Magneto Hydrodynamic Flow of a Viscous Incompressible Fluid Between a Parallel Flat Wall and a Long Wavy Wall

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Abstract. Magneto-hydrodynamic flow of a viscous incompressible slightly conducting fluid between a parallel flat wall and a long wavy wall has been studied. The velocity distribution, the coefficient of skin friction and temperature distribution has been evaluated. The effects of magnetic, suction and frequency parameters are investigated on velocity, the coefficient of skin friction and temperature distribution.

1. Introduction

Viscous fluid flow over a wavy wall has attracted the attention of relatively few researchers although the analysis of such flows finds application in such different areas as transpiration cooling of re-entry vehicles and rocket boosters, cross hatching on ablative surfaces and film vapourization in cumbustion chambers. Shankar & Sinha[1] have made a detailed study of the Rayleigh problem for a wavy wall. They have concluded that at low Reynolds numbers the waviness of the wall quickly ceases to be of importance as the liquid is dragged along by the wall, where at large Reynolds numbers the effects of viscosity are confined to a thin layer close to the wall and the known potential solution emerges in time. Vajravelu & Shastri[2] have devoted attention to the effect of waviness of one of the walls on the flow and heat transfer characteristics of an incompressible viscous fluid confined between two long vertical walls and set in motion by a difference in the wall temperatures. Lekoudis, Nayfeh, & Saric[3] have studied the compressible boundary layer flows over a wavy wall. Lessen & Gangwani[4] have investigated the effect of small amplitude wall waviness upon the stability of the laminar boundary layer.

In this paper, we study the magneto-hydrodynamic flow of a viscous, incompressible slightly conducting fluid between a parallel flat wall and a long wavy wall. X-axis is taken along the parallel flat wall and a straight line perpendicular to that as the Y-axis, so that the wavy wall is represented by $Y = \epsilon^* \operatorname{Cos} KX$ and the flat wall by $Y = 0$. The wavy and flat walls are maintained at constant temperatures of T_1 and T_0 respectively. We assume that the wave length of the wavy wall which is proportional to $1/K$

is large. The fluid is sucked through the wall $Y = 0$ with the constant suction velocity V_0. Taking the fluid to be of small conductivity with magnetic Reynolds number much less than unity, the induced magnetic field is neglected in comparison with the applied magnetic field[5].

2. Formulation and Solution of the Problem

In the absence of any input electric field the equations of momentum, continuity and energy are :

$$\rho\left(U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}\right)=-\frac{\partial P^*}{\partial X}+\mu\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right)-\sigma\mu_e^2 H_0^2 U \qquad (1)$$

$$\rho\left(U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}\right)=-\frac{\partial P^*}{\partial Y}+\mu\left(\frac{\partial V^2}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right) \qquad (2)$$

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \qquad (3)$$

$$\rho C_P\left(U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}\right)=K\left(\frac{\partial^2 T}{\partial X^2}+\frac{\partial^2 T}{\partial Y^2}\right)+\mu\phi \qquad (4)$$

where ρ is the fluid density, U and V are the velocity components along axes of co-ordinates, P^* the fluid pressure, μ the coefficient of viscosity, σ the electrical conductivity of the fluid, μ_e the magnetic permeability, H_0 the intensity of the magnetic field, C_P the specific heat of the fluid, T' the temperature, K the coefficient of thermal conductivity and

$$\phi=2\left[\left(\frac{\partial U}{\partial X}\right)^2+\left(\frac{\partial V}{\partial Y}\right)^2\right]+\left(\frac{\partial V}{\partial X}+\frac{\partial U}{\partial Y}\right)^2-\tfrac{2}{3}\left(\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}\right)^2$$

In the energy Eqn. (4), the Joule dissipation heat is assumed to be negligible[6].

The boundary conditions are

$$U=0,\ V=-V_0 \text{ where } V_0 \text{ is a constant} > 0$$

$$T=T_0 \text{ on } Y=0 \qquad (5)$$

$$U=0,\ V=0,\ T=T_1 \text{ on } Y=\epsilon^* \operatorname{Cos} KX \qquad (6)$$

Since the flat wall is infinite in length, $\dfrac{\partial U}{\partial X}=0$.

We obtain

$$V=-V_0 \qquad (7)$$

by integrating Eqn. (3) and using Eqn. (5).

We introduce the following non-dimensional equantities

$$u=\frac{U}{V_0},\ \eta=\frac{Y}{h},\ P=\frac{P^*h}{\mu_0 V}$$

$$v = \frac{V}{V_0}, \quad x = \frac{X}{h}, \quad T^* = \frac{T - T_0}{T_1 - T_0} \tag{8}$$

In view of Eqns. (7) and (8), Eqns. (1), (2) and (4) reduce to

$$\frac{\partial^2 u}{\partial \eta^2} + R\frac{\partial u}{\partial \eta} - Mu = \frac{\partial P}{\partial x} \tag{9}$$

$$0 = \frac{\partial P}{\partial Y} \tag{10}$$

$$\frac{\partial^2 T^*}{\partial \eta^2} + P \, . \, R\frac{\partial T^*}{\partial \eta} = -P \, . \, E\left(\frac{\partial u}{\partial \eta}\right)^2 \tag{11}$$

where

$$M = \frac{\sigma \mu_e^2 H_0^2 h^2}{\mu} \quad \text{(Magnetic parameter)}$$

$$P = \frac{\mu C_P}{K} \quad \text{(Prandtl number)}$$

$$E = \frac{V_0^2}{C_P(T_1 - T_0)} \quad \text{(Eckert number)}$$

$$R = \frac{V_0 h}{\nu} \quad \text{(Suction parameter)}$$

And in view of Eqn. (8), the boundary conditions (5) and (6) reduce to

$$\left.\begin{array}{l} u = 0, \; v = -1 \text{ at } \eta = 0 \\ u = 0, \; v = 0 \text{ at } \eta = \epsilon \operatorname{Cos} \lambda x \end{array}\right\} \tag{12}$$

where

$$\epsilon = \frac{\epsilon^*}{h} \quad \text{(non-dimensional amplitude parameter)}$$

$$\lambda = Kh \quad \text{(non-dimensional frequency parameter)}$$

From Eqn. (10), we observe that the fluid pressure P is independent of Y. We assume that the pressure gradient $\frac{\partial P}{\partial x}$ is a constant C. Solving the Eqn. (9) using the boundary conditions (12), we obtain the velocity distribution

$$u = \frac{C}{M}\left[\frac{\operatorname{Sinh}\{a(\epsilon \operatorname{Cos} \lambda x - \eta)\} + \operatorname{Sinh} a\eta}{\operatorname{Sinh}(a \, \epsilon \operatorname{Cos} \lambda x)} - 1\right] \tag{13}$$

where

$$a = \tfrac{1}{2}(R^2 + 4M)^{1/2}$$

The effects of magnetic and suction parameters on velocity distribution are shown in Figs. 1b and 2. It is observed that the velocity increases as the frequency parameter of the wavy wall increases as shown in Fig. 3.

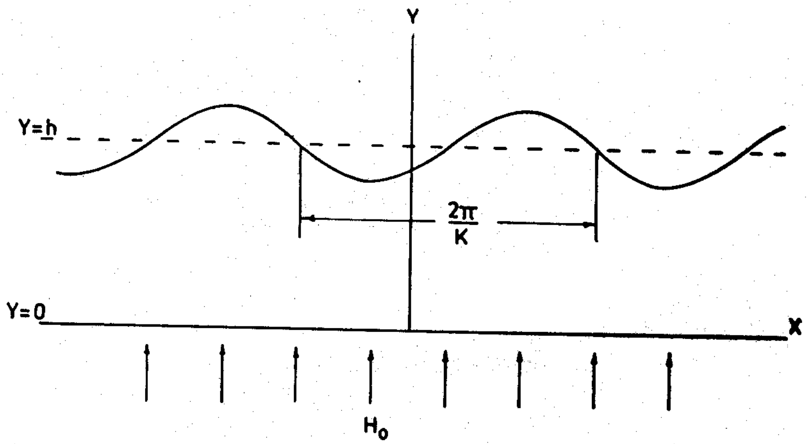


Figure 1a. Flow configuration.

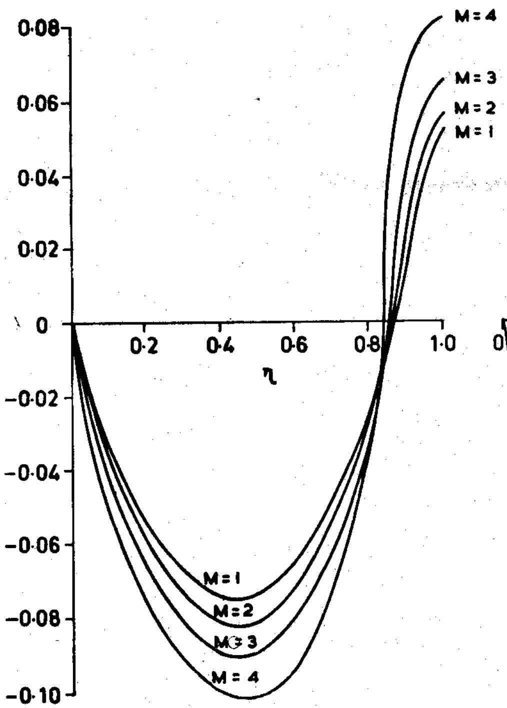


Figure 1b. Velocity distribution for different values of the magnetic parameter.

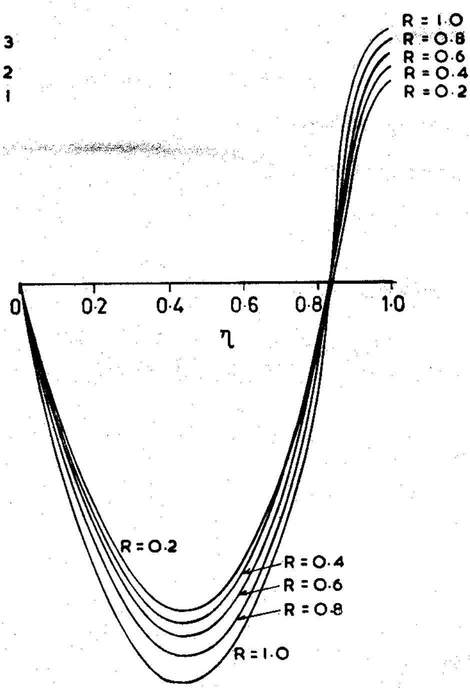


Figure 2. Velocity distribution for different values of the suction parameter.

Skin friction

The shearing stress at the flat wall is

$$\tau_0 = \frac{\mu U}{h}\left(\frac{\partial u}{\partial \eta}\right)_{\eta=0}$$

and the coefficient of skin friction is given by

$$C_f = \frac{2\tau_0}{\rho U^2} = \frac{2}{R_e}\left(\frac{\partial u}{\partial \eta}\right)_{\eta=0}$$

where R_e is the Reynolds number.

The coefficient of skin friction at the flat wall is

$$C_f = \frac{2Ca}{MR_e}\left[\frac{1 - \text{Cosh}\,(a\,\epsilon\,\text{Cos}\,\lambda x)}{\text{Sinh}\,(a\,\epsilon\,\text{Cos}\,\lambda x)}\right] \tag{14}$$

The effects of magnetic and suction parameters on the skin friction are examined. It is seen that the skin friction increases with the increase in the magnetic parameter (Fig. 4), whereas the skin friction decreases with the increase in suction parameter (Fig. 5). But Verma & Mathur[7] have observed that the coefficient of skin friction decreases as the magnetic or suction parameter increases when they considered magneto-hydrodynamic flow between two parallel plates, one in uniform motion and the other at rest with the uniform suction at the stationary plate. The effect of the frequency parameter of the wavy wall on the skin friction is shown in the Fig. 6.

Temperature Distribution

We evaluate the temperature distribution T^* by solving the equation

$$\frac{d^2T^*}{d\eta^2} + P \cdot R\,\frac{dT^*}{d\eta} = -\frac{P \cdot E \cdot C^2a^2}{M^2\,\text{Sinh}^2\,(a\,\epsilon\,\text{Cos}\,\lambda x)}\,[\text{Cosh}^2\,a\eta + \text{Cosh}^2\,\{a(\epsilon\,\text{Cos}\,\lambda x - \eta)\} - 2\,\text{Cosh}\,a\eta \times \text{Cosh}\,\{a(\epsilon\,\text{Cos}\,\lambda x - \eta)\}] \tag{15}$$

Using the boundary conditions

$$T^* = 0, \text{ at } \eta = 0$$
$$T^* = 1, \text{ at } \eta = \epsilon\,\text{Cos}\,\lambda x \tag{16}$$

Now the temperature distribution is

$$T^* = \frac{1}{(1 - e^{-a_2})} - \frac{e^{-PR\eta}}{(1 - e^{-a_2})} + \frac{P \cdot E \cdot C^2a^2}{(1 - e^{-a_2})\,M^2\,\text{Sinh}^2\,a_1}$$
$$\times [((\{e^{a_1} + 1 - e^{-a_2} - e^{-(a_1+a_2)} - e^{-PR\eta}\,\text{Sinh}\,a_1$$
$$- e^{a\eta}\,(1 + e^{-a_1})\,(1 - e^{-a_2})\} \div (a^2 + PR))$$
$$+ (\{e^{-a_1} + 1 - e^{-a_2} - e^{(a_1-a_2)} + e^{-PR\eta}\,\text{Sinh}\,a_1$$
$$- e^{-a\eta}\,(1 + e^{a_1})\,(1 - e^{-a_2})\} \div (a^2 - PR))$$
$$+ (\{e^{-(a_1+a_2)} - e^{a_1} + e^{-PR\eta}\,\text{Sinh}\,a_1$$
$$+ e^{2a\eta - a_1}\,(1 - e^{-a_2})\} \div (4a^2 + 2a\,PR))$$

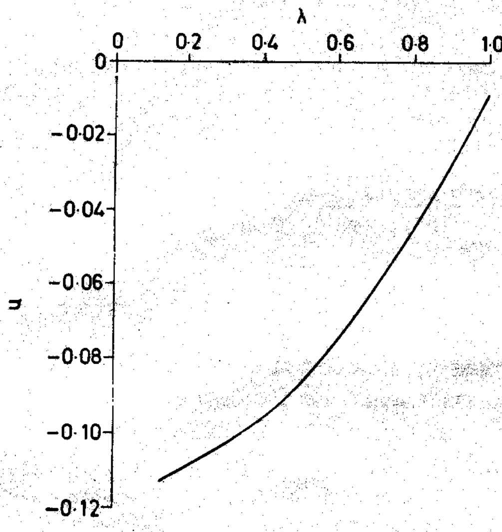
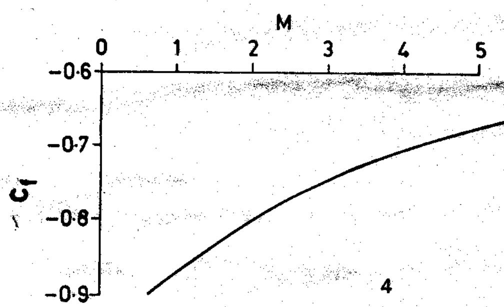
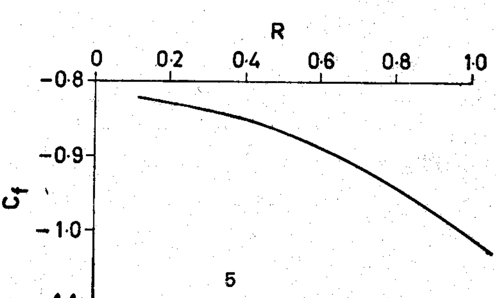


Figure 3. Velocity distribution vs. the frequency parameter of the wavy wall.





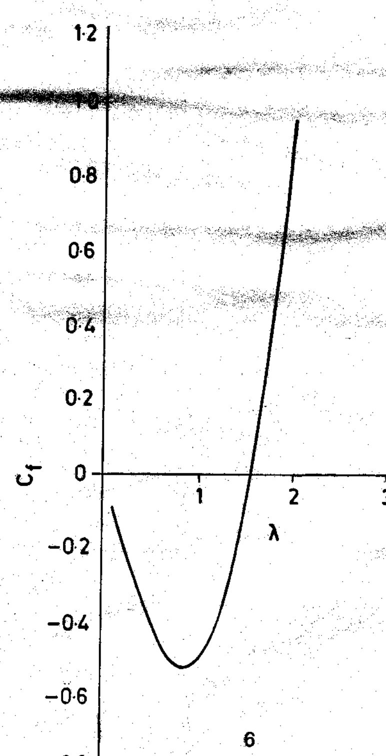


Figure 4. Coefficient of skin friction C_f against magnetic parameter.

Figure 5. Coefficient of skin friction C_f against suction parameter.

Figure 6. Coefficient of skin friction vs. frequency parameter of the wavy wall.

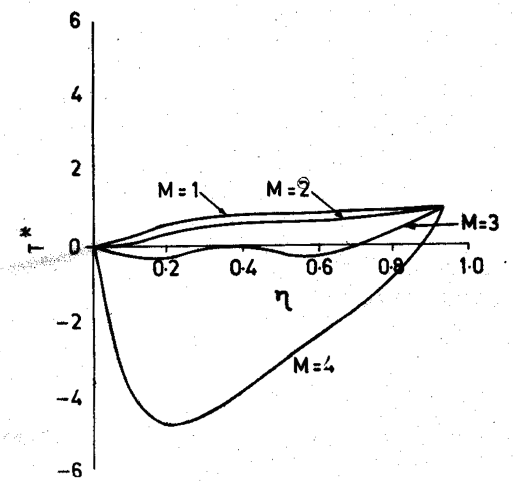


Figure 7. Temperature distribution for different values of the magnetic parameter.

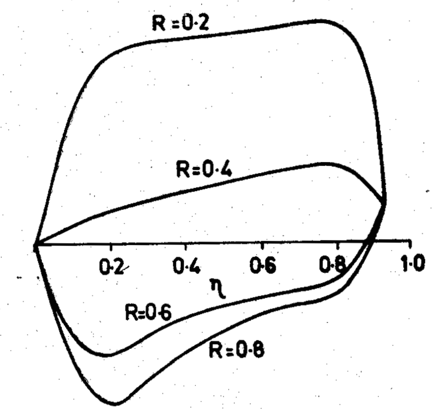


Figure 8. Temperature distribution for different values of the suction parameter.

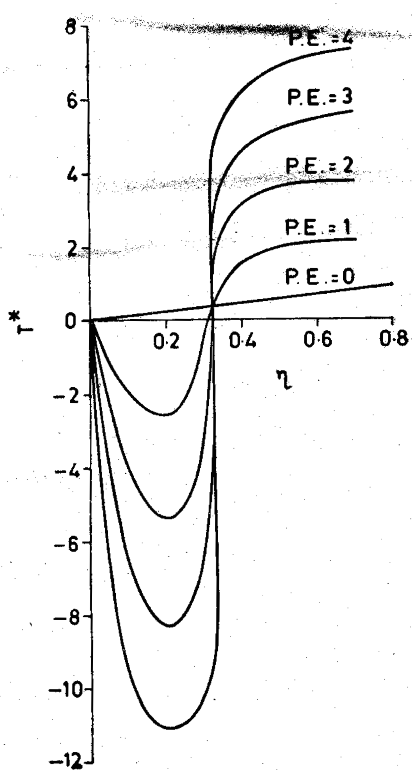


Figure 9. Temperature distribution at different points of the flow field for different values of P.E.

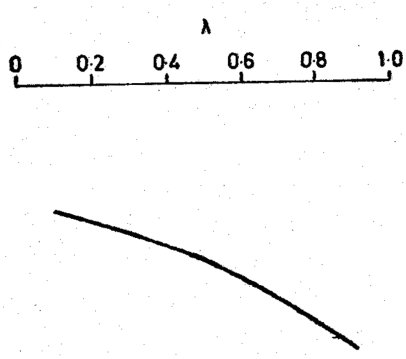


Figure 10. Temperature distribution vs. frequency parameter of the wavy wall.

$$+ (\{e^{(a_1-a_2)} - e^{-a_1} - e^{-PR\eta} \text{Sinh } a_1$$
$$+ e^{a_1} - 2a\eta (1 - e^{-a_2})\} \div (4a^2 - 2a\, PR))$$
$$+ (\{\epsilon \text{ Cosh } \lambda x (e^{-PR\eta} - 1) \text{ Cosh } a_1$$
$$+ \eta(1 - e^{-a_2}) \text{ Cosh } a_1\} \div PR)]$$

where

$$a_1 = a\epsilon \text{ Cos } \lambda x$$

$$a_2 = PR\, \dot{\epsilon} \text{ Cos } \lambda x$$

The effects of magnetic and suction parameters are investigated on temperature distribution. It is observed that the temperature increases as the magnetic parameter increases (Fig. 7), whereas the temperature decreases as the suction parameter increases (Fig. 8). The temperature distribution at various points of the flow field is obtained for different values of P. E. (Product of Prandtl and Eckert numbers) (Fig. 9). It is seen that the temperature decreases with the increase in the frequency parameter of the wavy wall (Fig. 10).

References

1. Shankar, P. N. & Sinha, U. N., *J. Fluid Mech.*, **77** (1976), 243.
2. Vajravelu, K. & Sastri, K. S., *J. Fluid Mech.*, **86** (1978), 365.
3. Lekoudis, S. G., Nayfeh, A. H. & Saric, W. S., *Phys. Fluids.*, **19** (1976), 514.
4. Lessen, M. & Gangwani, S. T., *Phys. Fluids.*, **19** (1976), 510.
5. Sparrow, E. M. & Cess, R. D., *Trans. ASME., J. Appl. Mech.*, **29** (1962), 181.
6. Soundalgekar, V. M. & Aranake, R. N., *Appl. Sci. Res.*, **34** (1978), 49.
7. Verma, P. D. & Mathur, A. K., *Natn. Inst. Sci., India*, **35A**, (1969), 507.