexible Link #2
9	Rigid Joint #1	10	Rigid Joint #2
11	Strain Gauge Amplifier Board	12	Strain Gauge Offset Potentiometer
13	Strain Gauge Connector	14	Base Plate
15	Link #1 End-Effector	16	Link #2 End-Effector
17	Joint #1 Limit Switches	18	Joint #2 Limit Switches

2-DOF Serial Flexible Link Robot Component Nomenclature

The -DOF Serial Flexible Link Robot system consists of the following hardware:

1. Two DoF Serial Flexible link (2DSFL)



2. AMPAQ-series Power Amplifier: Two-Channel Linear Current Amplifier. The 2DSFL robot is powered by a two-channel linear current Amplifier Package (AMPAQ) from the Quanser AMPAQ-series



3. Data Acquisition Card: Quanser's Q4 HIL board. The power amplifier and planar robot systems are designed to be fully compatible with the Q4 Hardware-In-the-Loop (HIL) board



4. ±15 VDC External Power Supply

It can provide the system with a maximum output power of 42 W at ± 15 VDC. It currently supplies power to the four joint position limit switches and the four strain gauge sensors.



5. Cables



The "Motor" cable carries the power leads from the power amplifier (AMPAQ) to one of the 2DSFL DC motors.

"Motor" Cable



The "Encoder" cable carries encoder signals and required DC power supply. One cable is used for each of the two 2DSFL motors.

"Encoder" Cable



"Analog" Cable



"Digital I/O" Cable



'15 VDC Power" Cable

The "Analog" cable comprises three sets of RCA male connectors. One connects an analog output of the data acquisition (e.g. Q8, Q4) terminal board to the power module for proper power amplification. Another carries the current sense signal from the AMPAQ to the Q8 terminal board, where the signal is then available to be monitored. The last set is used to connect the two strain gauge signals to the Q8 (or Q4) analog inputs.

The "Digital I/O" cable is used to connect from the AMPAQ "Enable" SCSI 16-pin Connector to the Q8 (or Q4) terminal board first header. This flat ribbon cable then carries the amplifier enable signals and can be used for extra digital signals. It is also used to connect from the Q8 terminal board second header to the 2DSFL limit switch connector (e.g. switch).

The "15 VDC Power" cable connects the external ± 15 VDC power supply to the 2DSFL "15 VDC Power Connector". It provides power to the system four limit switches and two strain gauge signal conditioning boards.

The Connection between the aforementioned components is as shown below



2DSFL Connections: Back Panel



Q8 Terminal Board Connections



Two-Channel AMPAQ Connections

Appendix C

F and **G** Matrices Formulation

Appendix C introduces the mathematical formulation of the parameters in G and F matrices, in MATLAB form, used for LQR controller design section 5.1.2.

```
G11 = 496/(320 * cos(theta2) - 855 * delta11 * sin(theta2) + 2970 * delta12
             sin(theta2) - 4852 * delta21 * sin(theta2) - 1884 * delta22
             * sin(theta2) + 39684)
G12 = -(453 * cos(theta2) - 42 * delta11 * sin(theta2) + 140 * delta12
             sin(theta2) - 240 * delta21 * sin(theta2) - 940 * delta22
             sin(theta2) + 537)/(2633 * (3000 * cos(theta2) - 855 * delta11)
             sin(theta2) + 2970 * delta12 * sin(theta2) - 4852 * delta21
             sin(theta2) - 1884 * delta22 * sin(theta2) + 3964))
G13 = (146 * delta11^2 * sin(theta2)^2 - 6170 * delta11 * delta12 * sin(theta2)^2
             + 20285 * delta11 * delta21 * sin(theta2)^2 + 7876 * delta11
             * delta 22 * sin(theta 2)^2 - 203 * delta 11 * sin(theta 2) - 358 * sin(2)
             * theta2) * delta11 + 41796 * delta12<sup>2</sup> * sin(theta2)<sup>2</sup> - 4966
             * delta12 * delta21 * sin(theta2)^2 - 192 * delta12 * delta22
             sin(theta2)^2 + 217 * delta12 * sin(theta2) + 5842744 * sin(2)
             * theta2) * delta12 + 69657 * delta21<sup>2</sup> * sin(theta2)<sup>2</sup> + 54094
             * delta21 * delta22 * sin(theta2)^2 - 1451 * delta21 * sin(theta2)
             -27 * \sin(2 * theta2) * delta21 + 1050 * delta22^2 * \sin(theta2)^2
             -5634 * delta22 * sin(theta2) - 107 * sin(2 * theta2) * delta22
             + 75 * \cos(theta2)^2 + 216 * \cos(theta2) - 431)/(2633 * (6604))
             * cos(theta2) - 1068117109533354131 * delta21 * sin(theta2)
             -414 * delta22 * sin(theta2) + 360) * (3000 * cos(theta2) - 855
             * delta11 * sin(theta2) + 2970 * delta12 * sin(theta2) - 4852
             * delta21 * sin(theta2) - 1884 * delta22 * sin(theta2) + 394))
G14 = -((6176 * delta11 * sin(2 * theta2))/2 - 95 * cos(theta2) - (896 * delta12)
             (2 * theta2))/2 + 1069 * delta21 * sin(2 * theta2) + 415
             * delta 22 * sin(2 * theta 2) - 71445 * cos(theta 2)^2 - 781
             (s \cos(theta2)^3 - 91858 * delta11^2 * \sin(theta2)^2)
G15 = \sin(theta2)^2 + 9409 * delta21 * delta22 * \sin(theta2)^2 - 5541 * delta21
             sin(theta2) - 2068 sin(2 theta2) delta21 + 650 delta22^2
             sin(theta2)^2 - 5634 * delta22 * sin(theta2) - 17 * sin(2 * theta2)
             * delta 22 + 851 * cos(theta 2)^{2} + 256 * cos(theta 2) - 439)/(268)
             *(304 * cos(theta2) - 106 * delta21 * sin(theta2) - 54 * delta22
             * sin(theta2) + 110) * (2460 * cos(theta2) - 635 * delta11
             sin(theta2) + 740 * delta12 * sin(theta2) - 953 * delta21
             * sin(theta2) ))
```

$$\begin{aligned} G16 &= delta22 * sin(theta2)^2 - 203 * delta11 * sin(theta2) - 35508 * sin(2 \\ &* theta2) * delta11 + 41796 * delta12^2 * sin(theta2)^2 - 466 \\ &* delta12 * delta21 * sin(theta2)^2 - 192 * delta12 * delta22 \\ &* sin(theta2)^2 + 217 * delta12 * sin(theta2) + 549 * sin(2 * theta2) \\ &* delta12 \end{aligned}$$

```
G26 = 388 * delta22 * sin(theta2) - 155 * delta11 * (sin(theta2))
             -sin(theta2)^3) + 132 * delta12 * (sin(theta2) - sin(theta2)^3)
             + 673 * delta21 * (sin(theta2) - sin(theta2)^3) + 2617 * delta22
             *(sin(theta2) - sin(theta2)^3) + 510 * delta11^2 * (cos(theta2))^3)
             -\cos(theta2)^{3} + 371 * delta12^{2} * (\cos(theta2))^{3}
             -\cos(theta_2)^3) + 740 * delta_{21^2} * (\cos(theta_2))^3)
             -\cos(theta_2)^3) + 111 * delta_{22}^2 * (\cos(theta_2))^3)
             -\cos(theta_2)^3) - 632 * delta_{11} * delta_{12} * sin(theta_2)^2 + 190
             * delta11 * delta21 * sin(theta2)<sup>2</sup> + 739 * delta11 * delta22
             sin(theta2)^2 - 162 * delta12 * delta21 * sin(theta2)^2 - 630
             * delta12 * delta22 * sin(theta2)^2 + 225 * delta21 * delta22
             sin(theta2)^2 - 4864 * delta11 * delta21^2 * sin(theta2)^3
G31 = 0
G32 = 0
G33 = 204 * cos(theta2) - 108 * delta21 * sin(2 * theta2) - 420 * delta22 * sin(2)
             * theta2) + 694 * cos(theta2)<sup>2</sup> + 118 * cos(theta2)<sup>3</sup> + 371
             * delta 11^{2} * sin(theta 2)^{2} + 26 * delta 12^{2} * sin(theta 2)^{2} + 289
             * delta 21^{2} * sin(theta 2)^{2} + 43 * delta 22^{2} * sin(theta 2)^{2} - 103
             * delta11 * sin(theta2) + 88 * delta12 * sin(theta2) + 10 * delta21
             * sin(theta2)
G34 = -362 * delta12 * delta22 * (cos(theta2) - cos(theta2)^3) + 5750
             * delta21 * delta22 * (cos(theta2) - cos(theta2)^3) + 172 * delta11
             * delta12 * delta21 * sin(theta2)^3 + 66 * delta11 * delta12 * delta22
             sin(theta2)^3 - 37 * delta11 * delta21 * delta22 * sin(theta2)^3
             + 32 * delta12 * delta21 * delta22 * sin(theta2)^3 - 540)/(341 * (77))
             * delta21^2 * sin(theta2)^2 + 605 * delta21 * delta22
             sin(theta2)^2 - 530 * delta21 * sin(theta2) - 456 * sin(2 * theta2)
             * delta 21 + 117 * delta 22^{2} * sin(theta 2)^{2} - 205 * delta 22
             sin(theta2) - 17721375 sin(2 theta2) delta22 - 298
             (scos(theta2)^2 + 243 * cos(theta2) + 90))
G35 = (170 * delta12 * sin(2 * theta2) - (405 * delta11 * sin(2 * theta2))/2 - 200
             (s \cos(theta2) + 3510 * delta21 * sin(2 * theta2) + 136 * delta22)
             sin(2 * theta2) + 600 * cos(theta2)^{2} + 192 * delta11
             sin(theta2) - 16400 * delta12 * sin(theta2) + 161 * delta21
             sin(theta2) + 6288 * delta22 * sin(theta2) - 460 * delta11
             * delta21 * sin(theta2)^2
```

$$\begin{array}{l} G36 = -1793*delta11*delta22*sin(theta2)^2 + 393*delta12*delta21 \\ * sin(theta2)^2 + 1528*delta12*delta22*sin(theta2)^2 \\ - 490)/(65*(77*delta21^2*sin(theta2)^2 + 60*delta21 \\ * delta22*sin(theta2)^2 - 53*delta21*sin(theta2) - 45639125 \\ * sin(2*theta2)*delta21+1175*delta22^2*sin(theta2)^2 - 205 \\ * delta22*sin(theta2) - 17721375*sin(2*theta2)*delta22 - 29 \\ * cos(theta2)^2 + 243*cos(theta2) + 900)) \\ \hline G41 = 0 \\ G42 = 0 \\ G43 = 0 \\ G44 = -125/(225750*cos(theta2) + 365113*delta21*sin(theta2) + 141771 \\ * delta22*sin(theta2) - 1250000) \\ G45(1170*cos(theta2) - 771*delta11*sin(theta2) + 65*delta12 \\ * sin(theta2) - 144)/(399*(225750*cos(theta2) + 365113 \\ * delta21*sin(theta2) + 1411*delta22*sin(theta2) - 1200)) \\ G46 = (91*cos(theta2) - 59*delta11*sin(theta2) + 510*delta12 \\ * sin(theta2) - 359)/(72057594037927936*(225*cos(theta2) \\ + 365113*delta21*sin(theta2) + 1411*delta22*sin(theta2) \\ - 1200)) \\ G51 = 0 \\ G52 = 0 \\ G53 = 0 \\ G54 = 0 \\ G55 = -cos(theta2)^3) - 632*delta11*delta12*sin(theta2)^2 + 190 \\ * delta11*delta21*sin(theta2)^2 + 739*delta11*delta22 \\ * sin(theta2)^2 - 162*delta12*sin(theta2)^2 - 630 \\ * delta12*delta22*sin(theta2)^2 + 225*delta21*delta22 \\ * sin(theta2)^2 - 4864*delta11*delta21*2*sin(theta2)^3 - 5 \\ * delta21*sin(theta2)^2 - 205*delta21*sin(theta2)^2 + 240 \\ * cos(theta2)^2 + 900)) \\ \end{array}$$

$$\begin{aligned} & G56 = 192 * delta12 * delta22 * sin(theta2)^2 + 217 * delta12 * sin(theta2) \\ & + 5842744 * sin(2 * theta2) * delta12 + 69657 * delta21^2 \\ & * sin(theta2)^2 + 54094 * delta21 * delta22 * sin(theta2)^2 - 1451 \\ & * delta21 * sin(theta2) - 27 * sin(2 * theta2) * delta21 + 1050 \\ & * delta22^2 * sin(theta2)^2 - 5634 * delta22 * sin(theta2) - 107 \\ & * sin(2 * theta2) * delta22 + 75 * cos(theta2)^2 + 216 * cos(theta2) \\ & - 431)/(2633 * (6604 * cos(theta2) - 106 * delta21 * sin(theta2) \\ & - 414 * delta22 * sin(theta2) + 360) * (3000 * cos(theta2) - 855 \\ & * delta11 * sin(theta2) + 2970 * delta12 * sin(theta2) - 4852 \\ & * delta21 * sin(theta2) + 1884 * delta22 * sin(theta2) + 394)) \end{aligned}$$

$$\begin{split} F2 &= cos(theta2) * (delta11_dot * (((325 * delta21_dot)/144 + (505 \\ * delta22_dot)/57) * ((657 * delta11)/1000 - (14 * delta12)/25) \\ &+ ((44 * delta11_dot)/360 + (337 * delta12_dot)/900) * ((1213 \\ * delta21) + (471 * delta22)/10000)) + delta12_dot * (((75 \\ * delta12_dot)/2882 + (583 * delta22_dot)/576) * ((657 \\ * delta11)/1000 - (14 * delta12)/25) - (301 * delta12_dot * ((1213 \\ * delta21)/10000 + (471 * delta22)/10))/2500) + theta1_dot \\ * (((1213 * delta21)/10000 + (471 * delta22)/10))/2500) + theta1_dot \\ * (((1213 * delta21)/10000 + (471 * delta12_dot)/200 + theta1_dot/2) \\ + (2652 * delta11 * delta12_dot)/144 - (2652 * delta12 \\ * delta11_dot)/144 + ((57 * delta11)/400 - (99 * delta12)/200) \\ * ((1213 * delta21_dot)/10000 + (3 * theta1_dot)/40 \\ + 1413/500))) + theta1_dot * sin(theta2) * ((279 \\ * delta11_dot)/10000 + (129 * delta12_dot)/4000 + (3 \\ * theta1_dot)/80) \end{split}$$

$$F3 = -\sin(theta2) * (theta1_dot * ((32 * delta21_dot)/72 - (26 * delta12_dot)/72 - (171 * theta1_dot) + (279 * theta2_dot)/10000 + 94/900) - delta12_dot * ((728 * delta12_dot)/288 + (44 * delta22_dot)/45) + delta11_dot * ((85 * delta21_dot)/288 + (337 * delta22_dot)/288) + theta2_dot * ((52 * delta11_dot)/288 - (450 * delta12_dot)/284 + (32 * delta21_dot)/72 + (50 * delta22_dot)/288 + (279 * theta2_dot)/20)) - theta2_dot * cos(theta2) * (((657 * delta11)/1000 - (14 * delta12)/25) * ((32 * delta21_dot)/14 + (50 * delta22_dot)/576 + (279 * theta2_dot)/20) - (265 * delta12 * theta1_dot)/144 + ((1213 * delta21) + (471 * delta22)/10) * ((4402 * delta11_dot)/180 + (334 * delta12_dot)/90 + (1029 * theta1_dot)/20 + (93 * theta2_dot)/500) + 32 * delta12 * delta21 * delta22 * sin(theta2)^3 - 540)/(341 * (77 * delta21^2 * sin(theta2)^2 + 605 * delta21 * delta22 * sin(theta2)^2 - 530 * delta21 * sin(theta2) - 456 * sin(2 * theta2) * delta21 + 117 * delta22^2 * sin(theta2)^2 - 205 * delta22 * sin(theta2)^2 + 243 * cos(theta2) + 90))$$

 $F4 = -\sin(\text{theta2}) * (\text{delta11}_\text{dot} * ((12 * \text{delta21}_\text{dot})/360 - (163))$ * delta22 dot)/144) + delta12 dot * ((12 * delta12 dot)/368 - (163 $* delta22_dot)/141) + theta2_dot * ((610 * delta11_dot)/284 - (903))$ * delta12_dot)/50 + (75 * delta21_dot)/12 + (583 * delta22_dot)/284 + (129 * theta2_dot)/8) + theta1_dot * ((263 * delta11_dot)/726 + (757 * delta21_dot)/167 + (297 * theta1_dot)/80 + (129 * theta2_dot)/4000 + 437/3602)) - theta2_dot * cos(theta2) * ((23 * delta11 * theta1_dot)/865 + ((657 * delta11)/1000 - (14 * delta12)/25) * ((875))* delta21_dot)/28 + (717 * delta22_dot)/58 + (129 * theta2_dot)) + ((1213 * delta21)/10000 + (471 * delta22)/10) * ((3) $* delta11_dot)/90 - (301 * delta12_dot)/1250 - (13)$ * theta1_dot)/200 + (43 * theta2_dot)/200) + - cos(theta2)^3) $+ 111 * delta 22^2 * (cos(theta2) - cos(theta2)^3) - 632 * delta 11$ * delta12 * sin(theta2)² + 190 * delta11 * delta21 * sin(theta2)² + 739 * delta11 * delta22 * sin(theta2)^2 - 162 * delta12 * delta21 $sin(theta2)^2 - 630 * delta12 * delta22 * sin(theta2)^2 + 225$ * delta21 * delta22 * sin(theta2)^2 - 4864 * delta11 * delta21^2 * sin(theta2)^3 $F5 = theta1_dot * sin(theta2) * ((32 * delta11_dot)/766 + (854))$ * delta12_dot)/176 + (1213 * theta1_dot)/20000) - theta2_dot ((657 * delta11)/1000 - (14 * delta12)/25) * ((2465)) $* delta11_dot)/846 + (638 * delta12_dot)/253) + (1213)$ * theta1_dot * ((57 * delta11)/400 - (99 * delta12/200)/10) + * delta11^2 * delta21 * sin(theta2)^3 - 733 * delta11 * delta22^2 $sin(theta2)^3 + 414 * delta12 * delta21^2 * sin(theta2)^3 - 39$ * delta11^2 * delta22 * sin(theta2)^3 - 734 * delta12^2 * delta21 $sin(theta2)^3 + 625 * delta12 * delta22^2 * sin(theta2)^3 - 285$ * delta12^2 * delta22 * sin(theta2)^3 - 870 * delta11 * delta12 $(\cos(theta2) - \cos(theta2)^3) + 1095 * delta11 * delta21$ $(\cos(theta2) - \cos(theta2)^3) + 42 * delta11 * delta22$ $(\cos(theta2) - \cos(theta2)^3) - 933 * delta12 * delta21$ $*(cos(theta2) - cos(theta2)^3)$ $F6 = theta1_dot * sin(theta2) * ((505 * delta11_dot)/25 + (692 * delta12_dot)/$ $642 + (471 * theta1_dot)/20) - theta2_dot * cos(theta2) * ((471 * theta1_dot * dot * dot + dot$ $((57 * delta11)/400 - (99 * delta12)/200))/100 + ((533 * delta11_dot)/231 +$ $(97 * delta12_dot)/115) * ((657 * delta11)/100 - (14 * delta12)/25)) +$ 5634 * $delta22 * sin(theta2) - 107 * sin(2 * theta2) * delta22 + 75 * cos(theta2)^2 +$ $216 * \cos(theta2) - 431)/(2633 * (6604 * \cos(theta2) - 1068117109533354131 *$

delta21 * sin(theta2) - 414 * delta22 * sin(theta2) + 360) * (3000 * cos(theta2) - 855 * delta11 * sin(theta2) + 2970 * delta12 * sin(theta2) - 4852 * delta21 * sin(theta2) - 1884 * delta22 * sin(theta2) + 394)