Def Sci J, Vol 32, No 3, July 1982, pp 215-218

Problem Concerning to a Sinusoidal Flux

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Received 30 March 1981

Abstract. A dynamical problem, concerning to semi-infinite medium subjected to a sinusoidal flux is studied. The consequential effect of this on the associated problem of thermal stresses is studied taking into account the inertial terms.

1. Introduction

Thermoelasticity describes the behaviour of elastic bodies under the influence of non-uniform temperature fields. Danilovskaya[1] solved the problem of determining thermal stresses in an elastic half-space arising after a sudden heating of its boundary by considering the effects of inertial terms using uncoupled theory. Further work on the effect of inertia was carried out by Mura[2,3], Sternburg & Chakravorty[4] and Nowacke[5].

In this paper, we consider a semi infinite medium ($x \geqslant 0$) subjected to a periodic flux, a Sinusoidal flux. The flux is introduced suddenly at time $t = 0$ and is allowed to continue. The consequential effect of this on the associated problem of thermal stress is studied taking into account the inertial terms since the body under consideration is massive. The flux is taken as one dimensional and the temperature changes in the transverse directions are neglected. As a result of this assumption the heat transport equation is one dimensional. It is further assumed that the displacement in the transverse directions are zero and so all the field equations of thermoelasticity are taken one dimensional. Integral transform method is used and the expression for the stress distribution is obtained in closed form.

2. Formulation and Solution of the Problem

The one dimensional Fourier heat conduction equation is

$$\frac{\partial^2 T}{\partial x^2} - \frac{1}{K_1} \frac{\partial T}{\partial t} = 0 \text{ for } \begin{matrix} x \geqslant 0 \\ t > 0 \end{matrix} \tag{1}$$

The boundary condition takes the form

$$\left.\begin{aligned} -K\frac{\partial T}{\partial x} &= 0 \quad \text{for } t < 0,\ x = 0 \\ &= Q_0 \sin\frac{\pi t}{t_0} \quad \text{for } t \geqslant 0,\ x = 0 \end{aligned}\right\} \tag{2}$$

and the regularity boundary condition is

$$T(x, t) = 0 \quad x \to \alpha \tag{3}$$

The initial condition is

$$T(x, t) = 0 \text{ for } t = 0 \tag{4}$$

Applying Laplace transform to (1) to (3), we get

$$\frac{d^2\bar{T}}{dx^2} - a^2\bar{T} = 0 \qquad \text{where } a^2 = \frac{p}{K_1} \tag{5}$$

$$-K\frac{d\bar{T}}{dx} = \frac{Q_0 b}{p^2 + b^2} \quad \text{where } b = \pi/t_0 \text{ for } x = 0 \tag{6}$$

and

$$\bar{T}(\alpha, p) = 0 \tag{7}$$

Where K is coefficient of thermal conductivity, K_1 is coefficient of thermal diffusity, and p is Laplace transform parameter.

From (5), (6) and (7), we get

$$\bar{T}(x, p) = \frac{Q_0 b\sqrt{K_1}}{K\sqrt{p}}\ \frac{\text{Exp}\left(-\sqrt{(p/K_1)\, x}\right)}{(p^2 + b^2)} \tag{8}$$

Inverting[6], we get

$$T(x, t) = \frac{Q_0 b\sqrt{K_1}}{K}\int_0^t \left[2\sqrt{(\tau/\pi)}\ \text{Exp}\left(-x^2/4K_1\tau\right) - (x/\sqrt{K_1})\ \text{Erfc}\left(x/2\sqrt{K_1\tau}\right)\right]\cos b\,(t - \tau)\, d\tau \tag{9}$$

The temperature displacement equation is

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{c_1^2}\ \frac{\partial^2 U}{\partial t^2} = m\,\frac{\partial T}{\partial x} \tag{10}$$

where $c_1^2 = (\lambda + 2\mu)/\rho$ and $m = \dfrac{(3\lambda + 2\mu)}{(\lambda + 2\mu)}\,\alpha$

Introducing the displacement potential ϕ, given by the relation

$$u = \frac{\partial\phi}{\partial x} \tag{11}$$

Equation (10) becomes

$$\frac{\partial^2\phi}{\partial x^2} - \frac{1}{c_1^2}\ \frac{\partial^2\phi}{\partial t^2} = mT \tag{12}$$

The boundary condition is taken as

$$u = \frac{\partial \phi}{\partial x} = 0 \text{ for } x = 0 \tag{13}$$

Initial conditions are taken to be

$$\phi(x, 0) = \dot{\phi}(x, 0) = 0 \tag{14}$$

Applying Laplace transform to (12), (13) and solving for ϕ, we get

$$\bar{\phi}\,(x, p) = \frac{mQ_0, bc_1^2}{Kp(p^2 + b^2)\,(p - c_1^2/K_1)} \left[\frac{\text{Exp }(-px/c_1)}{(p/c_1)} - \text{Exp }\frac{(-\sqrt{(p/K_1)\,x})}{(\sqrt{p/K_1})}\right] \tag{15}$$

we have

$$\frac{\partial \sigma}{\partial x} = \rho\,\frac{\partial^2 u}{\partial t^2} \tag{16}$$

From (13) and (16), we get

$$\sigma = \rho\,\frac{\partial^2 \phi}{\partial t^2} \text{ for } x \geqslant 0. \tag{17}$$

We will determine stress distribution with the boundary condition $\sigma = 0$ on $x = 0$. Applying the Laplace transform and using (14), we get

$$\begin{aligned}(\bar{\sigma})_1 &= p^2 \rho \bar{\phi} \\ &= \frac{mQ_0 bc_1^2 \rho}{K} \frac{p}{(p^2 + b^2)\,(p - c_1^2/K_1)} \left[\frac{\text{Exp }(-px/c_1)}{(p/c_1)} - \frac{\text{Exp}(-\sqrt{(p/K_1)\,x})}{(\sqrt{p/K_1})}\right]\end{aligned} \tag{18}$$

$(\bar{\sigma})_1$ is not vanishing on the boundary $x = 0$. So we will superpose a stress $(\bar{\sigma})_2$ such that $(\bar{\sigma})_1 + (\bar{\sigma})_2 = 0$, on $x = 0$ i.e we have to determine $\bar{\psi}$ such that $\bar{\phi} + \bar{\psi} = 0$ on $x = 0$. $\bar{\psi}$ is determined from

$$\frac{d^2\bar{\psi}}{dx^2} - \frac{p^2}{c_1^2}\,\bar{\psi} = 0 \tag{19}$$

Therefore $\quad \bar{\psi} = B(p) \text{ Exp } (-px/c_1) \quad (20)$

$B(p)$ is determined using $\bar{\phi} + \bar{\psi} = 0$ on $x = 0$

$$B(p) = -\frac{mQ_0 bc_1^2}{Kp(p^2 + b^2)(p - c_1^2/K_1)} \left[\frac{1}{(p/c_1)} - \frac{1}{\sqrt{(p/K_1)}}\right] \tag{21}$$

$$\bar{\sigma} = (\bar{\sigma})_1 + (\bar{\sigma})_2$$

$$= \frac{\rho m b Q_0 c_1^2 \sqrt{K_1 p}}{K(p^2 + b^2)(p - c_1^2/K_1)} [\text{Exp}(-px/c_1) - \text{Exp}(-\sqrt{(p/K_1)}\, x)] \quad (22)$$

Inverting[6], we get

$$\sigma(x, t) = -\frac{\rho m Q_0 b K_1^{3/2} c_1^4}{K(c_1^4 + K_1^2 b^2)} \int_0^t \left[\frac{1}{\sqrt{\pi\tau}} \text{Exp}(-x^2/4K_1\tau)\right]$$

$$[\text{Exp}\{(C_1^2/K_1)(t-\tau)\} = \cos b(t-\tau)$$

$$+ (K_1 b/c_1^2) \sin b(t-\tau)]\, d\tau \text{ for } t < x/c_1$$

$$= \frac{\rho m Q_0 b K_1 c_1}{K} \int_0^t [\text{Exp}\{(c_1^2/K_1)(t-\tau)\}$$

$$\text{Erf}\{(C_1/\sqrt{K_1})\sqrt{t-\tau}\}] \cos b(\tau - x/c_1)\, d\tau$$

$$- \frac{\rho m Q_0 b K_1^{3/2} c_1^4}{K(c_1^4 + K_1^2 b^2)} \int_0^t (1/\sqrt{\pi t}) \text{ Exp}(-x^2/4K_1\tau)$$

$$[\text{Exp}(c_1^2/K_1)(t-\tau) - \cos b(t-\tau)$$

$$+ (K_1 b/c_1^2) \sin b(t-\tau)]\, d\tau \text{ for } t > x/c_1 \quad (23)$$

Which is the required stress distribution.

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