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Radar Signal Recovery using Compressive Sampling Matching Pursuit Algorithm

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ABSTRACT

In this study, we propose compressive sampling matching pursuit (CoSaMP) algorithm for sub-Nyquist based electronic warfare (EW) receiver system. In compressed sensing (CS) theory time-frequency plane localisation and discretisation into a N×N grid in union of subspaces is established. The train of radar signals are sparse in time and frequency can be under sampled with almost no information loss. The CS theory may be applied to EW digital receivers to reduce sampling rate of analog to digital converter; to improve radar parameter resolution and increase input bandwidth. Simulated an efficient approach for radar signal recovery by CoSaMP algorithm by using a set of various sample and different sparsity level with various radar signals. This approach allows a scalable and flexible recovery process. The method has been satisfied with data in a wide frequency range up to 40 GHz. The simulation shows the feasibility of our method.

Keywords: Compressed sensing; Compressive sampling matching pursuit; Electronic warfare; Signal recovery

1. INTRODUCTION

Electronic warfare (EW) receiver systems are designed in wide or narrow band configurations with operating frequency range 0.5 GHz to 40 GHz to capture the electromagnetic scenario in real time with high probability of intercept. Current EW receiver systems work on digital receiver technology with Nyquist sampling, enabling digital processing by traditional signal processing software algorithms. There are two main reasons for improving the technology, First one is the Nyquist-rate for ultra wide band receivers, and is very large and gets limited by the speed of the analog to digital converter (ADC); and second, even if it is possible to sample a large bandwidth, the resulting digital dataset is very high; and this requires an enormous amount of transmission power to transmit for remote sensors applications[1].

The instantaneous bandwidth of digital receiver depends on sampling rate of ADC and must be wide enough in order to improve the probability of intercept. Modern signal processing approach of Sub-Nyquist sampling employs the techniques of compressed sensing theory and can be used for radar signal acquisition and reconstruction. CS theory implementation using random modulator pre-integrator (RMPI) is optimally matched for sparse signals. Sparsity is the model as a form of regularisation for taking advantage of reduced dimensionality to restrict or control the set of coefficient values which are allowed to produce an estimate of the data. Sparse signals can be represented as linear combinations of a small number of elementary functions belonging to a larger collection, or dictionary of functions.

Compressed sensing theory using RMPI states that a receiver signal can be sampled randomly without loss at a rate close to its information content and recover with stable recovery mechanism in the presence of noise and corruption. The RMPI provides with increased bandwidth and resolution without utilising superior ADCs below Nyquist rate. In this paper recovery and reconstruction of received radar signal from RMPI samples using advanced non linear algorithms (CoSaMP) is discussed. The simulated result of reconstructed signal is also presented.

2. RANDOM MODULATOR PRE-INTEGRATOR THEORY

The RMPI is a parallel architecture, performs three basic functions i.e. demodulation, low pass filtering and low rate sampling. These functions are implemented using a multiplier, integrator and low rate sampling ADC. However single channel RMPI provides the information required in digital form. The parallel channels are necessary to obtain the additional information like geometric and structural properties of the signal by projecting to union of subspaces [2, 3, 4]. The block diagram of EW receiver front end single channel configuration using Sub-Nyquist sampling is given in Fig. 1.

The input signal $x(t)$ is received through a Front End Amplifier (FEA) and antenna, where $x(n)$ is digitised version of $x(t)$. The RMPI first stage multiplies the input RF signal $x(t)$ and random discrete time sequence $P(n)$. The random discrete time sequence $P(t)$ of ± 1 with equal probability called as chipping sequence, which switches at Nyquist rate and spreads the spectrum over the entire band width, where $P(n)$ is digitised version of $P(t)$. The integration serves as a linear projector of

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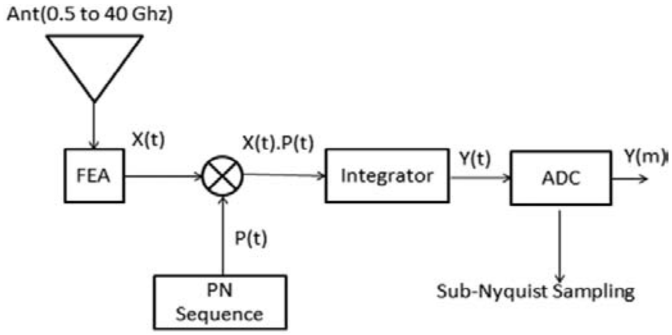

Figure 1. Block diagram of EW receiver front end.

randomly switched $x(t)$[5]. The integrator output can be sampled instantaneously every 'R' seconds, which is a fraction of the Nyquist rate to obtain a sequence of (compressing the signal's information) small number of measurements $Y(m) = (y_1, y_2, y_3, \ldots\ldots y_m)$ as defined in the matrix form

$$Y(m) = \begin{bmatrix} y_1 = & \varphi_{11}X_1 + \varphi_{12}X_2 + \ldots + \varphi_{1N}X_N \\ y_m = & \varphi_{M1}X_1 + \varphi_{M2}X_2 + \ldots + \varphi_{MN}X_N \end{bmatrix} \quad (1)$$

The sub-Nyquist sampled data $Y(m)$ is recovered by non linear signal recovery algorithm. The signal recovery algorithm uses restricted isometry (RIP) and coherence properties of measurement matrix as a measure of the signal. The recovery algorithm relies on linear programming and convex optimisation theory for extraction of sparse signal support and sparse coefficients required for signal reconstruction. Master clock driven control circuit is used to synchronise the process and reset the integrator after each frame.

3. RADAR SIGNAL RECOVERY

Any signal can be written as a weighted linear combination of sparse basis. Most radar signals are sparse in time and frequency domains and can be represented by a number of projections on a random basis. An s-sparse radar discrete-time signal vector $x(N\times 1)$ can be written as $x = \Psi\alpha$, where α is a sparse representation of coefficient vector x in the basis $\Psi(N\times M)$. The signal s-sparse means that x has s nonzero elements only[6].

At the receiver side it is critical to be able to reconstruct the original signal $x(t)$ from the compressed version. In this case a measurement matrix $\Phi(M\times N)$ and the output signal $Y(m\times 1)$ are collected such that: $Y = \Phi$x. The vector x needs to be recovered from the matrix Φ and measurement results $Y(m)$ with the constraint $M < N$. It is an undetermined system of equations, there are infinitely possible solutions for x signal. If the signal x is known to be sparse, then the sparsest solution would be acceptable. We can write the output signal $y = \Phi.x = \Phi.\Psi.\alpha$, where vector α should be determined and apply the constraint of sparsity on the vector α.

The RMPI receiver hardware digitises the randomly projected linear samples into a stream of compressed measurements m. The measurements m contain signals $x(n)$ drawn from one of a collection of low-dimensional subspaces (Sp) indexed by a parameter set $p = (p_1; p_2; : : : ; p_K)$. The collection of m measurements corresponding to d dimensional signal can be expressed mathematically using a measurement matrix Φ of size $m \times d$, containing the inner products between each pair of RMPI measurement vector $\Phi(n)$ in columns and basis functions $x(n)$ in rows. The first step in recovery is to design a proper sensing matrix and a recovery algorithm specific to the designed sensing matrix.

The sensing matrix Φ is to be constituted by chipping sequence, sub sampling rate and impulse response matrix of integrator. The sensing matrix Φ is with restricted isometry constant $\delta 2s \leq c$. The RMPI output vector samples of an arbitrary signal, contaminated with arbitrary noise is given by $y = \Phi x$ + error, for a given y $\in R^m$, we want to recover an approximately s-sparse vector x $\in R^N$ under the assumption that the vector of unknown errors $\in R^m$, where $x[n]$ has the form.

$$x[n] = \mu(t_n - \tau_0).\, A_0 \cos(2\Pi f_0 tn + \theta_0); \quad t_n = n/fnyq; \quad n = 1; \ldots.\, N; \quad (2)$$

The signal $x(n)$ is defined on the interval [0,T]. The instantaneous time samples are defined as $t_{n.}$ The RMPI output at the end of each time interval T after low rate sampling is given by

$$y(m) = \sum_{n=0}^{m} y(t)\delta(n - mT) = Y(mT) \quad (3)$$

For a given precision parameter η, the algorithm CoSaMP produces an s-sparse approximation $x(n)$ that satisfies Eqn. (3)

$$\| x(n) - x(n) \| 2 \leq C \,.\, \max\{\eta, (1/\sqrt{s}) \| x(n) - x(n)s/2 \| 1 + \| e \| 2 \quad (4)$$

where $x(n)s/2$ is a best $(s/2)$-sparse approximation to $x(n)$.

The $Y(m)$ samples are s-sparse and infinitely many solutions are possible. Finding the most suitable solution is difficult to reconstruct the original input signal for optimum solution by CS recovery algorithms. The sensing is carried out in discrete signal model by computing the linear projection $y(m) = \Phi(n)x(n)$ with a set of radar parameters corresponding to the subspace (Sp) will be searched. The subspace (Sp) contains a signal which comes closest to explaining the measurements $y(m)$. In the subspace no two s-sparse vectors can be mapped to the same low-dimension vector. The ambient data dimension is large (N) in many applications, the relevant information typically resides in a much lower dimensional space.

The block diagram of recovery and reconstruction of CS based receiver signal is as shown in Fig. 2. The information required for successful signal recovery is that the reconstruction algorithm uses the matrix Φ, the dictionary matrix Ψ and input compressed samples $Y(m)$ to produce the output vector α. The CS based signal recovery algorithm expands the RMPI output sequence $y(m)$ and divides the sequence into frames (small set of orthogonal vectors). The algorithm identifies the signal space (non zero terms) from the frame, discard the zero vectors and form the set of signal support vectors with higher amplitude. The transformation or dictionary matrix Ψ contain reference set of multiple signals i.e. elementary functions belong to a larger collection or dictionary of functions. The sampling matrix Φ contains the incoherent base vectors used for acquisition of the signal. The convex optimisation block multiply the sampling matrix and dictionary matrix (Column by row) to produce the sparse signal reconstruction matrix with base vectors $\tilde{A}$ exploiting the fact that $Y(m)$ will be sparse in the frame ($P=\Psi\Phi$) and the correct representation of $\tilde{A}$ that

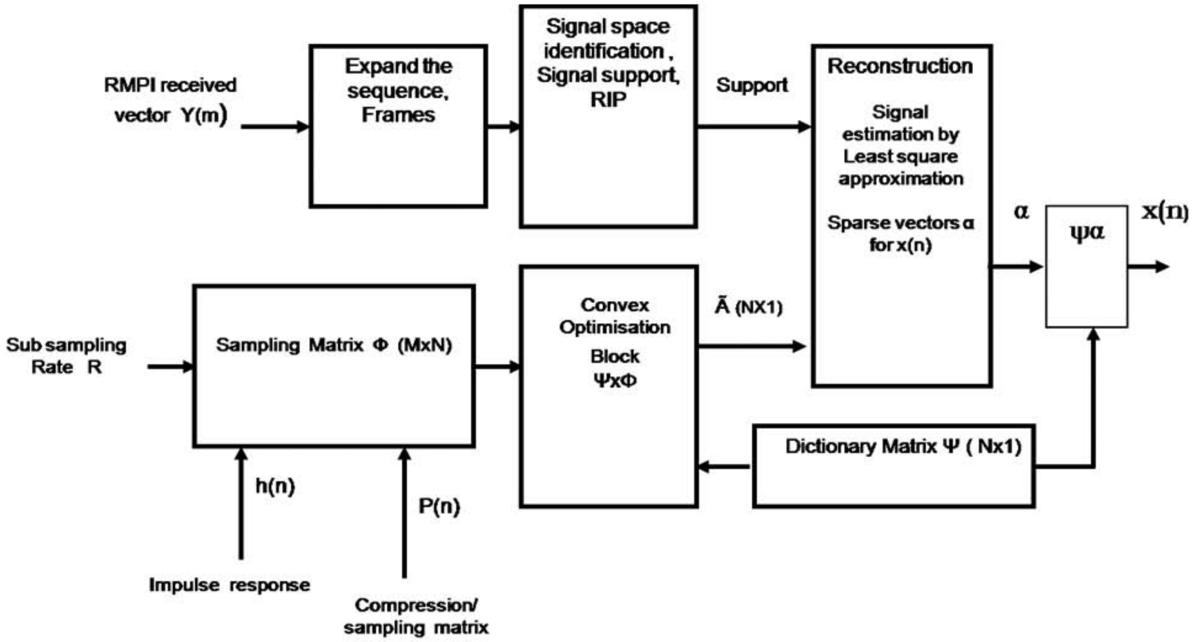

Figure 2. Recovery and reconstruction of CS based receiver signal.

yields our data $x = \Psi\alpha$. In CS theory the following equality for solving linear inverse problem is defined for reconstructing a signal from its compressed measurements $Y(m)$.

$$Y = \Phi x = \Phi\,\Psi\,\alpha = P\,\alpha; \quad P = \Phi\,\Psi; \tag{5}$$

where Φ and P are $M \times N$ measurement matrices in signal and transform domain respectively. Where $M << N$. Applying simple matrix inversion or inverse transformation techniques on compressed measurements $y(m)$ does not result in a sparse solution.

4. RECOVERY ALGORITHMS

The compressed samples collected from input signal with all its characteristics contain orthogonal bases very important to recover them. Since the collected samples are very few the recovery algorithms should be intelligent enough to recover the information. The recovery algorithms are categorised into 3 main groups, which are ℓ1-minimisation, greedy, and combinatorial algorithms. The most difficult part of signal reconstruction is to identify the locations of the largest components in the radar received signal achieving the stability, guarantees and fast reaction time. Other important algorithm properties such as storage requirements, ease of implementation and flexibility have to be compared for other applications. The main signal recovery algorithms used in compressed sensing are ℓ1-minimisation, orthogonal matching pursuit, iterative thresholding, regularised OMP, compressive sampling matching pursuit (CoSaMP) and subspace matching pursuit. Most of these methods calculate the support of the signal iteratively. Compute the quality of the output signals with error computation $||\Phi x - x||_2$ in approximating the samples and reflecting the actual approximation error[7,8,9].

4.1 Restricted Isometry Property

When the measurements are contaminated with noise or have been corrupted by some error such as quantisation, it will be useful to consider stronger conditions. Restricted isometry property (RIP) preserves the structure of the sparse and compressible signals. Matrix A satisfies RIP approximately preserves the distance between any pair of s-sparse vectors. RIP can be defined as M be an mxd sensing matrix, Then for every integer s and $0 < \epsilon < 1$, RIP satisfies the $(s; \epsilon)$, Restricted Isometry Property, if for every s-sparse vector Ã we have

$$(1 - \epsilon)\|\tilde{A}\|_2 \leq \|M\tilde{A}\|_2 \leq (1 + \epsilon)\|M\tilde{A}\|_2 \tag{6}$$

where $\|.\|$ denotes the Euclidean norm

4.2 Coherence

The mutual coherence of the N-dimensional orthonormal bases Φ and Ψ is the maximum absolute value for the inner product between rows of Φ and columns of Ψ of the bases:

$$\mu(\Phi\Psi) = \max_{1 \leq i,\ j \leq N} \langle \Phi i, \Psi j \rangle \tag{7}$$

The parameter μ is the measure of coherence to be close to minimum value, each of the measurement vectors must be spread out in the Ψ domain. A small value of μ(ΦΨ) indicates that Φ and Ψ are incoherent with each other, i.e., no element of one basis (Ψ) has a sparse representation in terms of the other basis (Φ). When both Φ and Ψ are orthogonal, the minimum number of measurements is required for perfect reconstruction.

4.3 CoSaMP Algorithm

The CoSaMP algorithm invokes iteratively to approximate the received signal using the RIP and incorporates several other ideas from the literature to accelerate the algorithm and to provide strong guarantees. Suppose that the sampling matrix Φ has restricted isometry constant $\delta_{2S} < 0.1$, for an s-sparse signal x, the vector $y = \Phi^*\Phi x$ can serve as a proxy for the signal because the energy for s components is same in x and y. Since the samples have the form $y = \Phi x$, we can obtain the proxy just by applying the matrix Φ* to the sample.

Compressive sampling matching pursuit performs the following activities with RMPI output samples. In identification, the current approximation induces a residual from the current samples, used to construct a proxy for the residual and locates the largest components of the proxy. As the algorithm progresses in support merger set of newly identified components are united with the set of components that appear in the current approximation as a tentative support for next approximation. In estimation on tentative support set the algorithm use least-squares to estimate the approximation of the radar signal on the merged set of components. In pruning the algorithm produces a new approximation by retaining only the largest entries in this least-squares signal approximation. In sample update the samples are updated so that they reflect the residual, the part of the signal that has not been approximated. This process is repeated until the halting criterion is triggered i.e., until we have found the recoverable energy in the signal or not.

4.4 Subspace Matching Pursuit Algorithms

The subspace matching pursuit (SP) algorithm is very similar to CoSaMP algorithm .The CoSaMP and SP methods differ in the last step, the CoSaMP algorithm takes as a new estimate the intermediate estimate is restricted to the new smaller support set, while SP solves a second least-squares problem restricted to this reduced support.

The Table 1 compares different algorithms with their performance[5].

5. CoSaMP SIMULATION AND RESULTS

The algorithm requires the sparsity level s as part of its input. The first method is to deduce the sparsity level s from the number m of measurements from m = 2s log N/s. The second method is to run CoSaMP using range of sparsity levels and to select the best approximation obtained. To limit the time and cost vary sparsity level s along G.P, say s = 1,2,4,8....., m, which increases the runtime by a factor no worse than O (log m). The quality of the output signal y, compute the error $\|\Phi y-u\|_2$ in approximating the samples and reflecting the actual approximation error. A priori the signal has to be simulated for sparsity level for correct recovery using CoSaMP algorithm. The samples have been sorted from highest amplitude to lowest amplitude in pruning[14].

5.1 Ultra Wideband Pulse

The CoSaMP algorithm is simulated in MATLAB 2013b with Ultra wideband pulse. The total numbers of 480 samples are used and 150 measurements have been considered for simulation. It is observed that the input signal sparsity level 9 is to be defined and considered twice the sparsity level of samples for correct recovery. Input impulse and the recovered pulse of UWP is depicted in Fig. 3(a). The error between these two pulses is about[10-12] 6.0 x 10^{-3}.

5.2 Random Pulse Spike

The CoSaMP algorithm is simulated in MATLAB 2013b with random pulse spikes. The total number of 512 samples

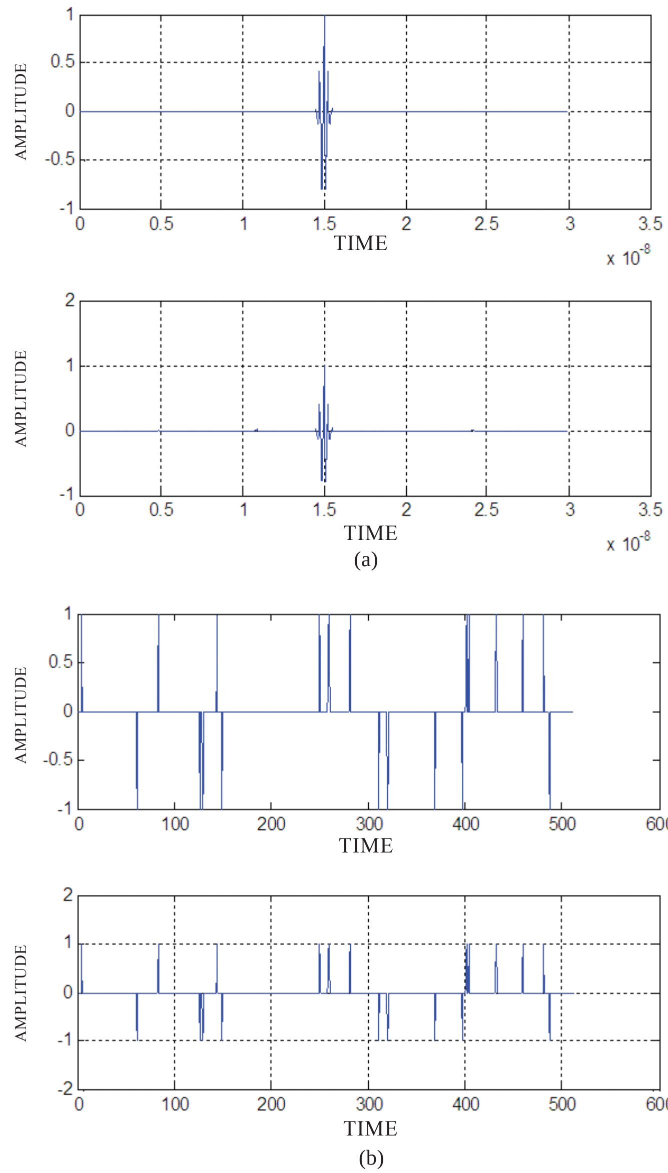

Figure 3. Original and reconstructed waveform of (a) UWP and (b) Random pulse spikes. [Units: Time(s), Amplitude(Volts)].

Table 1. Comparison of algorithm

Algorithm	Implementation/ Flexibility	Guarantees	Stability	Recovery time
ℓ1-norm minimisation	Simple	Uniform	Uniform	Slow, recover well at low sampling rate.
OMP	Simpler iterative	Not good	Far less stable	Fast, recovers with higher sampling rate.
ROMP	Iterative	Strong	Stable	Fast, compute second least-squares, restricted support vectors.
CoSaMP	Iterative	Strong	Stable	Fast

are used and 150 measurements have been considered for simulation. It is observed that the input signal sparsity level 20 is to be defined for correct recovery. The input pulse and the recovered pulse of random pulse spikes with error of 1.6089x10^{-15} is depicted in Fig. 3(b)[13].

5.3 Linear frequency Modulated Waveform

The CoSaMP algorithm is simulated in MATLAB 2013b with LFM waveform. The total number of 512 samples are used and 150 measurements have been considered for simulation. It is observed that the input signal sparsity level 50 is to be defined for correct recovery. The input pulse and the recovered pulse of LFM signal with error of 6.5 x 10^{-15}.is depicted in Fig. 4(a)[13].

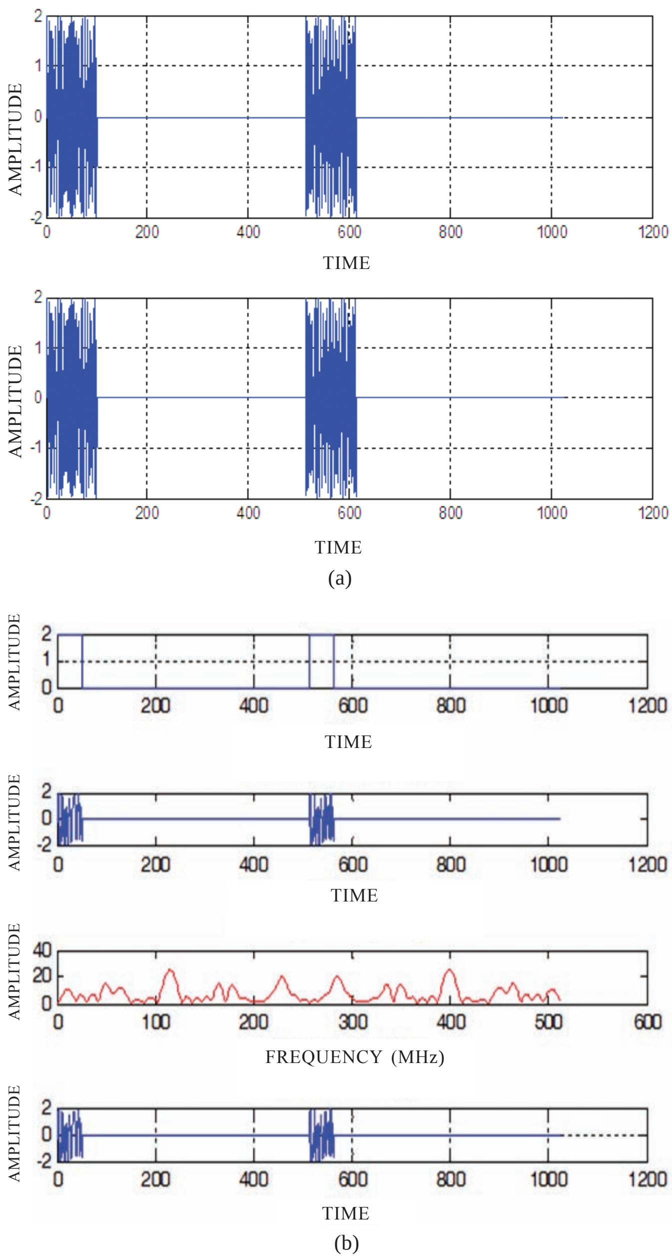

Figure 4. Original and reconstructed waveform of (a) Barker code pulse signals and (b) LFM pulse signals. [Units: Time(s), Amplitude(Volts)].

5.4 Barker Pulse Coded Waveform

The CoSaMP algorithm is simulated in MATLAB 2013b with barker coded pulse waveform. The total numbers of 512 samples are used and 200 measurements have been considered for simulation. It is observed that the input signal sparsity level 100 is to be defined for correct recovery. The input pulse and the recovered pulse of Barker coded signal with error of 3.22x10^{-14} is depicted in Fig. 4(b)[13,14].

6. CONCLUSION

The sub Nyquist sampling and compressed sensing based random modulator pre-integrator architecture eliminates high speed ADC technology. The RIP and coherent properties of the signal is exploited in recovery and reconstruction. The function of recovery and reconstruction algorithm is defined with comparative performance. The CS reconstruction algorithm CoSaMP is simulated in MATLAB 2013b with ultra-wideband pulse, random pulse spikes, LFM, Barker coded pulse waveforms and recovered successfully. The RMPI circuit implementation, recovery and reconstruction of the radar received signal be explored for use in EW receiver technology.

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