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Influence of Initial State Errors on Perturbation Guidance Accuracy

Xu Zheng*, Wuxing Jing, and Changsheng Gao

Department of Aerospace Engineering, Harbin Institute of Technology, Harbin - 150 001, China
*E-mail: zhengxu_hit@hotmail.com

ABSTRACT

The inertial navigation system is aligned and leveled before the launch of a long-range vehicle. However, the initial state errors caused by the non-uniformity of the Earth can influence the parameters in flight dynamics, which will bring about serious uncertainty for the impact point of a long-range vehicle. Firstly, this paper analyses the influence mechanism of initial state errors on nominal trajectory, navigation trajectory and guidance trajectory. Then, a propagation model of engine-cutoff state deviation caused by initial state errors is derived under the condition of without-guidance. On this basis, an accuracy analytical solution of initial state errors on perturbation guidance is finally proposed to obtain the real impact-point of the long-range vehicle. In the simulations, the influence properties of initial state errors on perturbation guidance is analysed, give influence regularities of single initial state error, and obtain the statistical properties of engine-cutoff state deviations and impact-point deviation by Monte Carlo technique. From the simulation results, it seems that the navigation state tracks the nominal state. However, the real impact-point deviation has not been truly eliminated, instead of the almost target-hit deviation calculated by navigation output. The proposed analytical guidance accuracy model can be rapidly computed to provide a compensation for guidance and control system to improve hit accuracy.

Keywords: Long-range vehicle; Inertial navigation system; Initial state errors; Perturbation guidance; Monte Carlo

NOMENCLATURE

ΔP_e	Initial state errors vector
C_P^I	Transform matrix from PCS to LICS
ΔX_K	Engine-cutoff state deviations in the case of without-guidance
ΔX_{NK}	Navigation state at engine-cutoff time
ΔL_1	Longitudinal deviation of without-guidance
ΔZ_1	Lateral deviation of without-guidance
ΔL_S	Longitudinal deviation of with-guidance
ΔZ_S	Lateral deviation of with-guidance

1. INTRODUCTION

In order to provide a reference in inertial space before launching a long-range vehicle, inertial navigation system (INS) is needed to conduct the operation of alignment and levelling[1]. Nevertheless, as a result of the complexities of the Earth's surface and internal structure, its inhomogeneity will bring about vertical deflection, which shows the difference between normal orientation of reference ellipsoid and perpendicular orientation of geoid[2]. Here we define vertical deflection and launch azimuth deviation as initial orientation errors (IOEs). Additionally, initial positioning errors (IPEs) will be produced in the case of the difference between nominal launch point and real launch point[3]. Thus, initial state errors (ISEs), involving IPEs and IOEs, will be engendered when establishing the dynamical model in launch inertial coordinate system (LICS).

In current engineering applications, ISEs are ordinarily neglected in flight hardware. Conversely, instrument errors[4-6] and spatial disturbing gravity[7-9] have been extensively considered in many research papers. Vathsal[10] provided an error model of the strapped down inertial navigation system in the state space format. However, with the improvement of the accuracy of inertial measurement unit (IMU), the ISEs present an increasing impact on aerospace tracking, telemetry and command[11]. Current research data show that, for long-range vehicles, impact-point deviations caused by ISEs have the same magnitude deviations caused by instrument errors. Therefore, ISEs are the non-negligible factors in affecting impact-point accuracy. In this case, study on ISEs has great significance in analysing and improving the impact-point accuracy of long-range vehicles.

At present, the study of ISEs is focused on the influence regularity of engine-cutoff state and impact-point in open-loop conditions. Bernstein[12] developed a technique called autocorrelation function of surface gravity anomalies to predict aircraft navigation errors induced by deflection model uncertainties. Wang[13] derived the relationship between target position deviation and vertical deflection. The results show that vertical deflection can lead to one km error for the long-range missile. Yang[14] unified the difference as the rotation and translation between standard launch coordinate system and actual one, and obtained the analytical formulas between impact-point deviation and ISEs. However, the effect of ISEs on guidance accuracy is lack of intensive studies, which is great

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significant to improve the impact-point accuracy. Jia[15] analysed the effect of vertical deflection on ballistic missile impact accuracy in the case of fixed-time engine-cutoff and fixed-range engine-cutoff, but not considering the azimuth and geodetic measurement error. Zheng[16] deduced an analytical model of positioning and orientation error on explicit guidance accuracy under the condition of independent astronomical measurements, and put forward an approximate analytical solution of estimating the hit deviation using standard trajectory parameter. However, when shortening the launch preparation time, the estimation model cannot be satisfied under the condition of independent astronomical measurements.

The previous literatures mainly focus on the influence of ISEs in the open-loop cases. To give the propagation law and influence magnitude of the ISEs under the circumstances of closed-loop, we firstly analyse the influence mechanism of the ISEs on nominal trajectory, navigation trajectory and guidance trajectory, deduce a propagation model of engine-cutoff deviations in the case of without-guidance, then propose an analytical estimation method of impact-point in the case of with- guidance. Finally, different scenarios, involving effect of ISEs and single ISE on guidance accuracy as well as Monte Carlo simulation, are simulated to obtain the influence laws.

2. INFLUENCE MECHANISM OF INITIAL STATE ERRORS ON TRAJECTORY PARAMETERS

The guidance system of a long-range vehicle generally uses the launch inertial coordinate system as the trajectory calculation system. The ISEs affect the initial position and coordinate orientation in the LICS, making a change of the navigation calculation frame. In addition, the ISEs will have an impact on the integral initial value and on the force in the flight process[17].

The flight trajectory of the long-range vehicle has changed under the influence of the ISEs, as shown in Fig. 1. Given the launch point and the target point, ‘Trajectory 1’ is a nominal trajectory without existence of ISEs, which is designed in the LICS based on standard ellipsoid. ‘Trajectory 2’ is the nominal trajectory with existence of ISEs, which is derived from the firing data in ‘Trajectory 1’. In the absence of guidance, impact-point deviation ΔL_1 , ΔZ_1 will be produced between the impact point of ‘Trajectory 2’ and the target point. In the actual launch case, the platform coordinate system (PCS) used in navigation calculation is an inertial coordinate system determined by plumb line before the launch of a long range vehicle, which is different from the LICS determined by standard ellipsoid. In the general case, the PCS is regarded as the LICS. Therefore, the states outputted by navigation are regarded as the states in the LICS. Thus, the navigation trajectory, namely ‘Trajectory 3’, will accurately hit the target by guidance system. Actually, because of the difference between PCS and LICS, navigation states are not the real states, thus the guidance and shutdown commands are not the commands ensuring accurately hit the target, forming a guidance trajectory named ‘Trajectory 4’.

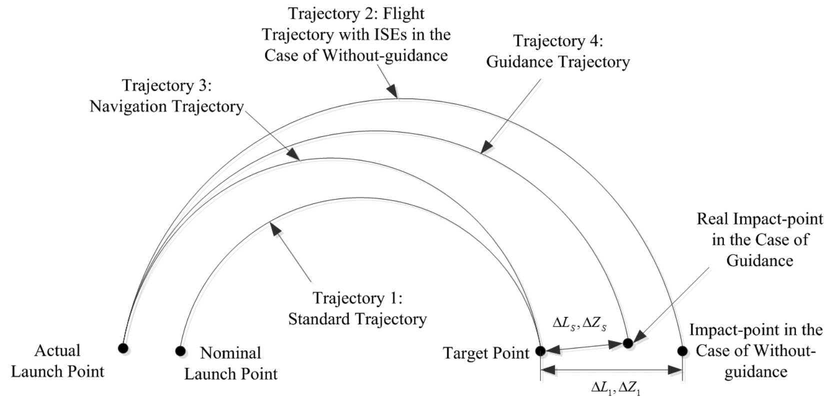

Figure 1. Effect of ISEs on flight trajectory.

The difference between the impact point of ‘Trajectory 4’ and target point is the impact-point deviation $\Delta L_S, \Delta Z_S$ in the case of guidance. Therefore, under the influence of the ISEs, the impact-point deviation cannot be necessarily eliminated when considering the guidance loop. On the contrary, the impact point probably has large deviation away from the target point when the long-range vehicle is guided by the wrong control instructions.

3. CALCULATION OF STATE DEVIATIONS

In order to obtain the influence magnitude of ISEs on perturbation guidance accuracy, an analytical propagation model is deduced to compute the engine-cutoff state deviations in the case of without-guidance.

The LICS $O_I - x_I y_I z_I$ adopts a standard ellipsoid as reference base, where axis $O_I y_I$ points to the outer normal direction. The PCS $O_P - x_P y_P z_P$ is established on inertial navigation system to perform actual positioning and orientation, where axis $O_P y_P$ points to the outer vertical direction, as shown in Fig. 2. For a nominal launch point O_I , λ_0 , B_0 , H_0 and A_0 respectively stand for geodetic longitude, geodetic latitude, elevation, and launch azimuth. For an actual launch point O_P , λ , B , H respectively stand for geodetic longitude, geodetic latitude, elevation, and λ_T , B_T , A_T respectively stand for astronomical longitude, astronomical latitude, astronomical azimuth.

The difference between the origins of LICS and PCS produces the IPEs, involving geodetic longitude error $\Delta\lambda_0$,

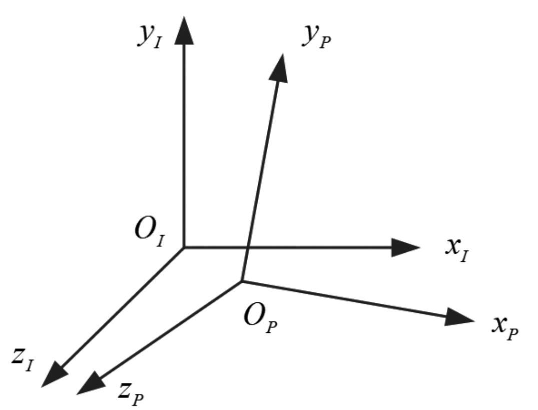

Figure 2. The difference between LICS and PCS.

geodetic latitude error ΔB_0 and elevation deviation ΔH_0. In addition, influenced by meridional direction component ξ and prime vertical direction component η in vertical deflection, three axes of the PCS are relatively rotated around the LICS, which leads to north-south direction deviation component ΔB_{T0}, east-west direction deviation component $\Delta\lambda_{T0}$ as well as launch azimuth deviation ΔA_0.

ΔB_{T0}, $\Delta\lambda_{T0}$ and ΔA_0 are given by[18]

$$\begin{cases}\Delta B_{T0} = B_T - B_0 = B + \xi - B_0 = \Delta B_0 + \xi \\ \Delta\lambda_{T0} = \lambda_T - \lambda_0 = \lambda + \eta\sec B - \lambda_0 = \Delta\lambda_0 + \eta\sec B\end{cases} \tag{1}$$

According to the previous descriptions, the IPEs, vertical deflection and launch azimuth deviation are combined into one vector to represent the ISEs as shown in Fig. 2, namely $\Delta P_e = [\Delta\lambda_0 \quad \Delta B_0 \quad \Delta H_0 \quad \xi \quad \eta \quad \Delta A_0]^T$.

Without the ISEs, navigation equation of a long-range vehicle in LICS is given by

$$\begin{cases}\dot{v}_I = g_I + \dot{W}_I \\ \dot{\rho}_I = v_I\end{cases} \tag{2}$$

where $\dot{\rho}_I$ stands for the derivative of position vector to time, $\dot{v}_I$ the derivative of velocity to time, v_I the velocity vector, g_I the gravitational acceleration and $\dot{W}_I$ the apparent acceleration. The terms above are all computed in LICS.

When considering the ISEs, navigation equation of a long-range vehicle in PCS is

$$\begin{cases}\dot{v}_P = g_P + \dot{W}_P \\ \dot{\rho}_P = v_P\end{cases} \tag{3}$$

where $\dot{\rho}_P$ stands for the derivative of position vector to time, $\dot{v}_P$ the derivative of velocity to time, v_P the velocity vector, g_P the gravitational acceleration and $\dot{W}_P$ the apparent acceleration. The terms above are indicated in PCS.

According to dynamic Eqns. (2) and (3), the ascent state deviation dynamic equation can be obtained in the case of without-guidance, that is

$$\begin{cases}\Delta\dot{v}_I = \Delta g_I + \Delta\dot{W}_I \\ \Delta\dot{\rho}_I = \Delta v_I\end{cases} \tag{4}$$

where Δg_I stands for the gravitational acceleration deviation, $\Delta\dot{W}_I$ the apparent acceleration projection deviation, $\Delta\rho_I$ the position deviation vector and Δv_I the velocity deviation vector.

The acceleration deviations in Eqn. (4) are measured in the LICS. The gravitational acceleration deviation is the difference between $C_P^I g_P$ and g_I. Generally, the apparent acceleration in the PCS is regarded as the acceleration in the LICS. Therefore, the apparent acceleration deviation is the difference between $C_P^I\dot{W}_I$ and $\dot{W}_I$. Thus, the expression of Δg_I and $\Delta\dot{W}_I$ are

$$\begin{aligned}\Delta g_I &= C_P^I g_P - g_I = g_P - g_I + \Delta C_P^I g_P \approx G_r\Delta\rho_I + \Delta C_P^I g_I \\ \Delta\dot{W}_I &= C_P^I\dot{W}_I - \dot{W}_I = \Delta C_P^I\dot{W}_I\end{aligned} \tag{5}$$

where C_P^I is the transform matrix from PCS to LICS, G_r is the partial derivative matrix of gravitational acceleration to position vector. Since the ISEs are quite small, we can assume that C_P^I equals the sum of unit matrix $E_{3\times3}$ and deviation matrix ΔC_P^I.

Hence, Eqn. (4) can be rewritten as

$$\begin{cases}\Delta\dot{v}_I = G_r\Delta\rho_I + \Delta C_P^I g_I + \Delta C_P^I\dot{W}_I \\ \Delta\dot{\rho}_I = \Delta v_I\end{cases} \tag{6}$$

wherein the IPEs are $\Delta\rho_0$, the initial velocity errors (IVEs) are Δv_0.

In inertial navigation system, gyroscopes and accelerations will drift along with time, which will bring about drift error of inertial instrument. The model of drift error is shown in Yang[11], *et al.* which can be increased in the established error dynamics model.

The expression of partial matrix can consult the Zheng[18], *et al*. The following parts gives the expressions of the $\Delta\rho_0$, Δv_0, and projection deviations $\Delta C_P^I g_I$, $\Delta C_P^I\dot{W}_I$.

(i) Initial Positioning Errors

IPEs is a function of λ_0, B_0, and H_0 in the nominal launch point. According to the geometric relationship of the spherical triangle, the expression of IPEs is

$$\Delta\rho_0 = \begin{bmatrix} R_e\cos B_0\sin A_0 & R_e\cos A_0 & 0 & \\ 0 & 0 & 1 & 0_{3\times3} \\ R_e\cos B_0\cos A_0 & -R_e\sin A_0 & 0 & \end{bmatrix}\Delta P_e \triangleq G_{\rho0}\cdot\Delta P_e \tag{7}$$

where R_e is the Earth's average radius, B_0 is the geodetic latitude at the launch point and A_0 is the launch azimuth.

(ii) Initial Velocity Errors

The initial velocity in the LICS is

$$v_0 = \omega_e\times R_0 \tag{8}$$

where ω_e is the Earth's rotation angular velocity vector in the LICS, R_0 is the position vector in LICS from geocentric point to nominal launch point.

Since the ISEs are small, the initial velocity errors can be obtained by the linearisation of equation (8), that is

$$\Delta v_0 = \omega_e\times\Delta R_0 + \Delta\omega_e\times R_0 \tag{9}$$

After detailed analysis and arrangement of ΔR_0 and $\Delta\omega_e$[18], the IVEs are

$$\Delta v_0 = G_{v0}\cdot\Delta P_e \tag{10}$$

Among which

$$\begin{aligned}G_{v0} = \Big[& \omega_e\times\frac{\partial R_0}{\partial\lambda_0} \quad \omega_e\times\frac{\partial R_0}{\partial B_0} + \frac{\partial\omega_e}{\partial B_0}\times R_0 \\ & \omega_e\times\frac{\partial R_0}{\partial H_0} \quad \omega_e\times\frac{\partial R_0}{\partial\xi} + \frac{\partial\omega_e}{\partial\xi}\times R_0 \\ & \omega_e\times\frac{\partial R_0}{\partial\eta} \quad \omega_e\times\frac{\partial R_0}{\partial A_0} + \frac{\partial\omega_e}{\partial A_0}\times R_0 \Big]\end{aligned} \tag{11}$$

(iii) $\Delta C_P^I g_P$, $\Delta C_P^I\dot{W}_I$

In Eqn. (6), ΔC_P^I is the transformation deviation from LICS to PCS, is given by

$$\Delta C_P^I = \begin{bmatrix} 0 & -\delta A_z & \delta A_y \\ \delta A_z & 0 & -\delta A_x \\ -\delta A_y & \delta A_x & 0\end{bmatrix} \tag{12}$$

wherein δA_x, δA_y, and δA_z are the approximate Euler angles from LICS to PCS, namely

$$\begin{cases} \delta A_x = \Delta\lambda_0 \cos B_0 \cos A_0 - \Delta B_0 \sin A_0 - \xi \sin A_0 + \eta \cos A_0 \\ \delta A_y = \Delta\lambda_0 \sin B_0 - \Delta A_0 + \eta \tan B_0 \\ \delta A_z = -\Delta\lambda_0 \cos B_0 \sin A_0 - \Delta B_0 \cos A_0 - \xi \cos A_0 - \eta \sin A_0 \end{cases} \tag{13}$$

Consequently, for an arbitrary vector $J \in R^{3\times 1}$, $\Delta C_P^I \cdot J$ can be transformed as

$$\Delta C_P^I J = G(J) \cdot \Delta P_e \tag{14}$$

where

$$G^T(J) = \begin{bmatrix} J_y \cos B_0 \sin A_0 + J_z \sin B_0 & -J_x \cos B_0 \sin A_0 - J_z \cos B_0 \cos A_0 & -J_x \sin B_0 + J_y \cos B_0 \cos A_0 \\ J_y \cos A_0 & -J_x \cos A_0 + J_z \sin A_0 & -J_y \sin A_0 \\ 0 & 0 & 0 \\ J_y \cos A_0 & -J_x \cos A_0 + J_z \sin A_0 & -J_y \sin A_0 \\ J_y \sin A_0 + J_z \tan B_0 & -J_x \sin A_0 - J_z \cos A_0 & -J_x \tan B_0 + J_y \cos A_0 \\ -J_z & 0 & J_x \end{bmatrix} \tag{15}$$

Thus, the acceleration projection deviations $\Delta C_P^I g_I$ and $\Delta C_P^I \dot{W}_I$ are

$$\Delta C_P^I \cdot \dot{W}_I = G(\dot{W}_I) \cdot \Delta P_e \tag{16}$$

$$\Delta C_P^I \cdot g_I = G(g_I) \cdot \Delta P_e \tag{17}$$

By rearranging the Eqn. (4), the state deviation equation is

$$\begin{bmatrix} \Delta\dot{v}_I \\ \Delta\dot{\rho}_I \end{bmatrix} = \begin{bmatrix} 0_{3\times 3} & G_r \\ E_{3\times 3} & 0_{3\times 3} \end{bmatrix} \begin{bmatrix} \Delta v_I \\ \Delta\rho_I \end{bmatrix} + \left\{ \begin{bmatrix} G(g_I) \\ 0_{3\times 6} \end{bmatrix} + \begin{bmatrix} G(\dot{W}_I) \\ 0_{3\times 6} \end{bmatrix} \right\} \Delta P_e \tag{18}$$

By solving state Eqn. (18), the engine-cutoff state deviations can be derived as

$$\begin{aligned} \begin{bmatrix} \Delta v_I(t_K) \\ \Delta\rho_I(t_K) \end{bmatrix} &= \Phi(t_K) \begin{bmatrix} G_{v0} \\ G_{\rho 0} \end{bmatrix} \Delta P_e + \\ &\int_0^{t_K} \Phi(t_K - \tau) \left\{ \begin{bmatrix} G(g_I) \\ 0_{3\times 6} \end{bmatrix} + \begin{bmatrix} G(\dot{W}_I) \\ 0_{3\times 6} \end{bmatrix} \right\} \Delta P_e d\tau \end{aligned} \tag{19}$$

Let

$$\begin{aligned} M_0 &= \Phi(t_K) \begin{bmatrix} G_{v0} \\ G_{\rho 0} \end{bmatrix} \\ M_g &= \int_0^{t_K} \Phi(t_K - \tau) \begin{bmatrix} G(g_I) \\ 0_{3\times 6} \end{bmatrix} d\tau \\ M_W &= \int_0^{t_K} \Phi(t_K - \tau) \begin{bmatrix} G(\dot{W}_I) \\ 0_{3\times 6} \end{bmatrix} d\tau \end{aligned} \tag{20}$$

where M_0 represents the propagation matrix of the engine-cutoff state deviations generated by IPEs and IVEs, M_g is the propagation matrix of gravitational acceleration projection deviation, M_W is the propagation matrix of apparent acceleration projection deviation.

Consequently, the engine-cutoff state deviations ΔX_K in the case of without-guidance are

$$\Delta X_K = \begin{bmatrix} \Delta v_I(t_K) \\ \Delta\rho_I(t_K) \end{bmatrix} = \left(M_0 + M_g + M_W \right) \Delta P_e \triangleq M_K \cdot \Delta P_e \tag{21}$$

Therefore, the relationship between impact-point deviation and ISEs is

$$\begin{aligned} \begin{bmatrix} \Delta L \\ \Delta Z \end{bmatrix} &= \begin{bmatrix} \dfrac{\partial L}{\partial X_K} & \dfrac{\partial Z}{\partial X_K} \end{bmatrix}^T \cdot M_K \cdot \Delta P_e \\ &\triangleq M_X \cdot M_K \cdot \Delta P_e \triangleq M_P \cdot \Delta P_e \end{aligned} \tag{22}$$

where $\dfrac{\partial L}{\partial X_K}$, $\dfrac{\partial Z}{\partial X_K}$ are respectively the partial derivative matrix of longitudinal and lateral direction to engine-cutoff states, M_P is the propagation matrix of impact-point deviation caused by the ISEs.

When considering the case of guidance, the navigation equation calculated by the onboard computer in the PCS is

$$\begin{cases} \dot{v}_N = g_N + \dot{W}_P \\ \dot{\rho}_N = v_N \end{cases} \tag{23}$$

where $\dot{\rho}_N$ is the derivative of position vector to time, $\dot{v}_N$ is the derivative of velocity vector to time, v_N is the velocity vector, g_N is the gravitational acceleration used by navigation calculation. When the onboard computer conducts recursive calculation, $\dot{W}_P$ in the LCS is adopted in navigation system.

The engine-cutoff states integrated the equation (23) is the navigation states X_{NK}. The difference between X_{NK} and X_K is the navigation error, namely $\Delta X_{NK} = X_{NK} - X_K$.

4. ANALYTICAL MODEL OF INITIAL STATE ERRORS ON PERTURBATION GUIDANCE

Due to the small amount of calculation, perturbation guidance is a common guidance for long-range vehicles, and it can basically meet the requirements of guidance accuracy. The ISEs are generally ignored in engineering applications. However, the difference between PCS and LICS results in inconsistency of the states in the two coordinate systems. Therefore, the guidance commands in the two coordinate systems are actually different when guaranteeing to accurately hit the target, which are generally regarded as the same values. In this case, it is necessary to carry out the research to explore the impact of ISEs on flight states and impact-point deviations under the condition of with-guidance, which is of great significance to improve the hit accuracy for long-range vehicles.

Suppose that X is the standard states in the LICS, and the trajectory is formed by a set of standard firing data. X_P is the uncontrolled states calculated by the standard firing data under the condition of actual launch. $\bar{X}_P$ is the uncontrolled states transformed from PCS to LICS. X_N is the states outputted by navigation system. Since the accelerometer installation is based on the local plumb line, vertical deflection causes the reference deviation for coordinate system. When a long-range vehicle is taken off, the accelerometer measures the components of apparent acceleration in the PCS. Therefore, navigation states, regarded as the parameters in the LICS, are employed in the guidance and control system. In this case, the engine-cutoff function satisfies that

$$\Delta L = \left(\frac{\partial L}{\partial X_{\hat{K}}}\right)^T \left(X_{NK} - X_{\hat{K}}\right) = 0 \tag{24}$$

Since the ISEs are quite small, we have

$$X_{NK} = X_{N\hat{K}} + \dot{X}_{\hat{K}}\left(t_K - t_{\hat{K}}\right) \tag{25}$$

where $X_{N\hat{K}}$ is the navigation state in the presence of ISEs at standard engine-cutoff time $t_{\hat{K}}$, $\dot{X}_{\hat{K}}$ is the derivative of trajectory state to time in the LICS at standard engine-cutoff time. Due to the small disturbance of ISEs, the derivation of state to time in the PCS is almost identical with the value in the LICS.

Substituting Eqn. (25) into Eqn. (24), we can have

$$\Delta L = \left(\frac{\partial L}{\partial X_{\hat{K}}}\right)^T \left[\left(X_{N\hat{K}} - X_{\hat{K}}\right) + \dot{X}_{\hat{K}}\left(t_K - t_{\hat{K}}\right)\right] = 0 \tag{26}$$

In Eqn. (26), the navigation state $X_{N\hat{K}}$ is directly used in the LICS. Consequently, the longitudinal deviation is still existed under the influence of ISEs. In actual condition, the longitudinal deviation is

$$\Delta L_S = \left(\frac{\partial L}{\partial X_{\hat{K}}}\right)^T \left(\overline{X}_{PK} - X_{\hat{K}}\right) \tag{27}$$

Similarly

$$\overline{X}_{PK} = \overline{X}_{P\hat{K}} + \dot{X}_{\hat{K}}\left(t_K - t_{\hat{K}}\right) \tag{28}$$

Thus, the Eqn. (27) can be rewritten as

$$\Delta L_S = \left(\frac{\partial L}{\partial X_{\hat{K}}}\right)^T \left[\left(\overline{X}_{P\hat{K}} - X_{\hat{K}}\right) + \dot{X}_{\hat{K}}\left(t_K - t_{\hat{K}}\right)\right] \tag{29}$$

Substituting Eqn. (26) into Eqn. (29), we can obtain

$$\Delta L_S = \left(\frac{\partial L}{\partial X_{\hat{K}}}\right)^T \left(\overline{X}_{P\hat{K}} - X_{N\hat{K}}\right) \tag{30}$$

Substituting the expression of $\Delta X_{N\hat{K}} = X_{N\hat{K}} - X_{\hat{K}}$ and $\Delta X_{\hat{K}} = \overline{X}_{P\hat{K}} - X_{\hat{K}}$ into equation (30), it can get

$$\Delta L_S = \left(\frac{\partial L}{\partial X_{\hat{K}}}\right)^T \left(\Delta X_{\hat{K}} - \Delta X_{N\hat{K}}\right) \tag{31}$$

In which, $\Delta X_{\hat{K}}$ can be rapidly computed by Eqn. (21).

The lateral deviation is

$$\begin{aligned}\Delta Z_S &= \left(\frac{\partial Z}{\partial X_{\hat{K}}}\right)^T \left(\overline{X}_{PK} - X_{\hat{K}}\right) \\ &= \left(\frac{\partial Z}{\partial X_{\hat{K}}}\right)^T \left[\left(\overline{X}_{P\hat{K}} - X_{\hat{K}}\right) + \dot{X}_{\hat{K}}\left(t_K - t_{\hat{K}}\right)\right]\end{aligned} \tag{32}$$

It can be obtained from Eqn. (26) that

$$t_K - t_{\hat{K}} = -\frac{\left(\dfrac{\partial L}{\partial X_{\hat{K}}}\right)^T \left(X_{N\hat{K}} - X_{\hat{K}}\right)}{\left(\dfrac{\partial L}{\partial X_{\hat{K}}}\right)^T \dot{X}_{\hat{K}}} \tag{33}$$

Substituting Eqn. (33) into Eqn. (32), the lateral deviation is

$$\Delta Z_S = \left(\frac{\partial Z_c}{\partial X_{\hat{K}}}\right)^T \left\{\left(\overline{X}_{P\hat{K}} - X_{\hat{K}}\right) - \dot{X}_{\hat{K}}\left[\frac{\left(\dfrac{\partial L}{\partial X_{\hat{K}}}\right)^T \left(X_{N\hat{K}} - X_{\hat{K}}\right)}{\left(\dfrac{\partial L}{\partial X_{\hat{K}}}\right)^T \dot{X}_{\hat{K}}}\right]\right\} \tag{34}$$

Substituting the expressions of $\Delta X_{N\hat{K}} = X_{N\hat{K}} - X_{\hat{K}}$ and $\Delta X_{\hat{K}} = \overline{X}_{P\hat{K}} - X_{\hat{K}}$ into Eqn. (34), we can get

$$\Delta Z_S = \left(\frac{\partial Z_c}{\partial X_{\hat{K}}}\right)^T \left\{\Delta X_{\hat{K}} - \dot{X}_{\hat{K}}\left[\frac{\left(\dfrac{\partial L}{\partial X_{\hat{K}}}\right)^T \Delta X_{N\hat{K}}}{\left(\dfrac{\partial L}{\partial X_{\hat{K}}}\right)^T \dot{X}_{\hat{K}}}\right]\right\} \tag{35}$$

Equations (31) and (35) are respectively the longitudinal deviation and lateral deviation caused by ISEs in the case of guidance. Thus, real impact-point deviations can be rapidly estimated by the derived analytical Eqns. (31) and (35), and it can save plenty of calculation time, which is great significant to conduct extensive simulations.

5. SIMULATION AND ANALYSIS

In the simulation part, the range of the long-range vehicle is 8000 km. The following parts are the influence regularities of ISEs in the case of guidance.

5.1 Effect of ISEs on Ascent Phase Parameters

In this part, values of ISEs are shown in Table 1. The vector ΔP_e presented in previous formulas is comprised of the units of ISEs. Thus, engine-cutoff state deviations $\Delta X_{\hat{K}}$ will be easily obtained by Eqn. (21). Launch point parameters are set as 117.3° E, 39.9° N, the elevation 10 m, and the launch azimuth A_0 30°.

Table 1. Values of ISEs

Items	Values	Items	Values
$\Delta\lambda_0('')$	5	$\xi('')$	15
$\Delta B_0('')$	5	$\eta('')$	15
$\Delta H_0(m)$	1	$\Delta A_0('')$	12

Under the influence of ISEs, the states in the case of guidance present different meanings. 'state 1' is the standard engine-cutoff state without ISEs in the absence of guidance, 'state 2' is the engine-cutoff state without ISEs in the case of guidance, 'state 3' is the engine-cutoff state with ISEs in the absence of guidance, 'state 4' is engine-cutoff navigation state with ISEs, and 'state 5' is the real engine-cutoff state with ISEs in the case of guidance. Table 2 shows the different kinds of engine-cutoff state deviations and impact-point deviations. ‘deviation 1’ is the difference between ‘state 2’ and ‘state 1’, ‘deviation 2’ is the difference between ‘state 3’ and ‘state 1’, ‘deviation 3’ is the difference between ‘state 4’ and ‘state 1’, and ‘deviation 4’ is the difference between ‘state 5’ and ‘state 1’.

Without the influence of ISEs, the vehicle makes effort to eliminate the impact of disturbance to continually approach the standard trajectory. The guidance state at engine-cutoff is ‘state 2’, and ‘deviation 1’ is shown in Table 2. We can get that the values of ‘deviation 1’ are not zeros and the engine-cutoff position deviations in the x, y direction are relatively larger than the deviation in the z direction. Despite the existence of ‘deviation 1’, the vehicle can be ensured to accurately hit the target when control function gives the shutdown command in the case of ‘state 2’. The longitudinal deviation and lateral deviation are respectively 0.4695 m and 0.2651 m, which indicates that the guidance system is effective.

In the presence of ISEs, ‘state 3’ is the engine-cutoff state without consideration of guidance, and ‘deviation 2’ is the difference between ‘state 3’ and ‘state 1’. In the previous study, $\Delta X_{\hat{k}}$ can be rapidly calculated through Eqn. (21). Among which, the engine-cutoff position deviations are induced by IPEs, IVEs and IOEs, and the engine-cutoff velocity deviations are induced by IVEs and IOEs. The longitudinal deviation and lateral deviation in the case of without-guidance are respectively 463.6 m and -17.1 m.

In the presence of ISEs, ‘state 4’ is the navigation state in the case of guidance. As navigation state is recognised by the onboard computer, the ‘state 4’ is ‘ approaching ‘state 1’. The resulting navigation state ‘deviation 3’ is close to ‘deviation 1’. Therefore, the vehicle believes it can accurately hit the target when satisfying the shutdown condition controlled by the computer. In this case, the longitudinal deviation and lateral deviation are respectively 0.4715 m and 0.3976 m.

However, when the onboard computer calculates the state in a recursive way, the actual launch parameters are considered to be same as the initial conditions of nominal trajectory. Meanwhile, it is believed that there is no reference difference between PCS and LICS. Under this circumstance, the ‘state 5’ is not identical with ‘state 4’. The ‘deviation 4’ calculated by the difference between ‘state 5’ and ‘state 1’ is the real state deviation in the case of guidance. It can be seen that ‘deviation 4’ is basically consistent with ‘deviation 2’ under the given simulation condition. The impact-point deviations in ‘deviation 4’ are respectively 458.6788 m and -22.6583 m. It seems that the navigation state tracks the nominal state. However, the real impact-point deviation is not truly eliminated.

The ‘deviation 2’ ~ ‘deviation 4’ in Figs. 3 and 4 are the ascent state deviations caused by ISEs. It can be seen that the curve of ‘deviation 2’ is smooth without consideration of guidance. The flight time in the first stage is 63.7 s, and the vehicle tracks the program attitude angle. When the velocity deviation appears as a result of disturbance, the control system governs the rudder to gradually decrease the velocity deviation. When the vehicle is flying at the second and third stages, the navigation state is tracking the standard trajectory by guidance and control system to make ‘deviation 3’ to be zero. Although the resulting navigation error is zero, the navigation state in fact does not represent the real state. Hence, the given guidance and control commands cannot guarantee to track on nominal state, and ‘deviation 4’ is still existed with the consideration of closed-loop.

Figure 5 is the deviation corrections of pitch angle and yaw angle. When the second stage (63.7 s) and the third stage (126.4 s) begin flight, the pitch channel and yaw channel are controlled under the drive of guidance. The attitude angle deviation and lateral deviation can be controlled to zero regardless of whether it has ISEs. The navigation deviation is eliminated by guidance and control commands to ensure the consistency between navigation state and nominal state. However, the vehicle receives wrong guidance and control instructions, and the pitch and yaw channel deviations still remain at engine-cutoff time.

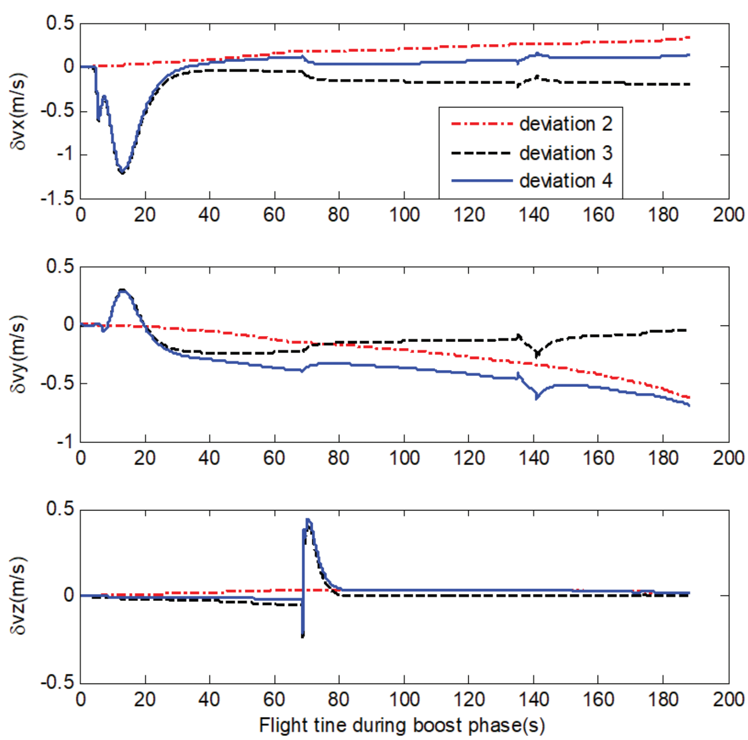

Figure 3. The ascent position deviations caused by ISEs.

Table 2. The engine-cutoff state deviations and impact-point deviations at different scenarios

Deviation	x(m)	y(m)	z(m)	Δv_x (m/s)	Δv_y (m/s)	Δv_z (m/s)	$\Delta L(m)$	$\Delta Z(m)$
Deviation 1	-19.7019	-19.2016	0.0894	0.0171	0.0080	0.0041	0.4695	0.2651
Deviation 2	228.0459	-41.8806	24.0624	0.3296	-0.6289	0.0149	463.6505	-17.1402
Deviation 3	-23.9507	-17.2305	0.0692	0.0123	0.0187	0.0025	0.4715	0.3976
Deviation 4	204.4182	-60.7742	23.9930	0.3471	-0.6254	0.0149	458.6788	-22.6583

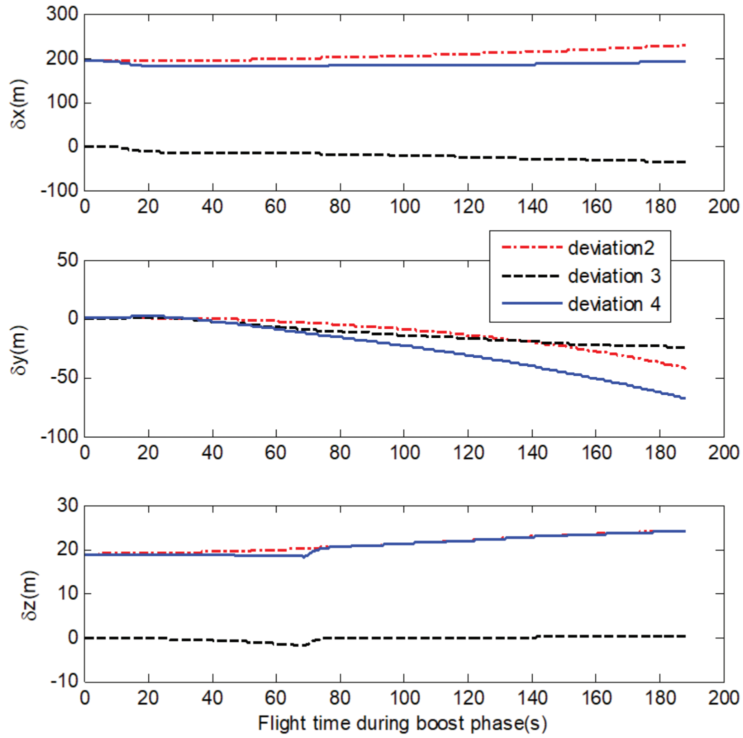

Figure 4. The ascent velocity deviations caused by ISEs.

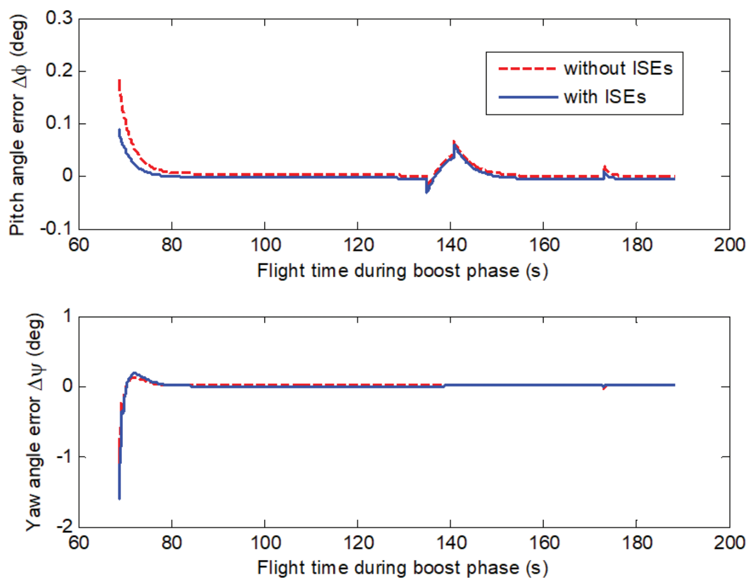

Figure 5. The deviation corrections of pitch and yaw angle.

5.2 Effect of Single ISE on Perturbation Guidance

In the previous section, the ISEs include geodetic longitude error, geodetic latitude error, elevation deviation, vertical deflection and launch azimuth error. In order to specifically analyze the effect of each item, Table 3 shows the simulation results of single ISE on guidance accuracy. Launch point parameters are same as the section 5.1.

In Table 3, when the single geodetic longitude error satisfies $\Delta\lambda_0 = 1''$, $\Delta\lambda_0$ mainly affects the position deviation in the x and z direction in 'deviation 2'. Because of the small change of coordinate reference caused by $\Delta\lambda_0$, the generated engine-cutoff velocity deviations are relatively smaller than that of vertical deflection. The case of $\Delta\lambda_0 = 1''$ will produce the longitudinal deviation 15.6795 m and lateral deviation -16.5736 m under the condition of without-guidance. In the presence of guidance, navigation state calculated by onboard computer cannot guarantee the exact consistency with standard engine-cutoff state. Thus, 'deviation 3' is formed and basically identical with other single disturbance. Although navigation state is not exactly same as standard state, the derived impact-point can ensure to hit the target when the engine shutdown at this navigation state. In fact, the real state 'deviation 4' is not navigation error in the case of guidance, and the longitudinal and lateral deviation are respectively 15.7096 m and -16.2561 m, which are very close to the condition of without-guidance. The geodetic latitude error ΔB_0 has a similar analysis result with $\Delta\lambda_0$. In the case of $\Delta H_0 = 1$ m, the real state deviation is similar to navigation deviation, and the impact-point deviations decrease compared with that of without-guidance.

In the case of effect of vertical deflection on guidance accuracy, the real impact-point deviations obviously decrease compared with the case of without-guidance. In the case of $\Delta A_0 = 1''$, the azimuth deviation mainly brings about the z direction state deviation and lateral deviation with the value of 30.5186 m. With the consideration of guidance, although the impact-point deviation calculated by navigation is close to zero, the real z direction state deviation is not eliminated and is close to the case of with-guidance. The real impact-point lateral deviation remains with the value of 30.5046 m.

In addition, we can obtain from Table 3 that $\Delta\lambda_0$, ΔB_0 and ΔA_0 will produce the obvious state deviations in the z direction. The real state deviations are different from navigation deviations in the case of guidance, and the impact-point deviations cannot be effectively wiped out. ΔH_0, ξ and η will lead to the state deviations in the x and y directions under the condition of with-guidance. The real state deviations can stay close to navigation deviations in the case of guidance, and the impact-point deviation can be evidently eliminated.

5.3 Monte Carlo Simulation

In this section, every ISE is considered to conduct a Monte Carlo simulation. Suppose that every ISE obeys the normal distribution, and the mean value μ and standard deviation σ are as shown in Table 4. The sensor noise obeys the normal distribution, and the standard deviation is 0.3 deg. The 1000 shooting times are simulated to obtain the influence regularities of ISEs. The impact-point deviations in the case of without-guidance are as shown from Fig. 6. The impact-point deviations obtained by navigation calculation are as shown from Fig. 7. The real engine-cutoff state deviations and impact-point deviations are as shown from Fig. 8.

Figure 6 are the impact-point deviation in the case of without-guidance. In Fig. 6, the longitudinal deviation is mainly scattered within 3 km, whose standard deviation is 910.5334 m. The lateral deviation is mainly scattered within 500 m, whose standard deviation is 164.5642 m. Figure 7 are the impact-point deviation in the case of guidance. The mean

Table 3. The effect of single ISE on guidance accuracy

Single item	Deviation	x(m)	y(m)	z(m)	Δv_x (m/s)	Δv_y (m/s)	Δv_z (m/s)	$\Delta L(m)$	$\Delta Z(m)$
$\Delta\lambda_0(1'')$	Deviation 2	13.1394	-0.7964	19.8487	0.0060	-0.0115	-0.0086	15.6795	-16.5736
	Deviation 3	-19.7807	-19.1696	0.0904	0.0170	0.0082	0.0041	0.4696	0.3506
	Deviation 4	-6.6413	-19.9660	19.9391	0.0230	-0.0034	-0.0045	15.7096	-16.2561
$\Delta B_0(1'')$	Deviation 2	26.2428	-0.3021	-16.2984	0.0006	-0.0026	0.0004	8.0448	8.6952
	Deviation 3	-19.7019	-19.2016	0.0894	0.0171	0.0080	0.0041	0.4695	0.2811
	Deviation 4	6.5139	-19.2016	-16.2288	0.0171	0.0080	0.0041	10.0065	9.2805
$\Delta H_0(1m)$	Deviation 2	-0.1678	3.8729	-0.0196	-0.0047	0.0240	-0.0003	26.0945	7.8345
	Deviation 3	-19.7019	-19.2016	0.0894	0.0171	0.0080	0.0041	0.4695	0.2811
	Deviation 4	-19.7019	-18.2016	0.0894	0.0171	0.0080	0.0041	4.0386	1.0351
$\xi(1'')$	Deviation 2	7.4717	-6.8731	-0.9753	0.0687	-0.0750	-0.0097	153.8611	0.4085
	Deviation 3	-19.8385	-19.1017	0.0891	0.0169	0.0084	0.0040	0.4694	0.3108
	Deviation 4	-18.5403	-20.7676	-0.7189	0.0294	-0.0157	-0.0038	12.9454	-9.7715
$\eta(1'')$	Deviation 2	4.6363	-4.2636	0.0859	0.0426	-0.0465	-0.0053	95.7767	0.7896
	Deviation 3	-19.8046	-19.1599	0.0907	0.0170	0.0082	0.0041	0.4696	0.3749
	Deviation 4	-18.9957	-20.1980	-0.2524	0.0248	-0.0068	-0.0071	7.7322	-12.9492
$\Delta A_0(1'')$	Deviation 2	-0.0253	0.0252	1.9679	-0.0003	0.0003	0.0285	2.7162	30.5186
	Deviation 3	-19.7222	-19.2254	0.0862	0.0171	0.0080	0.0039	0.4697	0.0937
	Deviation 4	-19.7232	-19.2254	2.0491	0.0171	0.0080	0.0323	3.3657	30.5046

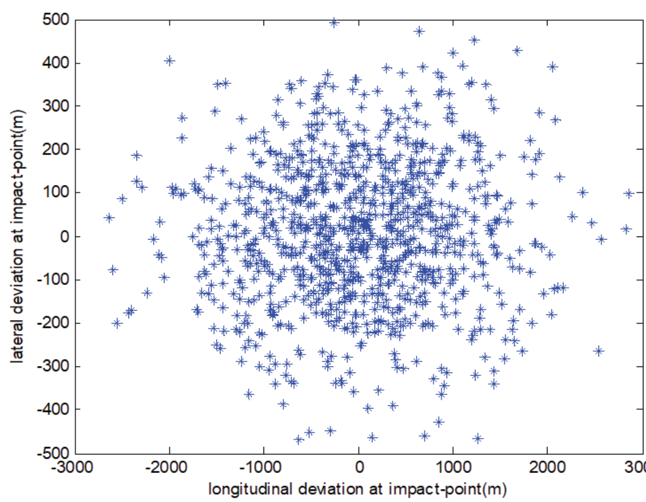

Figure 6. The impact-point deviation in the case of without-guidance.

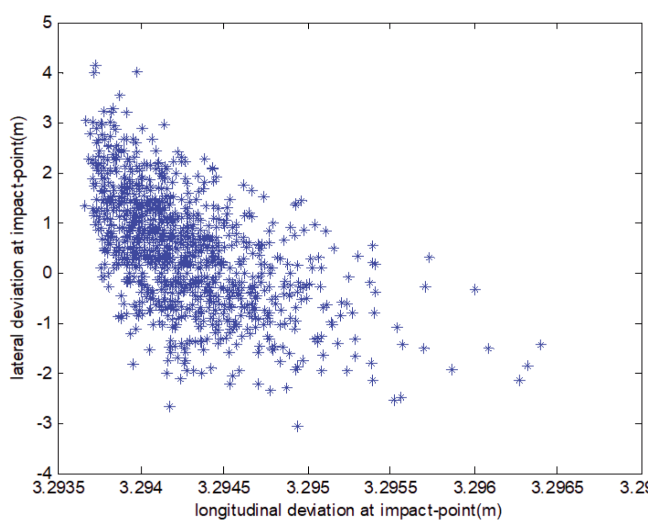

Figure 7. The impact-point deviation obtained by navigation calculation in the case of guidance.

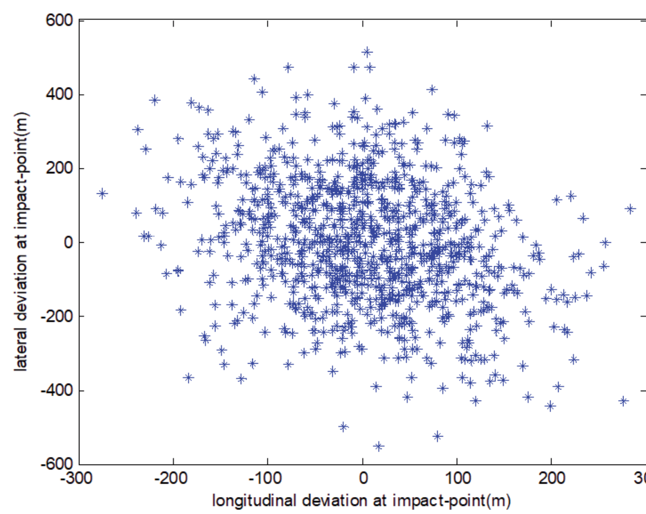

Figure 8. The real impact-point deviation in the case of guidance.

Table 4. The mean value and standard deviation

ISEs	μ	σ
$\Delta\lambda_0('')$	0	2
$\Delta B_0('')$	0	2
$\Delta H_0(1m)$	0	1
$\xi('')$	0	5
$\eta('')$	0	5
$\Delta A_0('')$	0	5

value of longitudinal deviation is 3.2943 m, and the standard deviation is 3.9867e-4 m. The mean value of lateral deviation is 0.4296 m, and the standard deviation is 1.1271 m. Figure 8 shows the impact-point deviation in the case of guidance. The real longitudinal deviation is mainly scattered within 300 m, whose standard deviation is 89.7009 m. The real lateral deviation is mainly scattered with 500 m, whose standard deviation is 167.5910 m.

Based on the analysis, the engine-cutoff position deviation is not evidently decreased, while the velocity deviation is reduced in the case of guidance. As the range is ceaselessly controlled during the boost phase, the real longitudinal deviation is obviously depressed compared to that of without-guidance, while the lateral deviation does not decrease.

6. CONCLUSIONS

(i) In this paper, we systematically analyse the influence mechanism of ISEs on nominal trajectory, navigation trajectory and guidance trajectory, which is of great reference value to obtain the influence characteristics of ISEs and to mprove the hit accuracy for long-range vehicles.

(ii) A rapid calculation model of engine-cutoff state uncertainty caused by ISEs is derived under the condition of without-guidance. On this basis, an accuracy analytical solution of ISEs on perturbation guidance is proposed to obtain the real impact-point of long-range vehicles. The proposed analytical guidance accuracy model can be rapidly computed to provide a compensation for guidance and control system to improve hit accuracy.

(iii) The navigation deviation is eliminated by guidance and control commands to ensure the consistency between navigation state and nominal state. However, the vehicle receives wrong guidance and control instructions. It seems that navigation state tracks the nominal state, but the real impact-point deviation has not been truly eliminated, instead of the navigation output target-hit deviation.

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CONTRIBUTORS

Dr Xu Zheng received his MS and PhD in Department of Aerospace Engineering from Harbin Institute of Technology in 2014 and 2018, respectively. He is currently an engineer in Nanjing Institute of Electronic Engineering. His research interests include nonlinear guidance and control, multidisciplinary design optimisation.
In the current study, he performed and simulated the rapid calculation model of engine-cutoff state uncertainty caused by ISEs.

Dr Wuxing Jing received MS and PhD from Harbin Institute of Technology in 1989 and 1994, respectively. He was a visiting professor at University of Glasgow, UK, from 2000 to 2001. He is currently a professor and PhD supervisor in Harbin Institute of Technology. His research interests are autonomous navigation, nonlinear guidance, dynamics and control of spacecraft.
In the current study, he performed and analysed the influence mechanism of ISEs on norminal trajectory, navigation trajectory and guidance trajectory.

Dr Changsheng Gao received MS and PhD from Harbin Institute of Technology in 2004 and 2007, respectively. His research interests are nonlinear guidance, dynamics and control of spacecraft.
In the current study, he performed and analysed the simulation results