Defence Science Journal, Vol. 69, No. 5, September 2019, pp. 464-468, DOI : 10.14429/dsj.69.14952

Modified Low Rank Approximation for Detection of Weak Target by Noise-space Exploitation in Through Wall Imaging

Mandar K. Bivalkar*, Bambam Kumar, and Dharmendra Singh

Department of Electronics and Communication Engineering, Indian Institute of Technology, Roorkee – 247 667, India
**E-mail:mbivalkar@somaiya.edu*

ABSTRACT

Low dielectric materials referred as weak targets are very difficult to detect behind the wall in through wall imaging (TWI) due to strong reflections from wall. TWI Experimental data collected for low dielectric target behind the wall and transceiver on another side of the wall. Recently several researchers are using low-rank approximation (LRA) for reduction of random noise in the various data. We explore the possibilities of using LRA for TWI data for improving detection of low dielectric material. A novel approach using modification of LRA with exploiting the noise subspace in singular value decomposition (SVD) to detect weak target behind the wall is introduced. LRA consider data has low rank in *f-x* domain for noisy data, local windows are implemented in LRA approach to satisfy the principle assumptions required by the LRA algorithm itself. Decomposed TWI data in the noise space of the SVD to detect the weak target adaptively. Results for modified LRA for detection of weak target behind the wall are very encouraging over LRA.

Keywords: Low-rank approximation; Noise space exploitation; Singular value decomposition; Target detection; Through wall imaging

1. INTRODUCTION

Through wall imaging (TWI) is an interesting area for key research in the recent years due to its numerous applications in military, calamity rescue and object detection etc. Electromagnetic signals in the frequency range 1 GHz-3 GHz provides good resolution in the down-range and cross-range[1]. Coherent summations from all sensor data are considered in delay-and-sum beamforming (DSBF) algorithm. DSBF is commonly used for imaging in the TWI. Different imaging algorithms for TWI and clutter removal techniques are discussed in[2]. Improvement in signal-to-noise ratio (SNR) in TWI are discussed in[3–5]. Wall clutter mitigation techniques are discussed in[6–8]. Images developed in TWI are not generally of acceptable level in terms of resolution, due to less number of pixels in beam formed image. Image enhancement techniques for TWI are proposed in[9–11]. Recently MUSIC algorithm for high resolution are discussed in[12–14]. Very few studies are available for detection of weak targets behind the wall in literature. Weak targets having dielectric constant near to the air dielectric constant (≈ 1), hence it will be difficult to detect, as mentioned by Gaikwad[15], *et al.*, they discussed about the clutter reduction for detection of metallic and low dielectric material such as Teflon behind brick wall. Statistics based adaptive algorithm for detection and identification of the target is proposed in[16].

This paper is about the detection of weak target such as Teflon behind brick wall using LRA. Reflections from the wall and background are often stronger than weak targets[17], hence detection of weak targets behind the wall become difficult but not impossible. Clutter reduction technique such as SVD is used frequently to improve signal-to-noise (SNR) in TWI. Spatial filtering is used to reduce the wall effect in[17] where in the spatial domain zero frequency along with low frequency components related to the wall reflections are suppressed. Spatial filtering technique cannot work in case of stationary targets behind the wall. Three antenna arrays at different antenna height are used parallel to the wall[18]. Two different arrays are used to give the difference between the received signals and to combat with wall reflections. This scheme does not mention the effect of the simple subtraction on the radar returns. Empirical LRA for seismic data is implemented in to reduce the random noise[18].

2. EXPERIMENTAL SET-UP AND SIGNAL MODEL

2.1 Experimental Set-up

Antenna array is placed in synthetic aperture radar (SAR) manner where array of antenna are placed at a stand-off distance of 57 cm in front of the wall and moved to different location horizontally and vertically at equal distance of 5 cm[15] to scan the whole wall. Reflection coefficients are measured for the wall and targets in the scene and pre- processing is carried out using following steps:

(i) Conversion from frequency domain to time domain
(ii) Conversion from time domain to spatial domain
(iii) Metal plate calibration
(iv) Velocity correction.

Received : 01 December 2018, Revised : 05 March 2019
Accepted : 20 April 2019, Online published : 17 September 2019

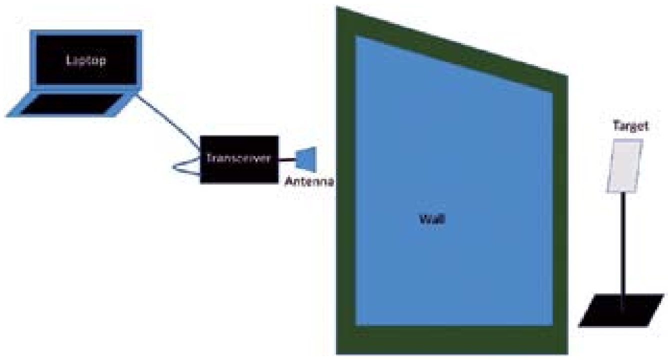

Figure 1. Set-up for TWI.

The set-up for which is shown in Fig. 1, transceiver is used to generate 201 stepped frequency continuous wave (SFCW) in the range 1 GHz-3 GHz.

2.2 Signal Modelling

The received signals from M antenna locations for Q no. of targets is given as Eqn (1)

$$x(n,t) = \sum_{q=0}^{Q-1} \sigma_q s(t - \tau_{n,q}) + Noise \qquad (1)$$

where, $s(t)$ is the transmitted signal from the radar which get convolved with wall transfer function, σ_q is the reflections received from the wall and targets. *Noise* is the additive noise, $\tau_{n,q}$ is the propagation delay between the n^{th} antenna position and the target Q. If we ignore the reflections from the wall then the signal path will be line-of-sight and two-way delay-time can be expressed as Eqn (2)

$$\tau_{n,q} = \frac{2}{c}\sqrt{(x_p - x_n)^2 + (z_p - z_n)^2} \qquad (2)$$

where, c is the speed of light. The co-ordinate (x_p, z_p) and (x_n, z_n) represents the Q^{th} target and n^{th} antenna position.

Figure 2 shows the geometry for TWI. DS-beamforming is the most popular and less complex imaging algorithm which does not consider the wave equation but require big computational power. Kirchhoff's imaging based on solving radar wave equation is proposed in[19]. Conventional DS-beamforming [20] used spatial frequency – domain, suppose the signal received at the antenna location is $z[m,n]$ of frequency f_n with delay $\tau_{q,m}$ then $z[m,n]$ can be represented as Eqn (3)

$$z[m,n] = \sum_{q=0}^{Q} \sigma_q \exp\{-j2\pi f_n \tau_{n,q}\} \qquad (3)$$

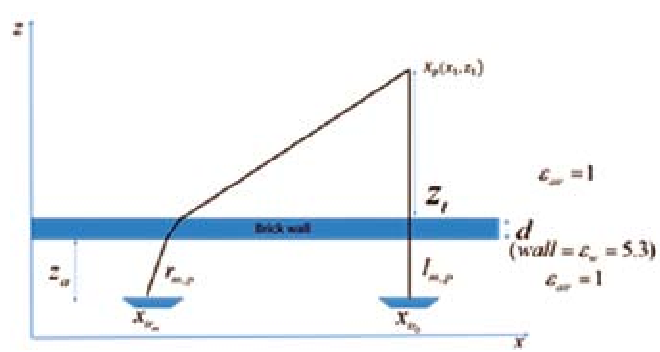

Figure 2. Geometry for TWI.

where m represents the spatial index and n represents the frequency index. Consider homogenous wall of thickness d with relative permittivity of the wall ε_r. The distance from the antenna and wall is (z_a) and from wall to the target (z_t). Velocity correction[20] for geometry shown in Fig. 2 is given by Eqn (4)

$$d_v = z_a + d\sqrt{\varepsilon_r} + z_t \qquad (4)$$

where d_v is the actual distance between the antenna and target after velocity correction. Delay $\tau_{n,q}$ by considering Eqn (4) can be given as Eqn (5)

$$\tau_{n,q} = \sqrt{(x_{tr_0} - x_{tr_n})^2 + (d_v + X_p)^2} \qquad (5)$$

Final image $s[k,l]$ can be recovered by DS – beamforming using Eqn (6)

$$s[x_k, z_l] = \frac{1}{NP}\sum_{n=0}^{N-1}\sum_{p=0}^{P-1} z[m,n]\exp\{j2\pi f_n \tau_{q,m}\} \qquad (6)$$

where $k*l$ are the number of pixels in the image, N is the total number of frequency points and P is the total number of scanned locations.

3. LOW-RANK APPROXIMATION

Low-rank approximation (LRA) is rank reduction based approach, it assumes that SAR data is low-rank after some rearrangement. There are different methods are proposed in the literature such as cadzow filtering, singular spectral analysis[21] and damped singular spectral analysis[22]. Among these newly developed methods, singular spectral analysis popularly known as low-rank approximation diverts the attention of researchers recently. LRA method is effective if the data is low-rank in nature, but when it is complex to make LRA effective it is implemented in local windows. Selection of the optimum rank for LRA in local windows is a challenging task.

Suppose the SAR data matrix D is composition for signal component S with added random noise then it can be represented as

$$D = S + Noise \qquad (7)$$

The optimum estimation for signal component S can be acquired by optimisation.

$$\min \|N\|_F^2 \; s.t.rank(s) = n \qquad (8)$$

where n is the rank constrained for data $S.\|\cdot\|_F$ is the Frobenius norm of D in this case effective solution for decomposition of D can be found using SVD and decomposition of the data matrix D can be represented by group of Eigen images. Low –rank component of S described with a few largest Eigen images represent the target and noise can be represented by other Eigen values. When the decomposition of *Noise* in to different frequency slices where Eigen images for the target are spread out is done, then the approach is called as LRA. In[7] TWI data for SVD is partitioned in to three Eigen spaces namely strongest reflections from antenna cross talk and wall reflection subspace, target subspace and noise subspace. This method is unable to detect weak target because target Eigen values are also spread over noise subspace. In this paper we exploit the noise space by LRA method and able to detect Teflon behind the wall successfully.

4. DETECTION OF WEAK TARGET USING MODIFIED LOW-RANK APPROXIMATION

In our experimental setup, we placed Teflon plate of dimensions 51 x 39 cm. behind the wall at a distance of 60 cm. A-scan for this arrangement is as shown in Fig. 3, where it observed that the target peak is very feeble and difficult to detect in comparison to other reflections.

The resolution for down range can be calculated by

$$\Delta R = c / 2K\Delta f \quad (9)$$

where K is the number of frequency points and Δf is step size. Cross range resolution can able to discriminate the target placed near to each other which can be given by

$$\Delta CR = \lambda R / D \quad (10)$$

where λ is the wavelength, R is the far field distance of the target and D is the aperture of the antenna. Cross range resolution at 2 GHz is 7.5 cm. All antenna positions from A-scan are obtained and average A-scan is calculated by

$$\tilde{s}_n = s_n - \frac{1}{N}\sum_{j}^{N} s_j \quad (11)$$

where, S_n is the A-scan at n^{th} position and N is the no. of antenna positions

Above technique is known as average trace subtraction and plot for reflections from the target are as shown in Fig. 4. From the plot we cannot determine the range-bin for the target since reflections from the target get obscured due to the other reflections in the scene.

B-scan matrix is collection of individual A-scan

$$S = [\tilde{s}_1, \tilde{s}_2, ..., \tilde{s}_n] \quad (12)$$

B-scan matrix is $M X N$ matrix, where M represents the range bins and N represents the antenna positions. Implementing SVD on B – scan matrix for D is

$$D = U * S * V^T \quad (13)$$

U and V are unitary matrices of left and right singular vectors. Singular values for S are in decreasing order i.e. $D_{11} \geq D_{22} \geq ... \geq D_{NN} \geq 0$ and then B-scan for whole image by using SVD can be given as and shown in Fig. 5.

$$D = \sum_{i=1}^{N} U_i S_i V_i^T \quad (14)$$

$S_i = S_{ii}$ is the i^{th} singular value for S, as discussed earlier few largest singular value in D represents reflections from the wall and remaining by target. B-scan image developed using SVD for Eqn (14) is as shown in Fig. 5, we can notice that only reflections from the wall can be traced and no target. Since the reflections from low dielectric targets are weak they become obscured due to strong reflections from the wall. Subspace projection based approach to mitigate wall effects is proposed in[23] where it is mentioned that location of the target, number and size of the target decides target subspace. The exact subspace for target subspace in case of weak target is difficult to locate since the eigenvalues for the target subspace are similar to the noise subspace. In this paper we proposed noise space exploitation using modified LRA algorithm to detect weak target behind the wall. Steps for traditional LRA algorithm are

(i) Calculate the SVD for data matrix D

$$D = U * S * V^T \quad (15)$$

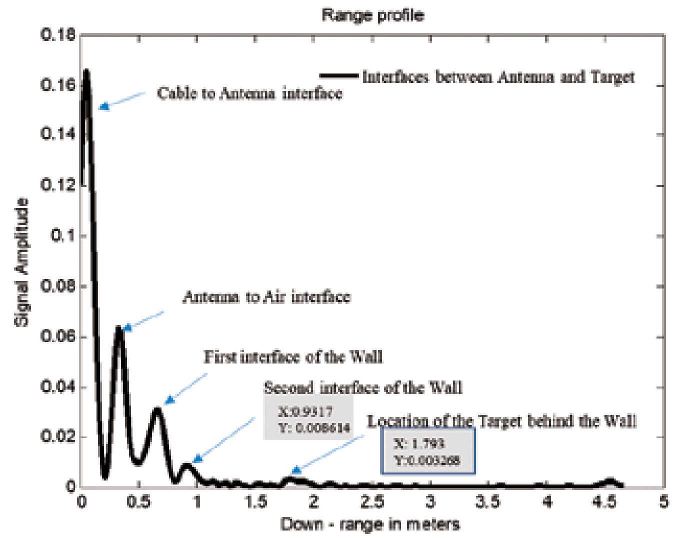

Figure 3. A- scan for teflon target.

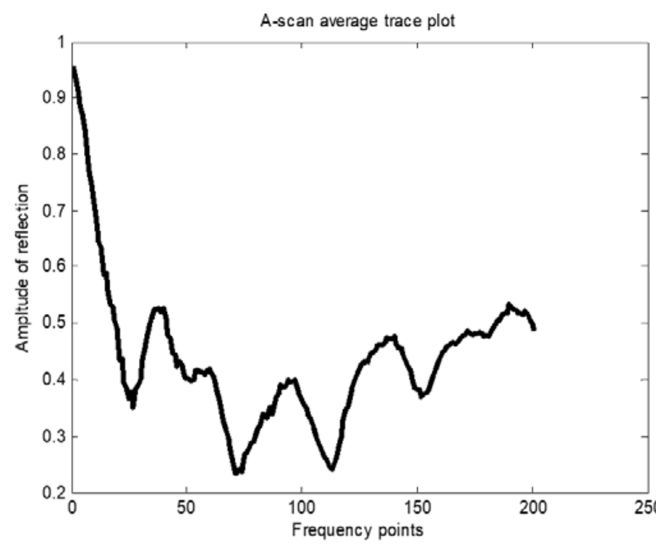

Figure 4. A-scan for teflon target after average trace sustraction.

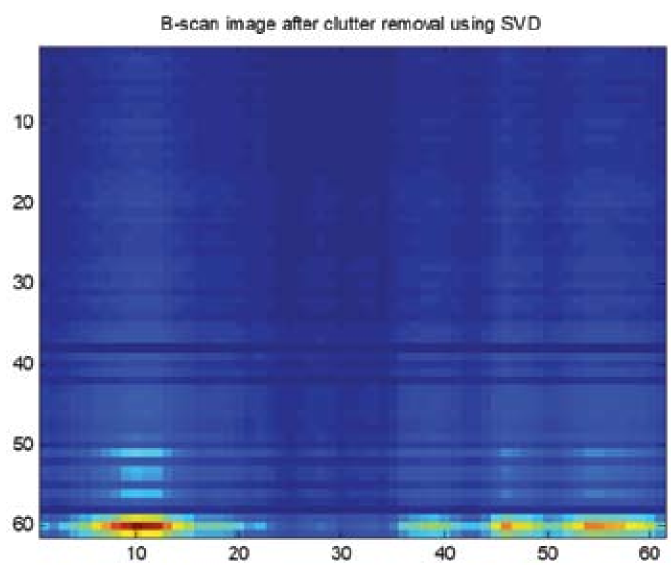

Figure 5. B-scan for weak target using SVD.

(ii) Select n largest diagonal singular values from the matrix S and set other values to zeros.

$$\widehat{S} = S(1:n,1:n) \tag{16}$$

(iii) Calculate LRA matrix

$$\widehat{D} = U\widehat{S}V^T \tag{17}$$

Figure 6 shows B-scan image for B-scan data matrix using LRA algorithm where also target is not visible. In[15] it is mentioned that largest eigenvalue corresponds to the wall reflection. When first eigen value which corresponds to wall is neglected modified matrix for $\widehat{S}$ can be given as

$$\widehat{S} = S(2:n,2:n) \tag{18}$$

As shown in Fig. 7 target is visible using modified LRA. The comparison for SVD, LRA, and modified LRA in terms of peak signal-to-noise ratio (PSNR) is given in Table 1. PSNR is used to calculate the distortion in the final image w.r.t. the input low resolution image and given by

$$PSNR = 10\log_{10}\frac{1}{MSE} \tag{19}$$

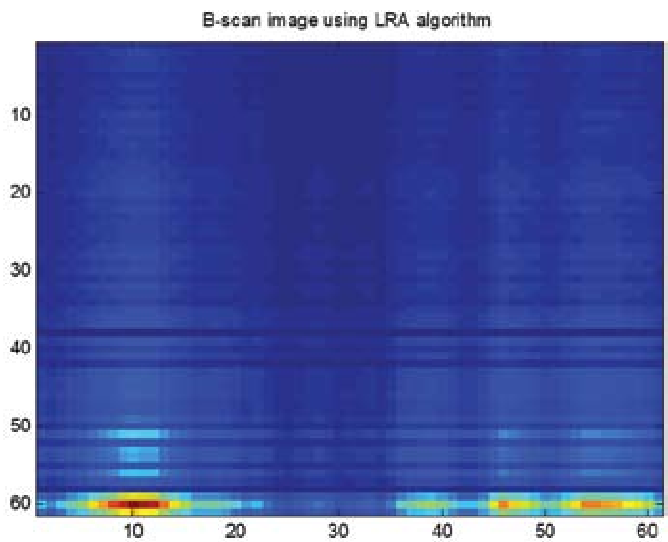

Figure 6. B- scan for weak target using LRA algorithm.

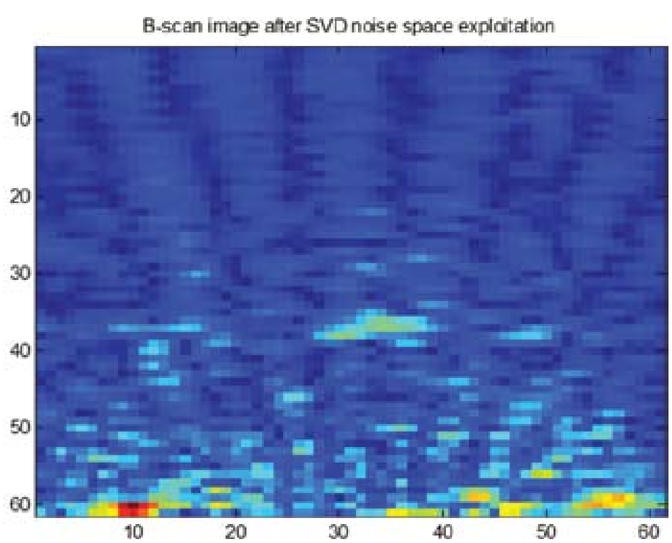

Figure 7. B- scan for weak target using modified LRA algorithm.

where

$$MSE = \frac{(O.I. - F.I.)^2}{(V.P.*\mathrm{H.P.})} \tag{20}$$

MSE Mean square error
OI Normalised image
FI Final image
VP Number of vertical scanning points
HP Number of horizontal scanning points

Table 1. PSNR values for different methods for noise reduction

Method	PSNR value
Singular value decomposition (SVD)	7.10 dB
Low-rank approximation (LRA)	7.23 dB
Modified LRA	9.20 dB

5. CONCLUSIONS

We have proposed a new modified low-rank approximation method to extract the target subspace from existing TWI data. This method gives improved de-noising performance, when the frame work for low-rank approximation is applied. The traditional LRA matrix decomposed adaptively into truly low-rank to detect weak target behind the wall. Our proposed approach can successfully applied for detection of weak targets in TWI which is not possible with general SVD approach. The experimental work shows that proposed approach gives superior performance over traditional SVD in the detection of weak targets.

REFERENCES

1. Baharian, M.; Rajabalipanah, H.; Fakheri, M.H. & Ali, Abdolali B. Removing the wall effects using electromagnetic complex coating layer for ultra-wideband through wall imaging. *Antennas Propag. IET Microw.*, 2017, **11**(4), 477-482. doi: 10.1049/iet-map.2016.0514.
2. Kidera, S.; Sakamoto, T.; Sato, T. High-resolution 3-D imaging algorithm without derivative operations for UWB through-the-wall radars. *In* IEEE Antennas and Propagation Society International Symposium, 2008, pp. 1-4.
3. Zhang, Y.; Xia, T. In-wall clutter suppression based on low-rank and sparse representation for through-the-wall radar. *IEEE Geosci. Remote Sens. Lett.,* 2016, **13**(5), 671-675. doi: 10.1109/LGRS.2016.2535161.
4. Yektakhah, B.; Dehmollaian, M. A method for cancellation of clutter due to an object in transceiver side of a wall for through-wall sensing applications. *IEEE Geosci. Remote Sens. Lett.,* 2012, **9**(4), 559–563. doi: 10.1109/LGRS.2011.2174771.
5. Solimene, R.; Cuccaro, A. Front wall clutter rejection methods in TWI. *IEEE Geosci. Remote Sens. Lett.,* 2014, **11**(6), 1158–1162. doi: 10.1109/LGRS.2013.2288491.
6. Tivive, F.H.C.; Amin, M.G.; Bouzerdoum, A. Wall clutter mitigation based on eigen-analysis in through-the-wall radar imaging. *In* 17th International Conference on Digital Signal Processing (DSP*)*, 2011, pp. 1–8.

7. Tivive, F.H.C.; Bouzerdoum, A. & Amin, M.G. An SVD-based approach for mitigating wall reflections in through-the-wall radar imaging. *In* IEEE RadarCon (RADAR),2011, pp. 519-524.
8. Ahmad, F.; Qian, J.; Amin, M.G. Wall Clutter Mitigation Using Discrete Prolate Spheroidal Sequences for Sparse Reconstruction of Indoor Stationary Scenes. *IEEE Trans. Geosci. Remote Sens.*, 2015, **53**(3), 1549–1557. doi: 10.1109/TGRS.2014.2345259.
9. Chen, X. & Chen, W. Double-layer fuzzy fusion for multiview through-wall radar images. *IEEE Geosci. Remote Sens. Lett.,* 2015, **12**(10), 2075–2079. doi: 10.1109/LGRS.2015.2448051.
10. Zhang, P. &Zhang, X. A Modified Phase Shift Migration Algorithm Applied to 2D and 3D Through-the-Wall Imaging. *IEEE J. Sel. Top. Appl. Earth Obs. Remote Sens.,* 2014, **7**(5), 1700–1708. doi: 10.1109/JSTARS.2013.2264719.
11. Riaz, M.; Muhammad; Ghafoor, A. Through-wall image enhancement based on singular value decomposition. Available at: https://www.hindawi.com/journals/ijap/2012/961829/. (Accessed on May 29, 2018).
12. Cho, Y.S.; Jung, H.K.; Cheon, C. & Chung, Y. S. Adaptive back-projection algorithm based on climb method for microwave imaging. *IEEE Trans. Magn.,* 2016, **52**(3), 1-4. doi: 10.1109/TMAG.2015.2479358.
13. Zhang, P.; Fei, P.; Wen, X. & Nian, F. A Novel Through-the-Wall Imaging Algorithm Combined with Phase Shift Migration and NUFFT. *Chin. J. Electron.,* 2017, **26**(5), 1096-1100. doi: 10.1049/cje.2017.08.002.
14. Klemm, M.; Craddock, I.J.; Leendertz, J.A. & Benjamin, R. Improved delay-and-sum beamforming algorithm for breast cancer detection. *Int. J. Antennas Propag.,* 2008, pp. 1-9. doi: 10.1155/2008/761402.
15. Gaikwad, A.N.; Singh, D. & Nigam, M.J. Application of clutter reduction techniques for detection of metallic and low dielectric target behind the brick wall by stepped frequency continuous wave radar in ultra-wideband range. *Sonar Navig. IET Radar,* 2011, **5**(4), 416-425. doi: 10.1049/iet-rsn.2010.0059.
16. Kumar, B.; Sharma, P. & Singh, D. Development of an efficient approach for MMW imaging system to identify concealed targets inside the book. *Microw. Opt. Technol. Lett.,* 2017, **59**(12), 2982–2990. doi: 10.1002/mop.30858.
17. Yoon, Y.S. & Amin, M.G. Spatial filtering for wall-clutter mitigation in through-the-wall radar imaging. *IEEE Trans. Geosci. Remote Sens.,* 2009, **47**(9), 3192–3208. doi: 10.1109/TGRS.2009.2019728.
18. Chen, Y.; Zhou, Y.; Chen, W.; Zu, S.; Weilin, Huang & Dong, Zhang. Empirical low-rank approximation for seismic noise attenuation. *IEEE Trans. Geosci. Remote Sens.,* 2017, **55**(8), 4696–4711. doi: 10.1109/TGRS.2017.2698342.
19. Yao, Q. &Qifu, W. Kirchoff migration algorithm for ground penetrating radar data. *In* 2012 International Conference on Computer Science and Electronics Engineering, 2012, **2**, pp. 396-398.
20. Ahmad, F.; Amin, M.G. & Kassam, S.A. A beamforming approach to stepped-frequency synthetic aperture through-the-wall radar imaging. *In* 1st IEEE International Workshop on Computational Advances in Multi-Sensor Adaptive Processing, 2005, pp. 24-27.
21. Qiao, T.; Ren, J.; Wang, Z.; Zabalza J.; Sun, M.; Zhao, H.; Li, S.; Benediktsson, J.A. ; Dai, Q. & Marshall, S. Effective denoising and classification of hyperspectral images using curvelet transform and singular spectrum analysis. *IEEE Trans. Geosci. Remote Sens.,* 2017, **55**(1), 119–133. doi: 10.1109/TGRS.2016.2598065.
22. Siahsar, M.A.N.; Gholtashi, S.; Torshizi, E.O.; Chen, W. & Chen, Y. Simultaneous denoising and interpolation of 3-D seismic data via damped data-driven optimal singular value shrinkage. *IEEE Geosci. Remote Sens. Lett.*, 2017, **14**(7), 1086–1090. doi: 10.1109/LGRS.2017.2697942.
23. Tivive, F.H.C.; Bouzerdoum, A. & Amin, M.G. A subspace projection approach for wall clutter mitigation in through-the-wall radar imaging. *IEEE Trans. Geosci. Remote Sens.,* 2015, **53**(4), 2108–2122. doi: 10.1109/TGRS.2014.2355211.

CONTRIBUTORS

Mr Bivalkar Mandar received MTech in Elecrtronics and Communication from Dr. Babasaheb Ambedkar Technological university, Lonere, Maharashtra in 2011 and pursuing PhD in Signal enhancement techniques for microwave imaging at the Indian Institute of Technology Roorkee (IIT Roorkee), Roorkee, India. His research interests include RF and microwave engineering, microwave and MMW imaging, and electromagnetic.
He developed an experimental set-up and carried out data collection and signal processing task.

Dr Bambam Kumar received his MTech in Integrated Electronics and Circuits from Indian Institute of Technology, Delhi in 2011 and completed the PhD in hidden object detection through Microwave and MMW imaging at the Indian Institute of Technology Roorkee (IIT Roorkee), Roorkee, India. He possesses more than 6 years of experience in teaching. He has received the Young Scientist award 2016 organised by UCOST, Uttarakhand. His research interests include RF and microwave engineering, microwave and MMW imaging, and electromagnetic.
He was involved in the data collection for an experimental set-up.

Dr Dharmendra Singh, received the PhD in Electronics Engineering from IIT (BHU) Varanasi, Varanasi, India. He has 24 year of experience in teaching and research. He was a Visiting Scientist Post-Doctoral Fellow with many universities in India and abroad. He is currently a Professor with the Department of Electronics and Communication Engineering, IIT Roorkee, Roorkee, India, and a Coordinator at RailTel-IIT Roorkee Centre of Excellence in Telecom, Roorkee. His research interests include microwave remote sensing, polarimetry, interferometry, and numerical modelling, through wall imaging and stealth technology.
He has provided the concept of the work, reviewed it and given shape to the manuscript.