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Influence of Constant Stresses on Transverse Surface Waves Between Two Elastic Media

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ABSTRACT

The influence of constant stresses on velocities of propagation of transverse surface wave between two elastic media is studied, assuming the constraints at contact surface to be elastic. It is observed that the elastic constant of constraint between two media is always influenced by the constant stresses.

1. INTRODUCTION

Kaliski[1] studied the existence of transverse waves between two elastic bodies when there is an external impedence. Bluestien[2], studied about the possibility of these waves in piezo-electric bodies when there is a thin layer of concentrated mass between two contact surfaces. Kaliski[3], showed that these waves may occur when there are elastic constraints at the contact surface between two homogeneous bodies. These results are new in the theory of surface waves and may find application as a model solution for other more complicated problems of the theory of surface waves in piezo-semi conductors and amplification of such waves. Kaczkowski[4], investigated the influence of constant stresses on the velocities of propagation of elastic waves and determined the magnitudes of the above stresses for which the state of equilibrium of a body becomes a state of neutral equilibrium.

In this paper, an attempt has been made to study the influence of constant stresses on the velocities of transverse waves between two elastic bodies, assuming the constraints at the contact surface to be elastic. The different ranges of elastic constraint are obtained.

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2. BASIC EQUATIONS AND BOUNDARY CONDITIONS

Let the x-axis be taken along the direction of the propagation of the surface wave and z-axis be directed towards the interior of the lower space. Stress-strain relations are given by

$$\sigma_{ij} = \lambda\, \Delta \delta_{ij} + 2\, \mu\, e_{ij},\ (i,j = 1, 2, 3) \tag{1}$$

where

$$\begin{aligned} e_{ij} &= \tfrac{1}{2}\,(u_{i,j} + u_{j,i}) \\ \Delta &= e_{kk} \end{aligned} \tag{2}$$

The equations of motion are[4]

$$\sigma_{ik,i} + \sigma^0_{ij}\, u_{k,ij}\, (1 - \delta_{ik})\, (1 - \delta_{jk}) = \rho\, \ddot{u}_k$$

where σ^0_{ik} are constant stresses, independent of time, and satisfy the equation of static equilibrium

$$\sigma^0_{ik,i} = 0$$

Here, the static state of strain and displacements produced due to the constant stresses are not considered and these constant stresses are assumed similar in both the half-spaces.

We use the notations λ, σ, ρ, u, u for $z > 0$ and λ_0, σ_0, ρ_0, μ_0, u_0 for $z < 0$.

For a transverse wave on the contact between two elastic bodies, the solution is independent of y and the displacement given by

$$u_2 = \begin{cases} u & \text{for} \quad z > 0 \\ u_0 & \text{for} \quad z < 0. \end{cases}$$

Across the interface the stress tensor must be continuous and the condition of contact of resistance[5] is

$$\text{Shearing Stress} = R \times \text{Slip} \tag{3}$$

where R is the elastic constant of constraint (proportionality factor) between the two bodies and slip is defined as the difference in the displacement of the material particles on either side of the interface. Hence the boundary conditions at $z = 0$ are

$$\begin{aligned} \sigma_{32} &= (\sigma_{32})_0 \\ \sigma_{32} + R\,(u - u_0) &= 0 \end{aligned} \tag{4}$$

If the slip tends to zero (when R is infinetely large) the interface will be bonded and when slip remains finite (when R is vanishingly small) the interface will be smooth. This can be seen from Eqn. 3. Therefore, there is only one empirical parameter R, to represent different degrees of bonding between the interfaces.

3. SOLUTION AND THE DISPERSION EQUATION

We assume the displacements

$$u = A\, e^{i(kx - pt - k\alpha z)} \qquad \text{for } z > 0$$
$$u_0 = A_0\, e^{i(kx - pt + k\beta z)} \qquad \text{for } z < 0 \tag{5}$$

in which k is the wave number, p is the frequency and α, β are complex constants which are to be determined. Using Eqn. (7), the Eqn. (2) reduces to

$$\mu\, \nabla^2 u_k + (\lambda + \mu)\, u_{i,ik} + \sigma^0_{ij}\, u_{k,ij}\,(1 - \delta_{ik})\,(1 - \delta_{jk}) = \rho\, \ddot{u}_k \tag{6}$$

Substituting the displacements (5) into Eqn. (6), we get

$$\mu[-k^2 - k^2\alpha^2] - \sigma^0_{11}\, k^2 + 2\sigma^0_{13}\, k^2\alpha - \sigma^0_{33}\, k^2\alpha^2 = -\rho\, p^2$$

$$\mu_0[-k^2 - k^2\beta^2] - \sigma^0_{11}\, k^2 - 2\sigma^0_{13}\, k^2\beta - \sigma^0_{33} k^2\beta^2 = -\rho_0\, p^2$$

Solving the above equations for α and β, we obtain

$$\alpha = \frac{\dfrac{\sigma^0_{13}}{\mu} \pm i\sqrt{\left(1 + \dfrac{\sigma^0_{33}}{\mu}\right)\left(1 - \dfrac{c^2}{b^2} + \dfrac{\sigma^0_{11}}{\mu}\right) - \left(\dfrac{\sigma^0_{13}}{\mu}\right)^2}}{\left(1 + \dfrac{\sigma^0_{13}}{\mu}\right)} \qquad \frac{\sigma^0_{33}}{\mu} \neq -$$

$$\beta = \frac{-\dfrac{\sigma^0_{13}}{\mu_0} \pm i\sqrt{\left(1 + \dfrac{\sigma^0_{33}}{\mu_0}\right)\left(1 - \dfrac{c^2}{b_0^2} + \dfrac{\sigma^0_{11}}{\mu_0}\right) - \left(\dfrac{\sigma^0_{13}}{\mu_0}\right)^2}}{\left(1 + \dfrac{\sigma^0_{33}}{\mu_0}\right)}, \qquad \frac{\sigma^0_{33}}{\mu_0} \neq -$$

where $b = \sqrt{\mu/\rho}$ and $b_0 = \sqrt{\mu_0/\rho_0}$ are the transverse wave speeds in two half-spaces.

As the surface wave decays as depth increases, we take the negative sign before the term under square root. Also the term under square root should be positive for the validity of the solution. Accordingly we have to assume the signs of the constant stress distribution.

Substituting Eqn. (5) into Eqn. (4), we get the dispersion equation

$$\frac{R_0}{\;} + \frac{\beta}{\;} = 0 \tag{7}$$

where

$$R_0 = R/\rho b^2 k, \quad n = b^2/b_0^2, \; n_1 = \rho/\rho_0$$

As this equation is not suitable to interpret the results, we discuss this in a particular case.

4. PARTICULAR SOLUTION

The solution (7) will be discussed in the particularly simple case $n = 1$ and $n_1 = 1$ i.e., in two similar half-spaces and $\sigma_{13}^0 = 0$. With this particular case a will be equal to β and the Eqn. (7) becomes

$$\frac{R_0}{\quad} + 1 = 0$$

Substituting the value of a in the above equation, we get

$$\xi = \sqrt{\left[1 + \frac{\sigma_{11}^0}{\mu} - 4R_0^2\left(1 + \frac{\sigma_{33}^0}{\mu}\right)\right]} \qquad (8$$

where $\xi = c/b$.

From the Eqn. (8), it is clear that the transverse waves will occur (Real solution fo ξ) if R_0 satisfies the condition

$$R_0 < \frac{1}{2}\sqrt{\frac{1 + \frac{\sigma_{11}^0}{\mu}}{1 + \frac{\sigma_{33}^0}{\mu}}} \qquad (9$$

The general frequency Eqn. (7) gives us, a similar but more complicated set of conditio for R_0. If the constant stresses σ_{11}^0, σ_{33}^0 vanish, the condition (9) tallies with the simil condition obtained in section (3).

If $\sigma_{33}^0 \to -\mu$, the Eqn. (8) gives,

$$\xi = 1\sqrt{1 + \frac{\sigma_{11}^0}{\mu}} \qquad (1$$

Here ξ is independent of R_0 i.e., R_0 does not influence the velocity propagatio Therefore the constant stresses of this order will nullify the effect of elastic constrain between the two elastic bodies.

If $\sigma_{33}^0 = \sigma_{11}^0$, the Eqn. (8) reduces to

$$\xi = \sqrt{\left[\left(1 + \frac{\sigma_{11}^0}{\mu}\right)\left(1 - 4R_0^2\right)\right]} \qquad (1$$

Case 1 : If the stress σ_{11}^0 is tensile, then the wave exists for $R_0 < 1/2$.

Case 2 : If the stress σ_{11}^0 is compressive and $\sigma_{11}^0 < -\mu$, then the waves will exist for $R_0 > 1/2$.

Therefore, unlike the condition of existence of waves ($R_0 < 1/2$) in section (3), the wider range of elastic constant of constraint ($R_0 > 1/2$) is obtained when the constant stresses are compressive.

Now, using the Eqn. (11) the relation ξ (quantity representing the velocity) and R_0 (elastic constant of constraint) is shown in Fig. 1. From this figure, it can be said that, for an increase in the elastic constant of constraint the velocity decreases when the constant stresses of order 10 μ (tensile) and increases when the constant stresses of order -10 μ (compressive). So the different orders of constant stresses restrict the nature of phase velocity as well as the ranges of elastic constant of constraints.

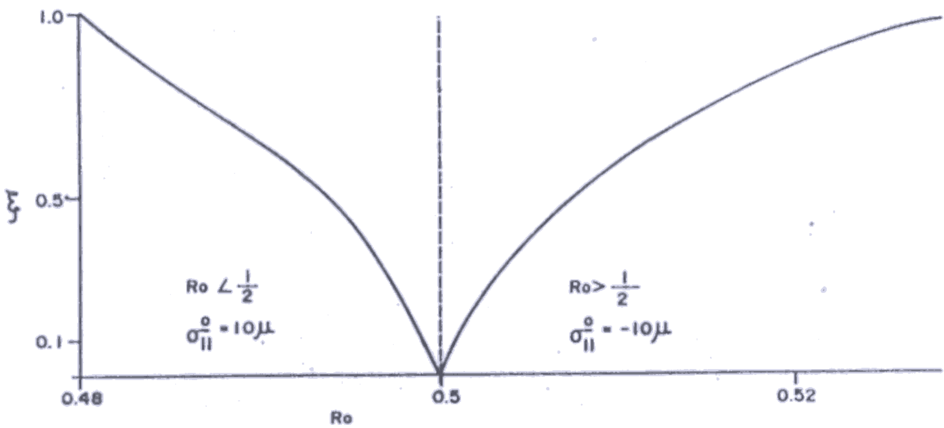

Figure Velocity *vs* elastic constraint

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