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Hall Effects on MHD Flow Through a Porous Straight Channel

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Abstract. The effect of Hall currents on the flow of a viscous incompressible slightly conducting fluid through a porous straight channel under a uniform transverse magnetic field is considered. The pressure gradient is taken as constant quantity and the case of steady flow is obtained by taking the time since the start of the motion to be infinite. Skin friction, temperature distribution and coefficients of heat transfer at both the plates have been evaluated. The effects of Hall parameter, magnetic parameter and Reynolds number on the above physical quantities have been investigated. Velocity distribution when the pressure gradient (i) varies linearly with time, and (ii) decreases exponentially with time has also been evaluated.

1. Introduction

The phenomenon of heat transfer is encountered in almost all branches of Technology. The MHD aspect of heat transfer in channel flow has been discussed by many researchers : the heat transfer problem in the case of fully developed flow by Siegel[1]. Alpher[2], Gershuni and Zukovitskii[3], Regirer[4] and Yen[5]; the heat transfer at the entrance region of the channel by Nigam and Singh[6]. All these papers examine the effect of magnetic field on heat transfer in a channel flow between conducting and non-conducting walls. MHD channel flows have a number of important applications such as MHD power generator, electromagnetic flow meter and electromagnetic accelerators. The last device is used extensively in connection with nuclear power reactor to pump liquid sodium as a coolant.

Verma and Mathur[7] have studied magnetohydrodynamic flow between two parallel plates, one in uniform motion and the other at rest with uniform suction at the stationary plate. They have observed the coefficient of skin friction decreases with the increase in Hartmann number. Satyaprakash[8] has obtained the exact solution of the problem of unsteady viscous flow through a porous straight channel. He has obtained the result that the velocity increases with time and tends ultimately towards the steady state at both points, as should have been the case in the presence of pressure gradient which remains constant for all times.

In this paper the effect of Hall currents on the flow of a viscous incompressible slightly conducting fluid through a porous straight channel under a uniform transverse magnetic field is considered. The pressure gradient is taken as constant quantity and the case of steady flow is obtained by taking the time since the start of the motion to be infinite. We have evaluated skin friction, the temperature distribution and coefficients of heat transfer at both the plates. We have investigated the effects of Hall parameter, magnetic parameter and Reynolds number on the above physical quantities. We have also evaluated velocity distribution when the pressure gradient (i) varies linearly with time, and (ii) decreases exponentially with time.

2. Formulation and Solution of the Problem

Unsteady two dimensional incompressible viscous flow through a straight channel with porous flat walls distant h apart in the presence of a uniform transverse magnetic field is considered. The lower plate is taken as X-axis and straight line perpendicular to that as Y-axis. It is assumed that the fluid is injected into the channel through the wall at $y = 0$ and sucked through the wall at $y = h$. Let u and v be the velocity components of the fluid at a point (x, y) in the direction of axes of coordinates respectively. It is assumed that the fluid is of small electrical conductivity with magnetic Reynolds number much less than unity so that the induced magnetic field can be neglected in comparison with the applied magnetic field. A uniform magnetic field of intensity H_0 in the direction of the Y-axis. Since the plates are infinite in length all physical quantities (except pressure) depend only on y and t. The equation of continuity $\nabla . \bar{q} = 0$ gives $v = 0$ where $\bar{q} = (u, v, w)$. The solinoidal relation $\nabla . \bar{H} = 0$ for the magnetic field gives $H_y = H_0 =$ constant everywhere in the fluid where $\bar{H} = (H_x, H_y, H_z)$. The conservation of electric charge $\nabla . \bar{J} = 0$ gives $J_y =$ constant, where $\bar{J} = (J_x, J_y, J_z)$. This constant is zero since $J_y = 0$ on the plates which are electrically non-conducting.

We shall assume that the induced magnetic field produced by the motion of the electrically conducting fluid is negligible, so that $\bar{H} = (0, H_0, 0)$. This assumption is justified since the magnetic Reynolds number is very small for the liquid metals. In the absence of an external electric field, the effect of polarization of the ionized fluid is negligible. We can also assume that the electric field[10] $\bar{E} = 0$. Under these assumptions the generalized Ohms law[11] is

$$\bar{J} + \frac{\omega_e \tau_e}{H_0} (\bar{J} \times \bar{H}) = \sigma \left[\mu_e (\bar{q} \times \bar{H}) + \frac{\nabla P_e}{e n_e} \right] \tag{1}$$

where σ, μ_e, ω_e, τ_e, e, n_e and P_e are respectively the electrical conductivity of the fluid, the magnetic permeability, the cyclotron frequency, the electron collision time, the electric charge, the number density of the electron and the electron pressure. In Eqn. (1) the ion slip and thermo-electric effects are neglected. Further for weakly ionized gases the electron pressure is negligible[12].

Thus the Eqn. (1) gives

$$J_x - \omega_e \tau_e J_z = 0 \tag{2}$$

$$J_z + \omega_e \tau_e J_x = \sigma \mu_e H_0 u \tag{3}$$

Solving the Eqns.(2) and (3), we obtain

$$J_x = \frac{\sigma \mu_e H_0}{(1 + m^2)} mu \tag{4}$$

$$J_z = \frac{\sigma \mu_e H_0}{(1 + m^2)} u \tag{5}$$

where $m = \omega_e \tau_e$ is the Hall parameter.

The equations of momentum, continuity and energy are respectively

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{\sigma \mu_e^2 H_0^2}{\rho(1 + m^2)} u \tag{6}$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \tag{7}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{8}$$

$$0 = K_T \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 \tag{9}$$

where ρ is the density of the fluid, ν the coefficient of kinematic viscosity, t the time measured since the start of the motion, p the pressure at a point (x, y), σ the electrical conductivity, H_0 the uniform applied magnetic field, μ the coefficient of viscosity, T the temperature and K_T the coefficient of thermal conductivity. In the energy Eqn. (9), the Joule dissipation heat is assumed to be negligible[13].

The initial and boundary conditions are :

$$t \leqslant 0,\ u = 0 \text{ and } v = 0 \text{ for } 0 \leqslant y \leqslant h$$

$$t > 0,\ u = 0 \text{ and } v = v_0, \text{ a constant} > 0 \text{ for } y = 0,\ h \tag{10}$$

at $y = 0,\ T = T_0$; at $y = h,\ T = T_1$

From the initial and boundary conditions (10), we may say that the velocity distribution is independent of x

Hence $$\frac{\partial u}{\partial x} = 0 \text{ and } \frac{\partial v}{\partial x} = 0 \tag{11}$$

On substituting $\frac{\partial u}{\partial x} = 0$ and using Eqn. (10) the Eqn. (8) yields $v = v_0$. Substituting $v = v_0$ in Eqns. (6) and (7), we obtain

$$\frac{\partial u}{\partial t} + v_0 \frac{\partial u}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma \mu_e^2 H_0^2}{\rho(1 + m^2)} u \tag{12}$$

and $$0 = \frac{\partial p}{\partial y} \tag{13}$$

Now we introduce the following dimensionless quantities :

$$u^* = \frac{u}{v_0},\; x^* = \frac{x}{h},\; y^* = \frac{y}{h},\; p^* = \frac{p}{\rho v_0^2}$$

$$t^* = \frac{t}{(h/v_0)},\; T^* = \frac{T - T_0}{T_1 - T_0} \tag{14}$$

In view of Eqn. (14), after dropping the superscripts* the Eqns. (11) to (13) and (9) reduce to

$$\frac{\partial u}{\partial x} = 0 \tag{15}$$

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{R}\frac{\partial^2 u}{\partial y^2} - Ku \tag{16}$$

$$0 = \frac{\partial p}{\partial y} \tag{17}$$

$$\frac{\partial^2 T}{\partial y^2} = -P.E\left(\frac{\partial u}{\partial y}\right)^2 \tag{18}$$

where $K = \dfrac{M}{1 + m^2}$

$M = \dfrac{\sigma \mu_e^2 H_0^2 h}{\rho v_0}$ (Magnetic parameter)

$R = \dfrac{v_0 h}{\nu}$ (Suction/injection parameter)

$P = \dfrac{\mu c_p}{K_T}$ (Prandtl number)

$E = \dfrac{v_0^2}{C_p(T_1 - T_0)}$ (Eckert number)

The initial and boundary conditions in dimension less form are

when $t \leqslant 0,\; u = 0$ for $0 \leqslant y \leqslant 1$ (19a)

when $t > 0,\; u = 0$ for $y = 0, 1$ (19b)

at $y = 0,\; T = 0$; at $y = 1,\; T = 1$ (19c)

It is observed that u is independent of x from Eqn. (15). Hence u is a function of y and t only. From Eqn. (17), we may say that p is independent of y. Therefore it follows from the Eqn. (16) that $\frac{\partial p}{\partial x}$ is a function of time only.

We assume that

$$\frac{\partial p}{\partial x} = -f(t)$$

The Eqn. (16) becomes

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial y} = f(t) + \frac{1}{R}\frac{\partial^2 u}{\partial y^2} - Ku$$

We define Laplace transform as

$$\bar{u} = \int_0^\infty u e^{-st} dt$$

with the inversion

$$u = \frac{1}{2\pi i}\int_{\gamma - i\infty}^{\gamma + i\infty} \bar{u} e^{st} ds$$

we denote

$$\bar{f}(s) = \int_0^\infty f(t)\, e^{-st} dt$$

In view of Eqns. (22) and (24), the Eqn. (21) is transformed into

$$\frac{d^2\bar{u}}{dy^2} - R\frac{d\bar{u}}{dy} - R(K + s)\,\bar{u} = -R\bar{f}(s)$$

The corresponding boundary conditions are

$$\bar{u} = 0 \text{ for } y = 0, 1$$

In view of conditions (26), the solution of Eqn. (25) is

$$\bar{u} = -\frac{\bar{f}(s)}{(K + s)\sinh \frac{1}{2}(R^2 + 4R(K + s))^{1/2}} \times \Big[e^{(R/2)y} \sinh \tfrac{1}{2}\{(R^2 + 4R(K + s))^{1/2}(1 - y)\} + e^{-(R/2)(1-y)} \sinh \tfrac{1}{2}\{(R^2 + 4R(K + s))^{1/2}\, y\}\Big] + \frac{\bar{f}(s)}{(K + s)} \qquad (27)$$

Case 1—Constant pressure gradient

Let us assume that the pressure gradient is a constant quantity. Hence let

$$\frac{\partial p}{\partial x} = -f(t) = -B,$$

where B is a positive constant

$$f(s) = \int_0^\infty Be^{-st}dt = \frac{B}{s}$$

By taking the inversion[14] of Eqn. (27), we obtain velocity distribution,

$$u = -\frac{B}{K\sinh\frac{1}{2}(R+4RK)^{1/2}}\Big[e^{(R/2)y}\sinh\tfrac{1}{2}\{(R^2+4RK)^{1/2}(1-y)\}$$
$$+ e^{-(R/2)(1-y)}\sinh\tfrac{1}{2}(R^2+4RK)^{1/2}y\Big] + \frac{B}{K} + \frac{Be^{-Kt}}{K\sinh\frac{R}{2}}$$
$$\times\Big[e^{-(R/2)(1-y)}\sinh\frac{R}{2}y + e^{(R/2)y}\sinh\frac{R}{2}(1-y) - \sinh\frac{R}{2}\Big]$$
$$+ 32BR\pi\sum_{n=1}^{\infty}\frac{ne^{(R/2)y}\sinh\pi y\{e^{-R/2}(-1)^n-1\}\exp\left\{-\frac{t}{4R}(R^2+4n^2\pi^2+4RK)\right\}}{(R^2+4n^2\pi^2+4RK)(R^2+4n^2\pi^2)}$$

The flow tends to be steady after a lapse of considerable time since the start of motion. In the case of steady state the velocity distribution is

$$u = -\frac{B}{K\sinh\frac{1}{2}(R^2+4RK)^{1/2}}\Big[e^{(R/2)y}\sinh\tfrac{1}{2}\{(R^2+4RK)^{1/2}(1-y)\}$$
$$+ e^{-(R/2)(1-y)}\sinh\tfrac{1}{2}\{(R^2+4RK)^{1/2}y\}\Big] + \frac{B}{K}$$

Now let us find the steady state solution directly from the equation of motion. After substituting B for $f(t)$, for steady state the Eqn. (21) becomes

$$\frac{d^2u}{dy^2} = R\frac{du}{dy} - KRu = -BR \tag{32}$$

The boundary conditions are same as those given in Eqn. (26). In view of the boundary conditions (26), the solution of Eqn. (32) is

$$u = -\frac{B}{K\sinh\frac{1}{2}(R^2+4RK)^{1/2}}\Big[e^{-(R/2)(1-y)}\sinh\tfrac{1}{2}\{(R^2+4RK)^{1/2}y\}$$
$$+ e^{(R/2)y}\sinh\tfrac{1}{2}\{(R^2+4RK)^{1/2}(1-y)\}\Big] + \frac{B}{K}$$

The results in Eqns. (31) and (33) are identical.

Skin friction

The shearing stress at the wall $y = 0$ is

$$\tau_o = \frac{\mu v_0}{h}\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

and the coefficient of skin friction is given by

$$\frac{2\tau_0}{\rho v_0^2} \quad \frac{2}{R}\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{34}$$

Hence the coefficient of skin friction at the wall $y = 0$ is

$$C_f = \frac{B}{K \sinh \frac{1}{2}(R^2 + 4RK)^{1/2}}\left[\frac{(R^2 + 4RK)^{1/2}}{R}\{e^{-R/2} + \cosh \tfrac{1}{2} \times (R^2 \quad 4RK)^{1/2}\} + \sinh \tfrac{1}{2}(R^2 + 4RK)^{1/2}\right]$$

$$\frac{Be^{-kt}}{K \sinh \frac{R}{2}}\left[e^{-R/2} - \cosh \frac{R}{2} \quad \sinh \frac{R}{2}\right]$$

$$64B\pi^2 \sum_{n=1}^{\infty} \frac{n^2\{e^{-R/2}(-1)^n \; - 1\} \exp\left\{-\frac{t}{4R}(R^2 + 4n^2\pi^2 + 4RK)\right\}}{(R^2 \quad 4n^2\pi^2 + 4RK)(R^2 + 4n^2\pi^2)} \tag{35}$$

The coefficient of skin friction at the wall y is

$$\frac{B}{K \sinh \frac{1}{2}(R^2 + 4RK)^{1/2}}\left[\frac{(R^2 + 4RK)^{1/2}}{R} \times \{e^{R/2} \quad \cosh \tfrac{1}{2}(R^2 + 4RK)^{1/2}\} + \sinh \tfrac{1}{2}(R^2 \quad 4RK)^{1/2}\right]$$

$$\frac{Be^{-Kt}}{K \sinh \frac{R}{2}}\left[\cosh \frac{R}{2} + \sinh \frac{R}{2} \quad e^{R/2}\right]$$

$$64B\pi^2 \sum_{n=1}^{\infty} \frac{n^2(-1)^n e^{R/2}\{e^{-R/2}(\;-1)^n - 1\} \exp\left\{-\frac{t}{4R}(R^2 + 4n^2\pi^2 + 4RK)\right\}}{(R^2 \quad 4n^2\pi^2 + 4RK)(R^2 + 4n^2\pi^2)} \tag{36}$$

Temperature distribution

From the Eqns. (18) and (33), we write

$$\frac{d^2T}{dy^2} = \frac{PEB^2}{K^2 \sinh^3 \alpha}\left[e^{-R(1-y)}\beta \cosh 2\alpha y + e^{Ry}\beta \cosh 2\alpha(1 \quad y) + \frac{R}{2}\alpha e^{-R1-y} \sinh 2\alpha y - RKe^{-(R/2)(1-2y)}\{\cosh \alpha + \cosh \alpha(1 \quad 2y)\}\right.$$

$$+ R\alpha\, e^{-(R/2)(1-2y)} \sinh \alpha(1-2y) - \frac{R}{2}\, \alpha e^{Ry} \sinh 2\alpha(1-y)$$

$$+ \frac{RK}{2} (e^{-R} + 1)\, e^{Ry} \Big] \qquad (37)$$

where $\alpha = \frac{1}{2}(R^2 + 4RK)^{1/2}$

$\beta = \frac{1}{4}(R^2 + 2RK)$

The boundary conditions are

at $\quad y = 0,\ T = 0;$ at $y = 1,\ T = 1 \qquad (38)$

Solving the Eqn. (37) using the boundary conditions (38), we obtain the temperature distribution.

$$T = \frac{B^2PE}{16K^4R^2 \sinh^2\alpha}\Big[a\,\{e^{-R} + \cosh 2\alpha + y\,.\,2\,.\sinh R$$

$$- e^{Ry}(e^{-R}\cosh 2\alpha y + \cosh 2\alpha(1-y))\} + b\,\{\sinh \alpha - y(e^R + 1)\sinh \alpha$$

$$- e^{Ry} \sinh \alpha(1-2y)\} + C\,\{\cosh \alpha + y(e^R - 1)\cosh \alpha$$

$$- e^{Ry} \cosh \alpha(1-2y)\} + d\,\{1 + y(e^R - 1) - e^{Ry}\}\Big] + y$$

where

$$a = \tfrac{1}{2}(R^2 + 2RK)^2 - 2R^2\alpha^2$$

$$b = 8R\alpha^3 e^{-R/2}$$

$$c = e^{-R/2}\left(4R^2\alpha^2 + 4\alpha^4 - \frac{R^4}{4}\right)$$

$$d = 8RK^3(1 + e^{-R} - 2e^{-R/2}\cosh \alpha)$$

Heat transfer coefficient

From the point of view of applications in technology it is of interest to know the rates of heat transfer at both the walls. The rate of heat transfer coefficient at the lower plate $y = 0$ is

$$q = \left(\frac{\partial T}{\partial y}\right)_{y=0}$$

$$= \frac{B^2PE}{16K^4R^2\sinh^2\alpha}\Big[a\,\{2\sinh R - R(e^{-R} + \cosh 2\alpha) + 2\alpha \sinh 2\alpha\}$$

$$+ b\,\{-(e^R + 1)\sinh \alpha - R\sinh \alpha + 2\alpha\cosh \alpha\}$$

$$+ C\{(e^R - 1)\cosh \alpha - R\cosh \alpha + 2\alpha \sinh \alpha\} + d\,\{e^R - R - 1\}\Big] + 1$$

The rate of heat transfer coefficient at the upper plate $y =$ is

$$q^* = \left(\frac{\partial T}{\partial y}\right)_{y=1}$$

$$= \frac{B^2 PE}{16K^4 R^2 \sinh^2 \alpha}\Big[a\,\{2 \sinh R - R(\cosh 2\alpha + e^R) - 2\alpha \sinh 2\alpha\}$$

$$+ b\,\{-(e^R + 1) \sinh \alpha + Re^R \sinh \alpha + 2\alpha e^R \cosh \alpha\}$$

$$+ c\,\{(e^R - 1) \cosh \alpha - Re^R \cosh \alpha - 2\alpha e^R \sinh \alpha\}$$

$$+ d\,\{e^R - Re^R - 1\}\Big] + 1$$

Case 2—Pressure gradient varies linearly with time

We now assume that

$$\frac{\partial p}{\partial x} \quad a_0 + a_1 t$$

Substituting this value in the Eqn. (16), we get

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial y} = a_0 + a_1 t + \frac{1}{R}\frac{d^2 u}{dy^2} - Ku$$

In view of the Eqn. (22), the Eqn. (43) is transformed into

$$\frac{d^2 \bar{u}}{dy^2} \quad R\frac{d\bar{u}}{dy} \quad R(K \quad s)\,\bar{u} \quad -R\left(\frac{a_0}{s} + \frac{a_1}{s^2}\right)$$

The boundary conditions are

$$\bar{u} = 0 \text{ for } y = 0,$$

The solution of the Eqn. (44), with the boundary conditions (45) is

$$\bar{u} \quad -\frac{\left(\frac{a_0}{s} + \frac{a_1}{s^2}\right)}{(K + s) \sinh \frac{1}{2} \{R^2 + 4R(K + s)\}^{1/2}}\Big[e^{(R/2)y} \sinh \tfrac{1}{2}$$

$$\times \{R^2 + 4R(K + s)\}^{1/2} (1 - y) + e^{-(R/2)(1-y)} \sinh \tfrac{1}{2}$$

$$\times \{R^2 + 4R(K + s)\}^{1/2}\, y\Big] + \frac{\left(\frac{a_0}{s} + \frac{a_1}{s^2}\right)}{(K + s)}$$

By taking the laplace inversion of Eqn. (46), we obtain the velocity distribution

$$u = \frac{a_0}{K}(1 - e^{-Kt}) + \frac{a_1}{K^2}(Kt - 1 + e^{-Kt}) - \frac{1}{K^2 \sinh^2 \alpha}\Big\{a_0 K \sinh \alpha$$

$$+ a_1\left(Kt \sinh \alpha - \frac{KR}{2\alpha} \cosh \alpha - \sinh \alpha\right)\Big\}\,\{e^{(R/2)y} \sinh \alpha(1 - y)$$

$$+ e^{-(R/2)(1-y)} \sinh \alpha y\}$$

$$- \frac{a_1 R}{2K\alpha \sinh \alpha} \left\{ e^{(R/2)y}(1 - y) \cosh \alpha(1 - y) + y e^{-(R/2)(1-y)} \cosh \alpha y \right\}$$

$$+ \frac{(a_0 K - a_1) e^{-Kt}}{K^2 \sinh \frac{R}{2}} \left\{ e^{(R/2)y} \sinh \frac{R}{2} (1 - y) + e^{-(R/2)(1-y)} \sinh \frac{R}{2} y \right\}$$

$$+ 8\pi \sum_{n=1}^{\infty} \frac{(a_0 \Upsilon_1 - a_1)\, n e^{(R/2)y} \sin n\pi y \{e^{-R/2}(-1)^n - 1\} \exp(-\Upsilon_1 t)}{\Upsilon_1^2 (R^2 + 4n^2\pi^2)}$$

where $\Upsilon_1 = \frac{1}{4R} (R^2 + 4n^2\pi^2 + 4RK)$

Case 3— Pressure gradient decreases exponentially with time

We take

$$-\frac{\partial p}{\partial x} = a_0 + \sum_{m_1=1}^{\infty} a_{m_1} e^{-m_1 t}$$

Substituting the value of Eqn. (48) in Eqn. (16), we get

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial y} = a_0 + \sum_{m_1=1}^{\infty} a m_1 e^{-m_1 t} + \frac{1}{R} \frac{d^2 u}{dy^2} - Ku$$

In view of Eqn. (22), the Eqn. (49) takes the form

$$\frac{d^2 \bar{u}}{dy^2} - R \frac{d\bar{u}}{dy} - R(K + s)\, \bar{u} = - R \left(\frac{a_0}{s} + \sum_{m_1=1}^{\infty} \frac{a m_1}{s + m_1} \right)$$

The corresponding boundary conditions are

$$\bar{u} = 0 \text{ for } y = 0, 1$$

The solution of the Eqn. (50), using the boundary conditions (51) is

$$\bar{u} = - \frac{\left(\frac{a_0}{s} + \sum_{m_1=1}^{\infty} \frac{a m_1}{s + m_1} \right)}{(K + s) \sinh \frac{1}{2} \{R^2 + 4R(K + s)\}^{1/2}} \Big[e^{(R/2)y} \sinh \tfrac{1}{2} \{R^2 + 4R(K+s)\}^{1/2}$$

$$\times (1 - y) + e^{-(R/2)(1-y)} \sinh \tfrac{1}{2} \{R^2 + 4R(K + s)\}^{1/2}\, y \Big]$$

$$+ \frac{\left(\frac{a_0}{s} + \sum_{m_1=1}^{\infty} \frac{a m_1}{s + m_1} \right)}{(K + s)}$$

Taking Laplace inversion of Eqn. (52), we obtain velocity distribution

$$u = \frac{a_0}{K}(1 - e^{-Kt}) + \sum_{m_1=1}^{\infty} \frac{am_1}{K - m_1}(e^{-m_1 t} - e^{-Kt})$$

$$\frac{a_0}{K \sinh \alpha}\{e^{(R/2)y} \sinh \alpha(1 - y) + e^{-(R/2)(1-y)} \sinh \alpha y\}$$

$$\sum_{m_1=1}^{\infty} \frac{am_1 e^{-m_1 t}}{(K - m_1)\sinh \eta}\{e^{(R/2)y} \sinh \eta(1 - y) + e^{-(R/2)(1-y)} \sinh \eta y\}$$

$$+ \left(\frac{a_0}{K} + \sum_{m_1=1}^{\infty} \frac{am_1}{(K - m_1)}\right) \frac{e^{-Kt}}{\sinh \frac{R}{2}} \left\{e^{(R/2)y} \sinh \frac{R}{2}(1 - y) + e^{-(R/2)(1-y)} \sinh \frac{R}{2} y\right\}$$

$$+ 8a_0\pi \sum_{n=1}^{\infty} \frac{ne^{(R/2)y} \sin n\pi y \{e^{-R/2}(-1)^n - 1\} \exp(-\gamma_1 t)}{\gamma_1(R^2 + 4n^2\pi^2)}$$

$$+ 8\pi \sum_{n=1}^{\infty} \sum_{m_1=1}^{\infty} \frac{am_1 ne^{(R/2)y} \sin n\pi y \{e^{-R/2}(-1)^n - 1\} \exp(-\gamma_1 t)}{(\gamma_1 - m_1)(R^2 + 4n^2\pi^2)} \tag{53}$$

where $\eta = \frac{1}{2}\{R^2 + 4R(K - m_1)\}^{1/2}$

3. Conclusions

We have obtained the velocity distribution in unsteady and steady cases, the coefficients of skin friction at both the walls, temperature distribution and the coefficient of heat transfer at both the walls. When the magnetic parameter M tends to zero our results regarding the velocity distribution in unsteady and steady cases coincide with those of Satyaprakash[8]. We have plotted a graph (Fig. 1) taking velocity distribution u against y for different values of m or M or R. It is observed that the velocity increases with the increase of Hall parameter m or suction/injection parameter R whereas it decreases with the increase in magnetic parameter M. In Fig. 2, we have drawn a graph taking velocity distribution u against time t. We have observed that the velocity increases with time and tends ultimately towards the steady state at both the points, as should have been the case in the presence of a pressure gradient which remains constant for all times. It is also observed that the steady state is obtained earlier than in the non-magnetic case. We can also conclude that the velocity at the point near the wall which is subjected to injection is less than that the velocity at the point near the wall which is subjected to suction. In Fig. 3, we have plotted a graph taking skin friction against m or M. We have seen that the skin frictions C_f and C_f^* at both

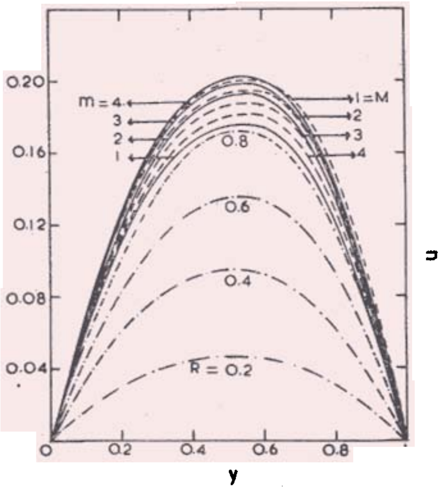

Figure 1. Velocity distribution against y for different values of m or M or R.

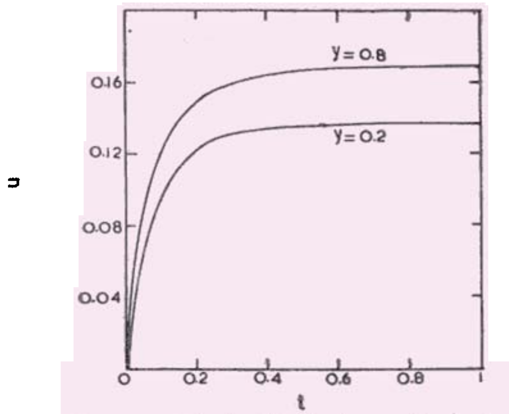

Figure 2. Velocity distribution against t for different values of y.

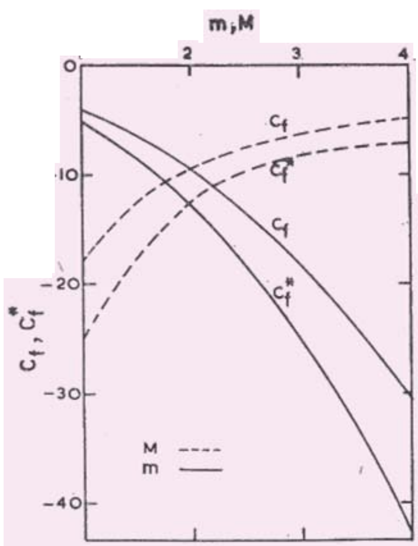

Figure 3. C_f , C_f^* plotted against m or M.

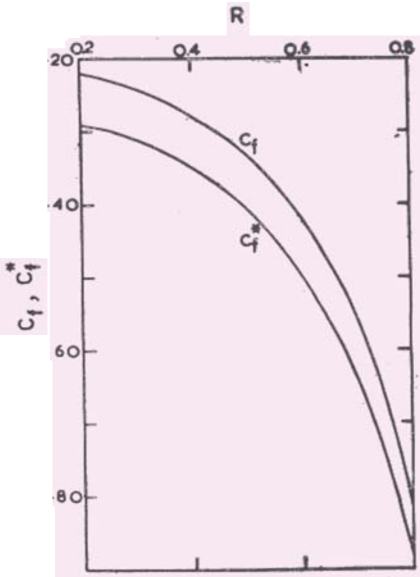

Figure 4. C_f , C_f^* plotted against R.

the walls decrease as m increases whereas they increase as M increases. But Verma and Mathur[7] have observed that the coefficient of skin friction decreases as the magnetic field strength increases at the stationary wall which is subjected to suction. We can also conclude that the coefficient of skin friction at the point near the wall which is subjected to suction is less than that at the point near the wall which is subjected to injection. In Fig. 4, we have shown the effect of suction/injection parameter R on skin frictions C_f and C_f^*. It is observed that the skin frictions C_f and C_f^* decrease with the increase in R. In Fig. 5, we have

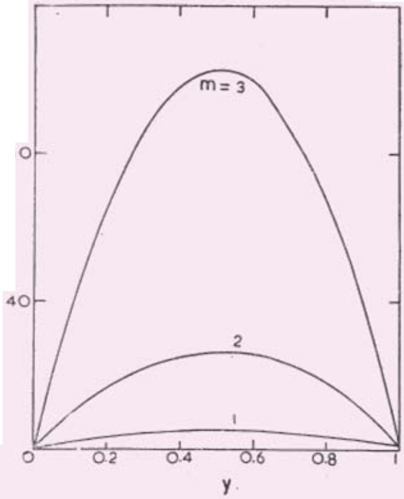

Figure 5. Temperature distribution against y for different values of m.

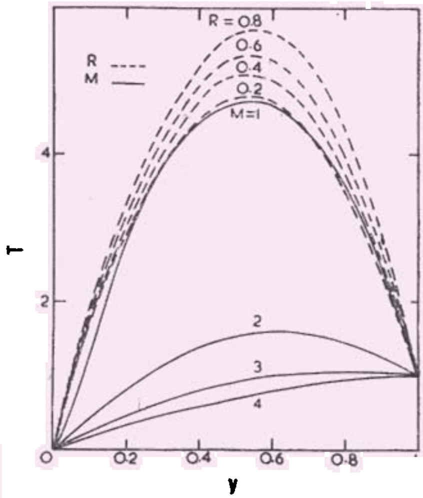

Figure 6. Temperature distribution against y for different values of M or R.

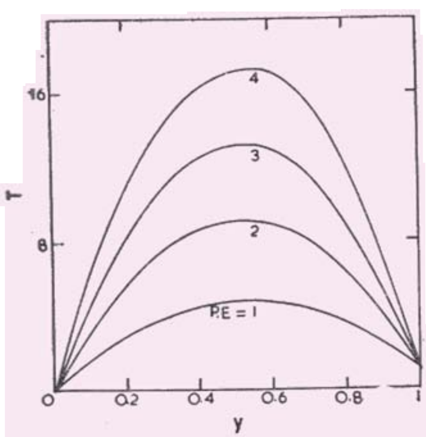

Figure 7. Temperature distribution against y for different values of PE.

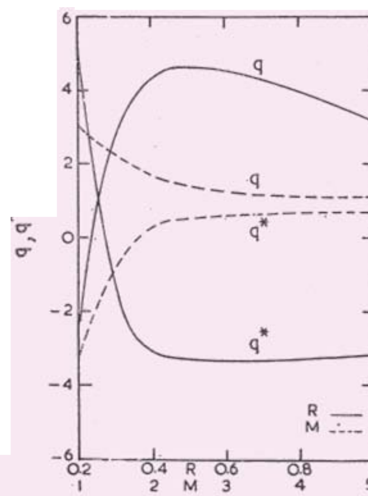

Figure 8. q q^* plotted against R or M.

plotted a graph taking temperature distribution against y for different values of m. We have seen that the temperature increases as m increases and this increment increases with the increase in m. In Fig. 6, we have drawn a graph taking temperature distribution against y for different values of M or R. It is observed that the temperature decreases with the increase in M whereas it increases as R increases. In Fig. 7, we have drawn the temperature distribution against y for different values of $P.E.$ (product of Prandtl and Eckert numbers). It is seen that the temperature increases with the increase in $P.E.$ In Fig. 8, the rates of heat transfer q and q^* are drawn against M or R. We have observed that q increases for small values of R and decreases for higher values of R whereas q^* decreases first upto $R = 0.6$ and increases for all values of $R > 0.6$. It is also observed that q decreases with the increase in M whereas q^* increases as M increases. Table 1 shows that q increases with the increase in Hall parameter m whereas q^* decreases as m increases.

Table 1.

m	$q \times 10$	$q^* \times 10$
	0.31437861	— 0.32240088
2	2.1091345	— 2.7498162
3	8.2728882	—11.447265
4	23.89082	—33.455854
5	55.949	—78.606016

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