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Creep Analysis of a Variable Thickness Rotating FGM Disc using Tresca Criterion

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ABSTRACT

The study investigates steady-state creep in a rotating *Al-SiC*$_p$ disc having different thickness profiles and reinforcement (*SiC*$_p$) gradients. The disc material is assumed to creep according to threshold-stress based law and yield following Tresca criterion. The stresses and strain rates in the disc are estimated by solving the disc equilibrium equation along with creep constitutive equations. It was observed that on increasing the disc thickness gradient, the radial stress decreases towards the inner radius but increases towards the outer radius, whereas the tangential stress decreases over the entire radius. With the increase in *SiC*$_p$ gradient in the FGM disc, the radial stress increases significantly throughout, however, the tangential stress increases towards the inner radius but decreases towards the outer radius. The strain rates in the disc reduce significantly over the entire disc radius and become relatively uniform with the increase in either disc thickness gradient or reinforcement gradient. Thus, the composite disc having higher thickness and higher reinforcement gradients exhibits lesser distortion.

Keywords: Creep, disc, functionally graded materials, particle gradient, thickness gradient

1. INTRODUCTION

Rotating disc is a widely used component in many engineering and structural devices such as steam and gas turbine rotors, flywheels, turbo generators, automotive brakes, internal combustion engines, turbojet engines, ship propellers, compressors, etc.[1-3]. In some of these applications, viz., turbine rotors and disc brakes, the disc is simultaneously subjected to elevated temperature and severe mechanical loading, and hence, vulnerable to creep.

Functionally graded materials (FGMs), a kind of composites, are obtained by mixing ceramic and metal in proportion that vary continuously as a function of position along certain dimension(s) of the structure[4,5]. FGMs can be manufactured by various techniques, viz., centrifugal casting, powder metallurgy, chemical vapour deposition, etc.[6]. FGMs find applications in components of advanced aircraft and aerospace engines, computer circuit boards, thermals barrier coatings, etc.[7], wherein the components have to sustain severe thermo-mechanical loading, as also observed in rotating discs.

Several investigators have analyzed elastic, elastic-plastic, and creep stresses and deformations in variable thickness rotating discs, which have been proved superior to constant thickness discs[8]. You[9], *et al.* analyzed elastic-plastic stresses and deformations in rotating disc with variable thickness and density using Runge-Kutta algorithm to calculate the deformations and stresses in various rotating disks. Jahed[10], *et al.* analyzed a variable thickness rotating inhomogeneous disc to optimize its weight. Bayat[11], e*t al.* obtained exact elastic solutions for rotating FGM discs having constant, parabolic, and hyperbolic convergent thickness profiles and observed that the FGM disc with parabolic or hyperbolic convergent thickness profile has lower stress and displacement profile. Bayat[12], *et al.* obtained thermo-elastic solutions for a rotating FGM disc with variable thickness under a steady temperature field. It was observed that FGM disc with concave thickness profile has smaller stresses and weight than a uniform thickness FGM disc. Nie and Batra[13] obtained exact solutions for thermo-elastic stresses and radial displacement (non-dimensional form) in the rotating disc made of rubber like FGM having radially-varying density, shear modulus, and coefficient of thermal expansion. The study also attempted to find the radial variation of shear modulus and coefficient of thermal expansion to achieve a constant value of either hoop stress or in-plane shear stress in the FGM disc. Bayat[14], *et al.* analyzed elastic stresses and displacement in metal-ceramic and ceramic-metal FGM discs for varying material grading indices and disc geometry. The results indicated that the radial displacement in the disc was reduced on employing ceramic-metal disc and concave thickness profile. Ali[15], *et al.* estimated non-dimensional, elastic stresses and displacement in a variable thickness, rotating FGM (*Al*-Ceramic-*Al*) disc. The material properties were represented by combination of two sigmoid FGMs and the effects of varying material grading indices and disc geometry were investigated on the stresses and displacements. Hassani[16], *et al.* analyzed stresses and strains in a variable thickness rotating disc with non-uniform material properties, subjected to thermo-elasto-plastic loadings.

The literature study revealed that the FGM disc having variable thickness has lower stresses and deformations than a constant thickness FGM disc. However, these studies pertain to elastic and thermo-elastic loadings. A rotating disc, in number of applications (as outlined earlier), is subjected to steady-state

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creep. In most of the studies pertaining to analysis of steady-state creep in rotating FGM disc, the disc thickness is assumed to be constant[1,17-19]. Only a few studies have been undertaken to analyze steady-state creep in FGM disc of variable thickness, wherein the researchers have observed that the disc weight[20] or strain rates in the disc could be significantly reduced using varying thickness profile[21]. However, these studies have assumed linear thickness profile of the disc and linear distribution of reinforcements. In this light, the present study investigates steady-state creep in rotating disc having various kinds of nonlinear thickness profiles and nonlinear distributions of reinforcement. The results are also estimated for constant thickness and uniform composite (non-FGM) discs.

2. DISC PROFILE AND DISTRIBUTION OF REINFORCEMENT

The study assumes a composite disc rotating at 15000 rpm with inner and outer radii, respectively as a (=31.75 mm), b(=152.4 mm). The disc thickness, $h(r)$, is assumed to vary nonlinearly, with radius r as

$$h(r) = h_b[r/b]^k \tag{1}$$

where h_b and k are respectively disc thickness at the outer radius and disc thickness index. By varying k, the thickness profile of the disc gets modified, except for $k=0$, which corresponds to constant-thickness disc. The disc dimensions and thickness profile as mentioned above, are similar to earlier works[16, 22].

For comparison, the volume of variable thickness disc was kept equal to a similar composite disc but of uniform thickness t(=25.4 mm),*i.e.* $\int_a^b 2\pi r h(r)dr = \pi(b^2 - a^2)t$. Therefore,

$$h_b = [(2+k)b^k(b^2 - a^2)t]/[2(b^{k+2} - a^{k+2})] \tag{2}$$

It was assumed that the FGM disc is made of *Al-SiC$_p$* composite[22], with *SiC$_p$* content, $V(r)$, decreasing nonlinearly from the inner to outer radii as

$$V(r) = V_b[r/b]^m \tag{3}$$

where m is the reinforcement gradation index. On varying m, the radial distribution of *SiC$_p$* in the FGM disc changes, except for $m=0$, which corresponds to disc having uniform *SiC$_p$* content.

If the total *SiC$_p$* content in variable thickness FGM disc and constant thickness uniform composite disc is equal, then,

$$\int_a^b 2\pi r V(r)h(r)dr = \pi(b^2 - a^2)tV_{av} \tag{4}$$

where V_{av} is the average *SiC$_p$* content in FGM disc.

Substituting $h(r)$ and $V(r)$, respectively from Eqns (1) and (3) in to above equation, one gets:

$$V_b = [(2+m+k)b^m(b^{k+2} - a^{k+2})V_{av}]/[(2+k)(b^{2+m+k} - a^{2+m+k})] \tag{5}$$

where V_b is the content of *SiC$_p$* at the outer radius.

Following the rule of mixture, the density of FGM disc at any radius (r) is given by[18]:

$$\rho(r) = \rho_m + 0.01(\rho_d - \rho_m)V_b(r/b)^m = A_\rho + B_\rho r^m \tag{6}$$

where

$$A_\rho = \rho_{m,} B_\rho = (\rho_d - \rho_m)V_b/(100b^m), \rho_m (= 2698.9\, kg/m^3)$$

is the density of *Al* and $\rho_d\ (= 3210\, kg/m^3)$ is the density[18] of *SiC$_p$*.

3. CREEP LAW

Material of the disc is assumed to undergo steady-state creep according to[1],

$$\dot{\bar{\varepsilon}} = [M(r)\{\bar{\sigma} - \sigma_0(r)\}]^n \tag{7}$$

where $\dot{\bar{\varepsilon}}$ is the effective strain rate, $\bar{\sigma}$ is the effective stress, $\sigma_0(r)$ is the threshold stress (*i.e.* the minimum level of stress required to initiate creep) and $M(r) = \left[\frac{1}{E}(A' \exp\frac{-Q}{RT})^{1/n}\right]$ is a creep parameter, where in A', n, Q, E, R and T denote respectively the structure dependent parameter, true stress exponent, true activation energy, temperature-dependent Young's modulus, gas constant and operating temperature. The details regarding parameters, as above, can be found in a work by Ma and Tjong[23].

In this study, the value of n is taken as 5, which corresponds to creep controlled by high temperature dislocation climb[22]. The values of parameters $M(r)$ and $\sigma_0(r)$ depend on particle size (P= 1.7 μ*m*), particle content $V(r)$ and operating temperature (T= 623 *K*) according to the following relations, as derived in earlier work[22],

$$M(r) = 0.0228 - 0.0088/P - 14.0267/T + 0.0322/V(r) \tag{8}$$

$$\sigma_0(r) = -0.084P - 0.023T + 1.185V(r) + 22.207 \tag{9}$$

In *Al-SiC$_p$*, significant creep is noticed at 623 *k*, which is well above homologous temperature (≅ 40 per cent of M.P. temperature of *Al-SiC$_p$*), that ranges above 700 °C, depending on *SiC$_p$* content. Further, for given values of P and T, the creep stresses and strain rates in the disc will vary on varying radial distribution of *SiC$_p$* in the disc, as attempted in this study.

4. ANALYSIS OF CREEP IN FGM DISC

It was assumed that the disc material is incompressible, stresses at any radius in the disc remain constant with time, elastic deformations(being small) are neglected and disc is under plane stress condition (*i.e.* axial stress (σ_z)=0), as its thickness being small compared to outer diameter. Similar assumptions were taken in earlier works[16,22,24].

The generalised constitutive equations for creep in an isotropic composite material under plane stress condition takes the following form when the reference frame is along the principal directions r, θ and z, directions[18],

$$\begin{aligned} \dot{\varepsilon}_r &= \dot{\bar{\varepsilon}}\left[2\sigma_r(r) - \sigma_\theta(r)\right]/(2\bar{\sigma}) \\ \dot{\varepsilon}_\theta &= \dot{\bar{\varepsilon}}\left[2\sigma_\theta(r) - \sigma_r(r)\right]/(2\bar{\sigma}) \\ \dot{\varepsilon}_z &= \dot{\bar{\varepsilon}}\left[-\sigma_r(r) - \sigma_\theta(r)\right]/(2\bar{\sigma}) \end{aligned} \tag{10}$$

where $\dot{\varepsilon}_r, \dot{\varepsilon}_\theta, \dot{\varepsilon}_z$ and σ_r, σ_θ are respectively the strain rates and stresses along radial, tangential, and axial directions of the disc, as indicated by the respective subscripts.

Assuming the material to yield according to Tresca criterion[25], the effective stress ($\bar{\sigma}$) in rotating disc, under biaxial state of stress, is given by

$$\bar{\sigma} = \sigma_\theta (\because \text{In a disc} : \sigma_\theta > \sigma_r > \sigma_z)^{26} \tag{11}$$

Substituting the values of $\dot{\bar{\varepsilon}}$ and $\bar{\sigma}$ respectively from Eqns (7) and (11) into the first equation amongst set of Eqns (10), one gets,

$$\dot{\varepsilon}_r = d\dot{u}_r/dr = [M(r)\{\bar{\sigma} - \sigma_0(r)\}]^n [2x(r) - 1]/2 \tag{12}$$

where $x(r) = \sigma_r(r)/\sigma_\theta(r)$, $\dot{u}_r$ is the radial deformation rate and u_r is the radial deformation.

Similarly the second equation amongst the set of Eqns (10) gives

$$\dot{\varepsilon}_\theta = \dot{u}_r / r = [M(r)\{\bar{\sigma} - \sigma_0(r)\}]^n [2 - x(r)]/2 \tag{13}$$

Dividing Eqn (12) by Eqn (13) and integrating between limits *a* to *r*, one obtains

$$\dot{u}_r = \dot{u}_a exp \int_a^r \Phi(r)/r\, dr \tag{14}$$

where $\dot{u}_a$ is the radial deformation rate at the inner radius and $\Phi(r) = [2x(r) - 1]/[2 - x(r)]$.

Substituting the value of $\dot{u}_r$ from Eqn (14) into Eqn (13) and simplifying, one gets,

$$\sigma_\theta(r) = \dot{u}_a^{1/n} \Psi_1(r)/M(r) + \Psi_2(r) \tag{15}$$

where $\Psi_1(r) = \Psi(r)^{1/n}; \Psi_2(r) = \sigma_0(r)$ and

$$\Psi(r) = 2/[r\{2 - x(r)\}]\exp \int_a^r \Phi(r)/r\, dr \,.$$

Multiplying Eqn (15) by $h(r)dr$ and integrating between limits *a* to *b*, one gets on simplifying

$$\dot{u}_a^{1/n} = [A_0 \sigma_\theta(avg) - \int_a^b h(r)\Psi_2(r)dr] / \int_a^b h(r)\Psi_1(r)/M(r)\, dr \tag{16}$$

where $A_0 = \int_a^b h(r)dr$ and $\sigma_\theta(avg) = (1/A_0)\int_a^b h(r)\sigma_\theta dr$ is the average tangential stress in disc.

Substituting the value of $\dot{u}_a^{1/n}$ from Eqn (16) into Eqn (15), one obtains

$$\sigma_\theta(r) = \Psi_1(r)[A_0\sigma_\theta(avg) - \int_a^b h(r)\Psi_2(r)\, dr]/[M(r) \int_a^b h(r)\Psi_1(r)/M(r)\, dr] + \Psi_2(r) \tag{17}$$

The force equilibrium equation for a variable thickness FGM disc, rotating at an angular velocity ω, is given by[22]

$$d/dr[rh(r)\sigma_r] - h(r)\sigma_\theta + \rho(r)\omega^2 r^2 h(r) = 0 \tag{18}$$

The disc is assumed to be connected to the shaft by means of splines where small axial movement is permitted. Therefore the following free-free condition applies[27],

$\sigma_r = 0$ at $r = a$ and $\sigma_r = 0$ at $r = b$.

Integrating Eqn (18) between limits *a* to *b* under the above boundary conditions, one gets

$$\sigma_\theta(avg) = \omega^2 h_b \begin{bmatrix} \{A_\rho(b^{3+k} - a^{3+k})/(3+k)\} + \\ \{B_\rho(b^{3+k+m} - a^{3+k+m})/(3+k+m)\} \end{bmatrix} / (A_0 b^k) \tag{19}$$

Integrating Eqn (18) between limits *a* to *r*, under the imposed boundary conditions, one obtains

$$\sigma_r(r) = \frac{1}{rh(r)}\left[\int_a^r h(r)\sigma_\theta dr - \frac{\omega^2 h_b}{b^k}\left\{\begin{array}{l} \dfrac{A_\rho(r^{3+k} - a^{3+k})}{(3+k)} + \\ \dfrac{B_\rho(r^{3+k+m} - a^{3+k+m})}{(3+k+m)} \end{array}\right\}\right] \tag{20}$$

Knowing the distributions of σ_r and σ_θ, $\dot{\varepsilon}_r$ and $\dot{\varepsilon}_\theta$ are estimated from Eqns (12) and (13).

5. RESULTS AND DISCUSSION

The stresses and strain rates in the disc have been estimated by an iterative process. The results are obtained for uniform composite discs with different thickness profiles, Discs D1-D4 (Table 1) and uniform composite disc of variable thickness (DS1) and FGM discs of variable thickness with different SiC_p gradients, *i.e.,* Discs DS2-DS5 (Table 2).The stress and strain rates in rotating disc could be significantly reduced by varying the disc thickness, with higher thickness near the inner radius[22]. Therefore, to achieve this kind of disc thickness profile, the index *k* was kept negative and its value was arbitrarily varied from 0 to -0.6.The value of gradation index *m* was varied from 0 to -1.8295, since for *m* = -1.8295, the content of SiC_p at the inner radius ($V_{\max}$) becomes 100 per cent (pure ceramic). The purpose of choosing negative values of *m* is to achieve radially decreasing SiC_p distribution in the FGM disc, which helps in minimizing creep deformations in FGM disc as compared to uniform composite disc[17].

Table 1. Uniform composite discs of variable thickness (V_{avg} = 20%, h_{avg}=25.4 mm)

Disc Notation	Thickness Index (*k*)	Disc Thickness (mm)		Thickness Gradient (mm)= (h_a-h_b)
		h_a	h_b	
D1	0	25.4	25.4	0.0
D2	-0.2	31.8	23.2	8.6
D3	-0.4	39.6	21.2	18.4
D4	-0.6	49.0	19.1	29.9

Table 2. Uniform composite and FGM discs of variable thickness (V_{avg}=20%, *k*=-0.6)

Disc Notation	Gradation Index (*m*)	Particle Content (vol%)		Particle Gradient (PG) = ($V_{\max}$-$V_{\min}$)
		$V_{\max}$	$V_{\min}$	
DS1	0	20	20	0
DS2	-0.5	33.1	15.1	18
DS3	-1.0	52.3	10.9	41.4
DS4	-1.5	78.6	7.48	71.12
DS5	-1.8295	100	5.67	94.33

5.1 Effect of Varying Disc Thickness Profile

The effect of varying disc thickness index (*k*) on creep behaviour of a uniform composite disc (*V*=20%) with four different thickness profiles (Table 1) shown in Fig. 1, is reported in this section. The average thickness (h_{avg}=25.4 mm) is kept the same for all the discs. With the decreases in *k* from 0 to -0.6, the disc thickness gradient, *i.e.,* the difference of maximum and minimum disc thickness, increases from 0 to 29.9 (Table 1).

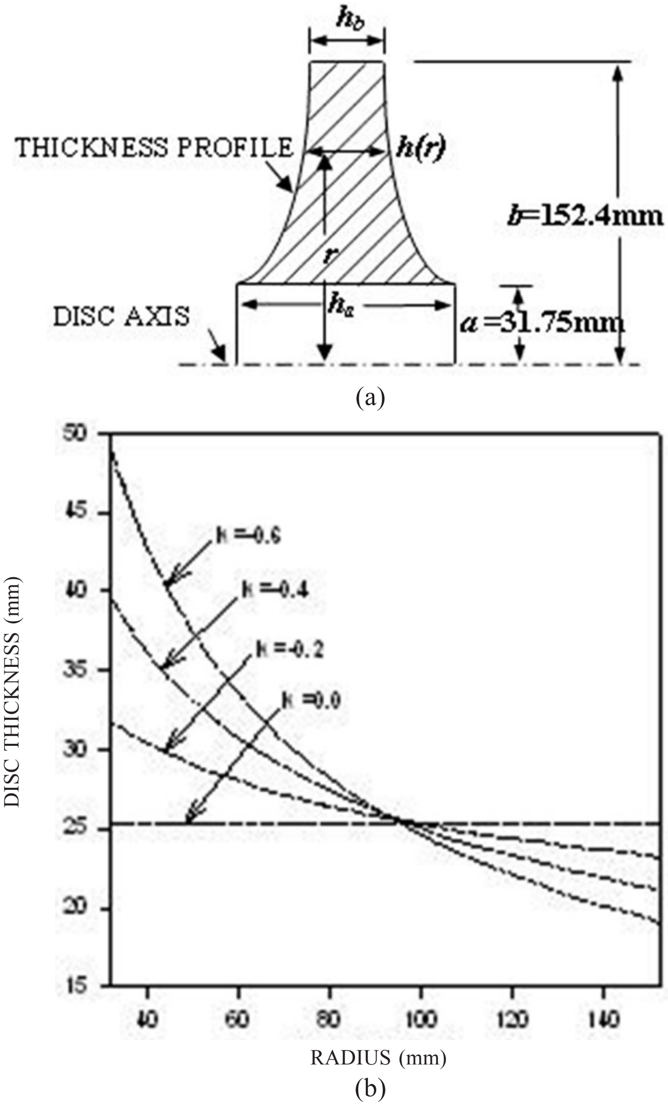


Figure 1. (a) Schematic showing disc dimensions and geometry, and (b) Variation of disc thickness.

5.1.1 *Effect on Stresses*

As compared to uniform thickness disc D1, the radial stress in variable thickness discs (D2-D4) decreases towards the inner radii but increases towards the outer radius [Fig. 2(a)]. The amount of decrease and increase noticed in radial stress increases with the decrease in *k*. The decrease observed in radial stress towards the inner radius is relatively higher than the increase observed towards the outer radius. It is important to mention that the maximum radial stress decreases slightly with the decrease in *k* and also the location of maximum radial stress is marginally shifted towards the outer radius. For example, the maximum radial stress in disc D4 (*k* =-0.6) decreases by about 2 per cent when compared to uniform thickness disc D1. Unlike radial stress, the tangential stress [Fig. 2(b)] decreases over the entire radius with the decrease in *k*. The tangential stress decreases slightly with the increasing radius. On increasing the disc thickness gradient, thickness increases near the inner radius, as a result the centrifugal force, owing to disc rotation, also increases due to increase in disc mass near the inner radius but there is simultaneous increase in cross-section area of the disc, which tends to reduce tangential stress near the inner radius. However, towards the outer radius, the reduced mass of disc, caused by reduced disc thickness with decreasing *k*, tends to reduce the tangential stress, in spite of reduced cross-sectional area. The decrease observed in variable thickness disc D4 is 19.44% and 13.93%, respectively at the inner and outer radii, as compared to uniform thickness disc D1.

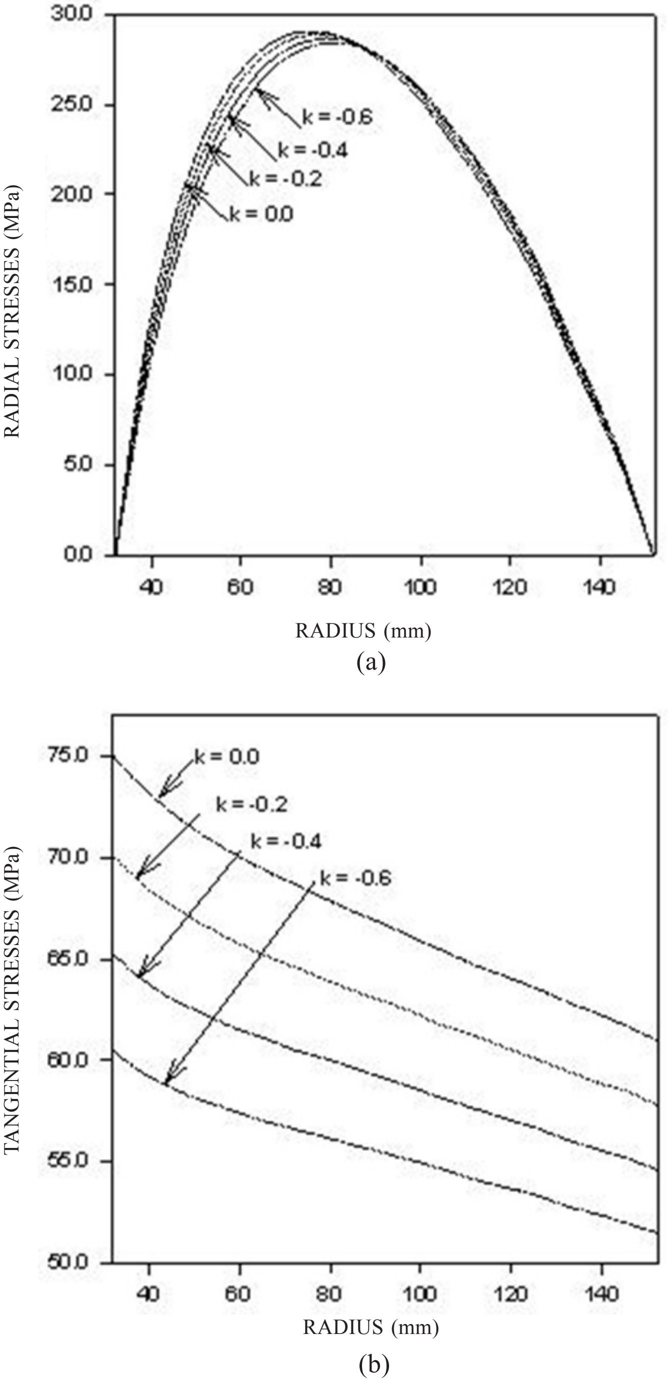
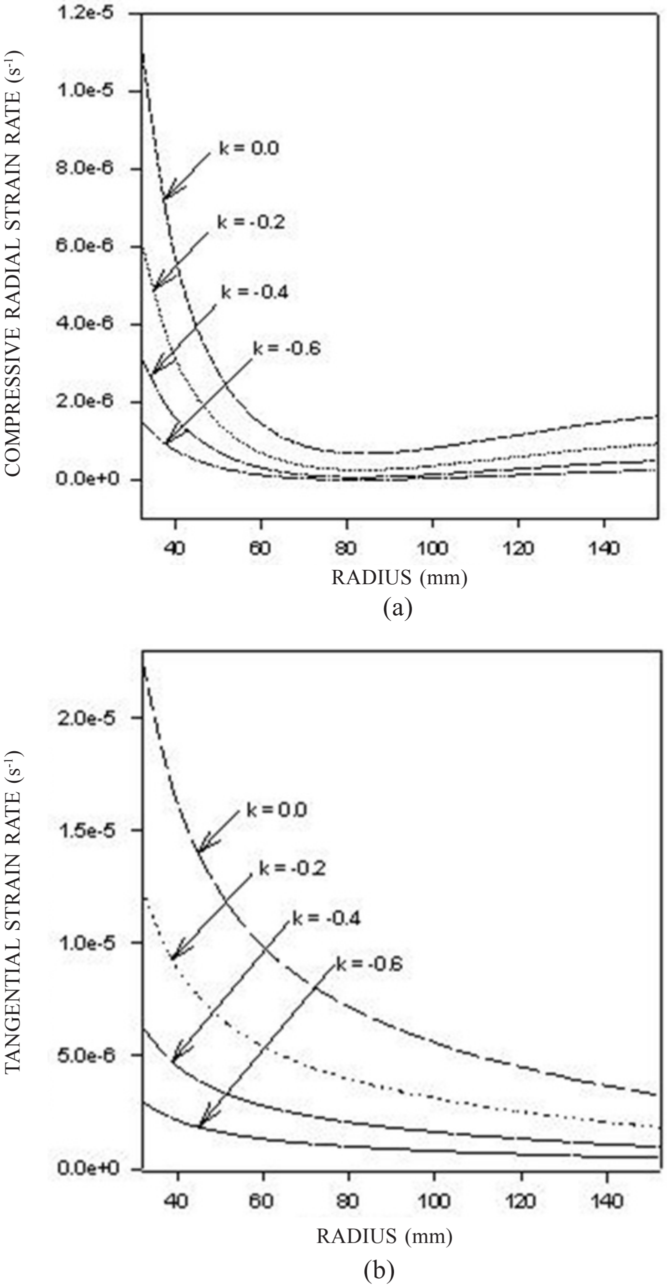


Figure 2. Variation of (a) radial and (b) tangential stresses in disc.

5.1.2 *Effect on Strain Rates*

The radial strain rate (compressive) decreases with the decrease in *k* over the entire disc radius [Fig. 3(a)]. The decrease observed is significantly higher near the inner radius than that observed towards the outer radius. The radial strain rate in disc D4 is lower by 9.81×10^{-6} and 1.39×10^{-6} at the inner and outer radii, respectively, when compared to disc D1.The effect of varying *k* on tangential strain rate [Fig. 3(b)] is similar to that observed for radial strain rate in Fig. 3(a). However, the decrease noticed in tangential strain rate is much higher than that observed for radial strain rate in the corresponding disc. The tangential strain rate in disc D4 is lower by 19.62×10^{-6} and 2.78×10^{-6} at the inner and outer radii, respectively, as compared to disk D1.It is evident from Eqns (12) and (13) that the reduction in strain rates with increasing disc thickness gradient is attributed to reduction in effective stress $(\bar{\sigma} = \sigma_\theta)$ with increasing disc thickness gradient. Further, with the increase in disc thickness gradient, the distribution of radial and tangential strain rates becomes relatively more uniform, and hence, reduces the chances of distortion in the disc.



(a)

(b)

Figure 3. Variation of (a) radial and (b) tangential strain rates in disc.

5.2 Effect of Varying Reinforcement Gradient

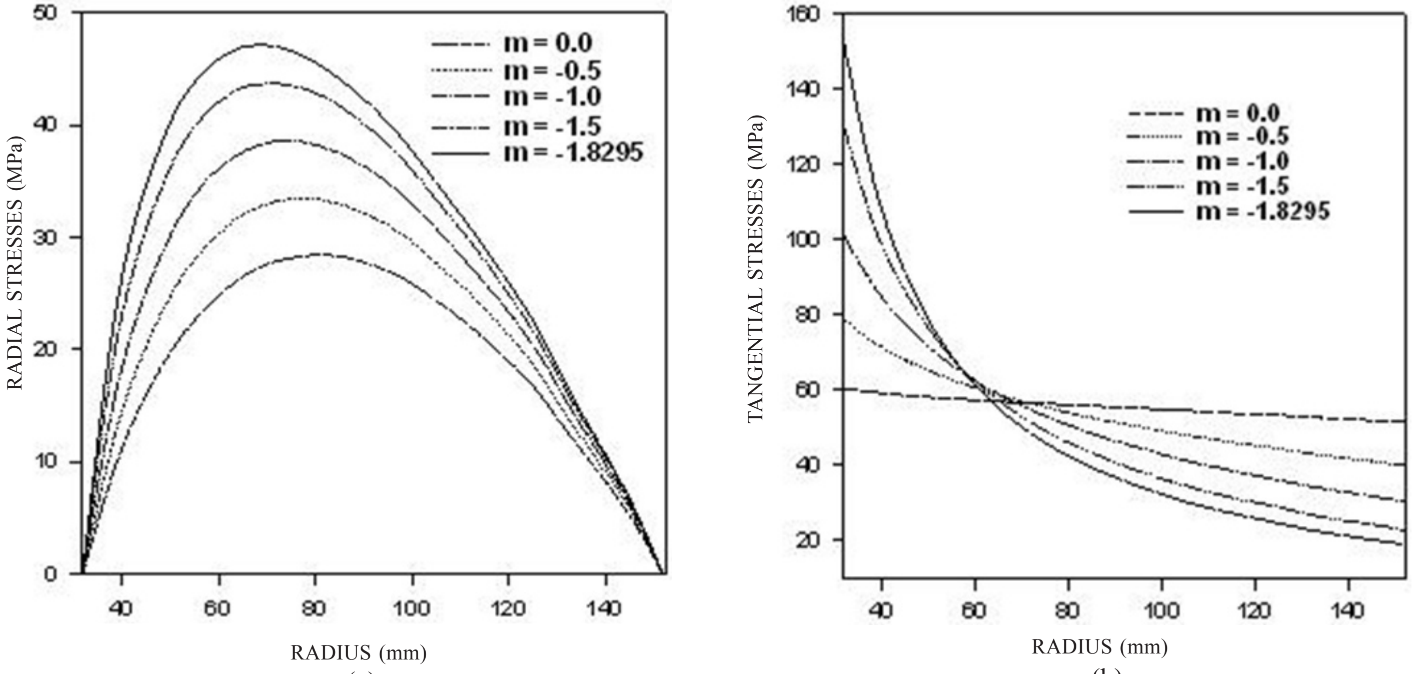
To investigate the effect of varying particle gradation index (*m*) on creep response of a variable thickness disc with *k*= -0.6, the results are estimated for four different FGM discs [DS2-DS5] (Table 2) and compared with a uniform composite disc DS1 (*m*=0). On decreasing *m* from 0 to -1.8295, the SiC_p content in the FGM discs (DS2-DS5) increases near the inner radius but decreases towards the outer radius, and as a result, the particle gradient (referred as PG hereafter) increases, when compared to uniform composite disc DS1(Table 2).

5.2.1 Effect on Stresses

As compared to uniform composite disc DS1, the radial stress [Fig. 4(a)] increases significantly throughout with the increase in PG in the FGM discs (DS2-DS5). The increase is more in the middle than that observed towards the inner and outer radii. Apart from increase in the magnitude of maximum radial stress, its location also shifts towards the inner radius as PG increases from 0 to 94.33 (Table 2).The maximum radial stress in the FGM disc DS5 is 18.6 MPa (65.5 per cent) higher as compared to uniform composite disc DS1. The tangential stress [Fig. 4(b)] increases towards the inner radius but decreases towards the outer radius when PG in the disc increases from 0 to 94.33.The increase observed in tangential stress towards the inner radius is significantly higher than the decrease observed towards the outer radius. The increase observed in tangential stress near the inner radius, with increasing PG, is due to increase in disc density near the inner radius, owing to higher amount of reinforcement, which leads to increase centrifugal load near the inner disc radius. However, towards the outer radius, the tangential stress decreases due to reduction in SiC_p content, which results in lesser centrifugal load. The increase and decrease observed in tangential stress in FGM disc DS5, respectively at the inner and outer radii, are 91.5 MPa (151.24%) and 32.5 MPa (63.11%) respectively, when compared to uniform composite disc DS1.

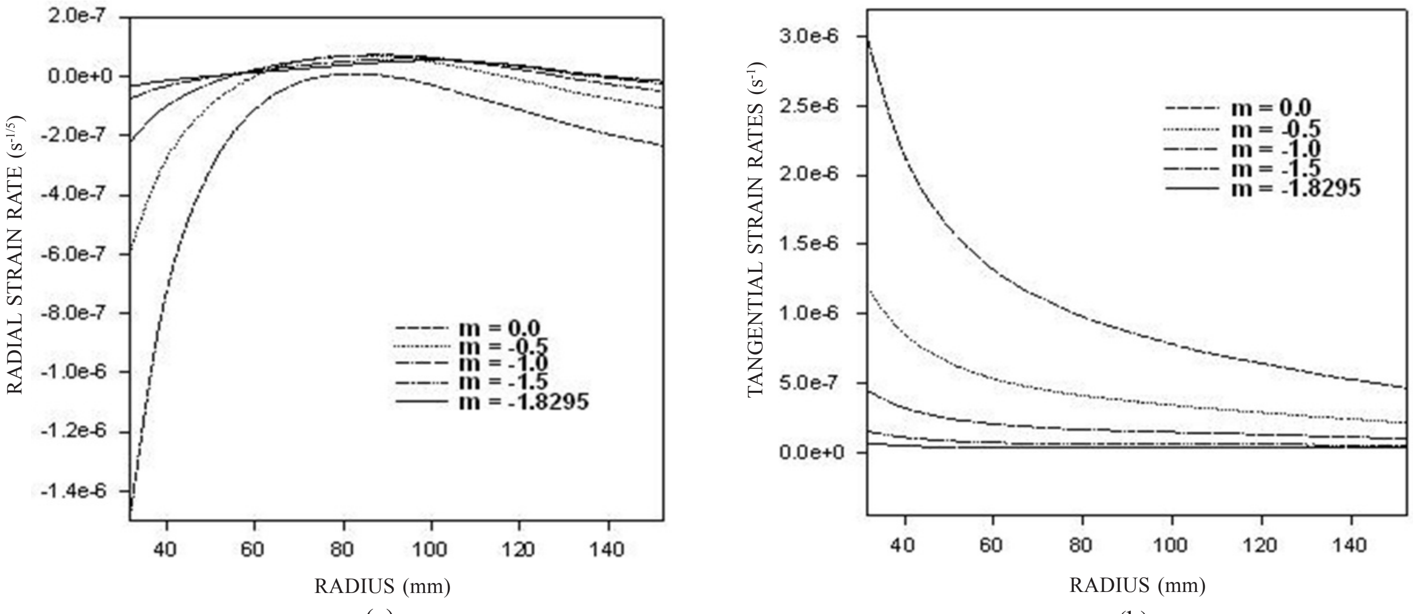
5.2.2 Effect on Strain Rates

The radial strain rate in the disc [Fig. 5(a)] reduces over the entire disc radius with the increase in PG from 0 to 94.33. The decrease in radial strain rate is observed with increasing radius, till one reaches near the middle of the disc, followed by slight increase with further increase in radius. When PG in the disc increases beyond 18, the radial strain rate changes its nature from compressive to small tensile for some regions in the middle of the disc. The reduction observed in radial strain rate in the FGM disc DS5, as compared to uniform composite disc DS1, is 1.45×10^{-6} (97.7%) and 2.18×10^{-7} (93.4%), respectively at the inner and outer radii. The tangential strain rate, [Fig. 5(b)], also decreases with the increase in PG. As compared to uniform composite disc DS1, the tangential strain rate in FGM disc DS5 is reduced by 2.91×10^{-6} (97.7%) and 0.435×10^{-6} (93.4%), respectively at the inner and outer radii. The FGM disc having higher PG, in spite of higher tangential stress, exhibits lower strain rates near the inner radius. It may be attributed to higher



(a) (b)

Figure 4. (a) Variation of radial and (b) tangential stresses in FGM discs.



(a) (b)

Figure 5. Variation of: (a) radial, and (b) tangential strain rates in FGM discs.

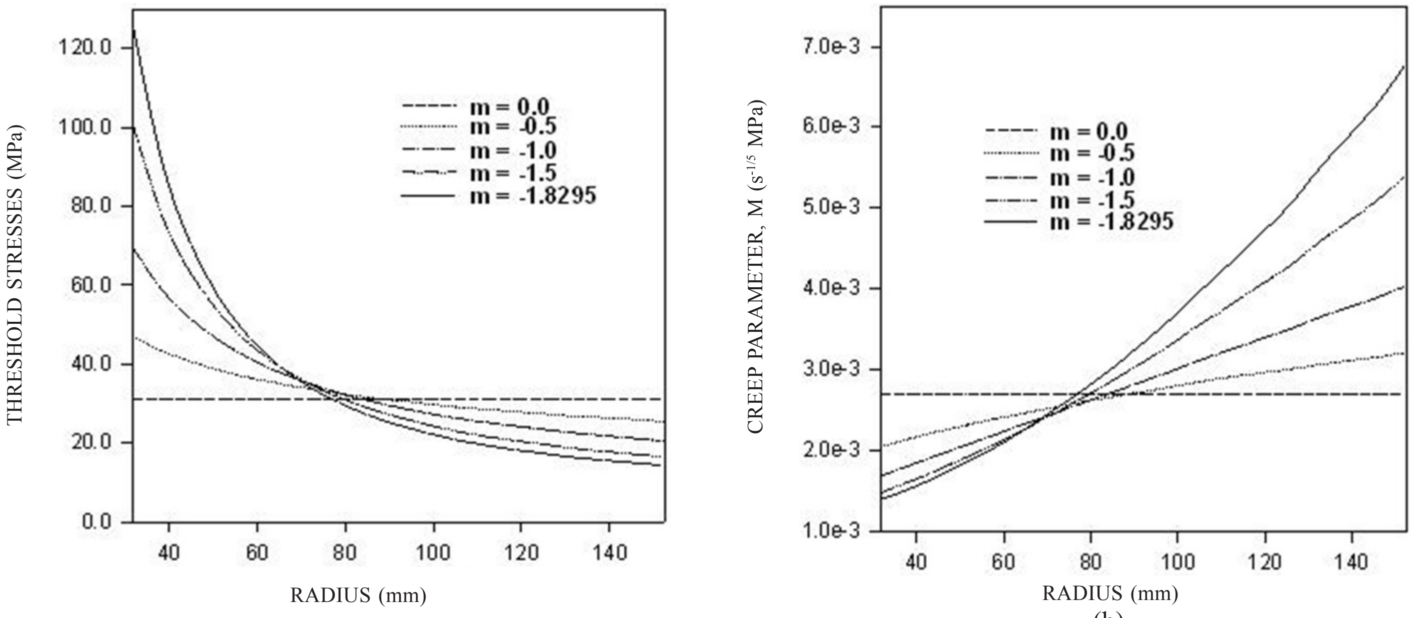
value of threshold stress and lower value of creep parameter *M*, shown in Figs. 6(a) and 6(b) near the inner radius of the FGM disc, due to higher SiC_p content, which dominates over higher value of effective stress (tangential stress) and causes strain rates to reduce near the inner disc radius. On the other hand, the lower value of threshold stress and higher value of creep parameter *M* show in Figs. 6(a) and 6(b) towards the outer radius of the FGM disc, owing to lower SiC_p content, dominates over the lower value of effective stress, resulting due to lower disc density, and reduces the strain rates again in the FGM disc having higher PG.

6. CONCLUSIONS

The study has led to the following conclusions

(i) On decreasing disc thickness index *k*, the radial stress decreases towards the inner radius but increases towards the outer radius, with relatively higher decrease towards the inner radius. The tangential stress decreases over the entire radius with the increase in disc thickness gradient, though the decrease observed reduces with the increasing disc radius.

(ii) The strain rates decrease throughout with the decrease in index *k*, with a significantly higher decrease near the



(a) (b)

Figure 6. Variation of (a) threshold stress and (b) creep parameter in FGM discs.

inner radius. On decreasing *k* from 0 to -0.6, the radial and tangential strain rates are reduced by more than one order of magnitude.

(iii) With the increase in SiC_p gradient in the FGM disc, the radial stress increases significantly throughout, with relatively higher increase in the middle. The tangential stress increases towards the inner radius but decreases towards the outer radius, with the increase in SiC_p gradient in the disc. The increase in tangential stress is more towards the inner radius. The maximum increase observed in radial and tangential stresses is respectively 65.5 per cent and 151.25 per cent, when SiC_p gradient in the disc increases from 0 to 94.33.

(iv) With increasing SiC_p gradient in the disc, the strain rates reduce throughout, with significantly higher decrease noticed near the inner radius. The order of strain rates in the disc is reduced by around two orders of magnitude, when SiC_p gradient in the disc increases from 0 to 94.33. On increasing SiC_p gradient in the FGM disc beyond 18, the nature of radial strain rate changes from compressive to small tensile in the middle of the disc.

(v) On increasing disc thickness gradient or particle gradient in the disc, the distribution of strain rates become relatively more uniform, thereby reducing the distortion in the disc.

(vi) Under given operating conditions, the uniform composite disc with thickness index (*k*) =-0.6 shows the lowest creep rates, which can be further reduced by employing FGM disc having radially varying SiC_p content, till it becomes pure ceramic (100 per cent SiC_p) at the inner radius.

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