COUPLE-STRESS IN A SEMI-INFINITE MEDIUM UNDER THE ACTION OF A DYNAMIC PRESSURE ON THE BOUNDARY

G. R. Tiwari

Shri G. S. Tech. Institute, Indore

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Effect of couple-stresses on elastic stress distribution has been investigated in a semi-infinite medium under the action of a dynamic pressure on the boundary. As a particular example this pressure has been assumed to be a pulse of pressure moving uniformly along the boundary. It is found that the effect of couple-stress on shear stresses is predominant on the boundary surface.

Classical theory of elasticity assumes the stress tensor to be symmetric, though it was pointed out much earlier by Cosserate brothers[1] that there may be physical phenomenon where the stress tensor is not symmetric. For example, when the couple-stresses are present, the shear stress on the surface $x =$ constant in the direction of y is not equal to the shear stress[2] on the surface $y =$ constant in the direction of x. In the present paper the effects of couple-stresses in a semi-infinite elastic medium under the action of a variable pressure on the boundary has been investigated. As a particular case a pulse of pressure moving uniformly along the boundary has been considered. The effect of couple-stress on shear stresses along the boundary, is proportional to the parameter of couple-stress for certain interval of time. This effect is predominant on the boundary surface.

COSSERATE'S EQUATIONS OF MOTION

The equations of motion in cartesian coordinates that hold in a stressed body when the couple-stresses are taken into account, besides the usual normal and shearing stresses[1,3] is written.

The consideration of forces along x and y axes gives the equations of motion[3,4].

$$\left.\begin{aligned} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} &= \rho \frac{\partial^2 u}{\partial t^2} \\ \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} &= \rho \frac{\partial^2 v}{\partial t^2} \end{aligned}\right\} \tag{1}$$

if the body forces are neglected.

The consideration of the moments yields[?-4]

$$\frac{\partial \mu_x}{\partial x} + \frac{\partial \mu_y}{\partial y} + \tau_{xy} - \tau_{yx} = 0 \tag{2}$$

if the body couples are omitted.

It is to be noted that the equation (2) does not contain inertia terms.

FUNDAMENTAL EQUATIONS

In a state of plane strain, the non-vanishing components (u, v) of the displacement vector are functions of x and y. The strain components

$$e_{xx} = \frac{\partial u}{\partial x}, \quad e_{yy} = \frac{\partial v}{\partial y}, \quad e_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \tag{3}$$

are associated with the stress components, while the local rotation component

$$\omega = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \tag{4}$$

is associated with the couple-stresses μ_x and μ_y. The couple-stress μ_x produces curvature K_x parallel to the x-axis and μ_y produces the curvature K_y parallel to y-axis. It has been shown by Mindlin[2] that

$$K_x = \frac{\partial \omega}{\partial x}, \quad K_y = \frac{\partial \omega}{\partial y} \tag{5}$$

so that

$$\frac{\partial K_x}{\partial y} = \frac{\partial K_y}{\partial x} \tag{6}$$

which is the compatibility equation between curvatures. Compatibility conditions between curvatures and the strain components are found by eliminating ω from (4) and (5) as

$$\left.\begin{aligned} K_x &= \frac{1}{2}\frac{\partial e_{xy}}{\partial x} - \frac{\partial e_{xx}}{\partial y} \\ K_y &= \frac{\partial e_{yy}}{\partial x} - \frac{1}{2}\frac{\partial e_{xy}}{\partial y} \end{aligned}\right\} \tag{7}$$

The curvatures have been assumed proportional to the couple stresses[2].

$$K_x = \frac{1}{4B}\mu_x, \quad K_y = \frac{1}{4B}\mu_y \tag{8}$$

where B is a modulus of curvature of the material. When $B = 0$, the classical results are obtained.

On substituting the values of K_x and K_y from (7) in (8) and using (3), we get

$$\left.\begin{aligned} \mu_x &= 4B\left[\frac{1}{2}\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) - \frac{\partial^2 u}{\partial x \partial y}\right] \\ \mu_y &= 4B\left[\frac{\partial^2 v}{\partial x \partial y} - \frac{1}{2}\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right] \end{aligned}\right\} \tag{9}$$

In the case of plane strain the normal strains (e_{xx}, e_{yy}) are related to the normal stresses $(\sigma_x; \sigma_y)$ as

$$\left.\begin{aligned} \sigma_x &= \lambda(e_{xx} + e_{yy}) + 2\mu e_{xx} \\ \sigma_y &= \lambda(e_{xx} + e_{yy}) + 2\mu e_{yy} \end{aligned}\right\} \tag{10}$$

where $\lambda = \dfrac{E\nu}{(1+\nu)(1-2\nu)}$ and $\mu = \dfrac{E}{2(1+\nu)}$ are Lame's elastic constants, E is the Young's modulus and ν is the Poisson ratio.

Symmetric Part

$$\tau_s = \frac{1}{2}(\tau_{xy} + \tau_{yx})$$

of the shear stresses (τ_{xy}, τ_{yx}) produces the shear strain e_{xy} and hence

$$e_{xy} = \frac{1}{\mu}\tau_s = \frac{1}{2\mu}(\tau_{xy} + \tau_{yx}). \tag{11}$$

Antisymmetric Part

$$\tau_A = \frac{1}{2}(\tau_{xy} - \tau_{yx}) \tag{12}$$

of the shear stresses produces the rotation ω, and the relation between τ_A and ω may be obtained with the help of equations (2), (5) and (8).

Also, the equations (2), (7) and (8) yield

$$\tau_{yx} - \tau_{xy} = 4B\left[\frac{1}{2}\left(\frac{\partial^2 e_{xy}}{\partial x^2} - \frac{\partial^2 e_{xy}}{\partial y^2}\right) + \frac{\partial^2 e_{yy}}{\partial x\partial y} - \frac{\partial^2 e_{xx}}{\partial x\partial y}\right] \tag{13}$$

From (11) and (13), we get on substituting from (5),

$$\left.\begin{aligned} 2\mu\left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right] &= \tau_{yx} + \tau_{xy}, \\ 2B\left[\nabla^2\left(\frac{\partial v}{\partial x}\right) - \nabla^2\left(\frac{\partial u}{\partial y}\right)\right] &= \tau_{yx} - \tau_{xy} \end{aligned}\right\} \tag{14}$$

These equations yield
$$\left.\begin{aligned} \frac{1}{\mu}\tau_{yx} &= (l^2\nabla^2 + 1)\frac{\partial v}{\partial x} - (l^2\nabla^2 - 1)\frac{\partial u}{\partial y} \\ \frac{1}{\mu}\tau_{xy} &= (l^2\nabla^2 + 1)\frac{\partial u}{\partial y} - (l^2\nabla^2 - 1)\frac{\partial v}{\partial x} \end{aligned}\right\} \tag{15}$$

where
$$l^2 = 2(1+\nu)\frac{B}{E} = \frac{B}{\mu}.$$

Substituting the values of σ_x, σ_y from (10) and the values of τ_{xy}, τ_{yx} from (15) in (1), we get

$$\left.\begin{aligned} &(\lambda+\mu)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x\partial y}\right) + \mu l^2\left(\nabla^2\frac{\partial^2 v}{\partial x\partial y} - \nabla^2\frac{\partial^2 u}{\partial y^2}\right) \\ &\qquad + \mu\nabla^2 u = \rho\frac{\partial^2 u}{\partial t^2}, \\ &(\lambda+\mu)\left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 u}{\partial x\partial y}\right) + \mu l^2\left(\nabla^2\frac{\partial^2 u}{\partial x\partial y} - \nabla^2\frac{\partial^2 v}{\partial x^2}\right) \\ &\qquad + \mu\nabla^2 v = \rho\frac{\partial^2 v}{\partial t^2} \end{aligned}\right\} \tag{16}$$

Let
$$u = \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y}\ , \ v = \frac{\partial \phi}{\partial y} - \frac{\partial \psi}{\partial x} \tag{17}$$

where ϕ and ψ are functions of x , y and t , then equation (16) yield

$$\frac{\partial^2 \phi}{\partial \tau^2} = \nabla^2 \phi\ , \quad \beta^2 \frac{\partial^2 \psi}{\partial \tau^2} = \nabla^2 \psi - l^2 \nabla^4 \psi \tag{18}$$

where $\beta^2 = c_1^2/c_2^2$, $c_1^2 = (\lambda + 2\mu)/\rho$, $c_2^2 = \mu/\rho$ and $\tau = c_1 t$.

From (18) can be determined ϕ and ψ if the boundary conditions are prescribed.

STATEMENT OF THE PROBLEM AND BOUNDARY CONDITIONS

The distribution of stresses in the semi-infinite elastic medium $y \geqslant 0$ when a variable pressure $P(x, \tau)$ is applied to the boundary $y = 0$ is considered. The x-axis is taken along the boundary and the y-axis pointing into the medium.

The boundary conditions are

$$\sigma_y = -P(x, \tau)\ , \ \tau_{yx} = 0,\ \mu_y = 0 \text{ on } y = 0 \tag{19}$$

where $P(x, \tau)$ is piecewise continuous and absolutely integrable in $(-\infty \leqslant x \leqslant \infty)$. Moreover, all the components of stress, displacement and couple-stress tend to zero as y tends to infinity.

SOLUTION OF THE PROBLEM

Performing over the equations (18), the Laplace transform, assuming that

$$\phi(x, y, 0) = \frac{\partial}{\partial \tau} \phi(x, y, 0) = \psi(x, y, 0) = \frac{\partial}{\partial \tau} \psi(x, y, 0) = 0 \tag{20}$$

we get
$$p^2 \phi_1 = \nabla^2 \phi_1 \text{ and } \beta^2 p^2 \psi_1 = \nabla^2 \psi_1 - l^2 \nabla^4 \psi_1 \tag{21}$$

where
$$\phi_1 = \int_0^\infty \phi e^{-p\tau}\, d\tau\ , \ \psi_1 = \int_0^\infty \psi\, e^{-p\tau}\, d\tau\ .$$

Now let
$$\left.\begin{aligned} \phi_2(\xi, y, p) &= \int_{-\infty}^{\infty} \phi_1(x, y, p)\, e^{i\xi x}\, dx \\ \text{and} \quad \psi_2(\xi, y, p) &= \int_{-\infty}^{\infty} \psi_1(x, y, p)\, e^{i\xi x}\, dx \end{aligned}\right\} \tag{22}$$

be the Fourier transforms of ϕ_1 and ψ_1 respectively. Multiplying equation (21) by $e^{i\xi x}$ and integrating with respect to x between the limits $(-\infty, \infty)$, we get

$$\frac{d^2\phi_2}{dy^2} = (\xi^2 + p^2)\,\phi_2\,,$$

$$l^2\left(\frac{d^2}{dy^2} - \xi^2\right)^2\psi_2 - \left(\frac{d^2}{dy^2} - \xi^2\right)\psi_2 = -\beta^2 p^2 \psi_2\,.$$

The solutions of these equations are

$$\begin{aligned}\phi_2 &= A\exp\{-y\,(\xi^2+p^2)^{\frac{1}{2}}\} + B_1\exp\{y\,(\xi^2+p^2)^{\frac{1}{2}}\},\\ \psi_2 &= A_1\exp\{y\,(\xi^2+p_1{}^2)^{\frac{1}{2}}\} + C\exp\{-y\,(\xi^2+p_1{}^2)^{\frac{1}{2}}\}\\ &\quad + C_1\exp\{y\,(\xi^2+p_2{}^2)^{\frac{1}{2}}\} + D\exp\{-y\,(\xi^2+p_2{}^2)^{\frac{1}{2}}\}\end{aligned} \tag{23}$$

where A, B_1, A_1, C, C_1, and D are constants, and

$$\begin{aligned}p_1{}^2 &= \frac{1}{2\rho^2}\{1 + (1-4\beta^2p^2l^2)^{\frac{1}{2}}\},\\ p_2{}^2 &= \frac{1}{2\rho^2}\{1 - (1-4\beta^2p^2l^2)^{\frac{1}{2}}\}\,.\end{aligned} \tag{24}$$

In order to satisfy the condition that the components of stress, displacement and couple-stress tend to zero as y tends to infinity, we take

$$B_1 = A_1 = C_1 = 0\,.$$

The inversion theorem for the Fourier transform[5] yields

$$\phi_1 = \frac{1}{2\pi}\int_{-\infty}^{\infty} A\exp\left\{-y\,(\xi^2+p^2)^{\frac{1}{2}}\right\}\exp(-i\xi x)\,d\xi\,,$$

$$\psi_1 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[C\exp\left\{-y\,(\xi^2+p_1{}^2)^{\frac{1}{2}}\right\} + D\exp\left\{-y\,(\xi^2+p_2{}^2)^{\frac{1}{2}}\right\}\right] \times \exp(-i\xi x)\,d\xi$$

Performing Laplace transform on (17) and substituting for ϕ_1 and ψ_1 from above we get

$$\bar{u} = \frac{-1}{2\pi}\int_{-\infty}^{\infty}\left[i\xi A\exp\left\{-y\,(\xi^2+p^2)^{\frac{1}{2}}\right\} + (\xi^2+p_1{}^2)^{\frac{1}{2}}C\exp\left\{-y\,(\xi^2+p_1{}^2)^{\frac{1}{2}}\right\}\right.$$
$$\left. + (\xi^2+p_2{}^2)^{\frac{1}{2}}D\exp\left\{-y\,(\xi^2+p_2{}^2)^{\frac{1}{2}}\right\}\right]e^{-i\xi x}\,d\xi\,,$$

$$\bar{v} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[-(\xi^2+p^2)^{\frac{1}{2}}A\exp\left\{-y\,(\xi^2+p^2)^{\frac{1}{2}}\right\} + i\xi C\exp\left\{-y\,(\xi^2+p_1{}^2)^{\frac{1}{2}}\right\}\right.$$
$$\left. + i\xi D\exp\left\{-y\,(\xi^2+p_2{}^2)^{\frac{1}{2}}\right\}\right]e^{-i\xi x}\,d\xi \tag{25}$$

where $\bar{u}$ and $\bar{v}$ are the Laplace transforms of u and v respectively.

Taking Laplace transform of (10), (15) and (19) and substituting for $\bar{u}$ and $\bar{v}$ from (25) we get

$$\frac{\bar{\sigma}_x}{2\mu} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Big[i\xi (\xi^2 + p_1^2)^{\frac{1}{2}} C \exp \Big\{ -y (\xi^2 + p_1^2)^{\frac{1}{2}} \Big\} + i\xi (\xi^2 + p_2^2)^{\frac{1}{2}} D \exp$$

$$\Big\{ -y (\xi^2 + p_2^2)^{\frac{1}{2}} \Big\} - \Big(\xi^2 - \frac{\lambda p^2}{2\mu} \Big) A \exp \Big\{ -y (\xi^2 + p^2)^{\frac{1}{2}} \Big\} \Big] e^{-i\xi x} \, d\xi ,$$

$$\frac{\bar{\sigma}_y}{2\mu} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Big[(\xi^2 + \frac{1}{2} \beta^2 p^2) A \exp \Big\{ -y (\xi^2 + p^2)^{\frac{1}{2}} \Big\} - i\xi (\xi^2 + p_1^2)^{\frac{1}{2}} C \exp$$

$$\Big\{ -y (\xi^2 + p_1^2)^{\frac{1}{2}} \Big\} - i\xi (\xi^2 + p_2^2)^{\frac{1}{2}} D \exp \Big\{ -y (\xi^2 + p_2^2)^{\frac{1}{2}} \Big\} \Big] e^{-i\xi x} \, d\xi ,$$

$$\frac{\bar{\tau}_{yx}}{2\mu} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Big[2i\xi (\xi^2 + p^2)^{\frac{1}{2}} A \exp \Big\{ -y (\xi^2 + p^2)^{\frac{1}{2}} \Big\} + \Big\{ 2\xi^2 + p_1^2 - l^2 p_1^4 \Big\}$$

$$C \exp \Big\{ -y (\xi^2 + p_1^2)^{\frac{1}{2}} \Big\} + \Big\{ 2\xi^2 + p_2^2 - l^2 p_2^4 \Big\} D \exp \Big\{ -y (\xi^2 + p_2^2)^{\frac{1}{2}} \Big\} \Big] e^{-i\xi x} \, d\xi ,$$

$$\frac{\bar{\tau}_{xy}}{2\mu} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Big[2i\xi (\xi^2 + p^2)^{\frac{1}{2}} A \exp \Big\{ -y (\xi^2 + p^2)^{\frac{1}{2}} \Big\} + \Big\{ 2\xi^2 + p_1^2 + l^2 p_1^4 \Big\}$$

$$C \exp \Big\{ -y (\xi^2 + p_1^2)^{\frac{1}{2}} \Big\} + \Big\{ 2\xi^2 + p_2^2 + l^2 p_2^4 \Big\} D \exp \Big\{ -y (\xi^2 + p_2^2)^{\frac{1}{2}} \Big\} \Big] e^{-i\xi x} \, d\xi ,$$

$$\bar{\mu}_x = \frac{B}{\pi} \int_{-\infty}^{\infty} \Big[i\xi p_1^2 C \exp \Big\{ -y (\xi^2 + p_1^2)^{\frac{1}{2}} \Big\} + i\xi p_2^2 D \exp$$

$$\Big\{ -y (\xi^2 + p_2^2)^{\frac{1}{2}} \Big\} \Big] e^{-i\xi x} \, d\xi ,$$

$$\bar{\mu}_y = \frac{B}{\pi} \int_{-\infty}^{\infty} \Big[p_1^2 (\xi^2 + p_1^2)^{\frac{1}{2}} C \exp \Big\{ -y (\xi^2 + p_1^2)^{\frac{1}{2}} \Big\} + p_2^2 (\xi^2 + p_2^2)^{\frac{1}{2}} D \exp$$

$$\Big\{ -y (\xi^2 + p_2^2)^{\frac{1}{2}} \Big\} \Big] e^{-i\xi x} \, d\xi , \tag{26}$$

where $\bar{\sigma}_x$, $\bar{\sigma}_y$, $\bar{\tau}_{yx}$, $\bar{\tau}_{xy}$, $\bar{\mu}_x$ and $\bar{\mu}_y$ are Laplace transforms of corresponding stress and couple-stress components.

The boundary conditions (19) yield

$$\left.\begin{aligned} \left(\xi^2+\frac{1}{2}\beta^2p^2\right)A - Ci\xi\left(\xi^2+p_1{}^2\right)^{\frac{1}{2}} - Di\xi\left(\xi^2+p_2{}^2\right)^{\frac{1}{2}} &= -\frac{P_2(\xi\,,p)}{2\mu}\,, \\ 2\,Ai\xi\left(\xi^2+p^2\right)^{\frac{1}{2}} + C\left(2\xi^2+p_1{}^2-l^2p_1{}^4\right) + D\left(2\xi^2+p_2{}^2-l^2p_2{}^4\right) &= 0\,, \\ Cp_1{}^2\left(\xi^2+p_1{}^2\right)^{\frac{1}{2}} + Dp_2{}^2\left(\xi^2+p_2{}^2\right)^{\frac{1}{2}} &= 0\,, \end{aligned}\right\} \quad (27)$$

where $$P_2(\xi\,,p) = \int_{-\infty}^{\infty} P_1(x\,,p)\, e^{i\xi x}\, dx$$

and $P_1(x\,,p)$ is the Laplace transform of $P(x\,,\tau)$.

Solving equations (27) we get

$$\left.\begin{aligned} A &= -\frac{g_1}{g_2}\,\frac{P_2(\xi\,,p)}{\mu} \\ C &= i\xi p_2{}^2\left(\xi^2+p_2{}^2\right)^{1/2}\left(\xi^2+p^2\right)^{1/2}P_2(\xi\,,p)/\mu g_2 \\ D &= -\,i\xi p_1{}^2\left(\xi^2+p_1{}^2\right)^{1/2}\left(\xi^2+p^2\right)^{1/2}P_2(\xi\,,p)/\mu\, g_2 \end{aligned}\right\} \quad (28)$$

where

$$\left.\begin{aligned} &= p_2{}^2\left(\xi^2+p_2{}^2\right)^{1/2}\left(2\xi^2+p_1{}^2-l^2p_1{}^4\right) \\ &\quad - p_1{}^2\left(\xi^2+p_1{}^2\right)^{1/2}\left(2\xi^2+p_2{}^2-l^2p_2{}^4\right), \\ g_2 &= p_2{}^2\left(\xi^2+p_2{}^2\right)^{1/2}\left\{\left(\xi^2+\frac{1}{2}\beta^2p^2\right)\right. \\ &\quad \left.\left(2\xi^2+p_1{}^2-l^2p_1{}^4\right) - 2\xi^2\left(\xi^2+p^2\right)^{\frac{1}{2}}\left(\xi^2+p_1{}^2\right)^{\frac{1}{2}}\right\} \\ &\quad - p_1{}^2\left(\xi^2+p_1{}^2\right)^{\frac{1}{2}}\left\{\left(\xi^2+\frac{1}{2}\beta^2p^2\right)\left(2\xi^2+p_2{}^2-l^2p_2{}^4\right)\right. \\ &\quad \left. - 2\xi^2\left(\xi^2+p^2\right)^{\frac{1}{2}}\left(\xi^2+p_2{}^2\right)^{\frac{1}{2}}\right\} \end{aligned}\right\} \quad (29)$$

Substituting the values of A, C and D in (25) and (26), the components of displacement, stress and couple stress on integrating and taking the inverse Laplace transform can be calculated. For showing the effect of couple stresses we calculate $\tau_{xy} - \tau_{yx}$.

From (26) we get

$$\begin{aligned} \bar{\tau}_{xy} - \bar{\tau}_{yx} = \frac{l^2}{\pi}\int_{-\infty}^{\infty}\Big[\, p_1{}^4C\exp\left\{-y\left(\xi^2+p_1{}^2\right)^{\frac{1}{2}}\right\} + p_2{}^4D\exp \\ \left\{-y\left(\xi^2+p_2{}^2\right)^{\frac{1}{2}}\right\}\Big]\, e^{-i\xi x}\, d\xi \end{aligned} \quad (30)$$

As a particular case a pulse of pressure moving uniformly with velocity v_0 along the boundary is considered, that is

$$P(x,\tau) = P\delta(x-\beta_1\tau)$$

where

$$\beta_1 = v_0/c_1$$

which gives

$$P_2(\xi, p) = P\delta(\xi\beta_1 + ip) \tag{31}$$

Substituting for C and D from (28) and for $P_2(\xi, p)$ from (31) in (30) and using the relation[5]

$$\int_{-\infty}^{\infty} f(\xi)\,\delta(\xi - a)\,d\xi = f(a)$$

we get
$$\bar{\tau}_{xy} - \bar{\tau}_{yx} = -\frac{\sqrt{2}}{4}\,\frac{lP}{\pi_\mu}\,\beta^2 \exp\left(\frac{-y}{\sqrt{2}l}\right)\frac{pe^{-xp/\beta_1}}{p+\gamma} \tag{32}$$

on neglecting terms containing higher powers of l, and writing

$$\gamma = \frac{1}{4}\,\beta_1(\beta^2\beta_1^2 - 2)^2(\beta_1^2 - 1)^{-\frac{1}{2}}(\beta^2\beta_1^2 - 1)^{-1}.$$

On taking inverse Laplace transform[6] we get from (32)

$$\tau_{xy} - \tau_{yx} = \begin{cases} \dfrac{\sqrt{2}}{4}\,\dfrac{l\beta^2\gamma}{\pi_\mu}\,P\exp(-y/\sqrt{2}.l)\,e^{-\gamma(\tau - x/\beta_1)}, & \tau > \dfrac{x}{\beta_1}, \\ 0, & 0 < \tau < x/\beta_1. \end{cases}$$

From this result it is obvious that the shear stress along the bounding surface ($y = 0$) is proportional to $l \exp\left\{-\gamma(\tau - x/\beta_1)\right\}$, provided τ is greater than x/β_1. In other words it can be said that the shear stress at any point on the bounding surface is proportional to l, the parameter of couple-stress. At any point inside the semi-infinite medium $\tau_{xy} - \tau_{yx}$ is proportional to $l \exp(-y/\sqrt{2}\,l.)$. That is, the effect of couple-stress is predominant on the bounding surface because the parameter, l, is generally small. When, $0 < \tau < x/\beta_1$ (or $0 < t < x/v_0$), $\tau_{xy} = \tau_{yx}$. Which is the classical result. It means that there is no effect of couple-stress on shear stresses when $0 < \tau < x/\beta_1$.

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