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Development of Mathematical Models, Simulating Vibration Control of Tracked Vehicle Weapon Dynamics

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ABSTRACT

Focuses upon development of the mathematical model, simulating the tracked vehicle weapon dynamics, integrated over a half car platform. Governing differential equations have been formulated for the weapon system using state space approach, simulating the elevation dynamics over a half vehicle chassis, and coded using Matlab. The elevation model of the weapon comprises 3 degrees of freedom, arising from the rotational dynamics of the drive, breech and muzzle, which has sequentially been coupled to the half car model. Thereafter, the backstepping, LQR and PID control techniques have been derived and incorporated into the state space matrix for the coupled dynamics model, in which the control parameters have been arrived at through various iterations. Comparative weapon dynamics response studies have been carried out between that obtained from the above control strategies and the passive model, over standard terrain conditions at specified speeds. The above study would form a very useful framework for implementation of alternate control strategies for weapon stabilisation in the full tracked vehicle.

Keywords: Weapon dynamics; Backstepping; PID; LQR; Ride dynamics

1. INTRODUCTION

The role of weapon of Main Battle Tank (MBT) is to fire at the target accurately irrespective of the motion of the tank or the target. The objective is that the weapon must point the target at all times in order to do so. Tanks are meant for cross-country terrains. These rough terrains act as the peaks and valleys of random nature. The weapon disturbances occur mainly due to these terrains. This causes the deviation of gun from the line of eyesight and the target flees away without being hit.

Wani[1], *et al.*, describes the weapon dynamics for azimuth and elevation drives for which the mathematical modelling were presented for the same. The responses for azimuth, elevation and coupled azimuth and elevation dynamics were carried out to study the gun behaviour for torque and Aberdeen Proving Ground disturbances. Using this as base for present work, a control algorithm has been developed for weapon dynamics. Ahmed[2], *et al.,* depicted the study to obtain a mathematical model for the passive and active suspensions systems for quarter car model and construct an active suspension control for a quarter car model subjected to excitation from a road profile using PID controller. Comparison between passive and active suspensions system were performed using road profile. The performance of the controller was compared with PID controller, and the passive suspension system. This paper gives the proper understanding for selection of the PID gains for half car as well as weapon system. Nagarkar[3], *et al.,* paper showed the analyses between passive suspension system and active suspension system using a linear quadratic regulator (LQR) controller using linear quarter-car model. The performance of the LQR controller was compared with the passive suspension system. The simulation results showed that the LQR controller improves vehicle ride comfort. LQR control technique has been derived and implemented for the present work. Meng[4], *et al.,* describes a method to optimise the weight matrix of the LQR controller by using the simulated annealing algorithm which utilises the random searching characteristics of the algorithm to optimise the weight matrices with the target function of suspension performance indexes. This method improves the design efficiency and control performance of the LQR control, and solves the problem of the LQR controller when defining the weight matrices. A simulation provided for vehicle active chassis control, suggested that the active suspension using LQR optimised by the genetic algorithm compared to the chassis controlled by the normal LQR and the passive one, showed better performance. Lin & Huang[5], developed a novel nonlinear backstepping design for the control of half-car active suspension systems to improve the inherent trade-off between ride quality and suspension travel and showed the comparative simulation between passive and non-linear backstepping control. It also suggested that to avoid damaging vehicle components and generating more passenger discomfort, the controller must be capable of preventing the suspension from hitting it

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travel limits. Controller designed with nonlinear filter showed the potential to achieve these conflicting control objectives by setting suspension to be soft when suspension travel is small, and it is adjusted to become stiff as it approaches the travel limits. Purdy[6], showed that stabilising an out of balance gun on a moving platform (tank or ship) was achieved using models of a balanced and out of balance gun on a main battle tank which was very difficult or impossible to achieve. The models of the guns used, included the effect of non-linear friction and out of balance. To improve the stabilisation of the out of balance gun, trunnion vertical acceleration feedforward was used. This is used to develop the elevation dynamics for WCS and later incorporating the control technique. Mathworks[7] has described the theoretical background behind Matlab programming in an elaborate manner.

The following points must be obeyed with respect to weapon control system (WCS):

(i) It must respond quickly
(ii) It must hit the target accurately irrespective of whether the target or tank in motion
(iii) It must be capable of withstanding any terrain disturbances.

In order to achieve the above mentioned points, the controllers namely, PID; Linear Quadratic Regulator (LQR) and Backstepping control techniques have been studied and implemented for WCS. The control can be achieved by adopting two strategies as follows:

- Indirect Control
- Direct control

1.1 Indirect Control

The suspension system of tank at present consists of the nitrogen gas and oil, which behave non-linearly. Implementation of control at suspension system minimises the terrain disturbances, which finally leads to minimisation of gun disturbance. The ride dynamics of the vehicle would be controlled which in turn takes care of the weapon stabilisation. In the present paper, a half car model has been considered to demonstrate the control over suspension system.

1.2 Direct Control

The control technique is implemented in the weapon itself to minimise the gun oscillations. In the present paper, elevation dynamics of the WCS has been considered. The location of controller is assumed to be between breech and the elevation drive.

The simulation is performed in MATLAB by prior formulation of the governing differential equations of motion for the half car, which in turn is sequentially coupled with the elevation dynamics, and solved using state space approach.

2. BACKGROUND

2.1 Suspension System

The half car is considered to have linear stiffness and damping characteristics, comprising totally 4 degree of freedom as shown in Fig. 1.

where y_{01} and y_{02} are the base excitations from the terrain.

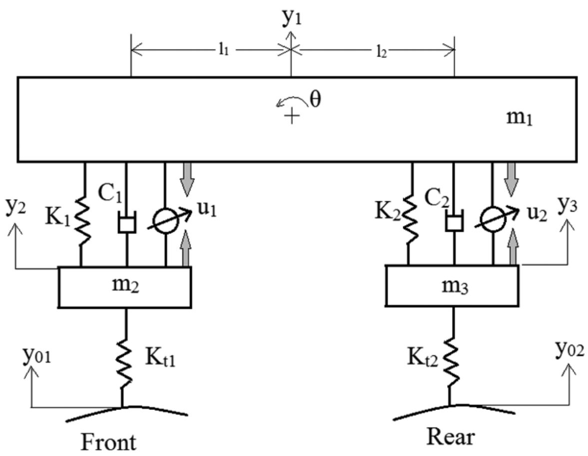

Figure 1. Half car schematic model.

2.2 Weapon System

In the present math model, the barrel is divided into 2 primary sections- muzzle (barrel front portion) and the breech (barrel rear portion), as shown in Fig. 2. The breech section in the weapon model is connected to the turret through the trunnion (which is considered to be a hinge joint with a certain amount of torsional damping between the breech and trunnion, which results from joint frictional effects), as shown in Fig. 2. The mass of the breech block and recoil systems is lumped to the breech section mass. The mathematical model for the gun barrel is formulated based on the lumped parameter flexible beam technique, in which the barrel length is divided into finite beam sections (for simplicity here 2 sections of the barrel are taken- muzzle and breech). Since the muzzle and breech are structurally coupled to each other, therefore a structural torsional spring and damper with appropriate torsional stiffness and damping is implemented between the muzzle and breech (as shown in Fig. 4). So, the barrel is not considered to be rigid. Considering the above, the muzzle and breech sections of the barrel have independent degrees of freedom.

The half car bounce and pitch responses serve as the input to the weapon dynamics model, in which the vibrations are transmitted to the muzzle through the breech by means of

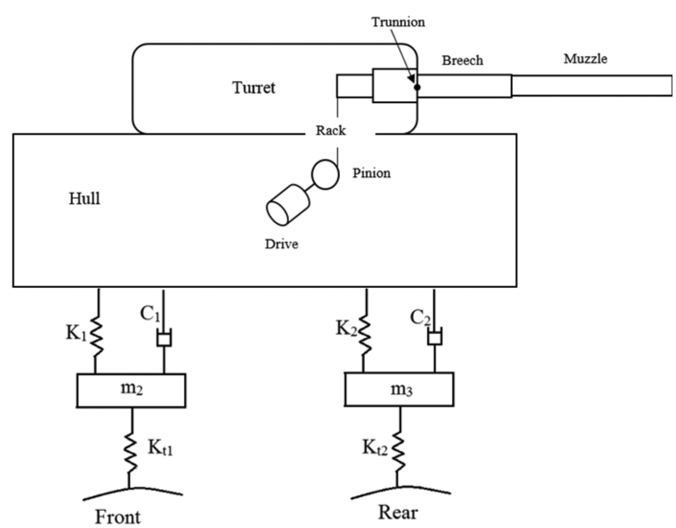

Figure 2. Gun Barrel mounted on half car suspension model.

structural coupling. The reference to the model has been taken from Purdy[6], based on which further control techniques have been implemented.

The elevation dynamics comprise 3 degree of freedom, arising from the angular displacements of breech, muzzle and drive. The drive in this case is an electric motor, which is having a rotational degree of freedom. This provides the required torque to elevate and depress the gun barrel to the desired angle through a rack and pinion arrangement, as shown in Fig. 2. The gun breech and muzzle, by virtue of being discrete beam sections have both angular and vertical degrees of freedom. The vertical degrees of freedom are eliminated by equations of constraint as described later in the paper.

Since for future, an All-Electric drive is being considered for gun control, therefore, the present model is accordingly developed. Figure 3 shows the elevation dynamics dimensions.

The rest of the paper has been arranged as; Mathematical model, control technique algorithms, Simulation results and comparison.

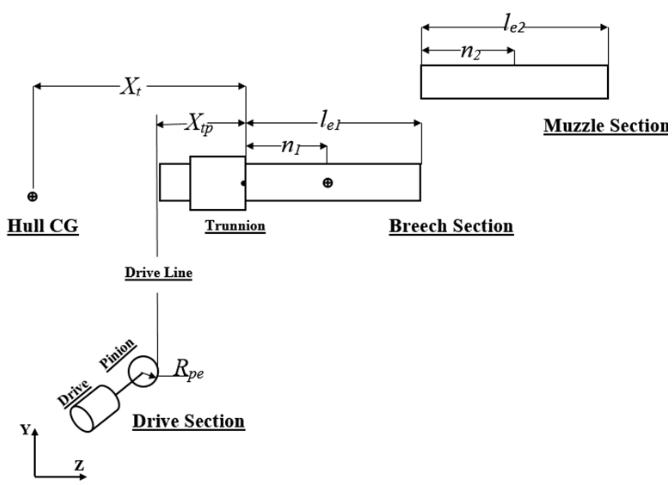

Figure 3. Elevation dynamics dimensions.

3. MATHEMATICAL MODELS

3.1 Suspension System

The suspension system for half car has been shown in Fig. 1.

The control forces u_1 and u_2 are applied at the front and rear sides, respectively see Appendix A.

The governing equation of motion for bounce is given by Eqn (1):

$$\begin{aligned} &m_1\ddot{y}_1 + K_1(y_1 - y_2 - l_1\theta) + K_2(y_1 - y_3 + l_2\theta) \\ &+C_1(\dot{y}_1 - \dot{y}_2 - l_1\dot{\theta}) + C_2(\dot{y}_1 - \dot{y}_3 + l_2\dot{\theta}) + u_1 + u_2 = 0 \end{aligned} \tag{1}$$

The pitch dynamics is described by Eqn (2):

$$\begin{aligned} &I\ddot{\theta} - K_1l_1(y_1 - y_2 - l_1\theta) - C_1l_1(\dot{y}_1 - \dot{y}_2 - l_1\dot{\theta}) \\ &+K_2l_2(y_1 - y_3 + l_2\theta) + C_2l_2(\dot{y}_1 - \dot{y}_3 + l_2\dot{\theta}) - u_1l_1 + u_2l_2 = 0 \end{aligned} \tag{2}$$

The governing equations of motion for unsprung mass are given in Eqns (3) and (4).

$$m_2\ddot{y}_2 + K_1(y_2 - y_1 + l_1\theta) + K_{t1}(y_2 - y_{01}) + C_1(\dot{y}_2 - \dot{y}_1 + l_1\dot{\theta}) - u_1 = 0 \tag{3}$$

$$m_3\ddot{y}_3 + K_2(y_3 - y_1 - l_2\theta) + K_{t2}(y_3 - y_{02}) + C_2(\dot{y}_3 - \dot{y}_1 - l_2\dot{\theta}) - u_2 = 0 \tag{4}$$

The Eqns (1) to (4) are converted to state-space form. The state vectors are derived for these 4 DOF system.

$$\begin{bmatrix} y_1 \\ \dot{y}_1 \\ \theta \\ \dot{\theta} \\ y_2 \\ \dot{y}_2 \\ y_3 \\ \dot{y}_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix}$$

Thus, the state equations are given in Eqns from (5) to (8):

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{1}{m_1}\left\{\begin{aligned} &K_1(x_1 - x_5 - l_1x_3) + K_2(x_1 - x_7 + l_2x_3) \\ &+C_1(x_2 - x_6 - l_1x_4) + C_2(x_2 - x_8 + l_2x_4) + u_1 + u_2 \end{aligned}\right\} \tag{5}$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = \frac{1}{I}\left\{\begin{aligned} &K_1l_1(x_1 - x_5 - l_1x_3) + C_1l_1(x_2 - x_6 - l_1x_4) \\ &-K_2l_2(x_1 - x_7 + l_2x_3) - C_2l_2(x_2 - x_8 + l_2x_4) + u_1l_1 - u_2l_2 \end{aligned}\right\} \tag{6}$$

$$\dot{x}_5 = x_6$$

$$\dot{x}_6 = -\frac{1}{m_2}\left\{K_1(x_5 - x_1 + l_1x_3) + K_{t1}(x_5 - y_{01}) + C_1(x_6 - x_2 + l_1x_4) - u_1\right\} \tag{7}$$

$$\dot{x}_7 = x_8$$

$$\dot{x}_8 = -\frac{1}{m_3}\left\{K_2(x_7 - x_1 - l_2x_3) + K_{t2}(x_7 - y_{02}) + C_2(x_8 - x_2 - l_2x_4) - u_2\right\} \tag{8}$$

The control algorithms are implemented into the Eqns (5) to (8), as discussed later in the paper.

3.2 Elevation Dynamics

The gun elevation dynamics comprise of 3 DOF, namely angular displacements of the drive, breech and muzzle, which are in turn sequentially coupled to the 4 DOF, pertaining to the ride model of the half car.

Equations of constraint for elevation dynamics-

$$\left.\begin{aligned} y_t &= y + \theta X_t \\ y_t &= y_1 - \theta_1 n_1 \\ y_t &= y_2 - \theta_2 n_2 - \theta_1 l_{e1} \end{aligned}\right\} \tag{9}$$

Rotational dynamics for the elevation drive-

$$I_{de}\ddot{\theta}_{de} - T_{de} + C_{de}\dot{\theta}_{de} + K_{de}\left(\theta_{de}R_{pe} + (\theta - \theta_1)X_{tp}\right)R_{pe} + uR_{pe} = 0 \tag{10}$$

Vertical and rotational motion of the Breech section for elevation dynamics-

$$m_4\ddot{y}_1 - f_y + K_{de}\left(\theta_{de}R_{pe} + \left(\theta-\theta_1\right)X_{tp}\right) + f_{12} + u = 0$$

$$I_1\ddot{\theta}_1 + C_{1p}\left(\dot{\theta}_1 - \dot{\theta}\right) + f_{12}\left(l_{e1} - n_1\right) + f_y n_1 - K_{12}\left(\theta_2 - \theta_1\right) - C_{12}\left(\dot{\theta}_2 - \dot{\theta}_1\right)$$
$$-K_{de}\left(\theta_{de}R_{pe} + \left(\theta-\theta_1\right)X_{tp}\right)\left(X_{tp}+n_1\right) - u\left(X_{tp}+n_1\right) = 0 \tag{11}$$

Figure 4 shows the free body diagram for elevation dynamics.

Vertical and rotational motion of the muzzle section for elevation dynamics -

$$m_5\ddot{y}_2 - f_{12} = 0 \tag{12}$$

$$I_2\ddot{\theta}_2 + f_{12}n_2 + K_{12}\left(\theta_2-\theta_1\right) + C_{12}\left(\dot{\theta}_2 - \dot{\theta}_1\right) = 0 \tag{13}$$

The state-space equation are derived for elevation dynamics in a similar way as that of suspension model.

Assuming,

$$\begin{bmatrix}\theta_2\\ \dot{\theta}_2\\ \theta_1\\ \dot{\theta}_1\\ \theta_{de}\\ \dot{\theta}_{de}\end{bmatrix} = \begin{bmatrix}x_9\\ x_{10}\\ x_{11}\\ x_{12}\\ x_{13}\\ x_{14}\end{bmatrix}$$

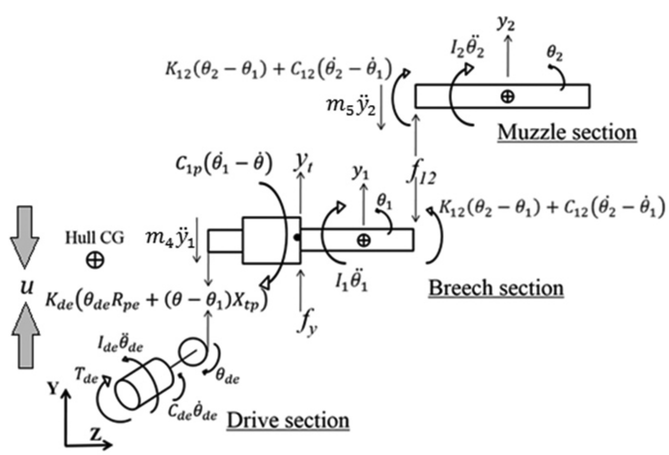

Figure 4. Elevation drive for WCS.

4. PID CONTROL DESIGN

PID is the most widely applied controller. The objective function can be controlled by varying the gains values K_P, K_I and K_D. If 'u' is the control variable, $y(t)$ and $f(t)$ are the actual and desirable output, then the tracking error is $e(t)=y(t)-f(t)$. The PID control law is given by:

$$u(t) = K_P e(t) + K_I\int e(t)dt + K_D\frac{d}{dt}e(t)$$

The proportional term of PID controller usually gives the controlled output without other terms. Integral term rejects the disturbance and derivative term provides the damping[2].

4.1 Suspension Model

The difference between the sprung and unsprung mass velocities is considered as the control parameter for front side of half car.

$$e(t) = \dot{y} - \dot{y}_1 - l_1\dot{\theta} = x_2 - x_6 - l_1x_4 \tag{14}$$

The control law is obtained as:

$$u_1 = K_{P1}\left(x_2 - x_6 - l_1x_4\right) + K_{I1}\left(x_1 - x_5 - l_1x_3\right) + K_{D1}\left(\ddot{x}_2 - \ddot{x}_6 - l_1\ddot{x}_4\right) \tag{15}$$

Similar steps are followed to obtain the control parameter for rear side.

$$u_2 = K_{P2}\left(x_2 - x_8 - l_1x_4\right) + K_{I2}\left(x_1 - x_7 - l_1x_3\right) + K_{D2}\left(\ddot{x}_2 - \ddot{x}_8 - l_1\ddot{x}_4\right) \tag{16}$$

4.2 Elevation Dynamics

The control parameter is the breech velocity, which is to be regulated.

$$e(t) = \dot{\theta}_1X_{tp} \tag{17}$$

The control equation is given by Eqn (18):

$$u = K_Px_4X_{tp} + K_Ix_1X_{tp} + K_D\dot{x}_4X_{tp} \tag{18}$$

5. LQR CONTROL DESIGN

Considering the state-space form as:

$$\dot{x} = Ax + Bu$$

The control vector 'u' in case of LQR control is decided based upon the quadratic performance index. The performance index is given by:

$$J_{LQR} = \int L\left(x,u\right)dt$$

where $L(x,u)$ is the quadratic or Hermitian function such that,

$$u = -L_{LQR}x$$

$K_{LQR}$is the LQR gain matrix. L_{LQR} leads to linear control.

The linear quadratic performance index is expressed as:

$$J_{LQR} = \int\left(x^TQx + u^TRu + 2x^TNu\right)dt \tag{19}$$

where, the first term on the right hand side signifies the difference in the error between initial and final state and second term signifies the energy expenditure on control signal[3].

The control 'u' is said to be optimal for:

$$u = -K_{LQR}x \tag{20}$$

where, $K_{LQR} = R^{-1}B^TP$

The K matrix can be obtained from the Algebraic Riccati matrix formula:

$$PA + A^TP - \left(PB+N\right)R^{-1}\left(B^TP + N^T\right) + Q = 0 \tag{21}$$

The K matrix can be obtained by using following code through MATLAB[7]:

$$\left[K,P\right] = LQR\left(A,B,Q,R,N\right)$$

6. BACKSTEPPING CONTROL DESIGN

The backstepping control design consists of two steps.

Elevation dynamics

We choose $z_1 = \theta_1 X_{tp}$ to regulate the breech displacement to minimise the muzzle displacement.

Step 1: The derivative of z_1 is computed as:

$$\dot{z}_1 = \dot{x}_{11}X_{tp} = x_{12}X_{tp} \tag{22}$$

$x_{12}$is used as the first virtual variable for which we choose the stabilising function as:

$$\alpha_1 = -c_1z_1 \tag{23}$$

where, c_1 is a positive design constant. The corresponding error state variable is defined as:

$z_2 = x_{12}X_{tp} - \alpha_1$, and the resulting error equation is

$\dot{z}_1 = -c_1 z_1 + z_2$

Step 2: The derivative of z_2 is given by:

$$\begin{aligned}
\dot{z}_2 &= \dot{x}_{12}X_{tp} - \dot{\alpha}_1 \\
&= -\frac{X_{tp}}{\left(I_1 + m_5 l_{e1}^2 + m_4 n_1^2\right)}\{m_5 n_2 l_{e1}\dot{x}_{10} + C_{1p}x_{12} - C_{1p}\dot{\theta} + \\
&\left(m_5 l_{e1} + m_4 n_1\right)\left(\ddot{y} + \ddot{\theta}X_t\right) - K_{12}x_9 + K_{12}x_{11} - C_{12}x_{10} + C_{12}x_{12} \\
&-K_{de}\left(x_{13}R_{pe} + \left(\theta - x_{11}\right)X_{tp}\right)X_{tp} + uX_{tp}\} + c_1\left[-c_1 z_1 + z_2\right]
\end{aligned} \tag{24}$$

The actual control inputs occur in the Eqn (24) and therefore the control law is given as in Eqn (25).

$$\begin{aligned}
u &= -\frac{1}{X_{tp}}\{\left\{\left(c_1^2 - 1\right)X_{tp}x_{11} - \left(c_1 + c_2\right)\left(X_{tp}x_{12} + c_1 X_{tp}x_{11}\right)\right\} \\
&\frac{1}{X_{tp}}\left(I_1 + m_5 l_{e1}^2 + m_4 n_1^2\right) + m_5 n_2 l_{e1}\dot{x}_{10} + C_{1p}x_{12} - C_{1p}\dot{\theta} \\
&+\left(m_5 l_{e1} + m_4 n_1\right)\left(\ddot{y} + \ddot{\theta}X_t\right) - K_{12}x_9 + K_{12}x_{11} - C_{12}x_{10} \\
&+C_{12}x_{12} - K_{de}\left(x_{13}R_{pe} + \left(\theta - x_{11}\right)X_{tp}\right)X_{tp}\}
\end{aligned} \tag{25}$$

where, c_2 is the positive design constant.

To analyse the system stability, the Lyapunov criteria[5] is considered as-

$$V = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2$$

The derivative of the Lyapunov function derived is given by Eqn (26):

$$\begin{aligned}
\dot{V} &= z_1\dot{z}_1 + z_2\dot{z}_2 \\
&= z_1\left[-c_1 z_1\right] + z_2[-\frac{X_{tp}}{\left(I_1 + m_5 l_{e1}^2 + m_4 n_1^2\right)}\{m_5 n_2 l_{e1}\dot{x}_{10} + C_{1p}x_{12} \\
&-C_{1p}\dot{\theta} + \left(m_5 l_{e1} + m_4 n_1\right)\left(\ddot{y} + \ddot{\theta}X_t\right) - K_{12}x_9 + K_{12}x_{11} - C_{12}x_{10} \\
&+C_{12}x_{12} - K_{de}\left(x_{13}R_{pe} + \left(\theta - x_{11}\right)X_{tp}\right)X_{tp} + uX_{tp}\} \\
&+c_1\left[-c_1 z_1 + z_2\right] + z_1] \\
&= -c_1 z_1^2 - c_2 z_2^2
\end{aligned} \tag{26}$$

The Eqn (26) shows that the error system is Globally Exponentially Stable according to Lyapunov theorem.

Alternatively, the closed loop error system is given by-

$$\begin{bmatrix}\dot{z}_1 \\ \dot{z}_2\end{bmatrix} = \begin{bmatrix}-c_1 & 1 \\ -1 & -c_2\end{bmatrix}\begin{bmatrix}z_1 \\ z_2\end{bmatrix}$$

The above 2x2 matrix is Hurwitz, and so the error system has a globally exponentially stable equilibrium at $(z_1, z_2) = (0, 0)$. Therefore, the defined objective of breech displacement minimisation is achieved.

7. SIMULATION RESULTS AND COMPARISON

7.1 Suspension System

The passive and controlled half car model equations of motion have been formulated and solved in Matlab using Runge Kutta explicit solver. The road wheels are applied with base excitation with suitable time delay (as shown in Fig. 5), as it negotiates the Aberdeen Proving Ground (APG) terrain at a vehicle speed of 30 kmph. The bounce and pitch dynamics for the passive and controlled half car models are shown in Figs. 6 and 7, respectively. Table 1 in Appendix A provides parameters and values for half car suspension system.

7.2 Elevation Dynamics

Table 2. in Appendix A provides parameters and values for elevation dynamics system. Here, as discussed earlier, the passive half car model is integrated with the elevation dynamics model, in which the control techniques are implemented.

Prior to application of the base excitation (as shown in Fig. 5), the elevation dynamics model is applied with a suitable drive torque input (as shown in Fig. 8) in order to position the gun at the desired angle.

The natural frequencies (Hz) of the half car system are found to be 1.17, 1.44, 9.35 and 9.34. The bounce and pitch acceleration responses are shown in the frequency domain in Figs 9. and 10, respectively.

The bounce and pitch behaviour of the sprung mass serve as inputs to the elevation dynamics model of the weapon system.

The details of inputs to the elevation model are:

- Torque – 2 s to 4.15 s
- APG – 5.54 s to 15.34 s

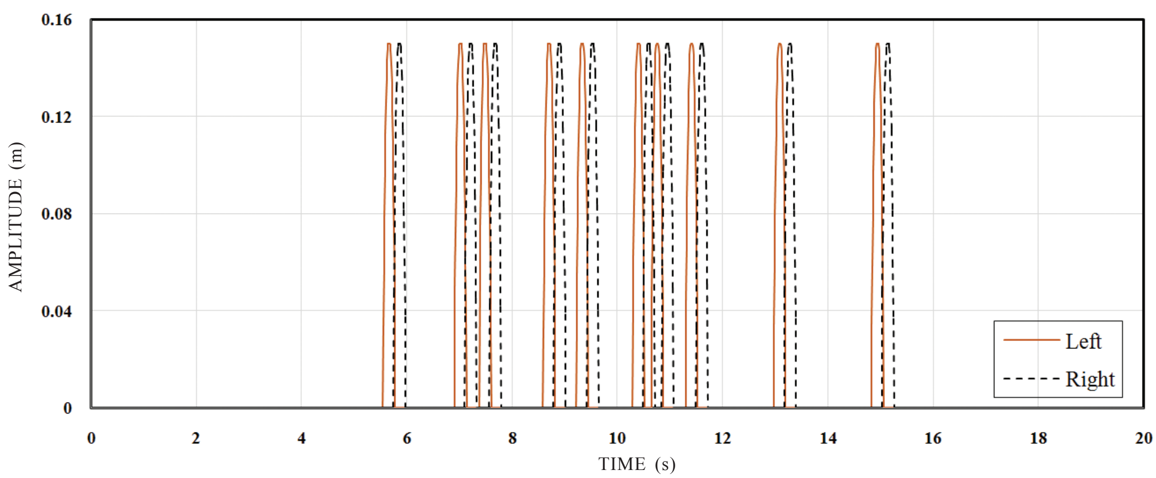

Figure 5. Base excitation of APG at speed 30 kmph.

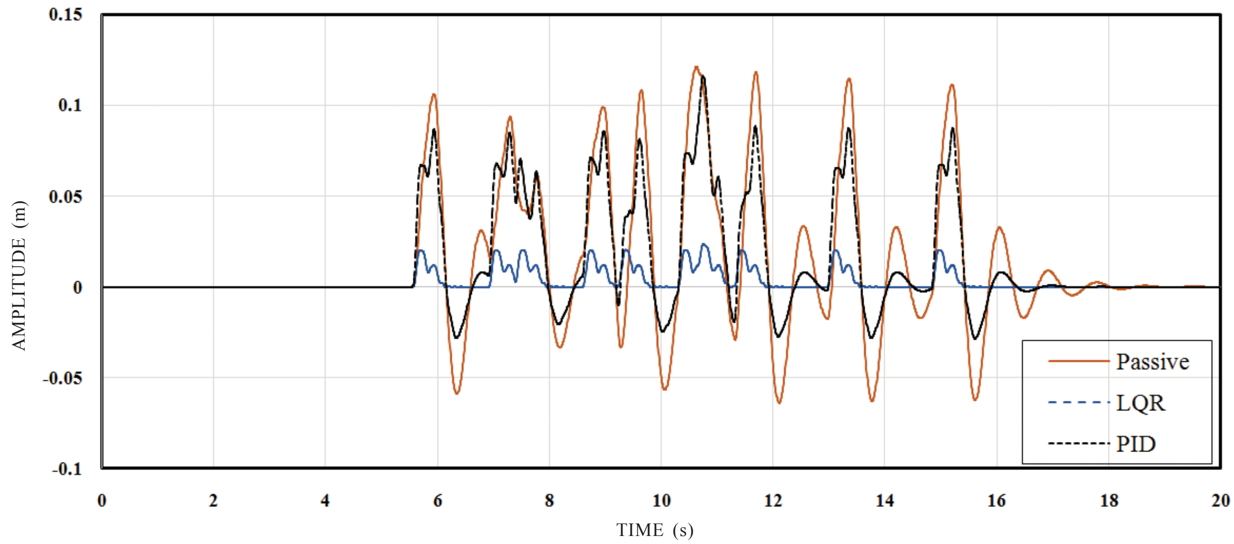

Figure 6. Bounce of sprung mass CG.

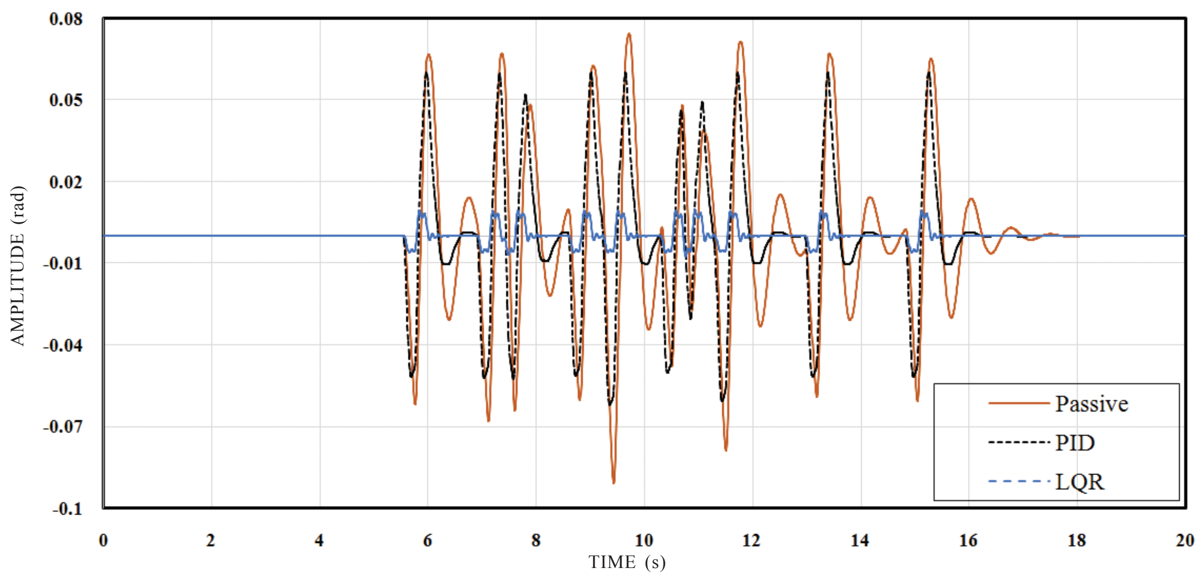

Figure 7. Pitch of sprung mass about CG.

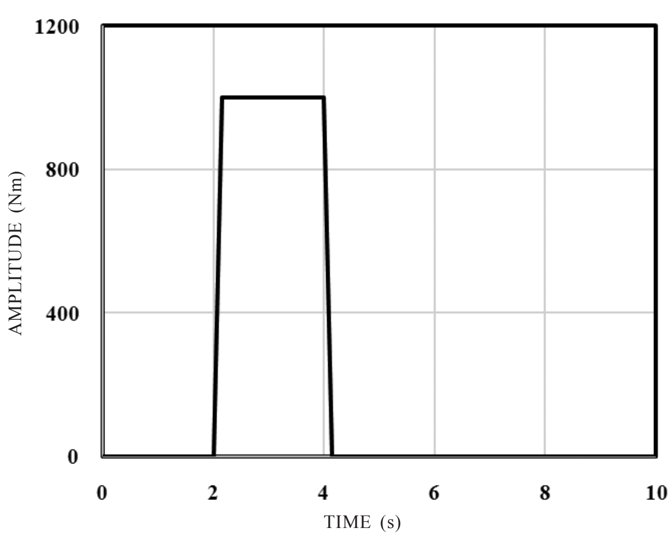

Figure 8. Plot of torque input to elevation drive section.

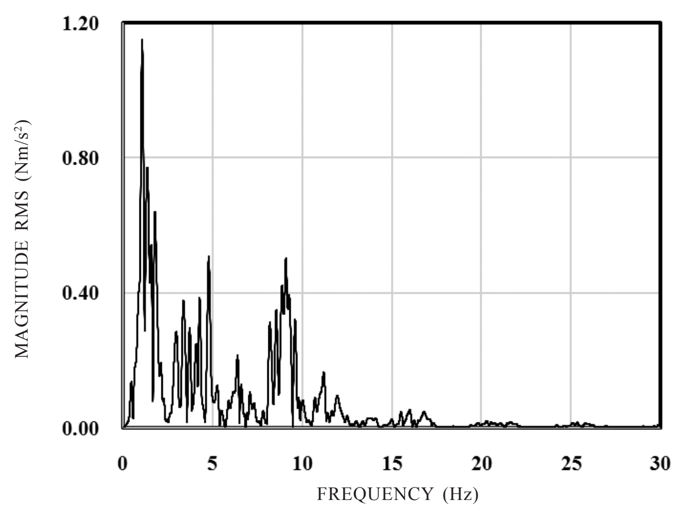

Figure 9. Plot for FFT of bounce acceleration of half car sprung mass at CG.

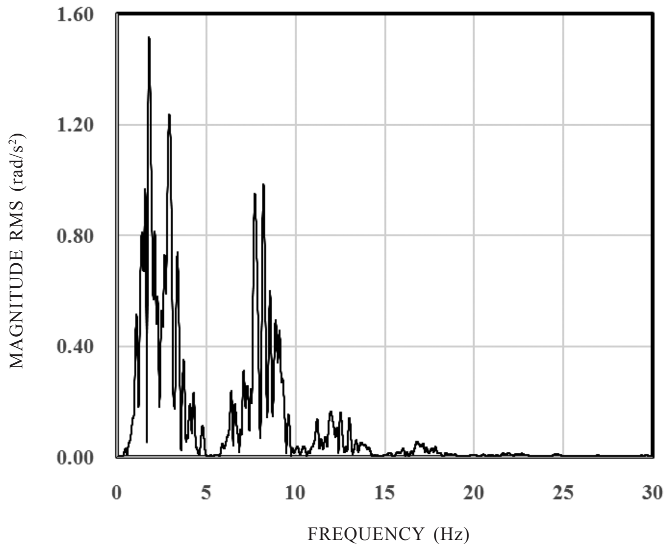

Figure 10. Plot for FFT of pitch angular acceleration of half car sprung mass about CG.

The comparative magnitude variation between the sprung mass pitch and breech angular displacements is shown in Fig. 11.

With respect to the weapon passive math model, the breech is connected to the turret through the trunnion, which is a hinge joint with a certain amount of torsional damping coefficient in between them (value taken from the Purdy[6]). Due to this, there is a very small amount of relative angular motion of the weapon platform with respect to the vehicle pitch, as observed from Fig. 11. With the same math model, as the value of the torsional damping is increased in between breech and trunnion, it would decrease the relative rotation of the weapon platform with respect to the vehicle motion, as shown in Fig. 12.

Incorporating higher value of torsional damping coefficient between breech and trunnion (300 kNms/rad), providing APG input to the half car model with vehicle moving at 30 kmph speed, it is observed from Fig. 12 that the relative magnitudes of pitch angular displacement response obtained from the half car ride model and breech angular displacement response are

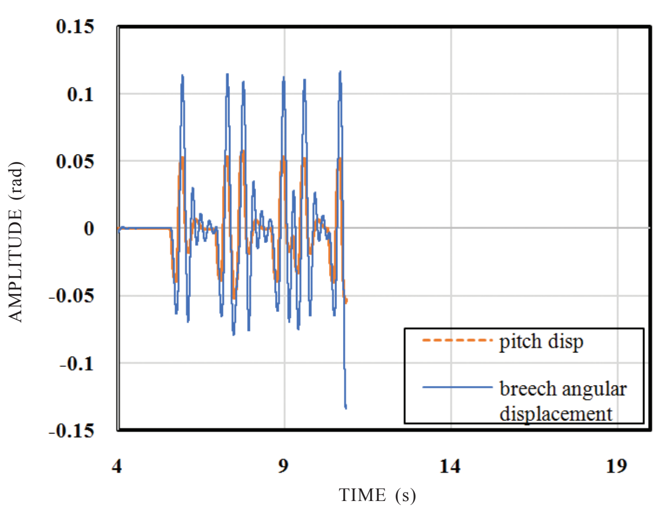

Figure 11. Half car pitch and breech angular displacement (with C_{1p}=1500 Nms/rad).

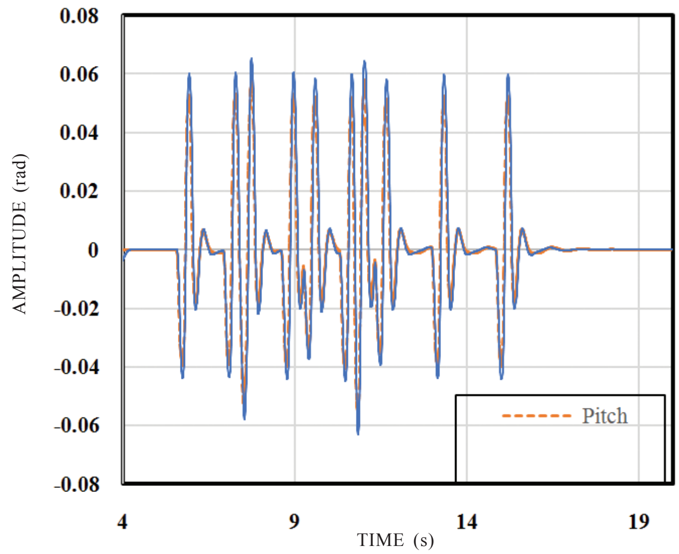

Figure 12. Comparison between half car pitch and breech angular displacement responses (with C_{1p}=300 kNms/rad).

closer compared to that obtained with relatively lesser value of torsional damping, as in Fig. 11.

In practice, the actual amount of torsional damping between the breech and trunnion may be determined from the experimental evaluation of energy loss per cycle during elevation and depression of the gun, which can be taken up in future to further fine-tune the inputs to the math model. Since the main focus of the paper is on implementing a suitable control technique for achieving the desired muzzle response over the standard terrain conditions, therefore, as of now, the values of torsional damping coefficient between the breech and trunnion is taken with reference from Purdy[6] (mentioned in Appendix A).

The integrated model for the weapon elevation and half car dynamics are solved in Matlab. The comparative muzzle and breech angular displacements after implementation of the control techniques are represented in Figs. 13 and 15, respectively.

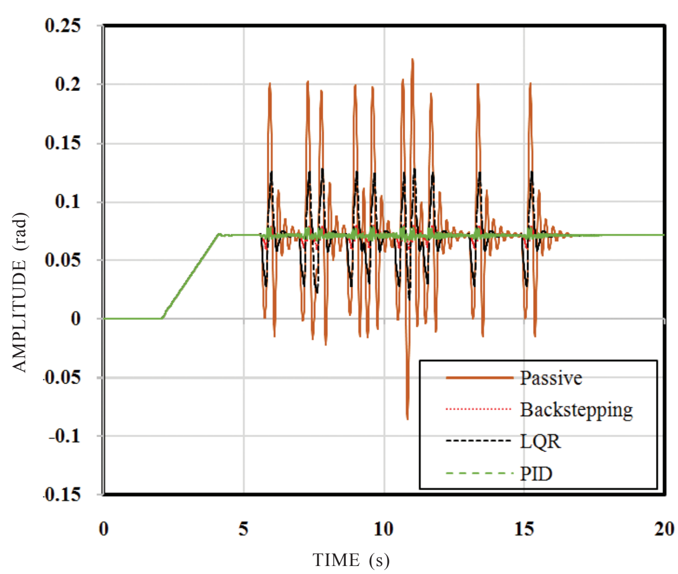

Figure 13. Muzzle angular displacement.

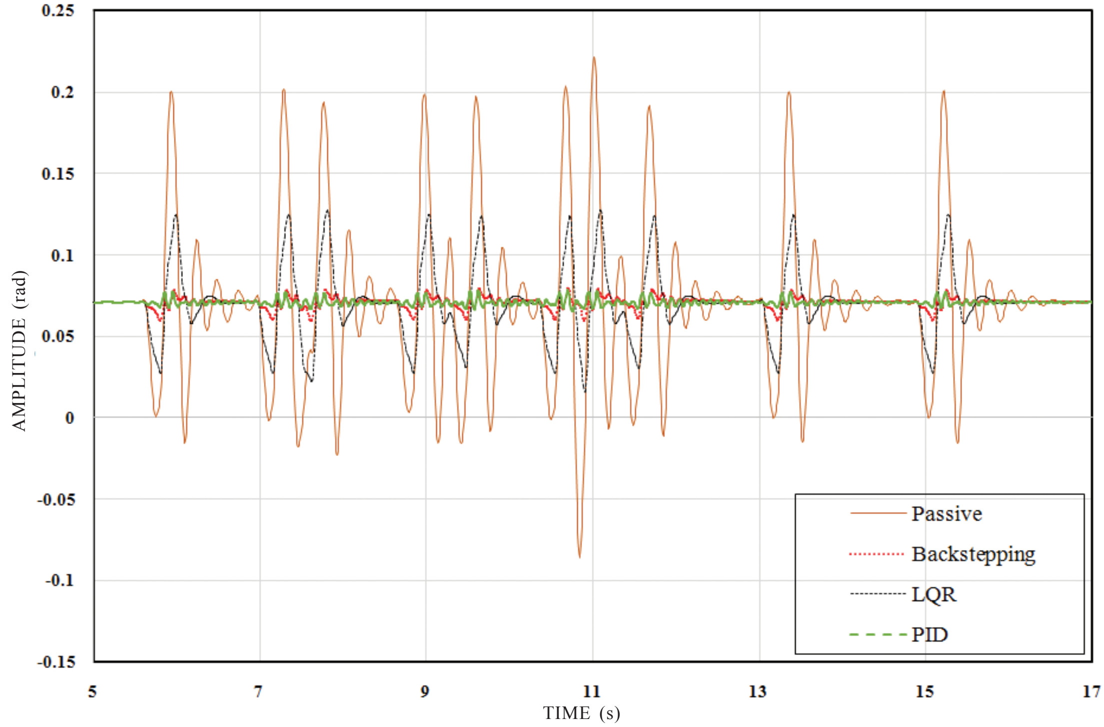

Figure 14. Enlarged time axis for muzzle angular displacement.

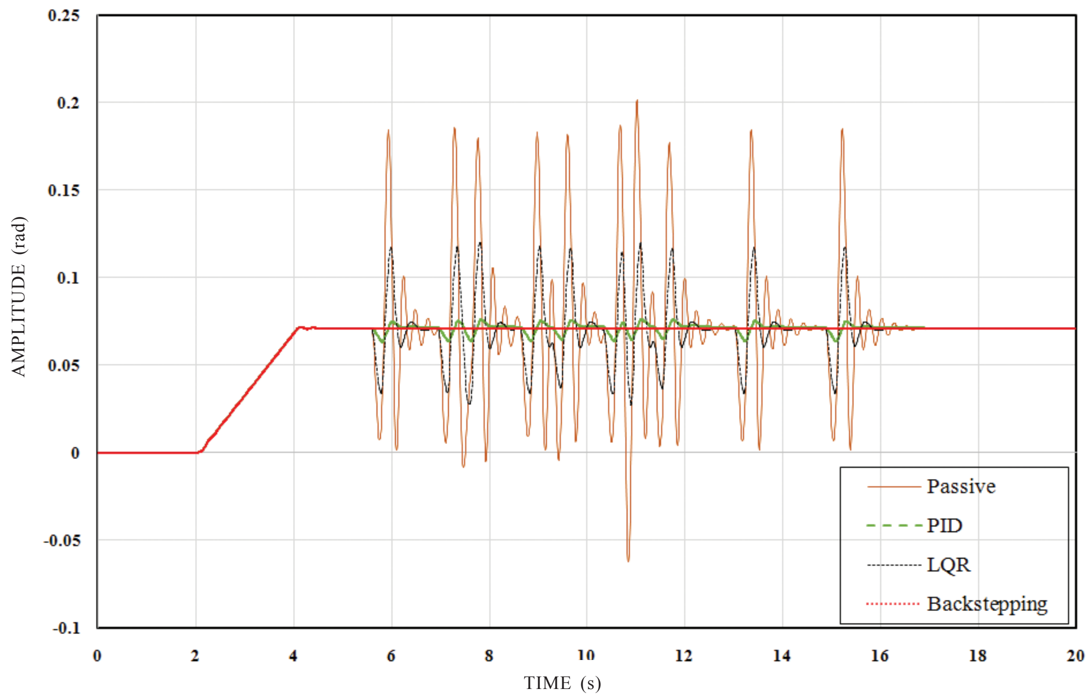

Figure 15. Breech angular displacement.

The enlarged plot of the Fig. 13 is shown in Fig. 14. The comparative muzzle angular acceleration responses from the passive and controlled system is shown in both time and frequency domains, as shown in Figs. 16 and 17, respectively.

8. CONCLUSIONS

The present work describes the comparative response analyses for the half car as well as the gun elevation dynamics with and without the implementation of the stated controller algorithms. The controller working range is set by the amount of weight (gain) values provided. By adopting certain tuning techniques such as Zinger-Nichols method in case of PID controller, tuned gain values may be used.

8.1 Suspension System

It is observed from the half car suspension system responses that, LQR control provides more damping characteristics spending lesser control effort than that of PID. This helps in estimating the amount of reduction of muzzle responses, when

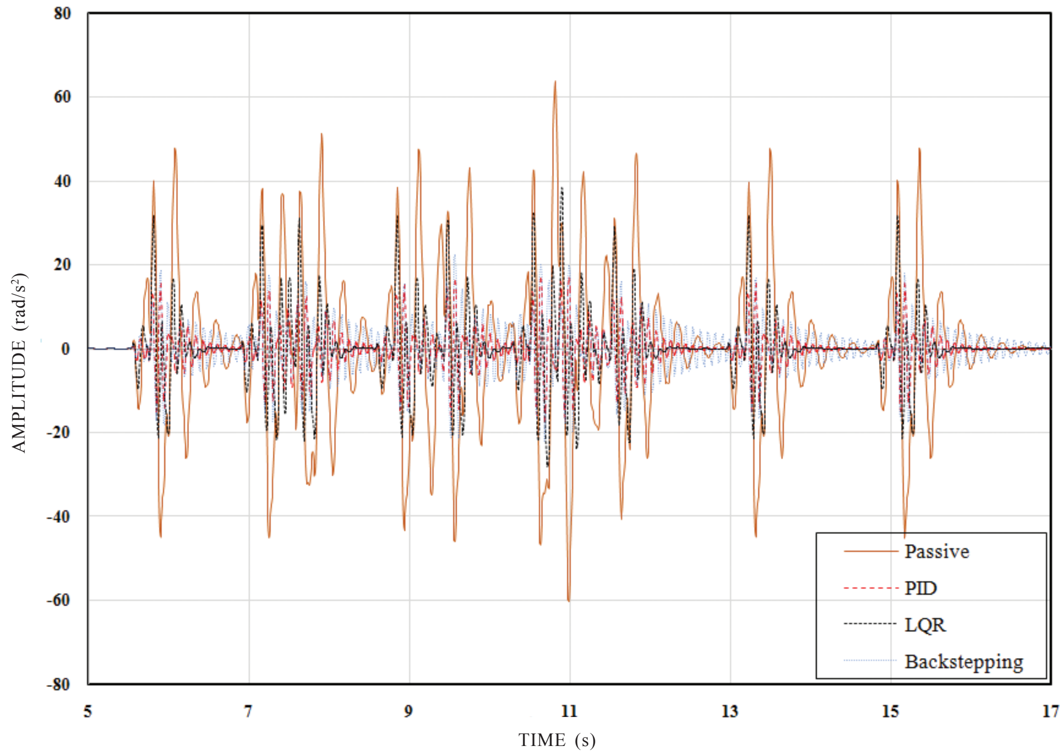

Figure 16. Plot of muzzle angular acceleration.

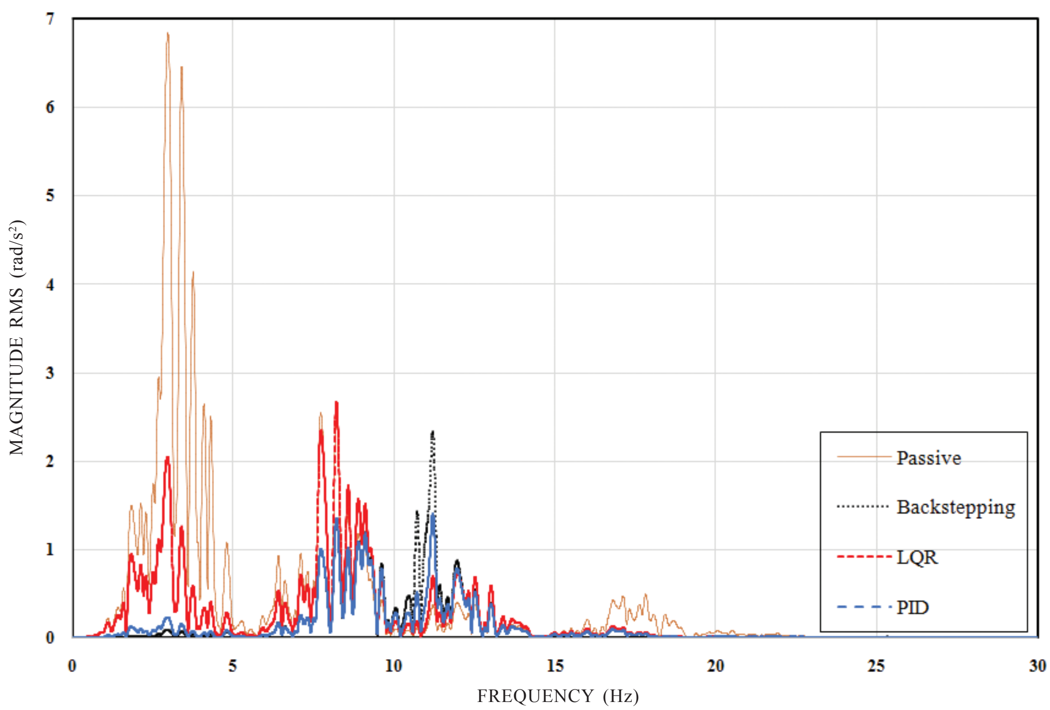

Figure 17. FFT plot with respect to muzzle angular acceleration.

the half car controlled model responses are fed as the input to weapon dynamics.

8.2 Weapon Dynamics

With respect to the weapon control system, it is observed that backstepping controller method provides lesser breech and muzzle angular displacement responses, compared to that of PID and LQR techniques. As a further study, it is observed that backstepping controller technique also requires lesser control effort for similar order of reduction, as compared to LQR and PID techniques. This study would form a very useful platform for implementation of alternate control techniques for the gun system. Furthermore, on the same lines the above control techniques can also be extended to non-linear suspension system, which would prove the purpose of weapon control.

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7. MATHWORKS; Help literature; Matlab R2013a.

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In the current study, he provided suggestions to the methodology and in the detailed discussion of the results of the current work.

Appendix A

Table 1. Parameters and corresponding values for half car suspension system

Half Car Suspension system Parameters		
m_1	Sprung mass of half car	575 kg
m_2 and m_3	Unsprung mass of wheel 1 and 2, respectively	60 kg
K_1	Spring stiffness between sprung mass and front wheel	16812 N/m
K_2	Spring stiffness between sprung mass and rear wheel	16812 N/m
C_1	Viscous damping between sprung mass and front wheel	1000 Ns/m
C_2	Viscous damping between sprung mass and rear wheel	1000 Ns/m
K_{t1}	Front tire wheel stiffness	190000 N/m
K_{t2}	Rear tire wheel stiffness	190000 N/m
l_1	Front side to sprung mass CG distance	2.2 m
l_2	Rear to sprung mass CG distance	2.6 m
I	Moment of Inertia of mass m1	769 kgm^2

Table 2. Parameters and values for elevation dynamics

Elevation dynamics system parameters		
X_t	Distance between Hull CG and Trunnion	1 m
X_{tp}	Horizontal distance between trunnion and center of pinion	0.75 m
K_{de}	Elevation driveline stiffness	6000kN/m
C_{de}	Elevation drive viscous damping coefficient	1.5 kNms/rad
R_{pe}	Pinion radius	0.04 m
I_{de}	Elevation drive inertia	0.5 kgm^2
C_{1p}	Trunnion torsional viscous damping coefficient	1.5 kNms/rad
m_4	Mass of cradle	2165 kg
I_1	Inertia of cradle	1090 kgm^2
l_{e1}	Length of gun barrel of breech section measured from trunnion	1.75 m
n_1	Distance between breech CG and trunnion	0.465 m
K_{12}	Torsional stiffness between the muzzle and breech sections (for elevation drive)	4000 kNm/rad
C_{12}	Viscous damping coefficient between the breech and muzzle section (for elevation drive)	2 kNms/rad
m_5	Mass of muzzle section	335 kg
I_2	Inertia of muzzle section	281 kgm^2
l_{e2}	Muzzle section length	3.25 m
n_2	Distance between muzzle CG and intersection point of muzzle and breech sections	1.319 m

(A) Suspension System

(i) PID gain values

$K_{P1} = 1000$
$K_{I1} = 5000$
$K_{D1} = 100$
$K_{P2} = 1000$
$K_{I2} = 0$
$K_{D2} = 0$

(ii) The LQR control gain matrices

$$Q_1 = diag(\begin{bmatrix} 1e10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix})$$

$$R_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

$$Q_2 = diag(\begin{bmatrix} 0 & 0 & 1e12 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(B) Elevation Dynamics System

(i) PID gain values

$K_P = 5000000$
$K_I = 0$
$K_D = 0$

(ii) LQR control gains are

$$Q = diag(\begin{bmatrix} 0 & 0 & 0 & 1e12 & 0 & 0 \end{bmatrix})$$

$$R = 0.1$$

(iii) Backstepping control design constants

$\epsilon = 0$
$c_1 = 25000$
$c_2 = 0$