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Recovery Ventilation and Oxygen Debt—A Mathematical Model for the Prediction of Recovery Ventilation

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Abstract. A mathematical model has been evolved for the estimation of recovery ventilation following an exercise. The model has been used to estimate recovery ventilation in moderate to heavy exercise for a period of 32 minutes. The model gives satisfactory predictions for persons of different age groups and under different environmental conditions thus establishing its universal applicability.

1. Introduction

Pulmonary ventilation has an effective and important role during exercise and recovery. During recovery it takes long time for ventilation to come back to pre-exercise base level. Stainsby and Barclay[1] stated that high ventilation continued for only a very short period of time but Welch *et al.*[2] found that hyperventilation and laboured breathing could persist for upto and above five minutes during recovery. For determination of oxygen debt the pulmonary ventilation has been taken into consideration for periods varying about 12 to 60 minutes after the cessation of exercise[3-11]. In field conditions and atheletic events the determination of oxygen debt during recovery poses a problem as it requires a well equipped laboratory. The measurement of recovery ventilation for long periods causes inconvenience to the subject. Katch *et al.*[12] in minute to minute recovery described ventilation by a two-component exponential equation. An attempt has been made in this paper to utilise the behaviour of exponentiality for the prediction of recovery ventilation with the help of a mathematical model. The relationship between recovery ventilation and oxygen debt has been evaluated with a view to suggest that recovery ventilation may be used as a measure of oxygen debt.

2. Derivation of Prediction Formula

The model function of recovery ventilation[1,12] is shown in Fig. 1 by the curve profile OC where time is plotted on X-axis and recovery ventilation is plotted on Y-axis.

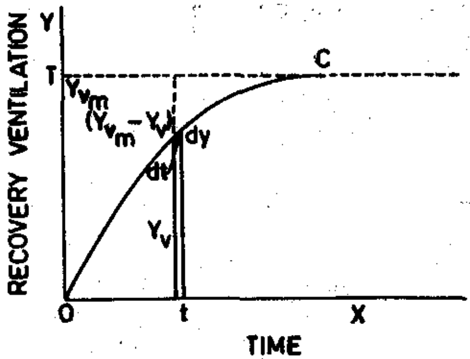

Figure 1. Recovery ventilation as a function of time t (Yv_m—total recovery ventilation, Y_v—recovery ventilation at instant t).

The rate of recovery ventilation above resting value $\left(\frac{dYv}{dt}\right)$ at instant t should vary directly as the recovery yet to be made i.e. $(Yv_m - Yv)$ where Yv_m is the total recovery ventilation and Yv is the recovery ventilation during the interval t.
Mathematically this means

$$dYv/dt = k(Yv_m - Yv) \tag{1}$$

where k is the rate constant characteristics of the recovery process. Equation (1) on integration yields

$$ln\ (Yv_m - Yv) = -kt + C$$

since $\quad Yv = 0$ at $t = 0$

$$ln\ Yv_m = C$$

substituting the value for C and removing logarithm

we obtain, $\quad Yv = Yv_m\,(1 - e^{-kt}) \qquad (2)$

This is a model of the recovery ventilation describing it as a single exponential recovery process. Katch *et al.*[13] have found that the recovery process of ventilation can be described by a two-component exponential equation. Sen Gupta *et al.*[10] have assumed that for practical purposes, the resultant of two functions exponential with respect to time but having different velocity constants, can be closely approximated by a single function which is exponential not with time but with respect to some fractional power of time over the period of recovery. Therefore, for the sake of generality, taking the fractional power of t as q, the general model of recovery ventilation may be modified to

$$Yv = Yv_m\,[1 - \exp\,(-kt^q)] \tag{3}$$

Let Yv_1, Yv_2 and Yv_3 be the values of Yv at times t_1, t_2 *and* t_3 respectively such that

$$t_2^q = 2t_1^q,\ \ t_3^q = 2t_2^q = 4t_1^q, \tag{4}$$

substitution of these values in Eqn. (3) yields

$$Yv_1 = Yv_m [1 - \exp(-kt_1^q)] \tag{5}$$

$$Yv_2 = Yv_m [1 - \exp(-2kt_1^q)] \tag{6}$$

$$Yv_3 = Yv_m [1 - \exp(-4kt_1^q)] \tag{7}$$

From Eqns. (5) and (6), we have

$$(Yv_m - Yv_1)^2 = Yv_m(Yv_m - Yv_2)$$

whence

$$Yv_m = Yv_1^2/(2Yv_1 - Yv_2) \tag{8}$$

Similarly from Equations (6) and (7) we have

$$Yv_m = Yv_2^2/(2Yv_2 - Yv_3) \tag{9}$$

Eliminating Yv_m between Eqns. (8) and (9) we have finally

$$Yv_3 = 2Yv_2 - (Yv_2/Yv_1)^2 (2Yv_1 - Yv_2) \tag{10}$$

Equation (10) can therefore be used to predict total recovery ventilation Yv_3 over a recovery period t_3 from values Yv_1 and Yv_2 measured over recovery periods t_1 and t_2.

Evaluation of q

The fractional power q can be evaluated from Eqn. (3) by writing it as

$$(Yv_m - Yv) = Yv_m [\exp(-kt^q)]$$

Taking logarithms to the base 10, we have

$$\log (Yv_m - Yv) = \log Yv_m - (k/2.303)\, t^q \text{ or}$$

$$\log [Yv_m/(Yv_m - Yv)] = \log (k/2.303) + q \log t \tag{11}$$

Thus $\log \{Yv_m/(Yv_m - Yv)\}$ plotted against time on a double log paper should yield a straight line, the slope of which gives the value of q.

3. Validation of Proposed Equation

Available Experimental Data

The fractional ventilation repayment data was available upto 30 minutes of recovery on three groups of subjects from our earlier studies. The physical characteristics of the subjects and environmental conditions are summarised in Table 1. Group 1 (8 subjects) and Group 2 (22 subjects) were observed under comfortable environmental conditions where as Group 3 (5 subjects) were observed under three different environmental conditions which may be characterised as comfortable, hot humid and very hot humid. Group 2 & 3 comprised of young healthy subjects while Group 1 comprised middle aged subjects.

Table 1. Physical and physiological characteristics of subjects and environments.

Group	No. of subjects	Age (Yr)	Ht (cm)	Wt (kg)	Environmetal conditions DB (°C)	WB (°C)	RH (%)	ET (°C)
1	8	44.9 ± 5.13	169.5 ± 5.70	65.8 ± 6.8	29.0	21.3	50	25.6
2	22	23.27 ± 1.93	165.3 ± 5.23	59.4 ± 3.4	27	22	60	24.4
3	5	21.8 ± 0.84	169.8 ± 2.06	60.16 ± 4.43	(i) Comfortable*			
					27	22	60	24.4
					(ii) Hot humid*			
					37	29	60	31.4
					(iii) Very hot humid*			
					40	32.5	60	33.9

*The five (5) subjects of Group 3 were separately subjected to three different environments (See Table 2C)

The subjects reported to the laboratory after a light breakfast. They were given rest for about 2 hours. Their resting ventilation was measured for 10 minutes. Then each subject was given exercise on a mechanically braked bicycle ergometer for different grades and durations on different days. The subjects breathed through a low resistance breathing valve and the expired air was collected in meteorological balloon and the volume was measured in a K.M. respirométer. The post exercise ventilation repayment was measured during recovery every minute for first 10 minutes and thereafter at intervals of 5 minutes upto 30 minutes. Tables 2A, 2B and 2C give recovery ventilation data for each subject.

Table 2A. Fractional recovery ventilation repayment for different rates of work.

Group	Environmental condition	Subject	Repayment (litres) 0–1 Min.	0–2 Min.	0–3 Min.	0–5 Min.	0–8 Min.	0–10 Min.	0–15 Min.	0–30 Min.
1	Comfortable	1	13.28	23.53	29.93	34.35	35.49	41.13	43.03	45.52
		2	52.13	78.85	94.14	117.58	137.23	157.04	184.94	218.24
		3	19.90	34.70	40.56	52.70	62.57	68.66	79.95	102.54
		4	57.78	91.32	119.86	143.64	162.19	182.77	215.42	264.43
		5	29.39	53.50	64.47	75.52	79.93	88.62	95.80	105.88
		6	9.85	17.00	19.04	25.20	33.00	35.84	44.61	66.59
		7	31.20	48.72	56.80	65.94	77.19	84.27	92.06	107.54
		8	29.61	39.13	46.60	52.26	56.07	60.61	63.31	65.80

Table 2B. Fractional recovery ventilation repayment for different rates of work.

Group	Environment condition	Subject	Repayment (litres) 0–1 Min.	0–2 Min.	0–3 Min.	0–5 Min.	0–8 Min.	0–10 Min.	0–15 Min.	0–30 Min.
2	Comfortable	1	26.40	45.98	60.40	72.80	80.50	86.75	98.36	121.50
		2	31.40	48.92	60.84	78.36	88.20	98.68	118.68	135.65
		3	28.72	46.48	89.50	54.40	69.80	101.54	125.30	165.45
		4	30.72	44.76	56.32	68.12	87.50	100.54	130.60	176.50
		5	33,50	51.30	63.84	82.40	99.40	110.14	128.36	173.68
		6	39.39	62.40	112.50	78.48	91.62	123.40	144.30	196.40
		7	38.54	58.65	75.05	96.05	114.30	126.76	152.52	205.40
		8	34.80	57.40	66.40	82.59	103.80	119.53	125.33	170.93
		9	18.88	34.03	43.02	52.88	62.48	69.50	80.67	104.19
		10	36.48	53.50	67.86	86.84	105.60	118.68	145.82	197.00
		11	42.85	62.80	79.80	95.68	109.36	118.18	143.38	168.46
		12	32.44	47.64	60.18	74.30	89.50	101.36	122.40	162.00
		13	29.46	46.35	58.40	72.68	86.40	95.82	114.28	160.80
		14	31.39	44.48	56.57	72.38	85.85	91.78	114.96	172.06
		15	30.68	47.80	58.52	74.84	87.55	95.86	114.36	155.27
		16	34.66	47.92	57.84	72.58	88.40	99.74	117.24	155.62
		17	37.86	52.40	64.18	83.77	98.80	106.74	124.62	168.60
		18	35.36	48.37	59.27	76.88	93.50	102.48	123.76	174.30
		19	29.62	44.38	54.30	68.28	78.68	83.60	92.28	133.64
		20	32.40	44.50	56.94	73.52	86.35	92.34	113.84	169.88
		21	33.14	53.60	96.70	67.84	84.30	103.68	120.42	151.90
		22	20.82	33.48	40.16	48.26	58.06	63.00	71.50	87.31

Table 2C. Fractional recovery ventilation repayment for different rates of work.

Group	Environmental condition	Subject	Repayment (litres) 0–1 Min.	0–2 Min.	0–3 Min.	0–5 Min.	0–8 Min.	0–10 Min.	0–15 Min.	0–30 Min.
3	Comfortable	1	13.32	16.87	18.09	18.19	18.62	19.40	20.06	20.50
		2	28.48	42.97	54.23	63.32	67.00	70.55	75.88	86.40
		3	13.21	15.54	17.32	20.21	23.21	23.88	27.67	30.02
		4	19.66	30.77	35.32	45.77	57.76	65.86	80.92	104.72
		5	17.77	23.98	29.30	30.27	30.94	31.78	32.02	32.34
3	Hot humid (H.H.)	1	15.55	21.17	26.90	34.57	38.32	40.02	51.77	66.41
		2	24.74	44.74	53.21	66.11	72.06	78.42	89.42	100.91
		3	16.10	21.97	24.62	29.76	31.60	35.23	37.78	38.84
		4	30.98	47.68	57.28	70.20	80.34	86.20	98.74	125.74
		5	13.13	18.85	21.55	25.35	32.37	35.37	38.84	52.46
3	Very hot humid (V.H.H.)	1	19.91	33.48	40.90	48.49	62.06	71.06	82.80	105.31
		2	30.86	46.80	54.34	70.28	85.95	94.30	111.36	148.90
		3	13.38	23.53	29.47	34.58	35.49	41.61	43.73	47.24
		4	30.62	48.92	59.66	70.42	86.20	92.36	114.88	148.44
		5	17.85	27.85	34.40	40.67	48.51	51.55	59.39	72.11

Evaluation of q from Experimental Values

The evaluation of q was done by actual plotting of Y against time t and only those data were utilised for which recovery was more or less complete. The data were analysed in terms of Eqn. (11) by plotting log $[Yv_m/(Yv_m - Yv)]$ against time on a double log paper separately for each individual exercise and a mean straight line fitted by hand in each case. The individual slopes varied from 0.40 to 0.64 with mean value of 0.56. However, for the purpose of the present paper, q has been taken to be 0.5 in order to simplify computational work.

Prediction of Recovery Ventilation

Taking q equal to 0.5, Eqn. (4) establishes the relative values of t_1, t_2 and t_3 viz $t_2 = 4\,t_1$ and $t_3 = 16\,t_1$ so that if t_3 is taken to be 32 minutes then $t_2 = 8$ minutes and $t_1 = 2$ minutes. For practical purposes the 32 minute value estimated from Equation (10) may be accepted as reasonably close to the 30 minute value actually observed.

Recovery ventilation over a period of 32 minutes was computed by substituting 2 minute and 8 minute values in Eqn. (10). In all, 45 values were estimated and plotted against observed values of recovery ventilation in Fig. 2. Reasonable good predictions are found with correlation coefficient $r = 0.99$ which is highly significant ($P < 0.001$).

Group 3 subjects were observed in three different environmental conditions i.e. comfortable, hot humid and very hot humid. The predictions of recovery ventilation were found satisfactory in the three environmental condions with 5 observations in each environment. The correlation coefficient r between observed and predicted

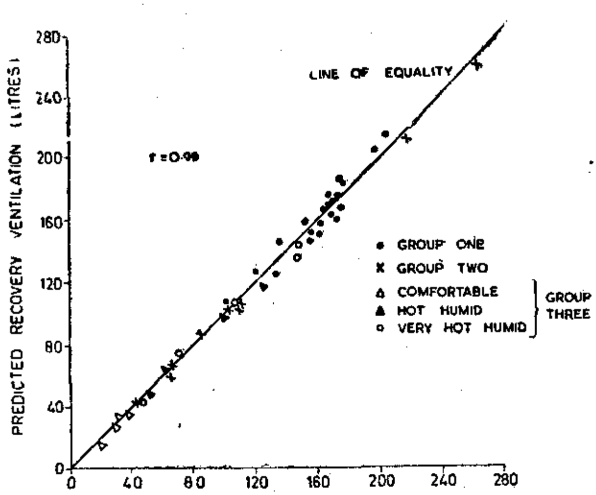

Figure 2. Comparison between recovery ventilation predicted from 2 observations (2 min & 8 min) and observed recovery ventilation up to 30 min.

values of recovery ventilation with 15 observations is 0.99 which is highly significant ($P < 0.001$). This establishes the general applicability and validity of the proposed model for different age groups and different environments.

Relationship Between Recovery Ventilation and Oxygen Debt

The data of recovery ventilation and oxygen debt were available from the studies of Welch *et al.*[2] and our present studies and have been shown in Table 3. The oxygen debt is well related with recovery ventilation. The correlation coefficient for 14 observations in the studies of Welch *et al.*[2] is found to be 0.96 which is highly significant ($P < 0.001$). The correlation coefficient for 23 observations in our studies is found to be 0.73 which is also highly significant ($P < 0.001$). This suggests that recovery ventilation may be used as a measure of oxygen debt.

Table 3. Recovery ventilation (litres) and oxygen debt (litres) during recovery following an exercise.

	Welch[2] *et al.* (1970)		Present study	
	O_2 debt (l)	Recovery VEBTPS (l)	O_2 debt (l)	Recovery VEBTPS (l)
	$r = 0.96$ ($P < .001$)		($r = 0.73$ ($P < .001$)	
1	8.9	174	2.901	45.52
2	3.4	97	4.748	215.24
3	6.9	140	3.429	102.54
4	6.8	141	7.212	264.43
5	6.4	147	5.614	99.88
6	6.2	135	2.375	64.59
7	4.8	118	2.800	102.54
8	3.8	105	3.531	65.80
9	3.7	101	1.105	20.50
10	1.8	42	1.436	86.40
11	1.6	26	0.985	30.02
12	2.1	67	1.336	104.72
13	1.8	47	1.284	32.34
14	3.3	104	1.781	66.41
15	—	—	2.933	100.91
16	—	—	1.364	38.84
17	—	—	1.615	125.74
18	—	—	1.630	52.46
19	—	—	1.977	105.31
20	—	—	2.573	148.90
21	—	—	1.545	47.24
22	—	—	2.115	148.44
23	—	—	1.906	72.11

4. Discussion

There is a general agreement on the exponential nature of recovery ventilation[1,13,11] but some workers have described the recovery process of ventilation by a two-component exponential equation[12]. In our present approach it has been assumed that the resultant recovery process is exponential with respect to some fractional power of time. The recovery ventilation process is exponential in nature, also follows indirectly from the studies of other workers[3,4,7,9] where they found oxygen consumption recovery process made of two exponential components i.e. alactacid and lactacid, oxygen debt and recovery ventilation eventually being measured for determination of oxygen debt.

In exercises light to moderately heavy, the recovery ventilation is spread over a limited period of 32 minutes and can be estimated from the observations with the help of 0–2 minutes and 0–8 minutes of recovery ventilation over the resting ventilation.

Thus, in situations where recovery ventilation is required, this method will be very helpful for predicting total recovery ventilation. The model has been found giving satisfactory predictions for persons of different age groups and under different environmental conditions thus establishing its universal applicability. Recovery ventilation may serve as a good measure of oxygen debt as the later is highly correlated with the former. Pulmonary ventilation has already been reported as a good measure and predictor of energy expenditure[15] during exercise.

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