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Multiplicative Error State Kalman Filter *vs* Nonlinear Complimentary Filter for a High Performance Aircraft Attitude Estimation

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ABSTRACT

Modern control law designs increasingly use aircraft attitude information to improve aircraft manoeuverability. Attitude information allows for gravity term compensations in the longitudinal as well as lateral directional control laws of a typical fighter aircraft. Methodologies and comparisons of multiplicative error state Kalman filter (MEKF) and nonlinear complimentary filter for estimation of attitudes of a high performance aircraft using its onboard autonomous sensors is presented. Shows a problem in pitch angle estimation beyond ± 80 deg in the MEKF and a solution is proposed for the same for the first time. Also presents novel aiding sensor modelling for the implementation of attitude heading reference system for this class of aircraft for the first time. The filter formulations are evaluated using full range manuoevering real flight test data.

Keywords: Attitude estimation, high performance aircraft, airdata, estimation, nonlinear complimentary filter, Kalman filter

1. INTRODUCTION

Attitudes are used to define a vehicle's orientation in space. The attitude heading reference system (AHRS) is a unit that computes vehicle's attitude. The attitude information provided by AHRS is typically employed in various applications such as vehicle navigation, guidance and control. The problem of estimating the attitudes of a high performance aircraft is considered for applications like gravity compensation in control laws[1] is studied. There are literatures that discuss the AHRS extensively[2-3]. But most of them address the attitude estimation without considering manoeuvering flights. This study addresses formulations that can handle full range manoeuvering flights such as inverted loops and full rolls. During these manoeuvers, the gravity compensation through attitude information in the longitudinal and lateral control laws of a typical fighter aircraft achieves improved performance.

The attitude information can be computed in the form of Euler angles, direction cosine matrix (DCM), and quaternion. Though the Euler angle formulation is more intuitive, it is not the choice of this study as they involve nonlinear and computationally intensive trigonometric functions and the angles are subjected to singularity problems for some rotations such as the gimbal lock. On the other hand, the quaternion and direction cosine matrices (DCMs) are commonly employed in aerospace applications. Quaternion is widely used in majority of attitude estimators[4,5], as they involve fewer elements to store in onboard memory than a DCM, and they are much easier to normalise. Another advantage of quaternion (or DCM) over Euler angles is their propagation equations are linear with respect to the quaternion and only depend on the gyro measured angular velocity. This leads to effective use of the angular rate sensors.

Therefore at the heart of AHRS discussed in this work, it is the three axis gyro whose measurements are used to propagate or predict the attitude dynamics in the form of Euler angles, DCM[6-7] or quaternion[3,4]. i.e., The gyro measurements are integrated to compute the attitudes. Since the gyro measurements contain random noise, bias and drifts, one cannot obtain the attitudes only using integration. The errors in the attitudes are then estimated using filters with aiding measurements. So the challenge is to model the aiding measurements within the filter framework, which arrests the growth of errors in attitudes. The formulations addressed in this study make use of only autonomous aiding sensors such as accelerometers, magnetometers and air data sensors. The global positioning system is not considered.

Two filter formulations are considered in this study. The multiplicative error state Kalman filter (MEKF) in the quaternion formulation and the nonlinear complimentary filter (NCF) in the DCM formulation. The Kalman filter formulation discussed in this study is free from singularity issues associated with covariance matrix. Primary motivation of this study is to explore MEKF and NCF mechanisations for full range manoeuvering flights using autonomous sensors. Although this form of the filters have been used in space applications but no literature is found particularly for dealing with highly manoeuvering flights. Especially, a problem faced in pitch angle estimation beyond ± 80 deg in the MEKF formulation

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is never discussed in any literature. Also, sensor matrix for autonomous attitude estimation is discussed for the first time considering the MEKF and NCF formulations. The filters are evaluated using simulated data and validated with real flight test data and conclusions are arrived. To validate the results comparisons are carried out against standard aided inertial navigation system called IN-GPS yielded attitudes. IN-GPS is connected to the avionics computer of the aircraft under consideration. This system offers attitudes with an accuracy of ±0.5 deg. Comparison of proposed AHRS results with standard IN-GPS is valid as IN-GPS is highly accurate in its formulation and the sensors used by the system. It uses GPS position and velocities as aiding measurements to arrest the error growth in attitudes. It is implemented in tightly coupled mechanism. Hence it is very accurate. However the disadvantage of IN-GPS is that it is not autonomous as it depends on GPS. The AHRS on the other hand is autonomous. Most aircraft will have both IN-GPS and AHRS for redundancy management. This study deals with AHRS formulation that can handle full range manoeuvers and the main contributions are:

- Design and implementation of filter methodologies for full range manoeuvers in the AHRS formulation using autonomous sensor matrix.
- A solution to overcome the problem of pitch angle estimation beyond ±80 deg.
- Comparison of MEKF with NCF.

2. MULTIPLICATIVE EXTENDED KALMAN

A quaternion $q=\left[q_0 \quad q_1 \quad q_2 \quad q_3\right]^T$ is an orientation quaternion which represents an orientation relative to a reference coordinate frame. In MEKF, the true quaternion q is represented as the quaternion multiplication of the estimated $\hat{q}$ quaternion and a small error quaternion δq.

$$q=\hat{q}\otimes\delta q \tag{1}$$

The foregoing definition is not common in other Kalman filtering problems, which typically model the error terms as additive. The additive error assumption is discussed by Beard & Mclain[8]. Adding two unit quaternion together does not produce another unit quaternion, which creates a problem that is normally dealt with by unwarranted renormalisation. Moreover the error covariance matrix becomes singular/undefined[3]. The multiplicative error formulation also has the physical meaning of providing a small rotation correction. This can easily be shown in the formulation of the composite DCM. C_n^b is a rotation matrix and describes the orientation of coordinate frame *n* with respect to the coordinate frame *b*. Further, rotation matrix can be written in terms of quaternion as

$$C_n^b(q)=C_n^b(\hat{q}\otimes\delta q)=C_n^b(\delta q)C_n^b(\hat{q})$$

$$C_n^b(q)=\begin{bmatrix} 2q_0^2+2q_1^2-1 & 2(q_1q_2+q_0q_3) & 2(q_3q_1-q_0q_2) \\ 2(q_1q_2-q_0q_3) & 2q_0^2+2q_2^2-1 & 2(q_3q_2+q_0q_1) \\ 2(q_3q_1+q_0q_2) & 2(q_3q_2-q_0q_1) & 2q_0^2+2q_3^2-1 \end{bmatrix} \tag{2}$$

Problem posed in this section is to estimate quaternion. The dynamics of quaternion is described by:

$$\dot{q}=\frac{1}{2}\omega\otimes q \tag{3}$$

where ω is the angular velocity and $\otimes$ represents quaternion multiplication.

This formulation uses an accelerometer and magnetometer as aiding measurement units. Typically for all the accelerated flights, modelling of the linear accelerations and centripetal acceleration terms (due to curved path of vehicle) are essential. These necessacitates velocity measurements from GPS or air data sensors. In this formulation, even for such flights, it is shown that the gravity vector and magnetic vector are the only aiding measurements.

The sensors used are modelled as follows:

The accelerometers measure the specific acceleration in the Body-Fixed Frame which is represented as

$$y_{acc}=C_n^b(q)\tilde{g}+b_{acc}+v_{acc} \tag{4}$$

$$y_{gyro}=\omega+b_{gyro}+v_{gyro} \tag{5}$$

$$y_{mag}=C_n^b(q)\tilde{m}+v_{mag} \tag{6}$$

where $\tilde{g}=[0 \quad 0 \quad g]^T$ and $\tilde{m}=[\cos\alpha \quad 0 \quad -\sin\alpha]^T$; g is the gravitational acceleration and α is the dip angle[9]. Sensor noises v_{acc}, v_{gyro} and v_{mag} are assumed to be zero mean white Gaussian noises and their variances are defined as follows:

$$E\left\{V_{acc}(t)V_{acc}(s)'\right\}=R_{acc}\delta(t-s) \tag{7}$$

$$E\left\{V_{gyro}(t)V_{gyro}(s)'\right\}=R_{gyro}\delta(t-s) \tag{8}$$

$$E\left\{V_{mag}(t)V_{mag}(s)'\right\}=R_{mag}\delta(t-s) \tag{9}$$

$$E\left\{V_{acc}(t)V_{gyro}(s)'\right\}=0,\; E\left\{V_{acc}(t)V_{mag}(s)'\right\}=0,\; E\left\{V_{gyro}(t)V_{mag}(s)'\right\}=0 \tag{10}$$

Accelerometer bias b_{acc} and gyro bias b_{gyro} are assumed to be constants. The accelerometer biases are not estimated as the sensors are limited in number and the bias states are unobservable. The gyro biases could be modelled as time varying first order Gauss Markov process if the sensors have higher drift rates. But for the class of aircraft discussed in the present study, sensors don't exhibit very high drift rates and constant bias model is found to be sufficient[1].

The process noise, measurement noise and filter covariance matrices for the filter are chosen as follows:

$$Q=diag\left(\left[1e-7\ 1e-7\ 1e-7\ 1e-13\ 1e-13\ 1e-13\right]\right);$$
$$R=diag\left(\left[1\ 1\ 1\ 0.000001\ 0.000001\ 0.000001\right]\right);$$
$$Phat=diag\left(\left[0.001\ 0.001\ 0.001\ 1e-7\ 1e-7\ 1e-7\right]\right);$$

where diag represents a diagonal matrix. Using quaternion algebra and sensor model that is discussed above, the indirect Kalman filter is formulated for attitude estimation problem.

2.1 Indirect Kalman Filter formulation

The objective of this filter is to estimate quaternion from y_{acc}, y_{gyro} and y_{mag} for full range manoeuvering flights. The filter is formulated in indirect Kalman filter formulation.

The first step of the filter is to compute quaternion:

$$\dot{\hat{q}} = \frac{1}{2} y_{gyro} \otimes \hat{q} \tag{11}$$

Because of the error in y_{gyro}, $\hat{q}$ also have orientation errors. δq is introduced to represent small errors in $\hat{q}$ and shown in Eqn. (1). It is to be noted that δq does not depend on angular velocity but it depends on gyro bias and noise, which is assumed to be small. Hence even if the rotations are large, the assumption of δq being small is valid. δq can be approximated as follows:

$$\delta q \approx \begin{bmatrix} 1 \\ q_e \end{bmatrix} \tag{12}$$

Therefore, in an indirect filter δq is estimated and orientation quaternion q is estimated using Eqn. (1)

The quaternion error dynamics from[9] is written as follows:

$$\dot{q}_e = \frac{1}{2}(\omega - y_{gyro}) + \frac{1}{2} q_e \times (\omega - y_{gyro}) - y_{gyro} \times q_e \tag{13}$$

Assuming that q_e and $(\omega - y_{gyro})$ are small, the second term on the right hand side can be ignored. Thus

$$\dot{q}_e = -y_{gyro} \times q_e - \frac{1}{2}(b_{gyro} - v_{gyro}) \tag{14}$$

The state vector $\mathbf{x}$ of indirect filter in discrete time is defined as

$$\mathbf{x} = \begin{bmatrix} q_e \\ b_{gyro} \end{bmatrix} \tag{15}$$

The state vector $\mathbf{x}$ is evolved according to the following discrete-time system model as

$$\mathbf{x}_k = \mathbf{A}(\mathbf{x}_{k-1}, \mathbf{u}_k) + v_k \tag{16}$$

where

$$\mathbf{A} = \begin{bmatrix} -\left[y_{k,gyro} \times \right] & -0.5\mathbf{I}3 \\ \mathbf{Z}3 & \mathbf{Z}3 \end{bmatrix},$$

$$\mathbf{Z}3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ \mathbf{I}3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ v_k = \begin{bmatrix} -0.5 v_{k,gyro} \\ v_{k,bgyro} \end{bmatrix} \tag{17}$$

and $\mathbf{u}_k$ is input to the filter which is gyro measurement. Whereas $v_{k,gyro}$ and $v_{k,bgyro}$ are gyro noise and small process noise in gyro bias respectively at k[th] time instant.

For update step, the measurement equations for the indirect Kalman filter are derived as follows:

From Eqn. (2), rotation matrix at k[th] time instant is

$$C^b_{n,k}(q) = C^b_{n,k}(\delta q) C^b_{n,k}(\hat{q}) \tag{18}$$

By replacing $\delta q \approx \begin{bmatrix} 1 \\ q_e \end{bmatrix}$, the rotation matrix $C^b_n(\delta q)$ is

$$C^b_n(\delta q) = \begin{bmatrix} 1 + 2q_{e,1}^2 & 2q_{e,1}q_{e,2} + 2q_{e,3} & 2q_{e,1}q_{e,3} - 2q_{e,2} \\ 2q_{e,1}q_{e,2} - 2q_{e,3} & 1 + 2q_{e,2}^2 & 2q_{e,2}q_{e,3} + 2q_{e,1} \\ 2q_{e,1}q_{e,3} + 2q_{e,2} & 2q_{e,2}q_{e,3} - 2q_{e,1} & 1 + 2q_{e,3}^2 \end{bmatrix} \tag{19}$$

Further, $q_{e,1}, q_{e,2}$ *and* $2q_{e,3}$ are assumed to be small, so the second order terms can be ignored. Hence, $C^b_n(\delta q)$ can be written as

$$C^b_n(\delta q) = \begin{bmatrix} 1 & 2q_{e,3} & -2q_{e,2} \\ -2q_{e,3} & 1 & 2q_{e,1} \\ 2q_{e,2} & -2q_{e,1} & 1 \end{bmatrix} = I - 2\left[q_e \times \right] \tag{20}$$

Substituting Eqn. (20) and Eqn. (18) into Eqn. (4) and Eqn. (6), measurement model of the indirect filter at k[th] time instant is given as

$$y_{k,acc} - C^b_{n,k}(\hat{q})\tilde{g} = 2\left[C^b_{n,k}(\hat{q})\tilde{g} \times \right] q_{e,k} + b_{acc} + v_{acc} \tag{21}$$

$$y_{k,mag} - C^b_{n,k}(\hat{q})\tilde{m} = 2\left[C^b_{n,k}(\hat{q})\tilde{m} \times \right] q_{e,k} + v_{mag} \tag{22}$$

In order to drive the Eqns (21) and (22), the following formulation is used

$$\left[q_e \times \right] C(\hat{q})\tilde{g} = -\left[C(\hat{q})\tilde{g} \times \right] q_e \tag{23}$$

Equations (16) and Eqns. (21) - (22) constitute the state model and measurement model respectively for the indirect Kalman filter. Figure 1, shows the overall filter architecture.

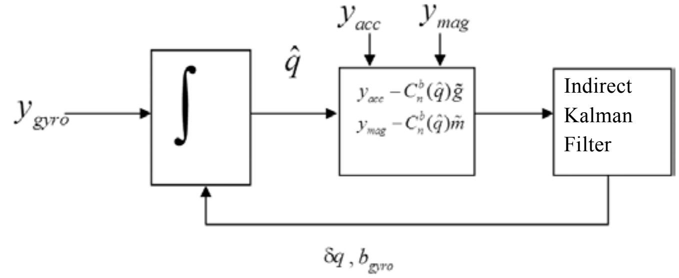

Figure 1. Filter architecture for MEKF.

2.2 Alternate Approach-Measurement Update using DCM

An alternate approach for measurement update in terms of DCM is presented here.

Instead of constructing δq as given by Eqn. (12), using q_e, the attitude error skew symmetric matrix ε is constructed as follows:

$$\varepsilon = \begin{bmatrix} 0 & -q_{e,3} & q_{e,2} \\ q_{e,3} & 0 & -q_{e,1} \\ -q_{e,2} & q_{e,1} & 0 \end{bmatrix}$$

The DCM propagation is performed from $\hat{q}$ Eqn. (11) and is denoted as $C^b_n(-)$. The filter updated DCM $C^n_b(+)$ is obtained as follows:

$$C^n_b(+) = (I + \varepsilon) C^n_b(-)$$

From $C^n_b(+)$, the quaternion are computed for the next propagation.

3. NONLINEAR COMPLIMENTARY FILTER

The nonlinear complementary filters have similar architecture to that of linear complementary filters. This family of filters are computationally efficient and are asymptotically stable. Moreover DCM based filter formulation is applicable to all orientation and avoids singularities and approximation errors[10-12]. The attitude kinematics of the true system using direction cosine matrix (DCM) is represented by

$$\dot{C}^n_b = C^n_b(\omega \times); \qquad C^n_b \in SO(3) \tag{24}$$

DCM is written in terms of rotation matrix that describes the orientation of the body coordinates frame *b* with respect to the inertial/navigation frame *n*. Rotation matrix C_b^n can be expressed as

$$C_b^n = \begin{bmatrix} \cos\theta\cos\psi & -\cos\phi\sin\psi+\sin\phi\sin\theta\cos\psi & \sin\phi\sin\psi+\cos\phi\sin\theta\cos\psi \\ \cos\theta\sin\psi & \cos\phi\cos\psi+\sin\phi\sin\theta\sin\psi & -\sin\phi\cos\psi+\cos\phi\sin\theta\sin\psi \\ -\sin\theta & \sin\phi\cos\theta & \cos\phi\cos\theta \end{bmatrix} \quad (25)$$

Consider the rotation kinematics given above in Eqn. (24) with measurements given in Eqns. (4), (6) and $k_p,\ k_I \succ 0$, the filter kinematics is given by

$$\begin{aligned} \dot{\hat{C}}_b^n &= \hat{C}_b^n\left(\left[(y_{gyro}-\hat{b}_{gyro})\times\right]+k_p\left[y_m\times\right]\right) \\ \dot{\hat{b}}_{gyro} &= -k_I y_m \end{aligned} \quad (26)$$

As gyro measurements contain bias and drift, orientation reference vectors are used for correction. It will detect orientation error by computing a rotation vector. Accelerometer and magnetometer outputs are used for calculation of reference vectors. Further, for both of the reference vector, the orientation error is calculated by taking the cross product of the measured vector with the vector that is estimated by the direction cosine matrix. Errors are compensated by providing a feedback loop back to the gyros.

y_m is the rotation error vector which is feed back through a proportional (k_p)plus integral (k_I) feedback controller to the gyros.

$$y_m = \left(\nu_a \times \hat{\nu}_a + \nu_m \times \hat{\nu}_m\right) \quad (27)$$

$$\nu_a \times \hat{\nu}_a = \frac{y_{acc}}{|y_{acc}|} \times C_b^{n^T}\frac{\tilde{g}}{|g|}, \quad (28)$$

$$\nu_m \times \hat{\nu}_m = \left(C_{b,11}^n \cos\psi_m - C_{b,21}^n \sin\psi_m\right)\begin{bmatrix} C_{b,31}^n \\ C_{b,32}^n \\ C_{b,33}^n \end{bmatrix} \quad (29)$$

$$\psi_m = \tan^{-1}\left(\frac{y_{mag}(2)}{y_{mag}(1)}\right) \quad (30)$$

where ν_a and ν_m are the vectorial measurements representing gravity and magnetic vector, respectively. Further, Euler angles are derived from the rotation matrix as

$$\psi = \tan^{-1}\left(\frac{C_{b,21}^n}{C_{b,11}^n}\right) \quad (31)$$

$$\theta = -\sin^{-1}\left(C_{b,31}^n\right) \quad (32)$$

$$\phi = \tan^{-1}\left(\frac{C_{b,32}^n}{C_{b,33}^n}\right) \quad (33)$$

3.1 Aiding Sensors for NCF

In the nonlinear complimentary filter formulation, gravity error vector calculated in Eqn. (28) does not hold good particularly in the highly manoeuvrable region. The reason being, in manoeuvrable flight, external acceleration is not equal to zero ($a_b \neq 0$) and needs to be compensated using aiding sensor. Therefore, accelerometer model given in Eqn. (4) is modified as

$$y_{acc} = C_n^b(q)\tilde{g} + b_{acc} + v_{acc} + a_b \quad (34)$$

a_b is the external/Instantaneous linear acceleration of the aircraft with respect to the inertial frame expressed in the body fixed frame is given by.

$$a_b = y_{gyro} * \mathbf{V} + \dot{\mathbf{V}} \text{ and} \quad (35)$$

$$\mathbf{V} = [u \quad v \quad w]^T \quad (36)$$

where $\mathbf{V}$ and $\dot{\mathbf{V}}$ represent the translational velocity and acceleration, respectively.

To recover an estimate of gravity vector for the NCF filter, a_b needs to be subtracted from the measured output, it requires velocity vector which can be calculated using air data sensor. NCF architecture is shown in Fig. 2.

Air data sensor measures airspeed (V_T), angle of attack (α) and sideslip angle (β). Body axis velocity components are related to air data sensor measurements by

$$u = V_T \cos\alpha\cos\beta \quad (37)$$

$$v = V_T \sin\beta \quad (38)$$

$$w = V_T \sin\alpha\cos\beta \quad (39)$$

Moreover $\dot{\mathbf{V}}$ is calculated using backward differentiation as

$$\dot{\mathbf{V}}(i) = \frac{\mathbf{V}(i)-\mathbf{V}(i-1)}{\partial T} \quad (40)$$

where ∂T is the sample time.

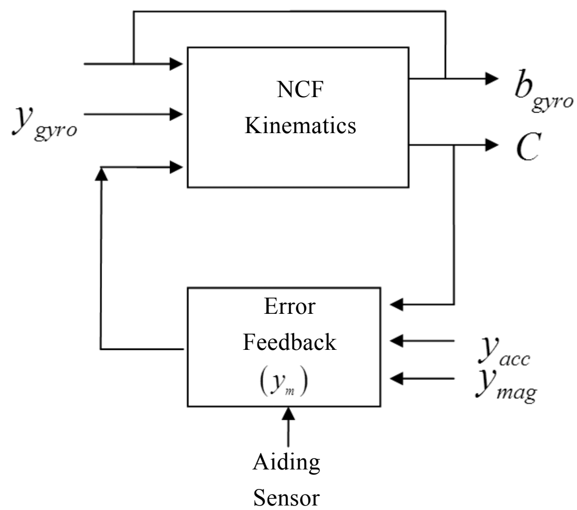

Figure 2. NCF architecture.

4. PERFORMANCE EVALUATION RESULTS

(i) Results of MEKF for simulated data. Also, the problem faced with Pitch 90 deg manoeuver with the standard MEKF approach[9] and its solution by the proposed method is discussed.
(ii) The MEKF implementation with quaternion and DCM error formulation measurement update are evaluated for flight test data.
(iii) An evaluation of MEKF with NCF.

4.1 Results of MEKF for Simulated Data

The AHRS formulation for the proposed MEKF is demonstrated using simulated data. The simulated data is generated from a real time Pilot in the Loop Simulator of a high performance aircraft. Two case studies are considered.

(i) Inverted loops

(ii) Full rolls

4.1.1 Inverted Loops

The inverted loop manoeuvre is performed by pulling the pitch stick until the aircraft touches pitch 90 deg. The comparison of estimated pitch and roll angle along with true vales are shown in Fig. 3. It is noted that at pitch angles beyond ± 80 deg, the filter estimates of the error quaternion diverge and hence the pitch angle. Below ± 80 deg, the filter converges and hence the pitch attitude settles in agreement with true vales. During pitch beyond ± 80 deg, the roll angle also is inaccurate. As discussed the introduction, this problem is first ever demonstrated in this study and a solution is proposed for this problem.

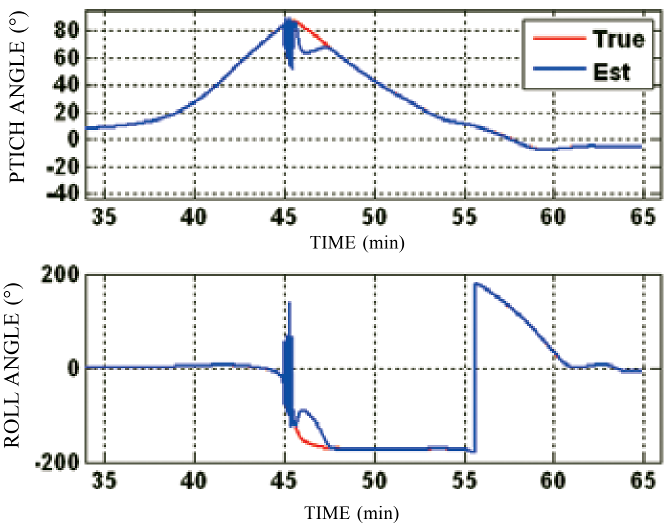

Figure 3. Problem in attitude for pitchbeyond 80 deg.

When pitch angle approaches ± 90 degrees, even the formulations such as quaternion, DCM pose problems while converting them back to Euler angles and is discussed in[13]. However, reference[13] or any other literature does not address this problem in an AHRS filter formulation using error states. The error state estimates of AHRS formulation also diverge, while pitch angle approaches ± 90 degrees.

The solution to the problem is as follows: When the estimated pitch angle, crosses ±80 deg, the error updates obtained from the filter is not given to propagated attitudes. i.e., during this period, only dead reckoning will be performed. Once this region is crossed, the error updates from the filter are used to correct the propagated attitudes. After applying the correction proposed as above, the plots of improved pitch angle and roll angle compared with their true vales are shown in Fig. 4.

4.1.2 Full rolls

The full roll manoeuvres are tested subsequently. The roll stick is deflected fully on the left side and then on the right side. This causes full 360 degree rolls and the algorithm is tested for extreme roll manoeuvres. In the full roll case the algorithm produced satisfactory output in both pitch and roll using the error state corrections provided by MEKF as shown in Fig. 5. Unlike the inverted loop case, no correction or modification is applied to MEKF for the full roll manoeuvres.

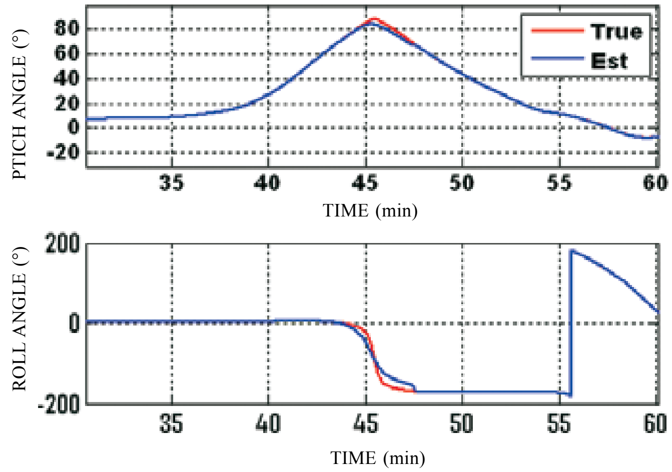

Figure 4. Improved estimation.

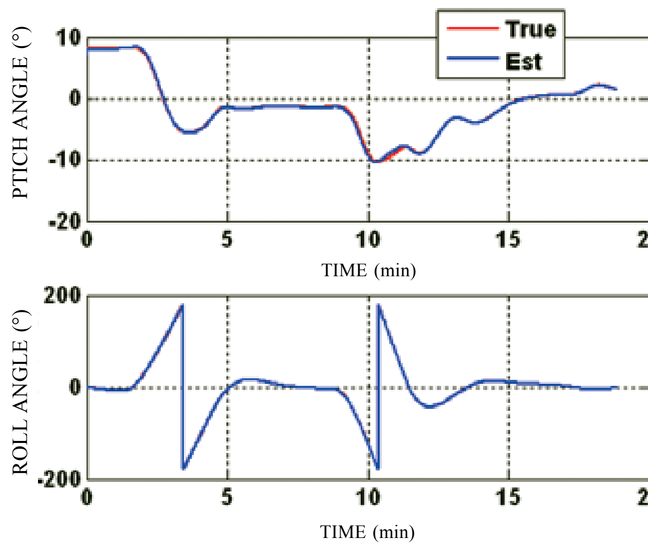

Figure 5. Full Roll estimation.

4.2 Results of MEKF for Flight Data

In any filter, the measurement model formulation is very important. Its structure varies depending upon the available aiding sensors. Much attention is not paid on the state model as in attitude estimation problems, the state models are standard. There are two measurement update models discussed in this paper.

The demonstration of MEKF for simulated data and pitch problem beyond ±80 degree are explained in section 3.1 already. In the present section, MEKF is evaluated using the flight test data of a high performance aircraft for full range aerobatic manoeuvres for the above mentioned measurement models.

4.2.1 Case-1 Full Roll Manoeuvers

In this manoeuver, the aircraft has three subsequent full rolls. The pitch angle varies about ±50 deg. The estimation is performed with quaternion and DCM model measurement updates. The estimated values are compared with onboard IN-GPS measured time histories of the aircraft. It is noted from Fig. 6, although both methods produced satisfactory results in comparison with onboard IN-GPS, the error values are less for the quaternion measurement update when compared to DCM measurement update. Table 1 presents the maximum pitch and roll errors obtained by both methods. The accuracy requirements of this AHRS filter are ±5 deg and ±15 in pitch angle and roll angle respectively. These errors are not steady state errors but occur due to time shift introduced by the filter.

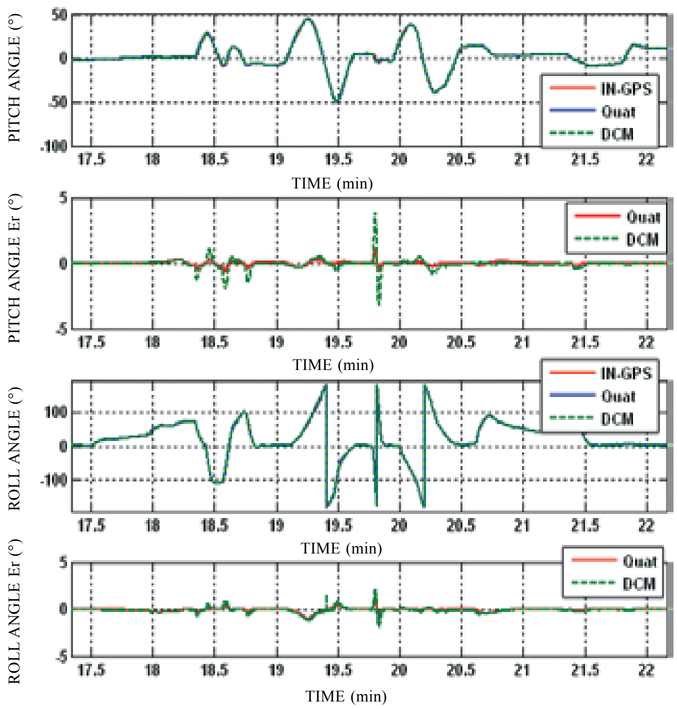

Figure 6. Comparison of MEKF results case1.

Table 1. Comparison of maximum pitch and roll angle errors

Case-1 Fast rolls	Max. Pitch angle Error (°)	Max. Roll angle Error (°)
Quaternion update method	1.1001	1.1116
DCM update method	3.8939	2.1026

4.2.2 Case-2 Inverted Loops with Full Rolls

In this section inverted loops with full rolls are taken for evaluation. The sortie consisted of two inverted loops where the pitch angle excursion is ±90 deg. It has been observed that both AHRS formulations namely quaternion and DCM produces larger errors beyond pitch angle ±80 deg in comparison with IN-GPS values as shown in Fig. 7 (Zoomed for one full inverted loop).

The results are shown in Fig. 8, for the entire sortie. It is noted that the problem shown in Fig. 7 is overcome as discussed in section 4.1.1. Again, it is seen from Fig. 8, that the quaternion update method outperformed the DCM method in flight data as well.

The maximum pitch angle errors obtained by the standard method and the proposed method are presented in Table 2.

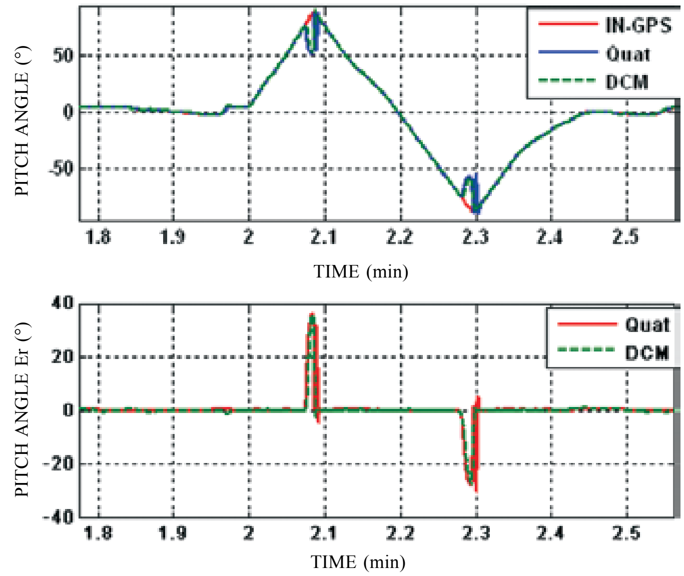

Figure 7. Comparison of MEKF results for pitch angle- case2 (Zoomed for one inverted loop).

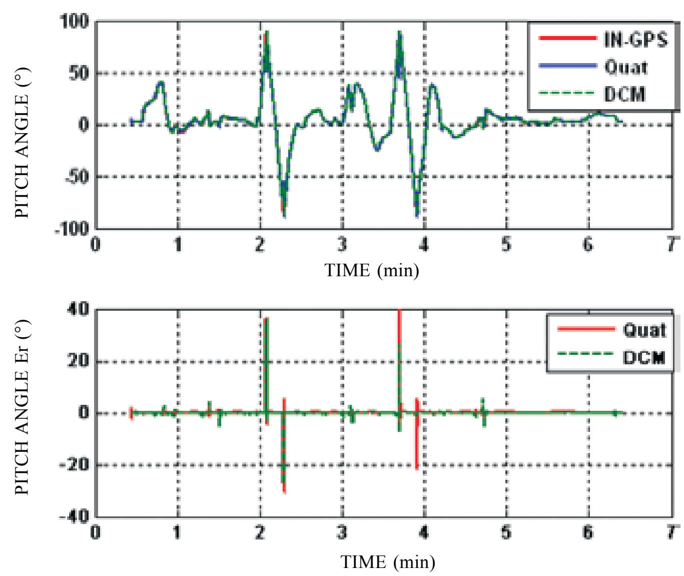

Figure 8. Comparison of MEKF results for pitch angle- case2 (Entire sortie).

Table 2. Comparison of pitch angle errors - standard formulation vs proposed method

Case-2 Inverted loops	Max. (°) Pitch angle Error (Standard method)	Max. (°) Pitch angle Error (Proposed method)
Quaternion update	40.2565	3.2665
DCM update	35.9422	6.0225

4.3 Evaluation of NCF Versus MEKF

In the recent days, the NCF has gained popularity in attitude estimation. The method does not involve any matrix inversions unlike the Kalman filters and hence is very efficient in minimising the onboard memory and computations. The high performance manoeuvers discussed earlier were evaluated using NCF. As discussed in section 3.1, the aiding sensor requirement was different for NCF when compared to MEKF. The MEKF made use of only the gravity and magnetic vector. However, the NCF needed air data parameters for modelling the linear accelerations and the component of centripetal accelerations. It can be seen in Figs. 9-10 that without air data velocities, the NCF produced huge errors.

It was found to be significantly reducing after modelling the linear and centripetal acceleration components. But, the MEKF outperformed NCF in terms of the pitch and roll angles with lesser onboard sensor requirements.

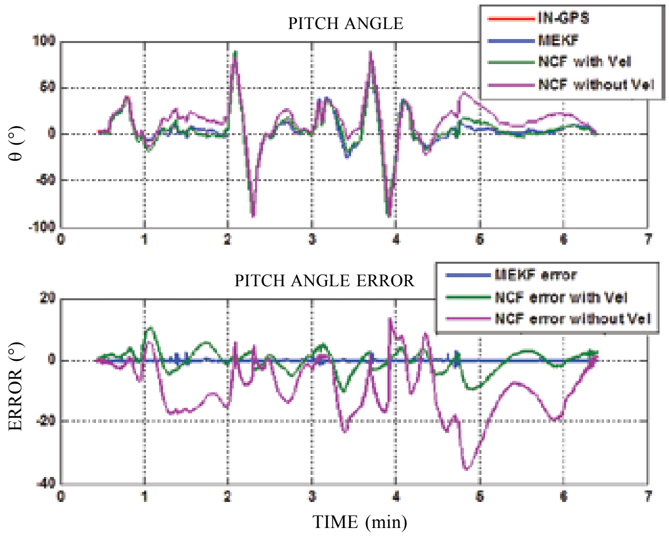

Figure 9. Comparison of MEKF results with NCF for pitch angle.

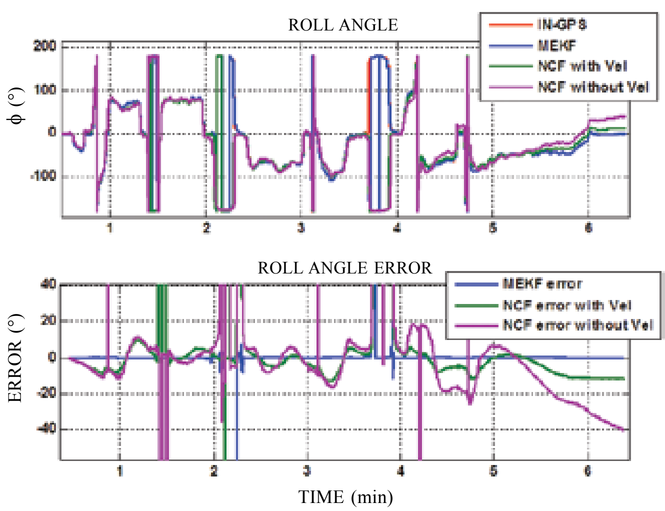

Figure 10. Comparison of MEKF results with NCF for roll angle.

5. CONCLUSIONS

The study presented methodologies and comparisons of Multiplicative Error state Kalman Filter (MEKF), and Nonlinear Complimentary Filter (NCF) for estimation of attitudes of a high performance aircraft using its onboard autonomous sensors. A solution is proposed for overcoming the problems faced for pitch angle estimation beyond ± 80 deg while using the standard MEKF formulations. The aiding sensor modelling for the filter implementations in both MEKF and NCF are discussed in detail for this class of aircraft. The filter formulations are evaluated using full range simulated and real flight test data as well. The MEKF works with inertial sensors and magnetometer alone whereas NCF requires air data sensors in addition to inertial sensors, magnetometer. Hence the MEKF outperforms NCF for full range manoeuvring flights and in terms of minimum number of sensors.

REFERENCES

1. Kamali, C.; Pashilakar, A.A.; Deodhare, Girish S.; & Saraf, Amitabh. Attitude estimation for high performance aircraft. AIAA GNC Conference, 2013-4958, 2013.
2. Demoz, Gebre-Egziabher. Design of multi sensor attitude determination systems. *IEEE Trans. Aerospace Electro. Sys.*, 2004, **40**(2), 627-649. doi: 10.1109/TAES.2004.1310010.
3. Lefferts, E.J. Kalman filtering for spacecraft attitude estimation. *Journal of Guidance*, 1982, **5**(5), (1982), pp. 417-429. doi: 10.2514/3.56190.
4. Raghavendra, P.K.; Kamali, C., Saraf, Amitabh & Suganya, S. Evaluation of a novel attitude estimation algorithm for a high performance fighter aircraft, ACODS 2014, pp. 26-33, doi: 10.3182/20140313-3-IN-3024.00192.
5. Kamali, C. Bayadi, Ramprakash; Deodhare, Girish S. & Patel, Vijay V. Pitch and Roll angle estimation using partial DCM model approach for a high performance aircraft. Indian Control Conference, ICC 2016-IEEE, Hyderabad, 2016, pp. 312-317. doi: 10.1109/INDIANCC.2016.7441145.
6. Dilshad, Nazia Ahsan & Jahan, Mohammed Kawser. An Improvement Strategy on Direction Cosine Matrix based Attitude Estimation for Multi-Rotor Autopilot. *Global J. Res. Eng.: F Electric. Electron. Eng.*, 2014, **14**(8), Version 1.0, Online ISSN: 2249-4596 & Print ISSN: 0975-5861.
7. Phuong, Nguyen Ho quoc. A DCM based orientation estimation algorithm with an inertial measurement unit and a magnetic compass. *J. Universal Comput. Sci.*, 2009, **15**(4), 859-876. doi: 10.3217/jucs-015-04-0859.
8. Beard, Randal W. & McLain, Timothy W. Small unmanned aircraft, theory and practice. Princeton University Press, 2011. doi: 10.2514/1.61067.
9. Hall, James K.; Knoebel, Nathan B. & McLain, Timothy W. Quaternion attitude estimation for miniature air vehicles using multiplicative extended Kalman filter. Paper No:1-4244-1537-3/08, IEEE, 2008. doi: 10.1109/PLANS.2008. 4570043.
10. Mahony, R.; Hamel, Tarek & Pflimlin, Jean-Michel. Nonlinear complementary filters on the special orthogonal group. *IEEE Trans. Automatic Control, Ins. Electric. Electron. Eng.*, 2008, **53**(5), 1203-1217. doi: 10.1109/TAC.2008.923738.
11. Hassen, Fourati. A Nonlinear Filtering Approach for the Attitude and Dynamic Body Acceleration Estimation Based on Inertial and Magnetic Sensors: Bio-Logging Application. IEEE sensors journal, 2011, **11**(1), 233-244, doi: 10.1109/JSEN.2010.2053353.
12. Kenneth Jensen. Generalized Nonlinear Complementary Attitude Filter. *J. Guidance, Control, Dynamics*, 2011,

34(5), 1588-1592. doi: 10.2514/1.53467.

13. Titterton, D.H. & Weston, J.L. Strapdown inertial navigation technology. Peter Peregrinus Ltd, 1997. doi: 10.1049/PBRA017E.

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