

# ARE PPP TESTS ERRATICALLY BEHAVED? SOME PANEL EVIDENCE

GUGLIELMO MARIA CAPORALE<sup>a</sup>, CHRISTOPH HANCK<sup>b\*</sup>

<sup>a</sup>Brunel University, London

<sup>b</sup>Universität Dortmund

October 5, 2006

## Abstract

This paper examines whether, in addition to standard unit root and cointegration tests, panel approaches also produce test statistics behaving erratically when applied to PPP. We show that if appropriate tests (which are robust to cross-sectional dependence and more powerful) are used, any evidence of erratic behaviour disappears, and strong empirical support is found for PPP. It appears therefore that recent advances in panel data econometrics might enable us to settle the PPP debate.

*Keywords:* Purchasing Power Parity (PPP), Real Exchange Rates, Erratic Behaviour, Panel Tests

*JEL Classification:* C12, C23, F31

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\*Corresponding author: Professor Guglielmo Maria Caporale, Brunel University, Uxbridge, Middlesex UB8 3PH, UK. Tel.: +44 (0)1895 266713. Fax: +44 (0)1895 269770. E-mail: Guglielmo-Maria.Caporale@brunel.ac.uk

# 1 Introduction

Purchasing Power Parity (PPP) is a key concept to the way international economists understand real exchange rate behaviour. Most of them would agree that PPP holds in the long run, if not continuously, at least in some form, and that therefore it represents a valid international parity condition [see, e.g., Taylor and Taylor, 2004, for a critical review of the PPP debate]. However, the available empirical evidence has not always been consistent with the PPP condition. Given the wide consensus on the theory, this failure of formal tests to provide support to PPP has mainly been attributed to flaws in the econometric approaches taken. Froot and Rogoff [1995], in particular, highlighted the limitations of the tests used in three successive stages in the time series literature on PPP. Initially, possible non-stationarities were overlooked. Then the null that the real exchange rate follows a random walk (long-run PPP being the alternative) was tested by means of unit root tests which are now well-known to have very low power; cointegration methods, subsequently used, suffered from similar problems. Recently, Caporale et al. [2003] have also argued that classical unit root tests are not informative about PPP. Specifically, they show that the type of stationarity exhibited by the real exchange rate cannot be accommodated by the fixed-parameter autoregressive homoscedastic models normally employed in the literature. In particular, they compute a recursive t-statistic, and show that it exhibits erratic behaviour, suggesting the presence of endemic instability, and of a type of non-stationarity more complex than the unit root one usually assumed. Similar results are reported in the case of trivariate cointegration tests by Caporale and Hanck [2006], who conclude that the observed erratic behaviour is therefore not due to arbitrarily imposed symmetry/proportionality restrictions.

In order to increase the power of tests of PPP, more recent studies have used panel methods [see, e.g., Wu, 1996, and Papell, 1997, 2002].<sup>1</sup> The present paper investigates whether erratic behaviour still occurs when panel approaches are taken. If erraticism is found to disappear once more powerful, panel tests are applied, one could then argue that the failure of earlier tests to give support to PPP theory was indeed due to their low power, rather than to incorrect assumptions about the dynamic features of the stochastic process of interest. In this case, panel tests, characterised by much higher power, could be seen as the way forward to settle the PPP debate. The layout of the paper is the following. Section 2 outlines the panel methods used.

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<sup>1</sup>See Caporale and Cerrato [2006] for a critical survey of the empirical literature testing PPP by means of panel methods. Another new development in the literature on real exchange rates is the modelling of nonlinearities [resulting, for instance, from transaction costs – see Taylor et al., 2001] in mean reversion. Some studies also allow for structural breaks [see, e.g., Papell, 2002].

Section 3 presents the empirical evidence. Section 4 summarises the main findings and offers some concluding remarks.

## 2 The Panel Tests

This section briefly describes the panel tests considered in this study. It is widely acknowledged that panels of exchange rate data are generally cross-sectionally dependent [O'Connell, 1998]. Panel unit root tests relying on the assumption of cross-sectional independence (see, e.g., Levin et al., 2002, Im et al., 2003, or Choi, 2001) will therefore suffer from size distortion, as recently demonstrated in, for instance, Hlouskova and Wagner [2006]. Accordingly, we focus on panel tests which are robust to the presence of cross-sectional dependence. More specifically, we consider the tests put forward by Choi [2006] and Phillips and Sul [2003].

### Choi [2006]

In the first step, the panel tests of Choi [2006] apply Elliott et al. [1996] GLS detrending to the panel, thereby removing cross-sectional dependence. In the second step, meta-analytic panel tests from, e.g., Choi [2001] can then be used [see also Maddala and Wu, 1999].

Choi [2006] assumes the following two-way error-component model

$$y_{it} = \beta_0 + x_{it} \quad (i = 1, \dots, N; t = 1, \dots, T),$$

where

$$x_{it} = \mu_i + \lambda_t + v_{it},$$

and

$$v_{it} = \sum_{l=1}^{p_i} \alpha_{il} v_{i(t-l)} + e_{it}$$

The test of a panel unit root is formulated as

$$H_0 : \sum_{l=1}^{p_i} \alpha_{il} = 1 \quad \forall i$$

against

$$H_1 : \sum_{l=1}^{p_i} \alpha_{il} < 1 \quad \text{for a non-zero fraction } \#i/N$$

The Elliott et al. [1996] GLS estimator of  $\beta_0$  is given by

$$\hat{\beta}_{0i} = \frac{y_{i1} + (1 - \frac{7}{T}) \sum_{t=2}^T y_{it} - (1 - \frac{7}{T}) y_{i(t-1)}}{1 + (T-1)(1 - (1 - \frac{7}{T}))^2}$$

Choi [2006] shows that demeaning  $y_{it} - \hat{\beta}_{0i}$  cross-sectionally gives, for large  $T$ ,

$$z_{it} := y_{it} - \hat{\beta}_{0i} - \frac{1}{N} \sum_{i=1}^N (y_{it} - \hat{\beta}_{0i}) \simeq v_{it} - v_{i1} - \bar{v}_{.t} + \bar{v}_{.1},$$

where  $\bar{v}_{.a} := \frac{1}{N} \sum_{i=1}^N v_{ia}$ . This expression is independent of  $\beta_0$ ,  $\lambda_t$  and  $\mu_i$ . Moreover,  $\bar{v}_{.t}, \bar{v}_{.1} \rightarrow_p 0$ . Hence,  $z_{it}$  is cross-sectionally independent.

In a second step, one applies meta-analytic panel tests to  $z_{it}$ . For instance, one can run Augmented Dickey-Fuller tests on  $z_{it}$ . Then, after having obtained the  $p$ -values of the test statistics<sup>2</sup>, one combines these into panel test statistics as follows:

$$P_m = -\frac{1}{\sqrt{N}} \sum_{i=1}^N (\ln(p_i) + 1) \quad (1)$$

$$Z = \frac{1}{\sqrt{N}} \sum_{i=1}^N \Phi^{-1}(p_i) \quad (2)$$

$$L^* = \frac{1}{\sqrt{\pi^2 N/3}} \sum_{i=1}^N \ln\left(\frac{p_i}{1-p_i}\right), \quad (3)$$

where  $\Phi$  is the standard normal cumulative distribution function. As  $N, T \rightarrow \infty$ ,  $P_m, Z, L^* \Rightarrow N(0, 1)$ . The tests are consistent because  $P_m \rightarrow_p \infty$  and  $Z, L^* \rightarrow_p -\infty$  under  $H_1$ .

### Phillips and Sul [2003]

Phillips and Sul [2003] work with the dynamic panel representation

$$y_{it} = \mu_i(1 - \rho) + \rho y_{i(t-1)} + \sum_{j=1}^{\ell_i} \phi_{ij} \Delta y_{i(t-j)} + u_{it}, \quad (4)$$

where  $t = 1, \dots, T$ ,  $i = 1, \dots, N$  and  $\rho \in (-1, 1]$ . They model cross-sectional dependence with a standard normal common time effect  $\theta_t$  which is allowed to affect the units of the panel heterogeneously:

$$u_{it} = \delta_i \theta_t + \epsilon_{it}.$$

The  $\epsilon_{it}$  are normal with mean zero and variance  $\sigma_i^2$ . Letting  $\mathbf{u}_t = (u_1, \dots, u_n)^\top$ ,  $\boldsymbol{\delta} = (\delta_1, \dots, \delta_n)^\top$  and  $\boldsymbol{\Sigma} = \text{diag}(\sigma_1^2, \dots, \sigma_n^2)$ , we have  $\text{Cov}(\mathbf{u}_t) = \boldsymbol{\Sigma} + \boldsymbol{\delta} \boldsymbol{\delta}^\top$ . To deal with the cross-sectional dependence in  $\mathbf{u}_t$ , Phillips and Sul [2003] suggest estimating the cross-section coefficients  $\boldsymbol{\delta}$  and

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<sup>2</sup>In practice, one can evaluate the numerical distribution functions obtained by MacKinnon [1994, 1996] via response surface regressions.

$\Sigma$  by computing  $M_T = \frac{1}{T} \sum_{t=1}^T \hat{\mathbf{u}}_t \hat{\mathbf{u}}_t^\top$ , where  $\hat{\mathbf{u}}_t$  is obtained from the residuals obtained under the null  $\rho = 1$  in (4), and iteratively solving the system of equations

$$\hat{\boldsymbol{\delta}} = (M_T \hat{\boldsymbol{\delta}} - \hat{\Sigma} \hat{\boldsymbol{\delta}}) / \hat{\boldsymbol{\delta}}^\top \hat{\boldsymbol{\delta}} \quad \hat{\sigma}_i^2 = M_{Tii} - \hat{\delta}_i^2.$$

Using the orthogonal complement matrix  $\hat{\boldsymbol{\delta}}_\perp$ , one then computes  $\mathbf{y}_t^+ = (\hat{\boldsymbol{\delta}}_\perp^\top \hat{\Sigma} \hat{\boldsymbol{\delta}}_\perp)^{-1/2} \hat{\boldsymbol{\delta}}_\perp^\top \mathbf{y}_t$ . Phillips and Sul [2003] show that the above transformation asymptotically removes the dependence in  $\mathbf{y}_t$  such that  $\mathbf{y}_t^+$  is cross-sectionally independent.

It follows that one can then perform panel unit root tests like the Fisher-type test

$$P = -2 \sum_{i=1}^{N-1} \ln(p_i) \quad (5)$$

using  $p$ -values from unit root tests applied to each series  $y_{it}^+$ ,  $i = \dots, N$ . In practice, one can obtain the  $p$ -values as described in the previous subsection. Under  $H_0$ ,  $P \Rightarrow \chi_{2(N-1)}^2$ .

### 3 Results

We now investigate whether using the panel unit root tests discussed above leads to erratic behaviour of the test statistics, namely whether there are frequent jumps from the rejection to the non-rejection region as new observations are recursively added to the sample. We use the dataset also employed by Taylor [2002], which includes annual data for the nominal exchange rate, CPI and the GDP deflator. This dataset is particularly useful for our purposes because it covers a long period, ranging from 1892 through to 1996. The countries contained in our panel are the following: Argentina, Australia, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Mexico, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland and the United Kingdom. We use the United States as the reference country throughout [see Taylor, 2002, for further details on data sources and definitions]. In order to investigate possible parameter instability, we create new time series resulting from the recursive estimation of the statistics from (1)–(3) and (5). That is, we use the first  $k$  observations to produce the first set of statistics, where we let  $k = 40$  to discard estimates which are likely to be affected by small-sample bias. We then add an extra observation to compute the second set of statistics based on  $k + 1$  data points, and repeat the process until all  $T$  available observations have been used to yield  $T - k + 1$  estimates of the test statistics.

We show the results obtained using CPI data to construct the real exchange rate series in Figures 1 to 4, where we plot the test statistic series against  $k$ , and the dashed lines indicate the appropriate critical values at the 5% level. <sup>3</sup>

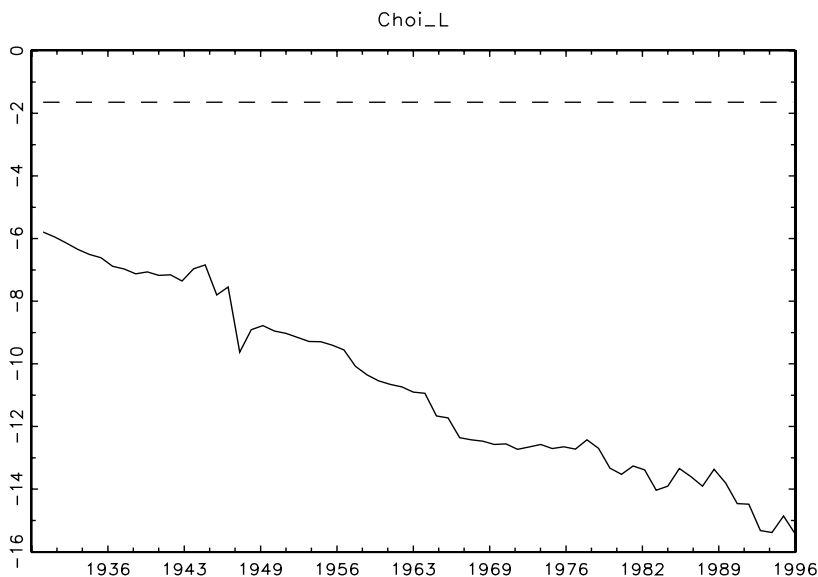


Figure 1: Test statistic series for  $L^*$  for various  $N$

It is fairly apparent that there is little evidence of erratic behaviour in the panel test statistic series. Rather, the test statistics seem to be approaching their respective probability limits under the alternative. In other words, it would appear that using suitably designed (i.e., robust to cross-sectional dependence) panel tests, with much higher power compared to standard unit roots, removes erraticism of the test statistics, and provides strong evidence in favour of PPP. Consequently, panel methods might enable us to solve the PPP puzzle [see Rogoff, 1996].

## 4 Conclusions

This paper has examined whether, in addition to standard unit root and cointegration tests, panel approaches also produce test statistics behaving erratically when applied to PPP. We have shown that if appropriate tests (which are robust to cross-sectional dependence and more powerful) are used, any evidence of erratic behaviour disappears, and strong empirical support is found for PPP. This suggests that power is the critical issue in testing PPP, rather than considering more complicated dynamic structures. Although nonlinear modelling also seems to be a very promising direction for future research on real exchange rates [see, e.g., Taylor and

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<sup>3</sup>The findings were very similar when the GDP deflator was used instead of the CPI series (they are not reported here for the sake of brevity).

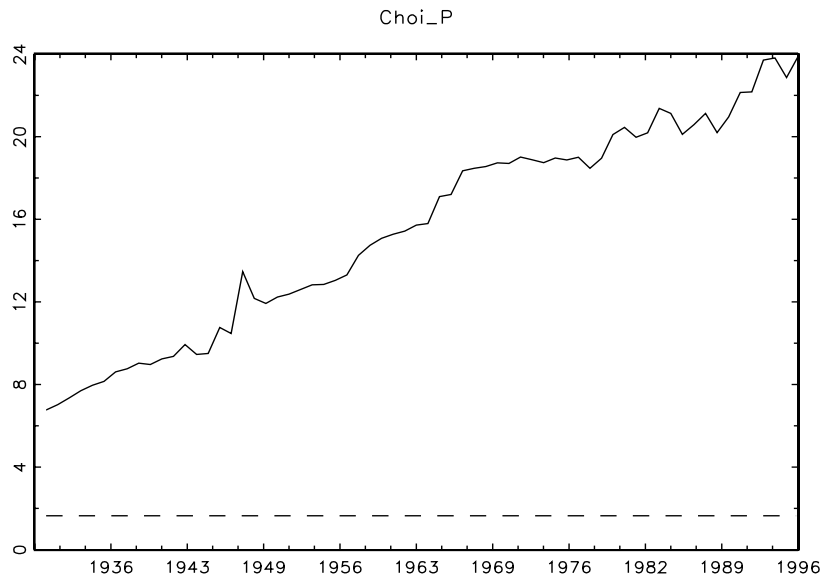


Figure 2: Test statistic series for  $P_m$  for various  $N$

Peel, 2000], addressing the power problem is confirmed here to be of crucial importance, and panel approaches appear to be able to provide conclusive evidence of the adequacy of PPP as a theory of real exchange rate determination, provided sufficiently long runs of data are used and cross-sectional dependence is tackled appropriately. It might be possible, after all, to settle the PPP debate exploiting recent advances in panel data econometrics.

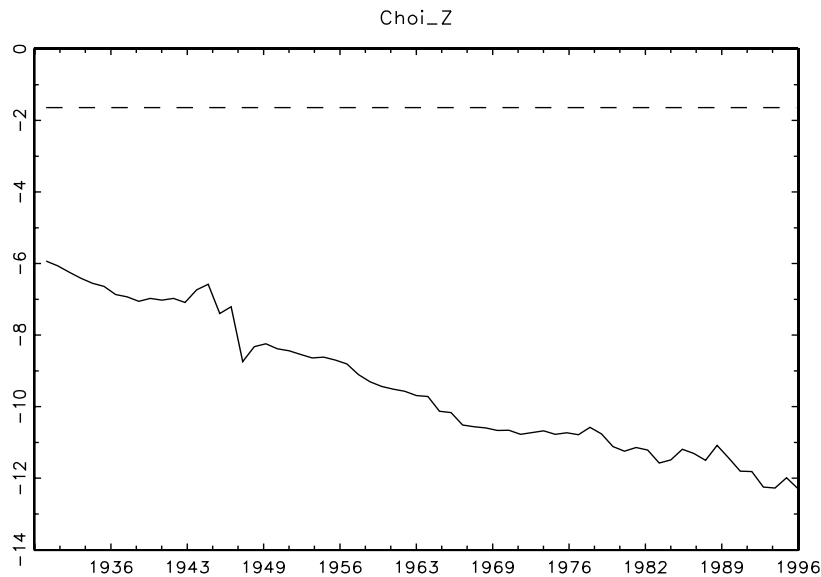


Figure 3: Test statistic series for  $Z$  for various  $N$

## References

- Caporale, G. M. and M. Cerrato: 2006, 'Panel data tests of PPP: a critical overview'. *Applied Financial Economics* **16**(1–2), 73–91.
- Caporale, G. M. and C. Hanck: 2006, 'Cointegration Tests of PPP: Do They Also Exhibit Erratic Behaviour?'. *CESifo Working Paper* (1811).
- Caporale, G. M., N. Pittis, and P. Sakellis: 2003, 'Testing for PPP: the erratic behaviour of unit root tests'. *Economics Letters* **80**, 277–284.
- Choi, I.: 2001, 'Unit Root Tests for Panel Data'. *Journal of International Money and Finance* **20**(2), 249–272.
- Choi, I.: 2006, 'Combination Unit Root Tests for Cross-Sectionally Correlated Panels'. In: D. Corbae, S. N. Durlauf, and B. E. Hansen (eds.): *Econometric Theory and Practice: Frontiers of Analysis and Applied Research: Essays in Honor of Peter C. B. Phillips*. Cambridge University Press, Chapt. 11, pp. 311–333.
- Elliott, G., T. Rothenberg, and J. Stock: 1996, 'Efficient Tests for an Autoregressive Unit Root'. *Econometrica* **64**(4), 813–836.
- Froot, K. A. and K. Rogoff: 1995, 'Perspectives on PPP and Long-Run Real Exchange Rates'. In: G. Grossman and K. Rogoff (eds.): *The Handbook of International Economics*, Vol. 3.



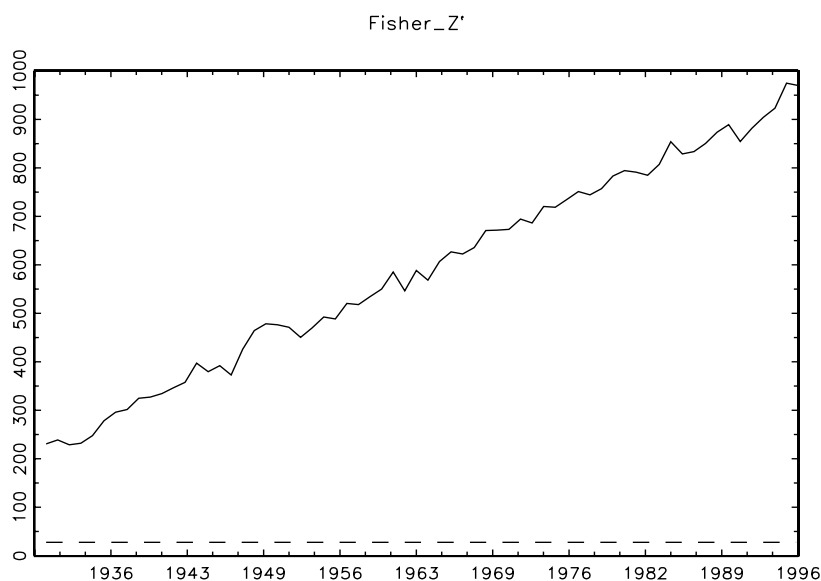


Figure 4: Test statistic series for  $P$  for various  $N$

Amsterdam: Elsevier, Chapt. 32, pp. 1647–1688.

Hlouskova, J. and M. Wagner: 2006, ‘The Performance of Panel Unit Root and Stationarity Tests: Results from a Large Scale Simulation Study’. *Econometric Reviews* **25**(1), 85–116.

Im, S., M. Pesaran, and Y. Shin: 2003, ‘Testing for Unit Roots in Heterogeneous Panels’. *Journal of Econometrics* **115**, 53–74.

Levin, A., C. Lin, and C.-S. J. Chu: 2002, ‘Unit Root Tests in Panel Data: Asymptotic and Finite-Sample Properties’. *Journal of Econometrics* **108**(1), 1–24.

MacKinnon, J. G.: 1994, ‘Approximate Asymptotic Distribution Functions for Unit-Root and Cointegration Tests’. *Journal of Business & Economic Statistics* **12**(2), 167–176.

MacKinnon, J. G.: 1996, ‘Numerical Distribution Functions for Unit Root and Cointegration Tests’. *Journal of Applied Econometrics* **11**(6), 601–618.

Maddala, G. and S. Wu: 1999, ‘A Comparative Study of Unit Root Tests with Panel Data and a New Simple Test’. *Oxford Bulletin of Economics and Statistics* **61**(S1), 631–652.

O’Connell, P.: 1998, ‘The Overvaluation of Purchasing Power Parity’. *Journal of International Economics* **44**(1), 1–19.

Papell, D. H.: 1997, ‘Searching for stationarity: purchasing power parity under the current float’. *Journal of International Economics* **43**, 313–332.

- Papell, D. H.: 2002, 'The great appreciation, the great depreciation, and the purchasing power parity hypothesis'. *Journal of International Economics* **57**(1), 51–82.
- Phillips, P. and D. Sul: 2003, 'Dynamic Panel Estimation and Homogeneity Testing under Cross Section Dependence'. *Econometrics Journal* **6**, 217–259.
- Rogoff, K.: 1996, 'The Purchasing Power Parity Puzzle'. *Journal of Economic Literature* **34**(2), 647–668.
- Taylor, A. and M. P. Taylor: 2004, 'The Purchasing Power Debate'. *Journal of Economic Perspectives* **18**(4), 135–158.
- Taylor, A. M.: 2002, 'A Century of Purchasing-Power Parity'. *The Review of Economics and Statistics* **84**(1), 139–150.
- Taylor, M. P. and D. Peel: 2000, 'Nonlinear Adjustment, Long-Run Equilibrium and Exchange Rate Fundamentals'. *Journal of International Money and Finance* **19**, 33–53.
- Taylor, M. P., D. Peel, and L. Sarnoc: 2001, 'Non-linear mean reversion in real exchange rates: towards a solution to the purchasing power parity puzzles'. *International Economic Review* **42**, 1015–1042.
- Wu, Y.: 1996, 'Are Real Exchange Rates Stationary? Evidence from a Panel Data Test'. *Journal of Money, Credit and Banking* **28**(1), 54–63.