A NOTE ON THE TEMPERATURE DISTRIBUTION IN A POROUS ANNULUS

Y. N. GAUR

M. R. Engineering College, Jaipur

(*Received* 11 *October* 1974; *revised* 5 *May* 1975)

Unsteady temperature distribution for flow of an incompressible viscous fluid in a porous annulus has been investigated under the assumption that the rate of injection of the fluid at the inner cylinder is equal to the rate of suction of the fluid at the outer one. Viscous dissipation has been neglected while rate of heat generation in the fluid has been accounted for. An expression for the temperature distribution has been obtained using Laplace transform technique and taking the temperatures of the boundaries to be functions of time.

Berman[1] has obtained an exact solution of the problem of steady flow of a viscous incompressible fluid through a porous annulus, when the rate of injection of the fluid at one boundary is equal to the rate of suction of the fluid at the other. Bhatnagar & Tikekar[2] have discussed the temperature distribution in a channel bounded by two coaxial circular pipes with time dependent boundary temperatures, while Gaur[3] has studied the unsteady temperature distribution for laminar flow in a porous straight channel.

Neglecting viscous dissipation, the energy equation in cylindrical polar co-ordinates, (r, θ, x), is

$$\rho C_v\left(\frac{\partial T}{\partial t}+\frac{v}{a}\frac{\partial T}{\partial \lambda}+u\frac{\partial T}{\partial x}\right)=\frac{\partial Q}{\partial t}+K\left[\frac{1}{a^2}\left(\frac{\partial^2 T}{\partial \lambda^2}+\frac{1}{\lambda}\frac{\partial T}{\partial \lambda}\right)+\frac{\partial^2 T}{\partial x^2}\right], \tag{1}$$

where u and v are the axial and radial velocities, λ $(= r/a)$ is the dimensionless radial co-ordinate, T is the temperature of the fluid, and 'a' and 'b' are the radius of the outer and inner cylinder respectively. The rest of the quantities have their usual meanings.

Under the assumption that the rate of injection of the fluid at the inner cylinder is equal to the rate of suction of the fluid at the outer one and these rates are uniform, T does not depend on x. The radial velocity is given by

$$v=\frac{v_a}{\lambda}=\frac{\sigma v_b}{\lambda}, \tag{2}$$

where $\sigma = b/a$, v_a and v_b are the constant velocities of suction and injection at outer and inner cylinder respectively.

Equation (1), therefore, becomes

$$\frac{\partial T}{\partial t}+\frac{v_a}{a\lambda}\frac{\partial T}{\partial \lambda}=\frac{1}{\rho C_v}\frac{\partial Q}{\partial t}+\frac{K'}{a^2}\left(\frac{\partial^2 T}{\partial \lambda^2}+\frac{1}{\lambda}\frac{\partial T}{\partial \lambda}\right), \tag{3}$$

where

$$K'=K/(\rho\ C_v)$$

and the initial and boundary conditions, are

$$\left.\begin{array}{ll} T=0,\ Q=Q_z, & t<0 \\ T=T_1(t), & \lambda=\sigma,\ t>0 \\ T=T_2(t), & \lambda=1,\ t>0 \end{array}\right\} \tag{4}$$

where $T_1(t)$ and $T_2(t)$ are some functions of time.

Adopting the usual Laplace transform technique and assuming that the temperature and rate of heat generation, remain bounded as $t \to \infty$, we can easily show that the present problem admits the following solution

$$T(\lambda, t) = \frac{1}{\rho C_v}\Big[Q(t) - Q_0\Big] + \lambda^{Pe/2}\,\sigma^{-Pe/2}\int_0^t S_1(u)\,F(t-u)\,du + \lambda^{Pe/2}\int_0^t S_2(u)\,G(t-u)\,du, \tag{5}$$

where

$$S_i(t) = T_i(t) + \frac{1}{\rho C_v}\Big[Q_0 - Q(t)\Big], \quad (i = 1, 2)$$

$$F(t) = L^{-1}\left[F(s) \equiv \frac{1}{D(s)}\left\{\begin{array}{l} I_{Pe/2}\left(\sqrt{\frac{a^2 s}{K'}}\right) K_{Pe/2}\left(\sqrt{\frac{a^2 s}{K'}}\,\lambda\right) - \\ - K_{Pe/2}\left(\sqrt{\frac{a^2 s}{K'}}\right) I_{Pe/2}\left(\sqrt{\frac{a^2 s}{K'}}\,\lambda\right) \end{array}\right\}\right]$$

$$G(t) = L^{-1}\left[G(s) \equiv \frac{1}{D(s)}\left\{\begin{array}{l} K_{Pe/2}\left(\sqrt{\frac{a^2 s}{K'}}\,\sigma\right) I_{Pe/2}\left(\sqrt{\frac{a^2 s}{K'}}\,\lambda\right) - \\ - I_{Pe/2}\left(\sqrt{\frac{a^2 s}{K'}}\,\sigma\right) K_{Pe/2}\left(\sqrt{\frac{a^2 s}{K'}}\,\lambda\right) \end{array}\right\}\right]$$

and

$$D(s) = I_{Pe/2}\left(\sqrt{\frac{a^2 s}{K'}}\right) K_{Pe/2}\left(\sqrt{\frac{a^2 s}{K'}}\,\sigma\right) - I_{Pe/2}\left(\sqrt{\frac{a^2 s}{K'}}\,\sigma\right) K_{Pe/2}\left(\sqrt{\frac{a^2 s}{K'}}\right)$$

where $I_{Pe/2}$ and $K_{Pe/2}$ are the modified Bessel functions of order $Pe/2$.

$F(s)$ admits simple poles at $s = -K'\,\alpha_j^2$, where α_j are the zeros of

$$J_{Pe/2}(a\,\alpha)\,Y_{Pe/2}(a\,\sigma\,\alpha) - J_{Pe/2}(a\,\sigma\,\alpha)\,Y_{Pe/2}(a\,\alpha).$$

Evaluating residues at these simple poles, we get :

$$F(t) = \sum_{j=1}^{\infty} K'\,\pi\,\alpha_j^2\left[\frac{J_{Pe/2}(a\,\alpha_j)\,Y_{Pe/2}(a\,\lambda\,\alpha_j) - J_{Pe/2}(a\,\lambda\,\alpha_j)\,Y_{Pe/2}(a\,\alpha_j)}{J^2_{Pe/2}(a\,\sigma\,\alpha_j) - J^2_{Pe/2}(a\,\alpha_j)}\right] \times$$

$$\times\; J_{Pe/2}(a\,\alpha_j)\,J_{Pe/2}(a\,\sigma\,\alpha_j)\exp(-K'\,\alpha_j^2\,t) \tag{6}$$

The poles of $G(s)$ are also the same ; hence proceeding in a similar manner, we obtain :

$$G(t) = \sum_{j=1}^{\infty} K' \pi \alpha_j^2 \left[\frac{J_{Pe/2}(a \lambda \alpha_j) Y_{Pe/2}(a \sigma \alpha_j) - J_{Pe/2}(a \sigma \alpha_j) Y_{Pe/2}(a \lambda \alpha_j)}{J^2_{Pe/2}(a \sigma \alpha_j) - J^2_{Pe/2}(a \alpha_j)} \right] \times$$

$$\times J_{Pe/2}(a \alpha_j) J_{Pe/2}(a \sigma \alpha_j) \exp(-K' \alpha^2_j t) \qquad (7)$$

Substituting these values of $F(t)$ and $G(t)$ from (6) and (7) in (5), we have :

$$T(\lambda, t) = \frac{1}{\rho C_v} \left[Q(t) - Q_0 \right] +$$

$$+ \lambda^{Pe/2} \sigma^{-P/2} \sum_{j=1}^{\infty} K' \pi \alpha_j^2 \left[\frac{J_{Pe/2}(a \alpha_j) Y_{Pe/2}(a \lambda \alpha_j) - J_{Pe/2}(a \lambda \alpha_j) Y_{Pe/2}(a \alpha_j)}{J^2_{Pe/2}(a \sigma \alpha_j) - J^2_{P/2}(a \alpha_j)} \right] \times$$

$$\times J_{P/2}(a \alpha_j) J_{Pe/2}(a \sigma \alpha_j) \int_0^t S_1(u) \exp\left\{-K' \alpha_j^2 (t-u)\right\} du +$$

$$+ \lambda^{Pe/2} \sum_{j=1}^{\infty} K' \pi \alpha_j^2 \left[\frac{J_{Pe/2}(a \lambda \alpha_j) Y_{Pe/2}(a \sigma \alpha_j) - J_{Pe/2}(a \sigma \alpha_j) Y_{Pe/2}(a \lambda \alpha_j)}{J^2_{Pe/2}(a \sigma \alpha_j) - J^2_{Pe/2}(a \alpha_j)} \right] \times$$

$$\times J_{Pe/2}(a \alpha_j) J_{Pe/2}(a \sigma \alpha_j) \int_0^t S_2(u) \exp\left\{-K' \alpha^2_j (t-u)\right\} du \quad ,$$

where $S_1(t)$ incorporates the boundary condition and the heat generation at the inner boundary and $S_2(t)$ at the outer. Thus given these boundary conditions and the rate of heat generation in the fluid, we can specify the solution completely.

The present investigation can be made use of for getting some idea of the flow of oil through oil filters in vehicles operated in desert areas. The oil filter used in a jeep or truck is a porous annulus and the boundaries change temperature with time, as the vehicle moves. Flow of exhaust gases through absorpter type of silencers, usually used in motor cycles, is another phenomenon, where the temperature changes at the boundaries take place with time.

ACKNOWLEDGEMENT

The author is thankful to Dr. P.D. Verma for his kind guidance during this investigation and also to Prof. P.L. Bhatnagar for encouragement.

REFERENCES

1. Berman, A. S., *J. Appl. Phys.*, **29** (1958), 71.
2. Bhatnagar, P. L. & Tikekar, V. G., *Proc. Camb. Phil. Soc.*, **62** (1966), 301.
3. Gaur, Y. N., *Def. Sci. J.*, **22** (1972), 5.