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Rotating Water Table for the Determination of Non-Steady Forces in a Turbine Stage Through Modified Hydraulic Analogy

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Abstract. Determination of non-steady forces in a real turbine stage is difficult due to the local flow conditions, for example high pressures, high temperatures and in-accessibility to the region etc. Experimentation in a real turbine is also prohibitive due to the costs involved. An alternate method of arriving at these non-steady forces through the use of modified hydraulic analogy is discussed. A rotating water table facility, developed and fabricated based on the principles of modified hydraulic analogy is described. A flat plate stage is simulated on the rotating water table, and the results obtained are presented.

1. Introduction

Majority of turbine blade failures are due to fatigue that occurs when a stage is operating at or near resonant conditions. The usual design practice is to tune a blade when one of its natural frequencies is close to any one of the possible excitation frequencies. With increase in size and speed of the machinery, it has now become imperative that some turbine stages operate around resonant conditions. To avoid fatigue of such blading, it has become necessary to determine the dynamic stresses due to non-steady forces acting on a turbine blade. The non-steady forces acting in a turbine are primarily due to the flow interaction between the stationary diaphragms and the rotating blades. Such interaction produces excitation at nozzle passing frequency and its harmonics.

Determination of non-steady forces resulting from such interaction in an actual turbine will be hampered by the local flow conditions like high pressures and high velocities and also the in-accessibility to the region of flow etc. These conditions do not allow the non-steady forces to be determined in a manner convenient for their accurate prediction. Further, it is difficult to visualise the process due to high velocities, and any predictions have to purely depend upon certain indirect measurements that are made. Further, experimentations in an actual turbine involves costly instruments that are to be specially designed and fabricated for the purpose. In this paper an alternate method of arriving at the non-steady forces in a turbomachine stage through a rotating water table based on principles of modified hydraulic analogy has been discussed. The existance of a mathematical analogy between the frictionless flow of a liquid over horizontal free surface and the isentropic compressible flow of a gas with a specific heat ratio $\gamma = 2$, has been known since a long time[1]. The validity of such an analogy has been proved for unsteady case by Loh[2], for one dimensional flow and Bryant[3], for two dimensional flow. The analogy had limited applications for quantitative studies, because no known gases exist having specific heat ratio equal to 2. For real gas flow situations, the analogy has to be suitably modified. Loh[2] suggested that by using non-rectangular cross sectional channels, an analogous system corresponding to a gas having any specific heat ratio can be produced. However, the construction of such channels for two dimensional flows is not possible. Several other authors have suggested correction factors to be applied to the data obtained on water table to correlate the same with gas dynamic solutions. Byrd and Williams[4] proposed correction factors for pressure, density and temperature ratios, without any correction on Mach number. Adams[5] proposed a correction factor for Mach number, however his corrections for pressure, temperature, and density are independent of the correction on Mach number. Using isentropic relations Schorr[6] suggested a correction factor for pressure ratio. Warner and Wicks have independently suggested (See reference 7) a correction factor for pressure ratio, for modeling a turbine stage on rotating water table by equating the critical pressure ratio of the stage operating with hypothetical gas and a real gas, without accounting for correction on Mach number. This correction was used to model a low pressure stage of U. S. Navy marine turbine by Rieger *et al*[7]. Rao[8] recognised that the correction of Mach number for modeling of real gases should be incorporated in the corrections of pressure and temperature also, and accordingly proposed a modified analogy through a series of five reports. In these reports, the modified analogy is established mathematically for isentropic subsonic, transonic, and supersonic flows. Where there is a shock in the flow, like normal shock in the divergent section of a nozzle and the oblique shock at the exit of a nozzle, the limitations of the analogy were discussed and it was shown that, if approach Mach number in supersonic flow is less than 1.7, the errors in modelling are of small order and the analogy can be used effectively. The effectiveness of this modified analogy was studied by Rao *et al.* for isentropic[9] and non-isentropic[10] flows, by simulating the compressible gas flow through a converging-diverging rocket nozzle of Back *et al*[11] on a flat water table.

Many attempts were made earlier to use the principles of hydraulic analogy for studying the problems in turbomachine stages, e.g., Owczarek[12] used a radial inflow water table to study a pressure wave interaction phenomenon between rotor row and stator row of a turbine. Meier[13] rotated a circular blade row in a tank of still water to study the pressure distributions on the blades of a cascade. Bomelburg[14] also made some studies on rotating blades, but instead of water, used kerosene oil as the working fluid to minimize the surface contamination effects. Rhomberg[15] described an experimental installation for investigating rotating transonic cascade with air. The flow patterns were photographed with the shadowgraph method at the cascade outlet.

Heen and Mann[16] have used the hydraulic analogy for the study of two dimensional flow in a partial admission turbine. Rieger *et al*[17] developed a large rotating water table apparatus which allows the gas flow conditions in a turbomachine stage to be modeled by the flow of water. However, only classical analogy was used in all these experiments, which limited the experimental results to be of qualitative nature.

Significant investigations to determine the non-steady blade forces in a turbine stage were carried out simultaneously by two groups, one at Indian Institute of Technology, New Delhi (Prof. J. S. Rao) and the other at Rochester Institute of Technology, New York (Prof. N. F. Rieger). The analytical thrust on this problem was initiated by Rao (see reference 7), who developed computer programs for the analysis of determination of non-steady forces in an elementary turbomachine stage. Experiments were conducted by Rieger *et al*[7] on a rotating water table designed for the purpose in which water flows through a stage of accurately scaled turbine nozzle and blade profiles. Similarity was maintained between the nozzle exit conditions of the prototype and water table model in their experiments.

In the following sections, modelling aspects of a turbine stage on a large rotating water table, developed and fabricated at Indian Institute of Technology, New Delhi are discussed. However, since the experimental facility is still under progress and in the absence of a provision of an actual turbine stage, a flat plate stage is initially modeled, and simulated on the water table. A brief description of the water table is given and the results obtained for the flat plate stage under consideration are presented.

2. Modelling Aspects

Before modelling any real gas flow through a turbine on water table, it is first necessary to determine the geometric conditions and quantities that are to be established on water table. They are briefly discussed below.

Stage Geometry

As it has been found in the work of Heen and Mann[16], the modelling of an axial flow stage by a moving cascade on a water table is very much experimentally involved, because of the steady flow conditions to be obtained on a water table with a finite moving cascade at constant velocity. Hence to a first approximation, the axial flow through a turbine stage can be substituted as a radial flow on water table by keeping the stator and rotar blades on two concentric circles having as large diameter as possible. Further the blade shapes in the prototype and the analogous model should be same. This ensures that

(a) The cross sectional areas of the flow boundaries in the original turbine and the equivalent 2-D model of the turbine on water table to be proportional.

(b) The surface slopes of the flow boundaries equal in both the prototype and model, giving equal flow directions at the boundaries.

Flow Conditions

The nozzle exit flow conditions are important in a turbine, since the steady and non-steady forces of the downstream blade depend upon them. As the final results that are to be obtained through stimulation on water table are the ratios of these two forces, it is essential that the nozzle exit conditions in both the prototype and the model are same.

Nozzle Exit Mach Number

One of the important parameters to be considered in the nozzle exit flow conditions is the Mach number. Since Froude numbers obtained on water table are not equal to the corresponding Mach numbers in the gas flow, while simulating a real turbines tage on water table, the Froude number at the nozzle exit can be obtained from the corresponding Mach number in the gas flow through modified analogy[8]. This can be accomplished as follows.

If M_N is the nozzle exit Mach number in the gas flow through the turbine stage, the corresponding Froude number on the water table F_N can be obtained from

$$F_N = C_{\bar{M}}.M_N \tag{1}$$

Where

$$C_{\bar{M}} = \frac{\left\{1 + \frac{F_N^2}{2}\right\}^{1.5}}{1.8371\left(\frac{2}{\nu+1}\right)\left(1 + \left(\frac{\nu-1}{2}\right)M_N^2\right)^{(\nu+1)/2(\nu-1)}} \tag{2}$$

The flow on the water table is to be adjusted with this value of Froude number at the exit of the nozzle blades.

Pressure Ratio in the Stage

The second important parameter to be taken into consideration while simulating the flow through a turbine stage on water table is the pressure ratio of the stage. The stage pressure ratio in a turbine is given by the ratio of nozzle inlet pressure and the blade exhaust pressure. The magnitudes of both the steady and unsteady forces will be affected by the stage pressure ratio, since this parameter determines the flow through the stage.

While simulating the turbine stage on water table the pressure ratio in the prototype turbine has to be again modified to obtain the corresponding pressure ratio on the water table. The modified pressure ratio can be obtained as follows.

If suffix 1 denotes the nozzle inlet conditions and suffix 2 denotes the rotor exit conditions then

$$\left(\frac{P_1}{P_2}\right)_W = C_P \left(\frac{P_1}{P_2}\right)_G \tag{3}$$

where subscripts W and G of the brackets indicate that the quantities are associated with water and Gas respectively. C_P, the correction factor is given by[8]

$$C_P = \left[\frac{1+\dfrac{F_2^2}{2}}{1+\dfrac{F_1^2}{2}}\right]^2 \left[\frac{1+\dfrac{\nu-1}{2}M_1^2}{1+\dfrac{\nu-1}{2}M_2^2}\right]^{\nu/(\nu-1)} \tag{4}$$

where

F_1 and F_2 are then Froude numbers on water table and
M_1 and M_2 are Mach numbers in the gas flow at nozzle inlet and rotor outle respectively.

The depths on the water table at nozzle inlet and rotor outlet should be adjusted such that the square of the ratio of these depths are equal to the pressure ratio obtained on the water table. As the flow on the water table would have been already adjusted for obtaining the required Froude number at the nozzle exit, the required pressure ratio on the table can be obtained by providing back pressure screens and only by adjusting the back pressure.

3. Description of the Rotating Water Table

The schematic diagram of the rotating water table for simulation of turbine stages is shown in Fig. 1.

Mechanical Construction

The rotor assembly consists of vertical main shaft supported by upper and lower ball bearings which in turn are supported by upper and middle frame assemblies which

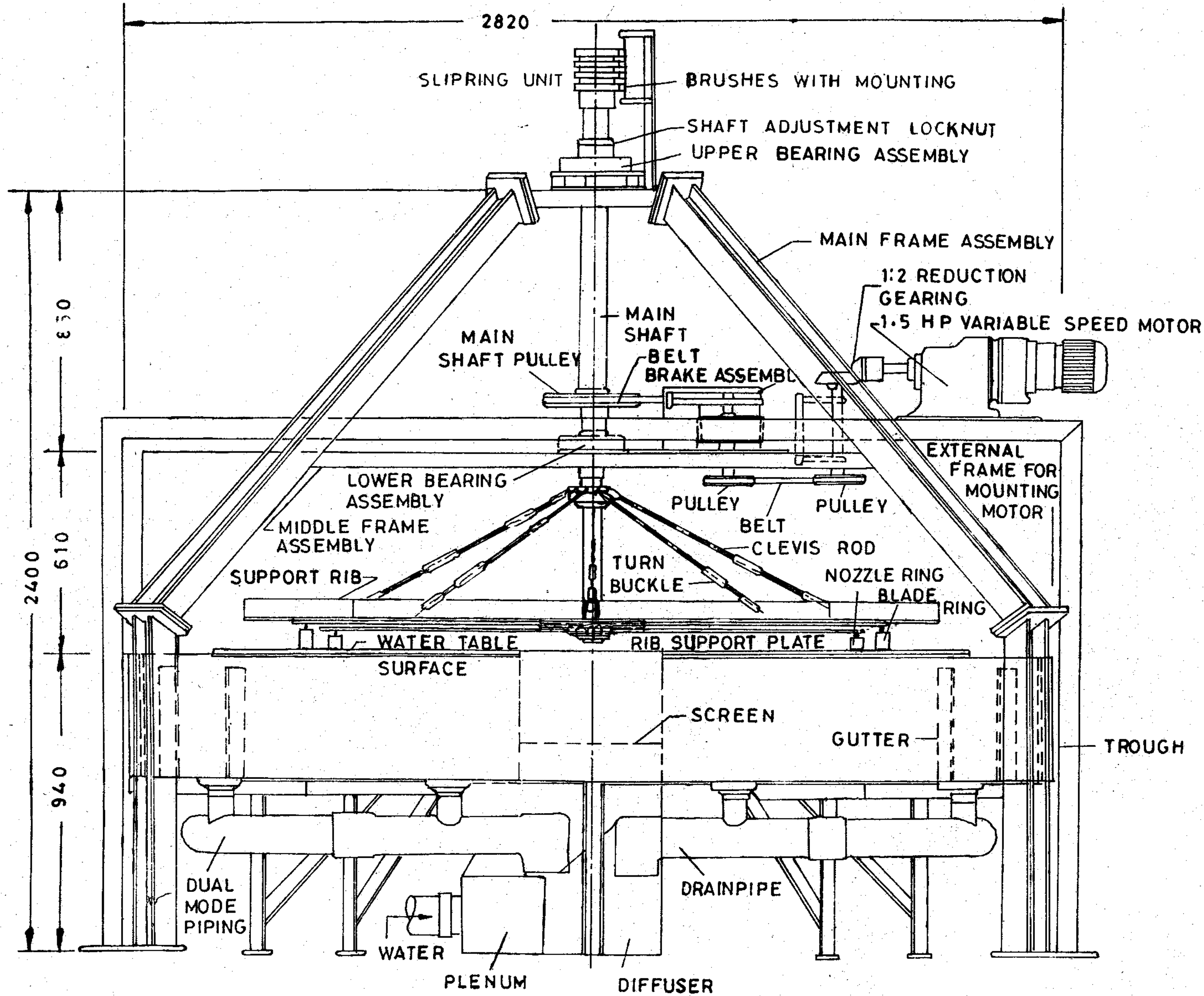

Figure 1. Rotating water table for simulation of turbine stages.

are welded steel structures. In the upper-most portion of the shaft a phosphor bronze slip ring unit is mounted. A pulley is fitted on the main shaft for receiving the drive from the drive mechanism. The shaft carries a perspex ring through eight radial ribs attached with clevis support rods to which moving blades are attached through blade rods above the water table surface. The levelling of the rotor support ring and adjustments for the blade clearances are obtained by the turnbuckles arranged in the clevis support rods. The rotor blades rotate concentric to the stationary nozzle blades at the speed assigned by the drive mechanism. The nozzle assembly consists of a perspex ring to which the nozzle or stator blades are attached. The ring along with the nozzle blades is supported by the water table surface. The actual water table on which the turbine stage is simulated rests on a hexagonal welded structure in the lower frame assembly. The upper, middle and lower frame assemblies are also supported through the welded joints by the main frame assembly, which is a large steel structure. The water table is made of 12 mm thick perspex sheet having approximate outer diameter of 250 cm, with inner diameter of 60 cm. A number of levelling screws arranged beneath the table enable it to be levelled accurately. A trough system consisting of a concentric channel surrounding the edge of the table is provided to collect the water flowing on the table. The trough system is connected to the sump through dual mode piping system. The rotor assembly is driven by a variable speed motor which is mounted on an external frame isolated from the main structure. The drive is transmitted from the motor shaft through bevel gearing to the viscous drive or brake mechanism which ultimately drives the rotor assembly.

Flow Circuit

The flow circuit for the water table is shown in Fig. 2. Water is supplied to water table apparatus from an over head tank through an orifice meter designed and

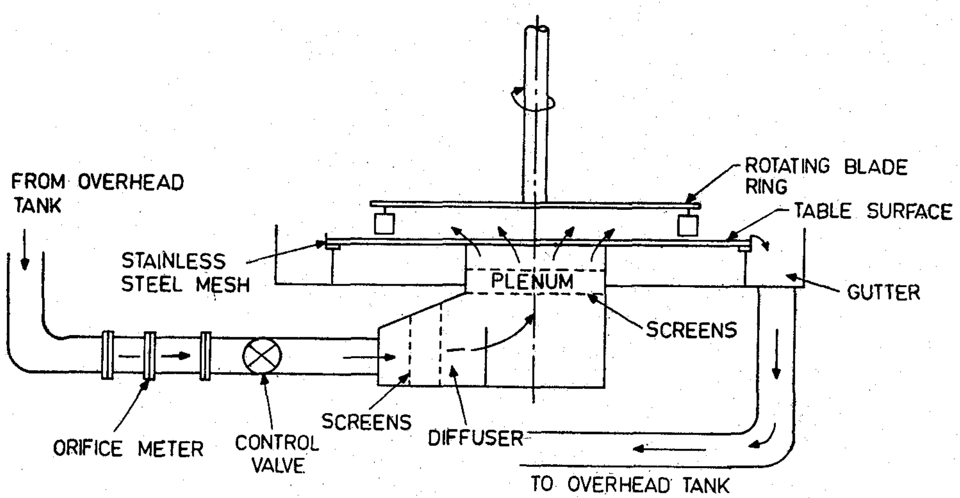

Figure 2. Flow circuit for the water table.

calibrated and is incorporated in the supply line to meter the amount of water passing on to the water table surface. The water after flowing through the orifice meter, a control valve and a filter fitted in the pipe line enters a plenum chamber (a vertical settling chamber) through a diffuser. Fine mesh screens fitted in the plenum and diffuser settle the disturbance in the incoming water. The water enters the table at the centre from the plenum chamber and flows on the levelled water table surface radially. The nozzle blades fixed to the nozzle ring direct the oncoming radial flow on to the concentric rotating blades. At the edge of the water table a fine stainless steel mesh is arranged for controlling back pressure. The water after passing through the back pressure screen is collected into the trough system to be transmitted to the sump via dual mode piping, from where it is pumped to the over head tank for recirculation. The depths on the water table are obtained by providing 3 mm wall taps in the table and connecting them to an inclined tube manometer. Wall taps are provided on 3 equiangularly located radial lines. The tappings are made before the nozzle and after the rotor blades. Additional tappings are also provided at the exit of the nozzles. The measured levels in the manometer before and after flowing the water on the table indicate the flow depths on the water table.

Instrumentation

The block diagram of the instrumentation used in the experiment on the rotating water table is given in Fig. 3 Semi-conductor strain gauges are mounted on the flat

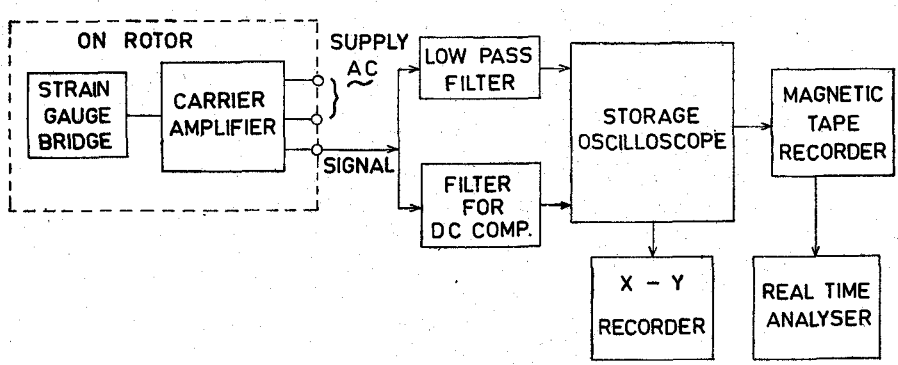

Figure 3. Block diagram for the instrumentation used on water table.

sections of blade support rods and the bending forces caused by the flow of water in two perpendicular directions (Axial and tangential) are measured using the instrumented blade rods. For measuring bending force in each direction two strain gauges were used and these are arranged for providing temperature compensation in the bridge circuit of a carrier amplifier unit. The carrier amplifier system is mounted on the rotor itself and consists of fully transistorized bridge amplifiers and demodulators which effect mechanical to electrical conversion of resistive transducer signals. During the conversion process, the electrical signals are amplified to a level adequate for driving, indicating or recording equipment. The resistive strain gauge elements are connected to circuit elements in the amplifier and demodulator units to form a full bridge network. The output signal from the amplifier unit is collected through the phosphor bronze slip ring unit and is passed on to a low pass active filter in order to pass only the desired low frequency signal (nozzle passing frequency). The filter is a two pole VCVS low pass filter using a precision op amp followed by a single pole integrator. The average DC level of the strain signal is obtained using a low pass active filter which produces a steady DC voltage representing the average value of the strain signal. Figures 4a and 4b show the filter circuits used for this purpose.

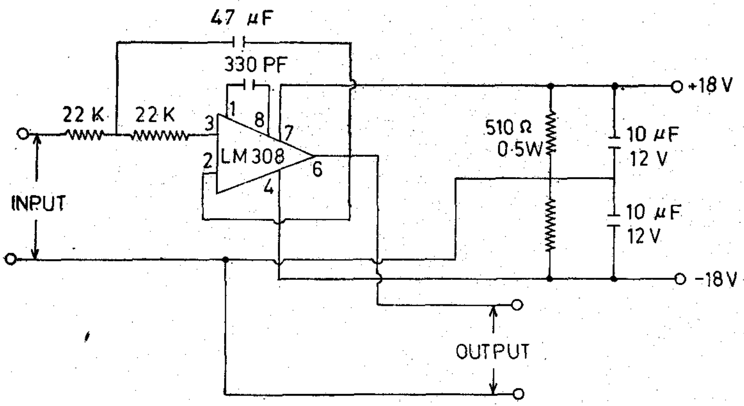

Figure 4(a) Filter circuit for low frequency components.

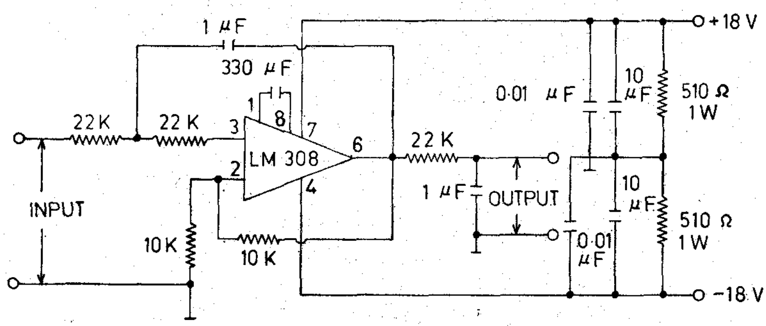

Figure 4(b) Filter circuit for DC component.

The output signals from the filter circuits are fed to a storage oscilloscope. The signals are analysed on the screen of the oscilloscope, stored and then transferred to an X-Y recorder for continuous recording of the signals on the charts. The signals are also passed on to a multi channel tape recorder for continuous recording and further analysis. The signals recorded on the tape recorder are finally analysed in a narrow band real time analyser.

4. Simulation of the Stage on Water Table

Stage Data :

For the purpose of conducting an experiment the following flat plate stage is considered

$\frac{P_1}{P_2}$	pressure ratio in the stage	2.15
M_N	nozzle exit Mach number	0.6
σ_s	Solidity ratio in the stator cascade	0.35
σ_r	Solidity ratio in the rotor cascade	0.64
C_v/C_s	Blade chord ratio	1
b'/C_r	Blade Axial gap/rotor chord ratio	0.35
$\frac{V_s}{u}$	nozzle exit velocity/blade velocity	0.33, 0.38, 0.41 0.44, 0.46
α_s	Stagger angle, stator cascade	0°
α_r	Stagger angle, rotor cascade	50°
$\frac{d_r}{d_s}$	Blade spacing ratio	

In order to obtain the geometric parameters of blades assumed, on the rotating water table, 25 nozzle blades and 50 rotor blades are mounted on concentric perspex rings. The inlet tips of the stator blades are arranged on a circle having a diameter of 157 cm. and the outlet tips of the rotor are on a circle having a diameter 182.4 cm. Both the stator and rotor blades are made of 2 mm aluminum sheet, having a height of 10 cm, and a chord of 7 cm. Each blade is fitted in a slot provided in a piece of aluminium flat having a threaded central hole to receive the blade rod. The stator stagger angle is approximated to 0° by keeping the blade in the radial direction on the water table. The rotor blades are arranged at an angle of 50° to the radial direction such that the condition of stagger angle for rotor blades to be $\alpha_r = 50°$, is satisfied. Fig. 5 shows the configuration diagram of the nozzle and the rotor blades on the water table.

The Mach number at the nozzle exit in the prototype is modified using Eqn. (1), to obtain the corresponding nozzle exit Froude number on the water table. The value of the Froude number obtained in present case is 0.577. The flow of water is adjusted such that this Froude number is obtained at the exit of the nozzle on the table. The flow under this condition was found to be 14.75 Kg/sec, with a height of 1.32 cm of the water flow at the nozzle exit. The velocity of water obtained at nozzle exit was 20.8 cm/sec. The water depth at the nozzle inlet for this flow was 1.41 cm. For the above flow with this depth, the nozzle inlet Froude number was found to be 0.57, and the rotor outlet Froude number 0.9306.

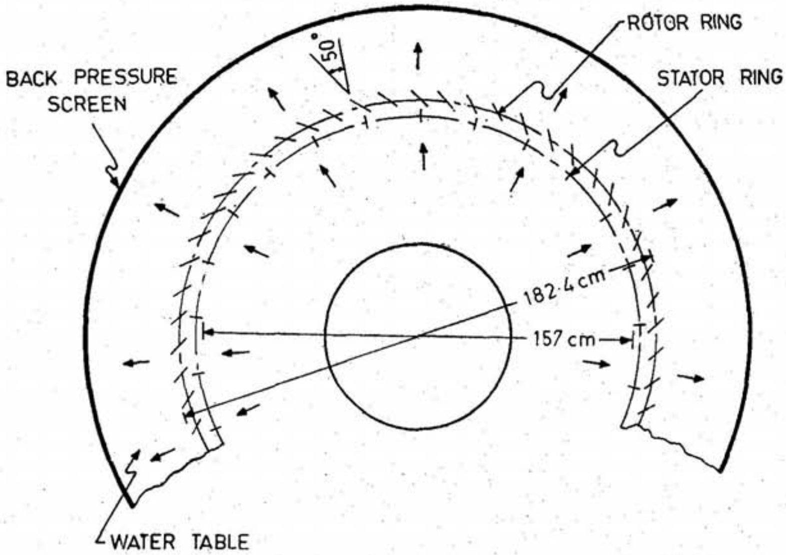

Figure 5. Configuration of the simulated stage on water table (part view).

To obtain the required pressure ratio on the water table, the pressure ratio of the prototype given is modified using Eqn. (3). However in order to determine the correction factor C_p in the Equation, Mach numbers at nozzle inlet and rotor outlet are required. These Mach numbers are obtained by applying correction factor given in Eqn. (2) on the Froude numbers at nozzle inlet and rotor outlet, and were found to be 0.5926 and 0.9360 respectively. Substituting the values of Froude numbers and Mach numbers thus obtained in Eqn. (4) the correction factor to be applied for obtaining the actual pressure ratio on the water table from the prototype pressure stage was found to be 1.0952 and the corresponding pressure ratio on the water table was found from Eqn. (3) to be 2.35. This must be equal to the square of the height ratio of nozzle inlet depth and rotor outlet depth on the water table. This condition was achieved on the water table by adjusting back pressure to obtain a depth of 0.92 cm at the rotor outlet on the table.

5. Test Procedure

Before commencement of the test, the table surface is cleaned perfectly. The back pressure screen at the edge of the table is checked and any fine particles collected there are removed with a brush. The levels of the water columns in the inclined tube manometer are noted when there is no flow of water. The strain gauge bridge is balanced and then the flow is commenced and the required velocity ratio viz, nozzle exit velocity/blade velocity is obtained by rotating the rotor at a required speed. Further velocity ratios required are obtained by keeping the stator exit velocity constant and changing the rotational speed of the rotor only. The rotor is allowed to

Figure 6. Sample chart recordings.

run in the established flow for some time to have steady conditions. The signals are then recorded for axial and tangential loadings alternately. Some representative samples of signals recorded are shown in Fig. 6. The signals in each case are recorded for nearly 30 minutes continuously at a suitable tape speed depending upon nozzle passing frequency under consideration. The DC levels of the signals are obtained on the screen of the oscilloscope. The back pressure screen is to be cleaned occasionally during the experiment, since any dust particles collected on the screen may cause partial arc effects and change the signal pattern considerably. The bridge balance also has to be checked during the experimentation.

6. Analysis of the Test Data

At each velocity ratio, the signals are recorded on magnetic tape for several revolutions of the water table as mentioned earlier, to obtain a representative sample for the

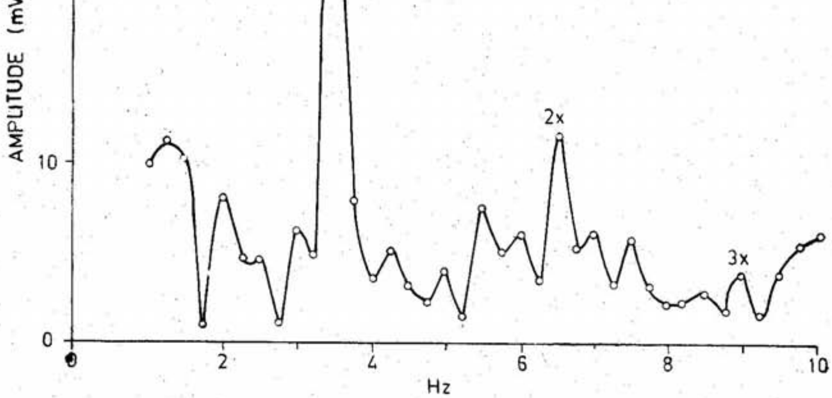

Figure 7. Frequency spectrum of the non-steady force.

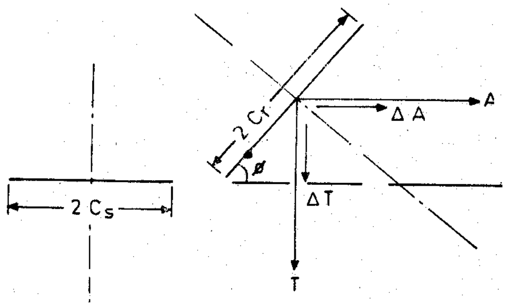

Figure 8. Resolution of unsteady force components.

entire force spectrum at the prescribed flow conditions. The signals were then analysed in the real time analyser to determine the magnitudes of each frequency component present in the flow throughout the frequency range. The frequency spectra obtained from such signal processing at different velocity ratios are plotted and the signal strengths corresponding to the 1st, 2nd and 3rd harmonics of the nozzle passing frequency are obtained. Fig. 7 shows a typical frequency spectrum obtained during the analysis of the signal on a real time analyser for a velocity ratio of 0.46. The signal strength is indicated in mV. The strengths of the signals correspond to the non-steady force magnitude in the respective harmonics. The corresponding DC levels of the signals are obtained by passing the relevant signal through the storage oscilloscope and by measurement of DC level directly from the steady bending forces acting on the moving blades. With the help of non-steady forces in axial and tangential directions obtained on water table, the non-steady lift acting on the rotor blade is obtained by the resolution of the force components in the lift direction as indicated in Fig. 8.

Table 1 gives the results obtained on water table for the stage gap ratio of 0.35, and the comparison of experimental results obtained for the first two harmonics of the rotor with those obtained for the flat plate stage through computer program[18], is shown in fig. 9.

Table 1 Comparison of unsteady lift ratios on the rotor blade. (zero stator inlet angle)

Velocity ratio	Unsteady lift (1st harmonic) Theoretical Inc.	Unsteady lift (1st harmonic) Theoretical Compr.	Unsteady lift (1st harmonic) Experimental	Unsteady lift (2nd harmonic) Theoretical Inc.	Unsteady lift (2nd harmonic) Theoretical Compr.	Unsteady lift (2nd harmonic) Experimental
0.33	0.1085	0.1010	0.0585	0.0097	0.0039	0.0275
0.38	0.1355	0.1295	0.0605	0.0171	0.0108	0.0305
0.41	0.1431	0.1369	0.0660	0.0195	0.0140	0.0285
0.44	0.1475	0.1425	0.0640	0.0205	0.0155	0.0315
0.46	0.1519	0.1458	0.0665	0.0209	0.0163	0.0340

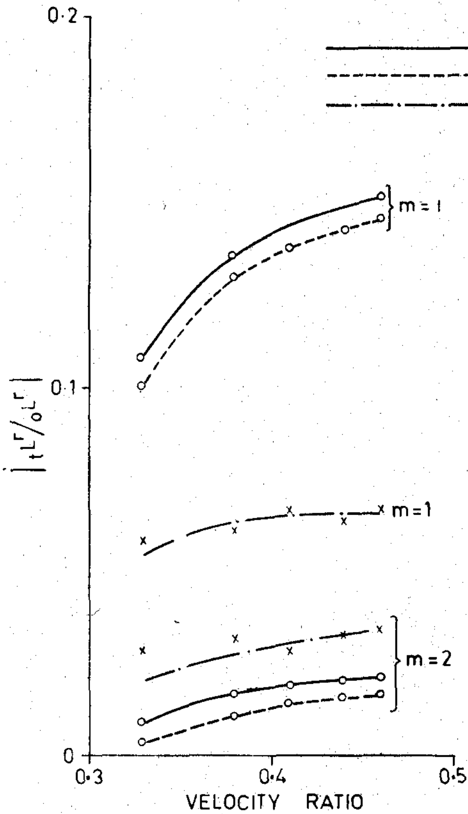

Figure 9. Comparison of unsteady lift forces on rotor blade. (zero stator inlet angle).

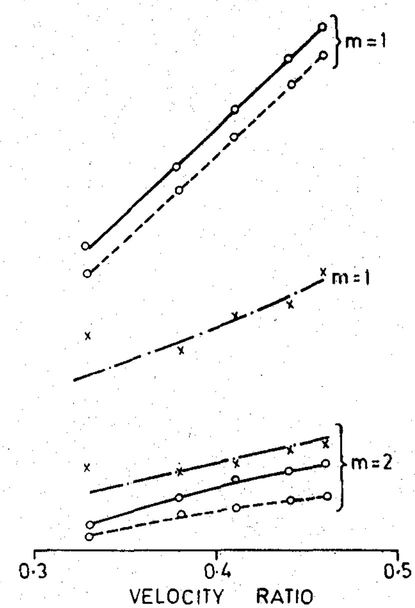

Figure 10. Comparison of unsteady lift forces on rotor blade (with 6° stator inlet angle).

The experimental results were compared with incompressible and compressible theoretical values in order to observe the effect of taking compressibility into account.

A similar experiment was conducted again, however, with nozzle blades having an incidence of 6° to the flow. The comparison of results for this case is shown in Fig. 10. It is evident from the Figs. 9 and 10, that the trends observed in the experiment are almost similar to the trends obtained from the theoretical results. However the non-steady lifts obtained on the water table are considerably less than the corresponding theoretical values in the 1st harmonic, and more in the 2nd harmonic. The effect of taking compressibility into account in both the cases seem to reduce the disparity between the theoretical and experimental results.

Rieger *et al*[7] have also compared their water table results with the values obtained from computer program for a typical blade stage. They have used classical analogy in modelling the stage. They have shown reasonably good correlation for very large stage gap ratios of the order 1 to 1.5. As the stage gap ratio decreased, their experimental results were in poor agreement, being 5 to 12 times lower than the theoretical values for a stage gap ratio of 0.5. The stage gap ratio used in this paper are more realistic, of a value equal to 0.35. Even for this ratio, experimental results observed with modified analogy are only half of the theoretical values. Thus the modified analogy seem to have improved the modelling considerably to obtain the non-steady force components. The difference between the experimental and theoretical results may be attributed to several other experimental and non-analogous effects. These are discussed below.

The perspex sheet with which the water table is made in the present set-up has slightly warped due to the prolonged exposure to different temperature conditions. This resulted in small variations in the blade clearances, (between the surface of the table and tips of the rotor blades), as the rotor rotated a full turn. Thus perfect geometric similarity could not be maintained on the water table for all the blades. This could be avoided by replacing the perspex table with an optically flat glass table.

The measurement of depth is approximate which can lead to errors in modelling. The velocities are also obtained based on these depths, thus the errors in depth measurement became cumulative. Also surface velocities are required for the analogy and therefore a laser doppler velocity measurement technique should be used to improve the accuracy substantially. The non-analogous effects which occur during the simulation process, e.g. surface tension effects can impose severe inaccuracies in analogous flow conditions. To preserve the analogy, surface tension effects must be reduced to a point where they become negligible. However, no attempt has been made in the present experimentation to control the surface tension effects.

Growth of the boundary layer on the water table may change the analogous conditions in a substantial manner. This may be reduced by having a table tilted slightly from the centre towards the periphery all around.

The depths on the water table should be very small to avoid vertical accelerations in the flow. The maximum depths obtained in the present experiment are, however, sufficiently small (of the order of 1.35 cm). Hence the effect of vertical accelerations may not be prominent in these particular cases.

7. Conclusions

A rotating water table is set up to determine the non-steady forces of a turbine stage using modified hydraulic analogy. The trends of the results obtained for a flat plate stage have been shown to be in good agreement with the theory. Further improvement in the surface quality of the table and the measurement techniques will be expected to give good results.

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