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Fusion of Redundant Aided-inertial Sensors with Decentralised Kalman Filter for Autonomous Underwater Vehicle Navigation

Vaibhav Awale[#] and Hari B. Hablani[*]

[#]*Department of Aerospace Engineering, Indian Institute of Technology Bombay, Mumbai - 400 076, India*
[*]*Department of Aerospace Engineering, Indian Institute of Technology, Gandhinagar - 382 424, India*
[*]*E-mail: hbhablani@iitgn.ac.in*

ABSTRACT

Most submarines carry more than one set of inertial navigation system (INS) for redundancy and reliability. Apart from INS systems, the submarine carries other sensors that provide different navigation information. A major challenge is to combine these sensors and INS estimates in an optimal and robust manner for navigation. This issue has been addressed by Farrell[1]. The same approach is used in this paper to combine different sensor measurements along with INS system. However, since more than one INS system is available onboard, it would be better to use multiple INS systems at the same time to obtain a better estimate of states and to provide autonomy in the event of failure of one INS system. This would require us to combine the estimates obtained from local filters (one set of INS system integrated with external sensors), in some optimal way to provide a global estimate. Individual sensor and IMU measurements cannot be accessed in this scenario. Also, autonomous operation requires no sharing of information among local filters. Hence a decentralised Kalman filter approach is considered for combining the estimates of local filters to give a global estimate. This estimate would not be optimal, however. A better optimal estimate can be obtained by accessing individual measurements and augmenting the state vector in Kalman filter, but in that case, corruption of one INS system will lead to failure of the whole filter. Hence to ensure satisfactory performance of the filter even in the event of failure of some INS system, a decentralised Kalman filtering approach is considered.

Keywords: Decentralised Kalman filter, inertial navigation, redundant navigation, autonomous underwater vehicle navigation

NOMENCLATURE

p	AUV position vector
v_e	Velocity reative to earth
a	Acceleration
f	Specific force vector
g	Gravity vector
ω	Angular rate vector
θ	Tangent top late form frame Euler angle three-tuple
b_a	Accelerometer r bias vector
b_g	Gyro bias vector
φ,θ,ψ	Roll, pitch, and yaw Euler angles
ϕ	Latitude
R_b^a	transformation matrix from coordinate system a to b

SUBSCRIPTS

t = tangent
p = platform
e = earth fixed frame

1. INTRODUCTION

The current major oceanic vehicles carry more than one set of inertial navigation system (INS). The primary reason to have multiple INS systems is to provide reliability in the event of failure. However before a failure occurs, navigation solution from different INSs may be fused to obtain a better estimate of position, velocity and attitude.

The submarine carries different navigation systems, each of them providing their optimal estimate of vehicle's navigation state. There is no luxury of having access to individual sensor measurements. Only the navigation system and their navigation solution are available. The main objective is fusing these different navigation solutions from different navigation systems to get best estimate of navigation solution. This leads us to the design of a decentralised Kalman filter, in which local filters provide local estimates and a master filter is used to combine all these local estimates to arrive at an optimal global estimate.

Study of multiple INSs systems for an oceanic vehicle is done by Christopher[5], *et al.* Development of a robust algorithm to account for delay in operation of acoustic sensors in an extended Kalman filter is done by Miller & Farrell[1]. The analysis presented herein is adopted from Farrell[2].

2. KINEMATIC MODEL

The platform frame represents a frame attached to the vehicle, and the IMU is mounted on the platform.

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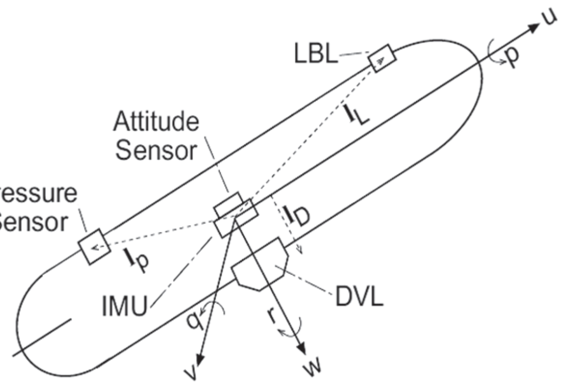

Figure 1. AUV sensor configuration depicting body frame *u*, *v*, and *w* axes; and sensor offsets[2] l_D, l_L, l_P.

The matrix $\Omega_{ab}^{c} = \left[\omega_{ab}^{c} \times\right]$ represents the skew symmetric form of ω_{ab}^{c} . ω_{ab}^{c} is angular velocity of frame *b* with respect to frame a as represented in frame *c*.

AUV will be manoeuvring locally in a ship relative coordinate system defined by LBL system, so we select a fixed tangent frame implementation[2]. The equations of motion are given by

$$\dot{p}^{t} = R_{p}^{t} v_{e}^{p} \tag{1}$$

$$\dot{v}_{e}^{p} = f^{p} + g^{p} - (\Omega_{ie}^{p} + \Omega_{ip}^{p}) v_{e}^{p} \tag{2}$$

$$\dot{\Theta} = \Omega_{E}^{-1} \omega_{tp}^{p} \tag{3}$$

$$\Omega_{E} = \begin{bmatrix} 1 & 0 & -\sin(\varphi) \\ 0 & \cos(\varphi) & \sin(\varphi)\cos(\theta) \\ 0 & -\sin(\varphi) & \cos(\varphi)\cos(\theta) \end{bmatrix} \tag{4}$$

In Eqn (1), position is found by integrating velocity with respect to earth fixed frame represented in tangent frame. Equation (2) gives the acceleration of platform frame with respect to earth fixed frame considering the Coriolis acceleration term. Equation (3) is used to find Euler angles from angular rate. The rotation sequence considered here is yaw-pitch-roll.

3. LOCAL FILTER

In this section we describe how the navigations state is estimated using an error state formulation of the Kalman filter given by Farrell[1], *et. al.* The IMU measurements are integrated continuously to obtain position, velocity and attitude, which are then corrected by aiding sensor measurements as and when they are available.

3.1 Inertial Measurement Unit

The inertial measurement unit (IMU) provides measurements related to the acceleration and angular rates of the AUV. The IMU outputs are reliably available at a known fixed rate. The IMU outputs will be integrated through the kinematic model to provide an estimate of the state of the AUV which includes attitude, velocity, and position. Due to uncertainty in the initial conditions and imperfections in the IMU, the INS estimate of the AUV state is also imperfect. The other on board sensor signals will be used, in complementary filter architecture, to correct the INS state estimate.

The IMU outputs are compensated for scale factor, temperature, and non-orthogonality by the manufacturer; therefore, the gyro and accelerometer measurements are modelled as[2]:

$$u_{a} = f^{p} + b_{a} + \eta_{a} \tag{5}$$

$$u_{g} = \omega_{ip}^{p} + b_{g} + \eta_{g} \tag{6}$$

where is b_a the accelerometer bias, $\eta_a \sim N(0, \sigma_g^2 I)$ ~ represents accelerometer measurement noise (Irepresents an identity matrix), ω_{ip}^{p} is the angular rate of the gyro relative to the inertial frame represented in platform frame, b_g represents the gyro bias, $\eta_g \sim N(0, \sigma_g^2 I)$ represents gyro measurement noise, and f^p is the specific force vector in platform frame. The IMU measurements are considered to be available at 100 Hz.

The bias vectors b_a and b_g are modelled as random constants plus random walks:

$$\dot{b}_{a} = w_{a} \tag{7}$$

$$\dot{b}_{g} = w_{g} \tag{8}$$

The driving noise vectors w_a and w_g are distributed according to $N(0, \sigma_{ba}^2 I)$ and $N(0, \sigma_{bg}^2 I)$, respectively.

$$\sigma_{a} = 0.01 \frac{m}{s\sqrt{s}}$$

$$\sigma_{ba} = 1 \times 10^{-4} \frac{m}{s^{2}\sqrt{s}}$$

$$\sigma_{g} = 0.116 \times 10^{-3} \frac{rad}{\sqrt{s}}$$

$$\sigma_{bg} = 1 \times 10^{-6} \frac{rad}{s\sqrt{s}}$$

3.2 Mechanisation Equations

The state vector for local filter consists of position, velocity, attitude, accelerometer and gyro bias and speed of sound. Speed of sound under water for this purpose is modelled as a first order Markov process as given in[1].

Given the kinematic equations summarised in Eqns (1)-(4), the inertial navigation system will propagate the state estimate through time using the equations[1]:

$$\dot{\hat{p}}^{t} = \widehat{R_{p}^{t}} \widehat{v_{e}^{p}} \tag{9}$$

$$\dot{\hat{v}}_{e}^{p} = \widehat{a_{p}} + \widehat{R_{t}^{p}} \widehat{g^{t}} - \left(\widehat{\Omega_{ie}^{p}} + \widehat{\Omega_{tp}^{p}}\right) \widehat{v_{e}^{p}} \tag{10}$$

$$\dot{\hat{\Theta}} = \widehat{\Omega_{E}^{-1}} \widehat{\omega_{tp}^{p}} \tag{11}$$

$$\dot{\hat{b}}_{a} = 0 \tag{12}$$

$$\dot{\hat{b}}_{g} = 0 \tag{13}$$

$$\dot{\hat{c}} = 0 \tag{14}$$

3.3 Error State Equations

The error state equations represent the expected value of the error between the true system and its estimate $\delta\dot{x} = \dot{x} - \dot{\hat{x}}$. The error state equations are derived in[1] and the final error state model is reproduced :

$$\delta\dot{x} = F\delta x + Gw \tag{15}$$

$$F = \begin{bmatrix} 0 & \widehat{R_p^t} & -\left[\widehat{v_e^t}\times\right] & 0 & 0 & 0 \\ F_{vp} & -\left(\widehat{\Omega_{le}^p} + \widehat{\Omega_{ip}^p}\right) & F_{vp} & -I & -\left[\widehat{v_e^p}\times\right] & 0 \\ F_{p\rho} & 0 & -\widehat{\Omega_{it}^t} & 0 & -\widehat{R_p^t} & 0 \\ & & 000000 & 0 & & \\ & & 000000 & 0 & & \\ & & 00000-\lambda_c & & & \end{bmatrix} \tag{16}$$

$$G = \begin{bmatrix} 0 & 0 & 0_0 & 0 \\ -I & -\left[\widehat{v_e^p}\times\right] & 0_0 & 0 \\ 0 & -\widehat{R_p^t} & 0^0 & 0 \\ 0 & 0 & I0 & 0 \\ 0 & 0 & 0I & 0 \\ 0 & 0 & 00 & 1 \end{bmatrix} \tag{17}$$

$$w = \left[\eta_a \eta_g w_a w_g w_c\right]^T$$

$$F_{vp} = \widehat{R_t^p}\left(\frac{\partial g^t}{\partial p} + \left[\widehat{v_e^t}\times\right]\left(\frac{\partial \omega_{ie}^t}{\partial p}\right)\Big|_{\hat{p}}\right) \tag{18}$$

$$F_{vp} = \widehat{R_t^p}\left(\left[g^t\times\right] + \omega_{ie}^t\left(\widehat{v_e^t}\right)^T - \left(\omega_{ie}^t\right)^T \widehat{v_e^t} I\right) \tag{19}$$

$$F_{\rho\rho} = \omega_{ie}\begin{bmatrix} \sin(\phi) \\ 0 \\ \cos(\phi) \end{bmatrix}\frac{\partial\phi}{\partial p} \tag{20}$$

3.4 Time Propagation

The error state and covariance matrix, $P^- = E\left\{\delta x^-, \delta x^{-T}\right\}$ are propagated by discretizing above mentioned error state equations:

$$\delta x_{k+1}^- = \Phi_k \delta x_k^+ \tag{21}$$

$$P_{k+1}^- = \Phi_k P_k^+ \Phi_K^T + Q_{dk} \tag{22}$$

3.5 Time Update Equations

The navigation state is not updated during the LBL interrogation period and the error state is accumulated using the time propagation equation. Once all LBL measurements arrive the navigation state is corrected using the error state and then the error state is set to zero. The update equations are Kalman filter equations given in[1,2].

$$K_k = P_k^- H_k^T \left(H_k P_k^- H_k^T + R_k\right)^{-1} \tag{23}$$

$$\delta y_k = y_k - h(\hat{x}_k^-) \tag{24}$$

$$\delta x_k^+ = \delta x_k^- + K_k(\delta y_k - H_k \delta x_k^-) \tag{25}$$

$$P_k^+ = (I - K_k H_k) P_k^- \tag{26}$$

$$\hat{x}^+ = \hat{x}^- + \delta x^+ \tag{27}$$

$$\widehat{R}_t^p(\widehat{\Theta}^+) = \widehat{R}_t^p(\widehat{\Theta}^-)\left(I - [\delta\rho\times]\right) \tag{28}$$

3.6 Aiding Sensor Models

The aiding sensors used for the autonomous underwater vehicle are magnetometer for azimuth angle aiding, Doppler velocity log for velocity aiding, pressure sensor for depth aiding, and long baseline transponder (LBL) sensor for position aiding. The aiding sensors are integrated with IMU measurements to obtain estimate of position, velocity and attitude. An error state formulation of the Kalman filter is used for this purpose as given in[1]. We follow the same approach here for formulating the local filters.

3.7 Attitude Update

The attitude sensor consists of three magneto resistive magnetic sensors, and a liquid filled two-axis tilt sensor. The raw measurements are normalised, linearised, and filtered internally to produce roll, pitch, and yaw information at 10 Hz. The sensor model is

$$y_e = \Theta + n_e \tag{29}$$

where sensor noise is distributed according to $N(0, diag(\sigma_\varphi^2, \sigma_\theta^2, \sigma_\Psi^2))$ with $\sigma_\varphi \approx 0.02\,rad, \sigma_\Psi \approx 0.1\,rad$

Predicted measurement is,

$$\hat{y}_e = \widehat{\Theta} \tag{30}$$

Also the measurement sensitivity matrix is given by,

$$H = \left[0\nabla^{-1}000 \quad 0\right] \tag{31}$$

where,

$$\nabla = \begin{bmatrix} \cos\theta\cos\Psi & -\sin\Psi & 0 \\ \cos\theta\cos\Psi & \cos\Psi & 0 \\ -\sin\theta & 0 & 1 \end{bmatrix}$$

The measurement noise matrix is given by:

$$R = E\left\{n_e n_e^T\right\} = diag(\sigma_\varphi^2, \sigma_\theta^2, \sigma_\Psi^2) \tag{32}$$

3.8 Doppler Velocity Log Update

The Doppler velocity log (DVL) emits encoded pulses from four transducers. The instrument measures the frequency shift of the reflected pulses to determine the relative velocity between the transducer head and the reflecting surface along each beam direction.

Let b_i for $i = 1...4$ denote a unit vector in the effective direction of the i-th transducer head. These directions are known in the platform frame. The i-th Doppler measurement is[2]

$$y_{Di} = (v_e^p + \omega_{ep}^p \times l_D^P)^T b_i^p + \eta_{Di} \tag{33}$$

where the reflecting surface is assumed to be stationary sea floor $\eta_{Di} \sim N(0, \sigma_{Di}^2)$, and l_D is the offset vector from the platform frame origin to the DVL transducer head $\sigma_{Di} \approx 0.021\ m/s$.

The measurement sensitivity matrix is given as,

$$h_{Di} = \left[0\left(b_i^p\right)^T 00\left(b_i^p\right)^T\left[l_D^p\times\right] \quad 0\right] \tag{34}$$

Measurement noise matrix is given by,

$$R_{Di} = \left(b_i^p\right)^T \left[l_D^P \times\right] \sigma_g^2 I \left[l_D^P \times\right]^T \left(b_i^p\right) + \sigma_{Di}^2 \quad (35)$$

3.9 LBL Update

The working of acoustic long baseline systems that provides round trip travel times from known locations is given by Farrell[1-2]. The corresponding measurement sensitivity matrix and measurement noise matrix are reproduced below:

$$y_{Li} = \frac{1}{c(t_0)} \| p_i - S(t_0) \| + \frac{1}{c(t_1)} \| s(t_i) - p_i \| + T_i + \eta_{Li} \quad (36)$$

where $c(t)$ is the speed of sound in water at time t, $\eta_{Li} \sim N\left(0, \sigma_L^2\right)$, and the AUV transceiver location $S(t)$ and vehicle position $p(t)$ are related by

$$S^t(t) = p^t(t) + R_p^t(t) l_L^P \quad (37)$$

$$h_{Li} = \left[D_i\left(t_0\right)\Phi\left(t_0, t_i\right) + D_i\left(t_i\right)\right] \quad (38)$$

$$D_i\left(t\right) = \left[\frac{\partial d_i}{\partial p}|_{\hat{x}(t)} \; 0 \frac{\partial d_i}{\partial p}|_{\hat{x}(t)} \; 00 \frac{\partial d_i}{\partial p}|_{\hat{x}(t)}\right] \quad (39)$$

$$d_i\left(t\right) = \frac{1}{c\left(t\right)} \| p_i - S\left(t\right) \| \quad (40)$$

$$R_{L_i} = \sigma_L^2 \quad (41)$$

3.10 Pressure Sensor Update

The measurement equations are given in[1].

$$y_p = s^T\left(p + R_p^t l_p^p\right) + b_p + \eta_p \quad (42)$$

where $s^T = \left[0, 0, s_p\right], s_p$ is a known scale factor, b_p is a known bias, l_p is the offset from the platform frame origin to the pressure sensor, and $n_p \sim N\left(0, \sigma_p^2\right)$. The measurement sensitivity matrix and the measurement noise matrix as given in[1] as reproduced below:

$$h_p = \left[s^T 0 - s^T \left[l_p^t \times\right] 00 \quad 0\right] \quad (43)$$

$$R_p = \sigma_p^2 \quad (44)$$

$$\sigma_p \approx 194 Pa$$

4. DECENTRALISED FILTER

Once estimates from the local filter consisting of one IMU unit and aiding sensors are available the next job is to combine these estimates from different filters. The state vector is truncated to only vehicle position, velocity and attitude for input to decentralised filter since the biases and speed of sound are used in correction of only local filters. When the state estimates and their associated covariances are known, they are used as inputs to decentralised Kalman filter which then updates its own states and the associated covariance matrix using the estimates from the local filter. The global estimates and their associated covariances are also propagated in time.

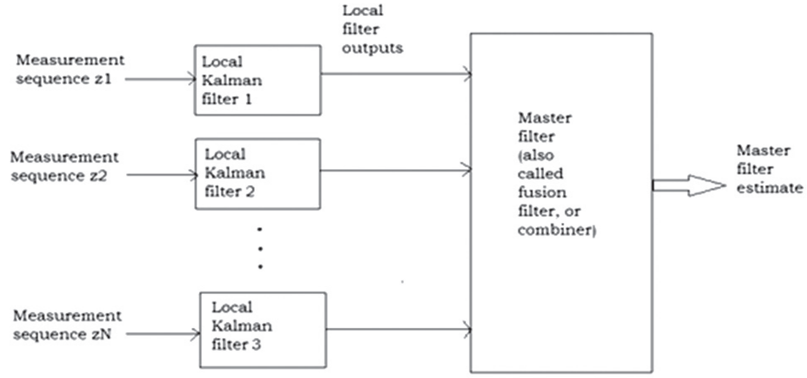

Figure 2. Decentralised Kalman filter with no feedback.

The global estimates thus obtained have lower covariance than variances of the local estimates. Also in the event of a failure of particular INS system, the covariance associated with those local estimates will be high, which can then be neglected or given a very low weight in computation of the global estimate. Thus this approach is shown to be robust to INS failures. In the paper, only two INS systems are considered for simulation, but the extension of the technique to more than two INS systems is evident.

Let

x_1= estimated state vector from local filter 1

x_2= estimated state vector from local filter 2

and let P_1 and P_2 be the associated covariance matrices respectively. Also let

m_1= state vector before measurement update from local filter 1

m_2= state vector before measurement update from local filter 2

and let M_1 and M_2 be the associated covariance matrices respectively.

Also for the global filter,

m = optimal estimate of x conditioned on both measurement streams up to but not including t_k.

M = covariance matrix associated with m.

In the decentralised filter the local filter estimates are treated as measurements, and the decentralised has its own global estimate which is propagated using mechanization equations. The global estimate of the decentralised filter is given in Brown and Hwang[3].

$$P^{-1} = \left(P_1^{-1} - M_1^{-1}\right) + \left(P_2^{-1} - M_2^{-1}\right) + M^{-1}$$

$$\hat{x} = P\left[\left(P_1^{-1}\hat{x_1} - M_1^{-1}m_1\right) + \left(P_2^{-1}\hat{x_2} - M_2^{-1}m_2\right) + M_2^{-1}m_2\right] \quad (45)$$

Local filters can pass their respective x_i, P_i^{-1}, m_i, M_i^{-1} on to the master filter which in turn can compute its global estimate. The local filters can do their own local projections and repeat the cycle at step k+1. Similarly master filter can project its global estimate and get a new m and M for next step.

5. SIMULATION RESULTS

The model for a single filter integrating DVL, LBL, and attitude and pressure sensors was developed and simulated using MATLAB and Simulink (R2012b). The sensor models

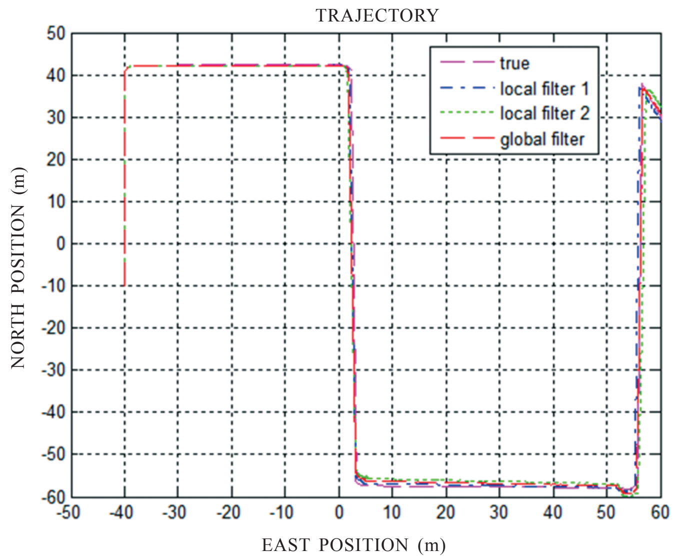

Figure 3. Trajectory of vehicle.

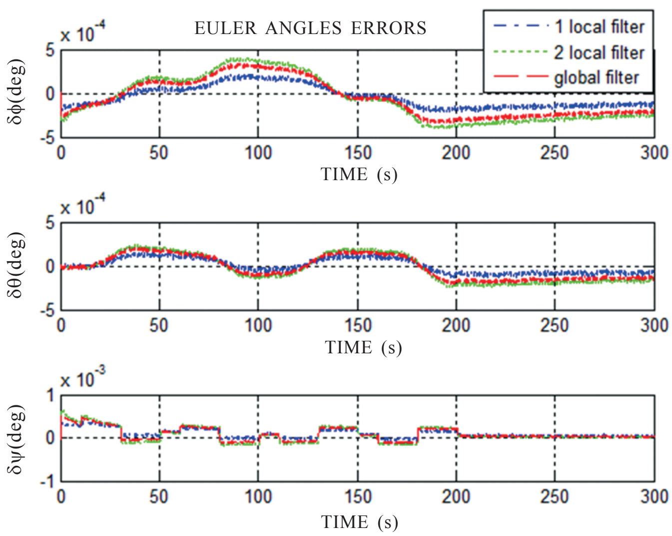

Figure 4. Euler Angle errors.

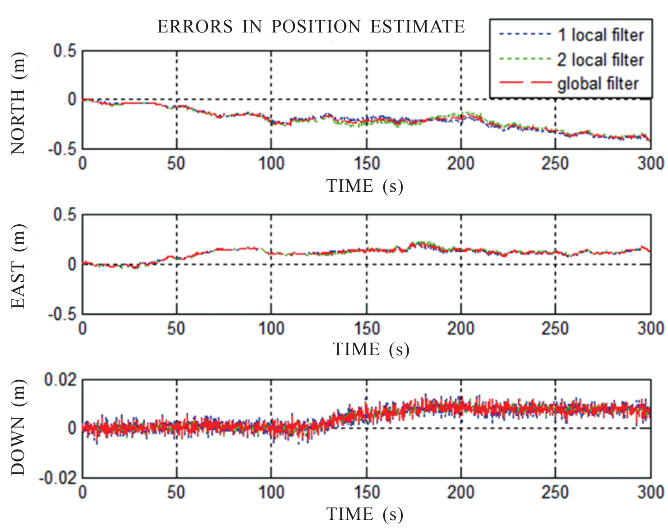

Figure 5. Position errors.

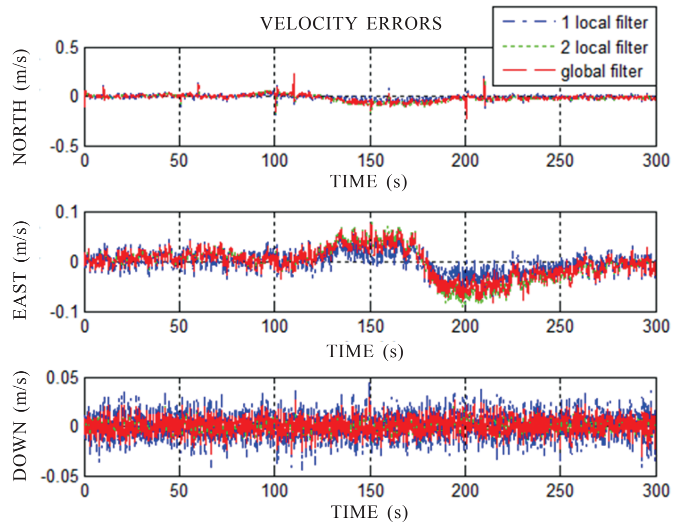

Figure 6. Velocity errors.

are similar to those presented above and in addition losses due to poor geometry and loss of line of sight are also considered.

The vehicle submerges 5 m in depth and executes a lawnmower search pattern. The trajectory is given in Fig. 3.

In the results we observe that estimate of decentralised filter follows the estimate of the local filter with least error. This is very helpful because if some local filter estimates are corrupt, the global filter with give it very less weight and the global estimate won't be affected much. The position, velocity and attitude errors are given in Figs. (4) to (6). As observed the estimate of decentralised Kalman filter is smoother than the local filter estimates. Also, the error is less.

6. CONCLUSIONS

It is observed that covariance and thus the uncertainty associated with global estimates is substantially lower than the covariance associated with the local estimates. Also, the global estimates tend to follow the best available local estimate and discard the local estimates with high uncertainty associated with them. High uncertainties in the local estimates indicate a sensor failure in that subsystem and hence the associated faulty sensor can be replaced quickly and further damage to navigation controlled. Thus this approach provides robustness to the system in terms of estimating and finding the faulty sensors quickly. Better fail safety techniques can also be implemented for quick detection of faults in the sensors.

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CONTRIBUTORS

Mr Vaibhav Awale is presently associated with Department of Aerospace Engineering, Indian Institute of Technology – Bombay, Mumbai.

Dr Hari B. Hablani received his MTech and PhD from Department of Aerospace Engineering, Indian Institute of Science, Bengaluru, India. Presently working as a Professor in the Department of Mechanical Engineering, Indian Institute of Technology –Gandhinagar, where he teaches and researches in the area of space flight navigation, guidance, and control. Earlier, he was a Professor in the Department of Aerospace Engineering, Indian Institute of Technology – Bombay and Kanpur during 2008-2014. He was with The Boeing Company (Rockwell International heritage) in Southern California, USA, during 1982 - 2008, where he was responsible for developing guidance, navigation, and control of spacecraft and strategic interceptors. He is a recipient of the Aeronautical Society of *India's Excellence in Aerospace Education Award*.