

## Optimal Operating Depth Search for Active Towed Array Sonar using Simulated Annealing

Sangkyum An<sup>@,\*</sup>, Keunhwa Lee<sup>#</sup>, and Woojae Seong<sup>@</sup>

<sup>@</sup>*Seoul National University, Seoul, 08826, Korea*

<sup>#</sup>*Sejong University, Seoul, 05006, Korea*

<sup>\*</sup>*E-mail: navy60@snu.ac.kr*

### ABSTRACT

In an active towed array sonar, it is important to find the optimal operation depth. Generally, the optimal depth can be chosen via numerical simulations for all sonar depths and this imposes great burdens of time and cost. In this paper, an efficient approach is proposed to find the optimal depth using the optimisation technique. First, the sonar performance function is newly defined as a measure of how well the active sonar might perform. This function depends on the properties of the ocean environment and the positions of sonar and underwater target. Then, the simulated annealing to find an optimal solution for maximising sonar performance is used. The optimised depth agrees well with the depth obtained from direct searching for all depths of source and receiver combinations, but its computational time is largely reduced.

**Keywords:** Optimal depth; Active towed array sonar; Probability of detection; Simulated annealing; Sonar performance function

### 1. INTRODUCTION

The active towed array sonar (ATAS) has been developed to overcome the drawback of the hull-mounted active sonar in the shadow zone. The ATAS consists of a varying depth active source and a towed receiver line array for reception<sup>1</sup>. In the ATAS, it is important to select operation depths of the source and receiver, which strongly affects the sonar performance.

Many studies on optimal depth have been done in the field of passive sonar. Ferla and Porter suggest the method for selecting the operation depth of passive sonar<sup>2</sup>. They define the detection radius (DR) to assess the performance of the passive sonar, and use it as a cost function for finding the optimal depth. They identify the optimal receiver depth as the depth where the probability of detection is highest in a range-integrated sense. Since DR is a function with single variable, its maxima can be rapidly found by direct calculation of the cost function. Sha and Nolte predict sonar detection performance for passive sonar<sup>3</sup>. They propose an analytical receiver operating characteristic expression that characterises the performance of the optimal Bayesian detector in the presence of ocean environmental and source position uncertainty. Gershfeld and Ingenito conduct a numerical simulation study to calculate the receiver depth that would maximise the signal-to-noise ratio for one of several source depths in various shallow water environments<sup>4</sup>. Domingo proposed a mathematical model that focuses on finding the optimal placement for wireless nodes in a three-dimensional

underwater wireless sensor network, whose communication is being affected by the shadow zone<sup>5</sup>.

For the ATAS, previous studies on optimal depth released in the open literatures cannot be found. Since target detection using the ATAS is a bi-static detection, the performance of ATAS is influenced by several parameters such as source depth, receiver depth, target depth, and target range. Thus, it is difficult to find the optimal depths of source and receiver in the ATAS while considering all the parameters.

The goal of this study is to develop an efficient method for finding the optimal depths of an active source and a towed line array in the ATAS using global optimisation algorithm. For optimisation, the sonar performance function (SPF) is formulated based on the sonar equation, which is a measure of how well the ATAS detects the target. The unknown target location is considered probabilistically. The optimal depth of the source and receiver can be found at the peak of the SPF. Rather than directly calculating the SPF for all depths of source and receiver, simulated annealing (SA) optimisation algorithm is applied to find the optimal depth efficiently.

### 2. SONAR PERFORMANCE PREDICTION

#### 2.1 Sonar performance function

To find the optimal operating depth of source and receiver in ATAS, a useful measure that quantifies the performance of ATAS is necessary. In general, the quantity 'range of the day' is often used as a criterion parameter that represents the sonar performance, based on the probability of detection. Range of

the day indicates a range where the probability of detection is 50 per cent.

In active sonar, the probability of detection is as given by a log-normal distribution<sup>2</sup> :

$$p_D(SE) = \frac{1}{\sqrt{2\pi\sigma}} \int_{-\infty}^{SE} \exp\left[-\frac{x^2}{2\sigma^2}\right] dx \quad (1)$$

where  $SE$  is the signal excess,  $p_D$  is the probability of detection, and  $\sigma$  is the standard deviation of  $SE$ .

The  $SE$  represents the degree to which the source signal stands out from the ambient noise with an  $SE$  of 0 dB implying a 50 per cent probability of detection<sup>2</sup>. From the active sonar equation, the  $SE$  is given by<sup>6</sup>,

$$SE = FOM - (TL_1 + TL_2) \quad (2)$$

where  $FOM$  is the figure of merit and  $TL_1$  and  $TL_2$  are respectively the transmission loss from the source to the target and from the target to the receiver.

Although range of the day provides the physical range obtained from the probability of detection, it remarkably varies with the receiver depth, the target depth, and the range of target and ATAS. Because of such variability, there is an ambiguity in calculating the optimal location with range of the day.

In this study, the SPF is proposed based on the probability density function (PDF) of detection, the PDF of target depth and the PDF of relative range as follows:

$$SPF(Z_R, Z_S) = \int_0^\infty \int_0^D p(Z_T) p(R) p_D(Z_R, Z_S, Z_T, R) dZ_T dR \quad (3)$$

where  $D$  is water depth at range  $R$ ,  $p(Z_T)$  is the PDF of target depth  $Z_T$ ,  $p(R)$  is the PDF of relative range,  $p_D$  is the PDF of detection,  $Z_R$  is the receiver depth and  $Z_S$  is the source depth.

Eqn (3) is defined in order to consider both location of target and probability of detection for evaluate the performance of ATAS. When the location of the target is unknown, the probability of detection is useless. In this case, Eqn (3) provides more rational information with the stochastic approach. For a localised target [ $p(Z_T) p(R) = \delta(Z_T - Z_0) \delta(R - R_0)$ ], it is seen that Eqn (3) becomes the probability of detection naturally. It is noted that Eqn (3) is not the cumulative density function (CDF).

The PDFs of the target depth and relative range will depend on the situation being considered. For example, if

the range of the target is known, the PDF of relative range is set to be a delta function with the singularity at specific relative range. Otherwise, it is reasonable to assume that  $p(R)$  follows a uniform distribution. The PDFs of the target depth is considered to be a normal distribution since the underwater target has a tendency to use an underwater channel.

Simulation of SPF is performed for the following ocean environment. The sound speed profile and water depth are as given in Fig. 1. The sea floor is composed of silt and clay with the density and sound speed of 1470 kg/m<sup>3</sup> and 1585 m/s<sup>7</sup>. The frequency of the source is 3.5 kHz. Transmission losses in Eqn (2) are computed using the acoustic propagation model Bellhop<sup>8</sup>. The standard deviation in Eqn (1) and the FOM in Eqn (2) are respectively set at 8 dB and 130 dB. Figure 2 shows SPF for all source and receiver depths in two cases where the target location is unknown and when only target range is given. The PDF of target depth is considered to be the normal distribution with the mean of 100 m and the standard deviation of 20 m. The PDF of the relative range of target is chosen as the uniform distribution. As shown in Figs. 2(a) and 2(b), the ambiguity surfaces of SPF are different for the two cases. But, it is clear that there is a global optimal location.

To find the optimal location by the direct calculation of

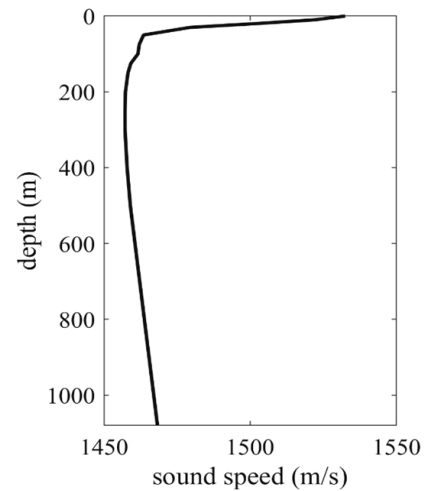


Figure 1. Sound speed profile in the East Sea of Korea during the summer.

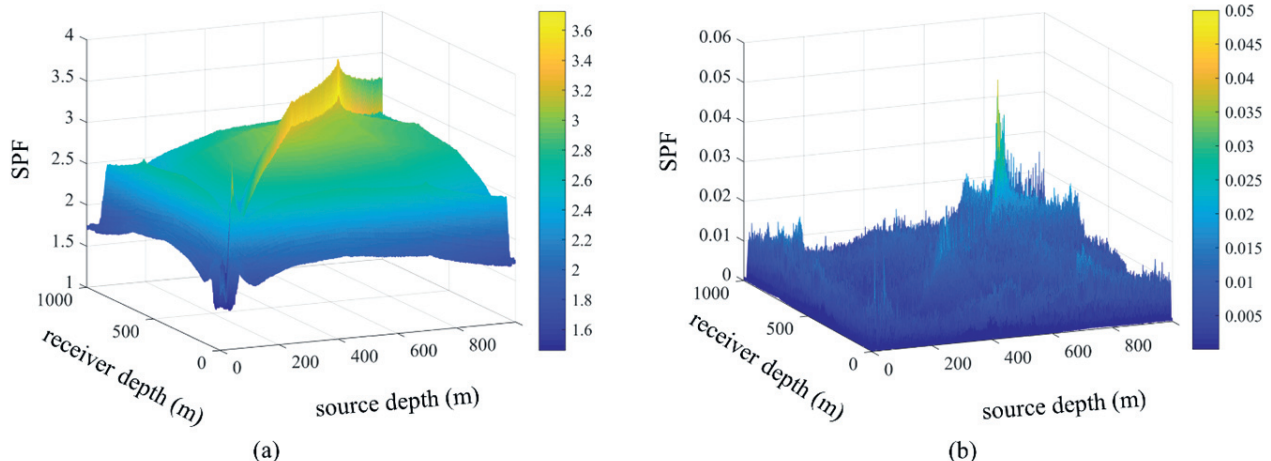


Figure 2. Sonar performance function as a function of receiver depth and source depth when (a) target location is unknown and (b) only target range is known.

Eqn (3) for all source, and receiver depths, much computations would be required. Even if the relative range is known, dividing each depth into 1000 points, a total of  $10^6$  calculations is needed to obtain the ambiguity surface of SPF. Therefore, it is necessary to introduce an optimisation method to find the optimal locations.

### 3. OPTIMAL DEPTH SELECTION USING SIMULATED ANNEALING

As described in the previous section, the direct calculation of SPF for all depths of source and receiver is computationally intensive. This section present how to obtain the optimal depth efficiently using a global optimisation algorithm.

#### 3.1 Simulated Annealing for the Optimal Depth

Simulated Annealing (SA) is a global search method that find its inspiration in the physical annealing process studied in statistical mechanics<sup>9</sup>. Since SA is characterised by ease of implementation and fast convergence, it is usually employed as an optimisation method to find an optimal solution for military problem<sup>10,11</sup>. Basically, SA calculates the object function at a point, and then finds a new point with a higher value of object function with the probability. This process is repeated until the optimal point is reached. The flow chart of the SA algorithm is as given in Fig. 3.

An optimisation is performed for the ocean environment used in the previous section. Figure 4 shows the optimisation result symbolised by red asterisks on the ambiguity function of SPF. It is clear that the optimised depths are consistent with those of the direct calculation of SPF for both cases. However, the time cost is reduced about a thousand times as listed in Table 1.

#### 3.2 Optimal Depth in the East Sea of Korea

Assuming that the active source and receiver array in ATAS are to be operated at the same depth, optimal depths are calculated in the region of East Sea of Korea between 130°E and 135°E longitude and 36°N and 41°N latitude using the proposed method. Sound speed profile at each location is calculated from temperature and salinity data taken in the NOAA World Ocean Database<sup>12</sup>.

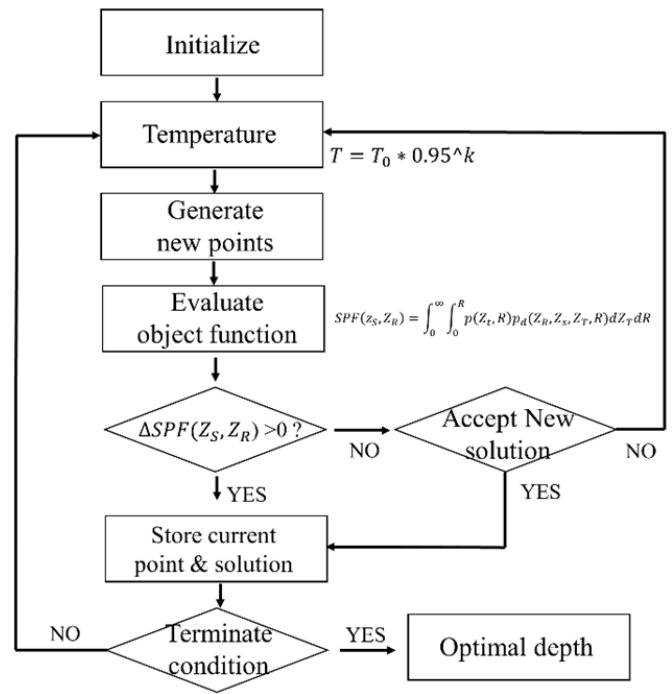


Figure 3. Flow chart of the simulated annealing process.

Table 1. Comparison of the direct calculation and optimisation results.

Case	Item	Number of calculations	Optimal depth (m)	SPF
Fig. 4(a)	SA optimisation	810	728	3.521
	Direct calculation	1,000,000	728	3.520
Fig. 4(b)	SA optimisation	741	215	4.384
	Direct calculation	1,000,000	215	4.387

Bathymetry is obtained from the US Geological Survey (USGS), Global 30 Arc-Second Elevation (GTOPO30) model<sup>13</sup>. Figure 5 shows the bathymetry and the sound speed in the corresponding region. The optimal depths are calculated with 0.25° resolution. Assume that the mean depths of the target are 200 m and 400 m with the standard deviation of 70 m.

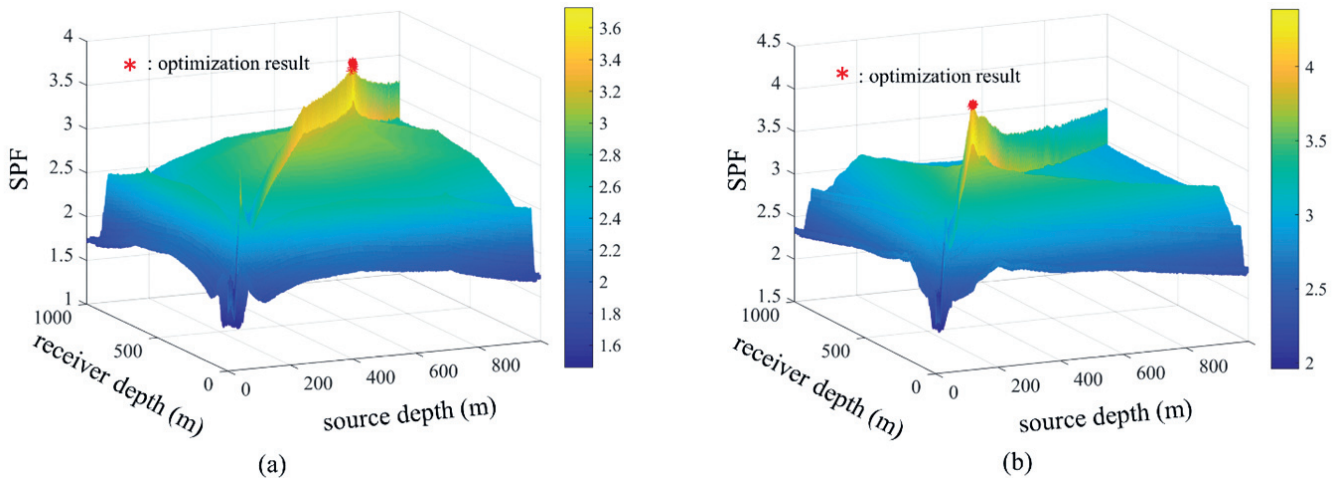


Figure 4. Sonar performance function as a function of receiver depth and source depth with a mean target depth of (a) 100 m and (b) 300 m. The result obtained by the optimisation algorithm is marked with a red asterisk

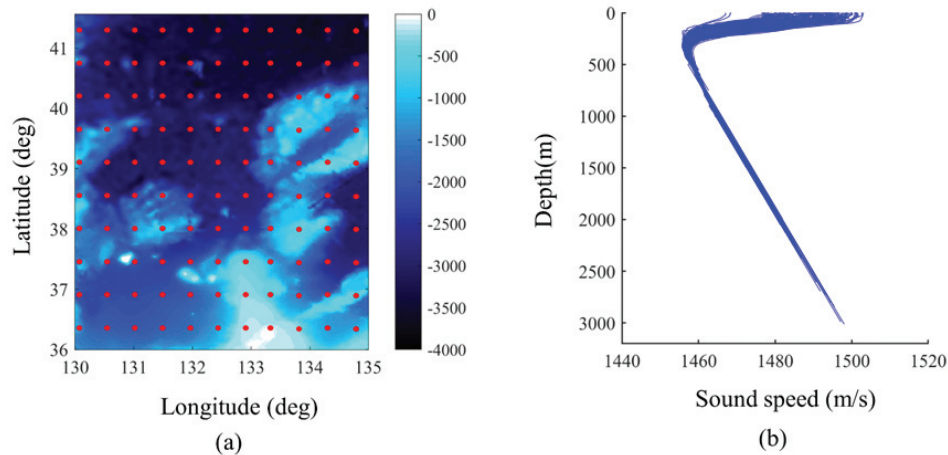


Figure 5. (a) Bathymetry plot for longitude and latitude where red dots represent the ATAS locations for the calculation of its optimal depth and (b) sound speed profiles in the corresponding region.

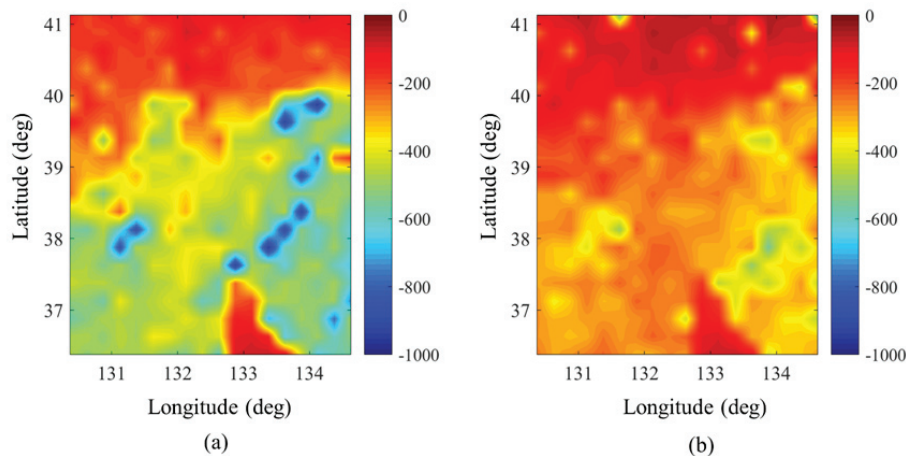


Figure 6. Optimal depth of the ATAS when the source depth and the receiver depth are the same; (a) mean target depth of 200 m and (b) mean target depth of 400 m.

The results are plotted in Fig. 6. Figure 6(a) plots the result for the mean of 200 m and Fig. 6(b) shows the result for the mean of 400 m. The unit of the colour bar is meters. As shown in Fig. 6, the optimal depth depends on the sonar operation region. When the target is moved near the depth of 200 m in the lower region of Fig. 6(a), the sonar has to be operated at greater depth. As the depth of target increases, the optimal depth of the sonar becomes shallower, as shown in Fig. 6(b). Finally, it is noted that it would take about thousand times computation to obtain the same figure as Fig. 6 doing the direct calculations as listed Table 1.

#### 4. CONCLUSIONS

By operating the ATAS at optimal depth in the ocean, the probability of detection of an underwater target will increase. An efficient method is presented to find the optimal depth of ATAS using SA. The SPF is suggested and used to evaluate the sonar performance with the PDFs of detection, target depth, and relative range. The calculations of the proposed method are about one thousand times less than in the direct calculation of the SPF for all depths. With some numerical examples, it is shown that our method can find a global optimal depth and would be a valuable tool in recommending the optimal depth for the ATAS operator.

#### REFERENCES

1. Kumar, A.; George, J. & Jagathyraj, V. P. Requirements analysis of an integrated sonar suite for surface ships: systems engineering perspective. *Def. Sci. J.*, 2014, **64**(4), 366-370. doi: 10.14429/dsj.64.4486
2. Ferla, C.M. & Porter, M.B. Receiver-depth selection for passive sonar systems. *IEEE J. Ocean. Eng.*, 1991, **16**(3), 267-278. doi: 10.1109/48.90883
3. Sha, L. & Nolte, L.W. Bayesian sonar detection performance prediction with source position uncertainty using SWellEx-96 vertical array data. *IEEE J. Ocean. Eng.*, 2006, **31**(2), 345-355. doi: 10.1109/JOE.2006.875263
4. Gershfeld, D.A. & Ingenito, F. Optimum depth of propagation in shallow water. Naval Research Lab. Rep. 8741, 1983. doi: 10.1109/WD.2009.5449707
5. Domingo, M.C. Optimal placement of wireless nodes in underwater wireless sensor networks with shadow zones. *Wireless Days (WD), 2009 2nd IFIP. IEEE*, 2009, 1-6.
6. Urlick, R.J. Principles of underwater sound. McGraw-Hill, New York, 1983.

7. Kim, D.C.; Kim, S.J.; Seo, Y.K.; Jeong, J.H.; Kim, Y.E. & Kim, G.Y. Physical properties of Southeastern Yellow Sea Mud (SEYSM): Comparison with the East Sea and the South Sea mudbelts of Korea. *J. Kor. Soc. Ocean.*, 2001, **5**(4), 335-345.
8. Porter, M.B. The BELLHOP manual and user's guide: Preliminary draft. <http://oalib.hlsresearch.com/Rays/HLS-2010-1.pdf>. (Accessed on 8 August 2018).
9. Aarts, E. & Korst, J. Simulated annealing and Boltzmann machines: a stochastic approach to computing. Wiley, Chichester, 1989.
10. Bisht, S. Hybrid genetic-simulated annealing algorithm for optimal weapon allocation in multilayer defence scenario. *Def. Sci. J.*, 2004, **54**(3), 395-405. doi: 10.14429/dsj.54.2054
11. Stodola, P. & Mazal, J. Model of optimal cooperative reconnaissance and its solution using metaheuristic methods. *Def. Sci. J.*, 2017, **67**(5), 529-535. doi: 10.14429/dsj.67.10530
12. Boyer, T.P.; Antonov, J.I.; Baranova, O.K.; Coleman, C.; Garcia, H.E.; Grodsky, A.; Johnson, D.R.; Locarnini, R.A.; Mishonov, A.V.; O'Brien, T.D.; Paver, C.R.; Reagan, J.R.; Seidov, D.; Smolyar, I.V. & Zweng, M.M. World Ocean Database 2013. Silver Spring, MD, NOAA Printing Office, 2013.
13. Global 30 Arc-Second Elevation. U.S. Geological Survey's Earth Resources Observation and Science (EROS) Center. <http://ita.cr.usgs.gov/GTOPO30> (Accessed on 8 August 2018).

## CONTRIBUTORS

**Dr Sangkyum An** received his BS in Electrical Engineering from Naval Academy, Changwon, Korea in 2006. And he received his MS and PhD in Naval architecture and Ocean Engineering from Seoul National University, Seoul, Korea in 2011 and 2019, respectively. Currently, he is serving as lieutenant commander in ROK Navy. His current research area: underwater signal processing and underwater experimental technique. In the current study, he has carried out the design and implementation of the algorithm. He has also contributed in writing the research paper.

**Dr Keunhwa Lee** received his BS and PhD in Naval architecture and Ocean Engineering from Seoul National University, Seoul, Korea in 1996 and 2006, respectively. Currently, he is an associate professor at Sejong University, Seoul, Korea. His current research area: Acoustic modeling/data-driven modeling, Ocean ambient noise, sonar signal processing. In the current study, he has contributed in enhancing the proposed algorithm. He has also revised the manuscript based on the reviewer comment.

**Dr Woojae Seong** received the BS and MS in Naval architecture and Ocean Engineering from Seoul National University, Seoul, Korea in 1982 and 1984, respectively, and the PhD(Ocean Engineering) from Massachusetts Institute of Technology, Cambridge, United States in 1991. Currently, he is a Professor at Seoul National University. His current research area: sound propagation/reverberation modeling in underwater, ocean sediment acoustics, underwater signal processing, machine learning applications and localisation of cavitation noise. In the current study, he has supervised to conduct the research and write the paper.