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Multivariable *H*-Function with Application to Temperature Distribution in a Moving Medium

A. K. SINGH & Y. N. PRASAD

Banaras Hindu University, Varanasi-221005

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Abstract. The Meijer's *G*-function has been obtained for the first time as a solution of a differential equation governing a heat conduction problem. The problem of temperature distribution in a moving medium between $x = -1$ and $x = 1$ and having variable velocity and variable thermal conductivity is considered. Due to a general character of the *G*-function, many known and unknown results may be derived as particular cases. The modified multivariable *H*-function to obtain a particular solution has been employed.

1. Introduction

Prasad & Singh[1] have recently defined the modified multivariable *H*-function with the help of Mellin-Barnes type multiple contour integrals and discussed its analyticity. We, however, omit details and enumerate here few of their results which are of frequent use in our present discussion.

If we let

$$V_i = \sum_{j=1}^{n} \alpha_j^{(i)} - \sum_{n+1}^{p} \alpha_j^{(i)} - \sum_{j=1}^{q} \beta_j^{(i)} + \sum_{j=1}^{M^{(i)}} \delta_j^{(i)} - \sum_{M^{(i)}+1}^{Q^{(i)}} \delta_j^{(i)} + \sum_{j=1}^{N^{(i)}} \gamma_j^{(i)}$$

$$- \sum_{j=N^{(i)}+1}^{P^{(i)}} \gamma_j^{(i)} - \sum_{j=1}^{R} \lambda_j^{(i)}, \qquad i = 1,\ldots, r, \tag{1}$$

then for $V_i > 0$, the multiple contour integral defining the multivariable function $H[x_1,\ldots,x_r]$ is absolutely convergent and defines the *H*-function analytic in the sectors given by

$$|\arg x_i| < \tfrac{1}{2}\pi V_i, \qquad i = 1,\ldots, r, \tag{2}$$

points $x_i = 0$ being tacitly excluded.

$$H[x_1,\ldots,x_r] = O(|x_1|^{\alpha_1}\ldots|x_r|^{\alpha_r}) \qquad \text{for } (x_1,\ldots,x_r) \to (0,\ldots,0),$$

where

$$\alpha_i = \min Re[d_j^{(i)} \big/ \delta_j^{(i)}], \qquad j = 1,\ldots,M^{(i)},\ i = 1,\ldots,r. \tag{3}$$

Also for $n = 0$,

$$H[x_1,\ldots,x_r] = O(|x_1|^{\beta_1}\ldots|x_r|^{\beta_r}) \text{ when } (x_1,\ldots,x_r) \to (\infty,\ldots,\infty),$$

where

$$\beta_i = \max Re[C(_j^{(i)} - 1) \big/ \gamma_j^{(i)}], \qquad j = 1,\ldots,N^{(i)},\ i = 1,\ldots,r. \tag{4}$$

2. Statement of the Problem and Governing Equations

Let us consider a medium (say bar) moving in the direction of its length (i.e. x-axis) between the limits $x = -1$ to $x = 1$. The bar is supposed to be so thin that the temperature at all points of the section may be taken to be the same. We also assume that conductivity of the medium is a function of position and is proportional to $(1-x^2)$. Further, the velocity of the medium is supposed to be a function of position and proportional to $[\alpha + \gamma - \beta + (\alpha + \beta - \gamma)x]$. Lateral surface of the medium (i.e. bar) and its two ends ($x = \pm 1$) are supposed to be insulated.

Remark 1. Our suppositions regarding variable velocity and variable thermal conductivity are not without practical interest since similar situations may arise in the case of continuous snow fall over hill surface of constant slope.

The differential equation satisfied by the temperature $\theta(x, t)$ at any time t in a solid medium with conductivity K, density ρ, specific heat C and with no generation of heat within the medium is given[2] as

$$\rho C \frac{\partial \theta}{\partial t} + \frac{\partial}{\partial x}\left(-K\frac{\partial \theta}{\partial x}\right) = 0. \tag{5}$$

But in our problem the medium is moving with velocity u in the direction of x-axis and, therefore, while calculating the rate at which heat crosses any plane, a convective term $\rho C\theta u$ must be added to the part due to conduction. Eqn. (5) then assumes the form

$$\rho C \frac{\partial \theta}{\partial t} + \frac{\partial}{\partial x}\left(-K\frac{\partial \theta}{\partial x} + \rho C\theta u\right) = 0. \tag{6}$$

If both conductivity K and velocity u of the medium are functions of positions and (or) temperature, Eqn. (6) becomes

$$\rho C\frac{\partial \theta}{\partial t} + \rho C u\frac{\partial \theta}{\partial x} + \rho C\theta\frac{\partial u}{\partial x} - K\frac{\partial^2 \theta}{\partial x^2} - \frac{\partial \theta}{\partial x}\frac{\partial K}{\partial x} = 0. \tag{7}$$

Remark 2. In deriving the heat conduction equation in the first draft of our paper the term $\rho C\partial \dfrac{\partial u}{\partial x}$ falling in the left hand side of Eqn. (7) was missing.

If

$$K = C_1(1-x^2), \tag{8}$$

and

$$u = C_2[\alpha + \gamma - \beta + (\alpha - \gamma + \beta)x], \tag{9}$$

where C_1 and C_2 are constants of proportionality, $\alpha > -1, \beta > \gamma = 1$, then Eqn. (7) reduces to

$$\frac{\partial\theta}{\partial t} + C_2[\alpha+\gamma-\beta+(\alpha-\gamma+\beta)x]\frac{\partial\theta}{\partial x} + \frac{2C_1}{\rho C} \times \frac{\partial\theta}{\partial x} + C_2(\alpha-\gamma+\beta)\theta - \frac{C_1}{\rho C}(1-x^2)\frac{\partial^2\theta}{\partial x^2} = 0. \tag{10}$$

Let us, for further simplicity, assume $C_2 = \dfrac{C_1}{\rho C}$ which transforms Eqn. (10) into

$$\frac{1}{C_2}\frac{\partial\theta}{\partial t} + [\alpha+\gamma-\beta+(\alpha-\gamma+\beta+2)x]\frac{\partial\theta}{\partial x} + (\alpha-\gamma+\beta)\theta - (1-x^2)\frac{\partial^2\theta}{\partial x^2} = 0. \tag{11}$$

Let the initial temperature distribution in the medium be given by

$$\theta(x, 0) = f(x). \tag{12}$$

Results used in the Sequel

$$\int_{-1}^{1}(1-x)^{\rho}(1+x)^{\sigma} G^{1;2}_{2;2}\left[\tfrac{1}{2}(x-1)\,\middle|\,\begin{matrix}1+\nu, & -\nu-\alpha-\beta\\ 0, & -\alpha\end{matrix}\right]dx$$

$$= \frac{2^{\rho+\sigma+1}\,\Gamma(\sigma+1)\,\Gamma(\nu+1)}{\Gamma(\nu+\alpha+1)}\sum_{N=0}^{\nu}\frac{\Gamma(-\nu+N)\,\Gamma(\nu+\alpha+\beta+1+N)\,\Gamma(\rho+1+N)}{N!\,(\alpha+1)_N\,\Gamma(\rho+\sigma+2+N)}, \tag{13}$$

provided that $Re(\rho) > -1$, $Re(\sigma) > -1$.

$$\int_{-1}^{1}(1-x)^{\alpha}(1+x)^{\beta} G^{1,2}_{2,2}\left[\tfrac{1}{2}(x-1)\,\middle|\,\begin{matrix}1+\nu, & -\nu-\alpha-\beta\\ 0, & -\alpha\end{matrix}\right] G^{1,2}_{2,2}$$

$$\cdot\left[\tfrac{1}{2}(x-1)\,\middle|\,\begin{matrix}1+\mu, & -\mu-\alpha-\beta\\ 0, & -\alpha\end{matrix}\right]dx$$

$$= 0 \text{ when } \mu \neq \nu,\ Re(\alpha) > -1,\ Re(\beta) > -1. \tag{14}$$

$$\int_{-1}^{1} (1-x)^{\rho} (1+x)^{\beta} G_{2,2}^{1,2}\left[\tfrac{1}{2}(x-1)\,\middle|\, \begin{matrix} 1+\nu, & -\nu-\alpha-\beta \\ 0, & -\alpha \end{matrix}\right] G_{2,2}^{1,2}$$

$$\cdot \left[\tfrac{1}{2}(x-1)\,\middle|\, \begin{matrix} 1+\nu, & -\nu-\rho-\beta \\ 0, & -\rho \end{matrix}\right] dx$$

$$= \frac{2^{\rho+\beta+1}\,\Gamma(\beta+\nu+1)\,\Gamma(\alpha+\beta+2\nu+1)\,\Gamma(\rho+\beta+\nu+1)[\Gamma(-\nu)\Gamma(\nu+1)]^2}{\nu!\,\Gamma(\rho+\beta+2\nu+2)\,\Gamma(\nu+\alpha+1)}, \quad (15)$$

provided $Re\,(\rho) > -1$, $Re\,(\beta) > -1$.

$$\int_{-1}^{1} (1-x)^{\rho}\,(1+x)^{\sigma}\,G_{2,2}^{1,2}\left[\tfrac{1}{2}(x-1)\,\middle|\, \begin{matrix} 1+\nu, & -\nu-\alpha-\beta \\ 0, & -\alpha \end{matrix}\right]$$

$$\cdot\, H\left[z_1(1-x)^{\delta_1}(1+x)^{\mu_1},\ldots, Z_r(1-x)^{\delta_r}(1+x)^{\mu_r}\right] dx \quad (16)$$

$$= \frac{2^{\rho+\sigma+1}\,\Gamma(\nu+1)}{\Gamma(\nu+\alpha+1)} \sum_{N=0}^{\nu} \frac{\Gamma(-\nu+N)\,\Gamma(\nu+\alpha+\beta+1+N)}{N!\,(\alpha+1)_N}\; F(\rho, \sigma; N),$$

where

$$F(\rho, \sigma; N) = H^{0,\, n+2\,;\,0\,:\,(M',N')\,;\ldots;\,(M^{(r)},N^{(r)})}_{p+2,\, q+2\,:\,R\,:\,[p',Q']\,;\ldots;\,p^{(r)},\,Q^{(r)}]}\left[\begin{matrix}(-\sigma;(\mu_i)_{i=1,\ldots,r}), \\ \{(b_q;(\beta_q^{(i)})_{i=1,\ldots,r})\},\end{matrix}\right.$$

$$(-\rho-N;(\delta_i)_{i=1,\ldots,r}),\ \{(a_p;(\alpha_p^{(i)})_{i=1,\ldots,r})\} : :$$

$$(-\rho-\sigma-1-N;\ (\delta_i+\mu_i)_{i=1,\ldots,r}) : \{(l_R;(U_R^{(i)}\lambda_R^{(i)})_{i=1,\ldots,r})\} :$$

$$\left.\begin{matrix}\{(C'_{p'},\gamma'_{p'})\};\ldots;\{(C^{(r)}_{p^{(r)}},\gamma^{(r)}_{p^{(r)}})\} \\ \{(d'_{Q'},\delta'_{Q'})\};\ldots;\{(d^{(r)}_{Q^{(r)}},\delta^{(r)}_{Q^{(r)}})\}\end{matrix}\,\middle|\, z_1\,2^{\delta_1+\mu_1},\ldots, z_r\,2^{\delta_r+\mu_r}\right]. \quad (17)$$

Result (16) is valid if $Re\left(\rho + \sum_{i=1}^{r} \delta_i\alpha_i\right) > -1$, $Re\left(\sigma + \sum_{i=1}^{r} \mu_i\alpha_i\right) > -1$; $\delta_i \geqslant 0$, $\mu_i > 0$ (or $\mu_i \geqslant 0$, $\delta_i > 0$) for $i = 1,\ldots, r$; $|\arg z_i| < \frac{1}{2}\pi V_i$, $V_i > 0$ where α_i and V_i are respectively given by Eqns. (3) and (1).

Remark 3. Proofs of the results (13)–(15) are based upon the hypergeometric series representations[3] of the G-function (or G-functions) occurring in the integrand and appeal to the known relationships[4]. Result (16) is the direct consequence of the result (13) and the definition of the multivariable H-function.

3. Solution and Analysis

Assuming the solution of the differential Eqn. (11) as

$$\theta(x, t) = X(x)\,T(t), \quad (18)$$

Equation (11), consequently, reduces to

$$\frac{1}{C_2T}\frac{dT}{dt} = (1-x^2)\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{x}[\beta-\alpha-\gamma-(\alpha-\gamma+\beta+2)\,x]\frac{dX}{dx} + (\gamma-\alpha-\beta). \tag{19}$$

Evidently, each side of Eqn. (19) equals a constant. Assuming the constant to be $(\nu+1)(\gamma-\alpha-\beta-\nu)$, Eqn. (19) yields

$$(1-x^2)\frac{d^2X}{dx^2} + [\beta-\alpha-\gamma-(\alpha-\gamma+\beta+2)x]\frac{dX}{dx} + \nu(\nu+\alpha+\beta-\gamma+1)\,X = 0, \tag{20}$$

and

$$\frac{dT}{dt} - C_2(\nu+1)(\gamma-\alpha-\beta-\nu)\,T = 0. \tag{21}$$

It is not much difficult to exhibit[3] that the solution of Eqn. (20) is

$$X = C_3\, G^{1,2}_{2,2}\left[\tfrac{1}{2}(x-1)\,\middle|\,\begin{matrix}1+\nu,\ \gamma-\nu-\alpha-\beta\\ 0,\ -\alpha\end{matrix}\right], \tag{22}$$

and that of Eqn. (21) is

$$T = C_4 \exp\{C_2(\nu+1)(\gamma-\alpha-\beta-\nu)t\}, \tag{23}$$

where C_3 and C_4 are constants of integration.

A general solution may, therefore, be given as

$$\theta(x,t) = \sum_{\nu=0}^{\infty} A_\nu \exp\{C_2(\nu+1)(\gamma-\alpha-\beta-\nu)\,t\}\, G^{1,2}_{2,2}\left[\tfrac{1}{2}(x-1)\,\middle|\,\begin{matrix}1+\nu,\ \gamma-\nu-\alpha-\beta\\ 0,\ -\alpha\end{matrix}\right], \tag{24}$$

which on appeal to the initial condition Eqn. (12) leads to

$$f(x) = \sum_{\nu=0}^{\infty} A_\nu\, G^{1,2}_{2,2}\left[\tfrac{1}{2}(x-1)\,\middle|\,\begin{matrix}1+\nu,\ \gamma-\nu-\alpha-\beta\\ 0,\ -\alpha\end{matrix}\right]. \tag{25}$$

To determine the constants $A_\nu(\nu = 0, 1, \ldots)$ we multiply both sides of Eqn. (25) by $(1-x)^\alpha(1+x)^{\beta-\gamma}\, G^{1,2}_{2,2}\left[\tfrac{1}{2}(x-1)\,\middle|\,\begin{matrix}1+\nu,\ \gamma-\nu-\alpha-\beta\\ 0,\ -\alpha\end{matrix}\right]$ and integrate, thereafter, with respect to x between the limits $x = -1$ to $x = 1$. In the right hand side we interchange the order of summation and integration which is justifiable due to absolute convergence of the integral and the series involved therein. Finally, appeal to the results (14) and (15), consequently, demands

$$A_\nu = \frac{2^{\gamma-\alpha-\beta-1}(\alpha-\gamma+\beta+2\nu+1)\,\Gamma(\nu+\alpha+1)}{\Gamma(\nu+\beta+1)\,\Gamma(\nu+1)\,\Gamma(-\nu)\,\Gamma(-\nu)\,\Gamma(\nu+\alpha-\gamma+\beta+1)} \cdot \int_{-1}^{1}(1-x)^\alpha(1+x)^{\beta-\gamma}\, G^{1,2}_{2,2}\left[\tfrac{1}{2}(x-1)\,\middle|\,\begin{matrix}1+\nu,\ \gamma-\alpha-\beta-\nu\\ 0,\ -\alpha\end{matrix}\right] f(x)\,dx. \tag{26}$$

Combining Eqn. (26) with Eqn. (24), we obtain a general solution as

$$\theta(x, t) = 2^{\gamma-\alpha-\beta-1}$$

$$\sum_{\nu=0}^{\infty} \frac{(\alpha+\beta+2\nu+1-\gamma)\,\Gamma(\nu+\alpha+1)}{\Gamma(\beta+\nu-\gamma+1)\,\Gamma(\nu+1)\,\Gamma(-\nu)\,\Gamma(-\nu)\,\Gamma(\nu+\alpha-\gamma+\beta+1)}$$

$$. \exp\{C_2(\nu+1)(\gamma-\alpha-\beta-\nu)\,t\}\, G_{2,2}^{1,2}\left[\tfrac{1}{2}(x-1)\,\middle|\, \begin{matrix} 1+\nu, & \gamma-\nu-\alpha-\beta \\ 0, & -\alpha \end{matrix}\right]$$

$$. \int_{-1}^{1} (1-x)^{\alpha}\,(1+x)^{\beta-\gamma}\, G_{2,2}^{1,2}\left[\tfrac{1}{2}(x-1)\,\middle|\, \begin{matrix} 1+\nu, & \gamma-\nu-\alpha-\beta \\ 0, & -\alpha \end{matrix}\right] f(x)\,dx, \tag{27}$$

provided that $f(x)$ is such that right hand side of Eqn. (27) makes sense.

4. Example

If the initial temperature distribution in the medium is given by

$$f(x) = (1-x)^{\rho}\,(1+x)^{\sigma}\, H[z_1(1-x)^{\delta_1}(1+x)^{\mu_1},\ldots,z_r(1-x)^{\delta_r}(1+x)^{\mu_r}], \tag{28}$$

substituting this value for $f(x)$ in Eqn. (27) and utilizing the result (16), the solution to the problem is given by

$$\theta(x,t) = 2^{\rho+\sigma} \sum_{\nu=0}^{\infty} \sum_{N=0}^{\nu} \frac{\Gamma(\alpha-\gamma+\beta+2\nu+2)\,(-\nu)_N\,(\alpha+\beta-\gamma+\nu+1)_N}{\Gamma(\alpha+\beta-\gamma+2\nu+1)\Gamma(\beta-\gamma+\nu+1)\Gamma(-\nu)\,N!\,(\alpha+1)_N}$$

$$. \exp\{C_2(\nu+1)(\gamma-\alpha-\beta-\nu)\,t\}\, G_{2,2}^{1,2}\left[\tfrac{1}{2}(x-1)\,\middle|\, \begin{matrix} \nu+1, & \gamma-\nu-\alpha-\beta \\ 0, & -\alpha \end{matrix}\right]$$

$$F(\rho+\alpha,\ \sigma+\beta;\ N), \tag{29}$$

where $F(\rho + \alpha,\ \sigma + \beta;\ N)$ is given by Eqn. (17). Result (29) is valid under the conditions $Re\left(\rho + \alpha + \sum_{i=1}^{r} \delta_i\,\alpha_i\right) > -1$, $Re\left(\sigma + \beta + \sum_{i=1}^{r} \mu_i\,\alpha_i\right) > \gamma - 1$ and $\left|\arg z_i\right| < \frac{1}{2}\pi V_i$, $V_i > 0$ (α_i and V_i being usual).

Remark. We have already mentioned that the G-function[3] is of very general character and includes, as its special cases, many of the elementary functions as well as polynomials. And, thus, the main reason for considering the G-function instead of usual functions is to unify as well as generalize the previous results. Moreover, separate treatments for a large number of cases may be avoided by this single treatment of the problem considered herein. Similar arguments may be given for considering the initial temperature distribution in the medium in terms of multi-variable H-function.

5. Particular Cases

(*i*) When the medium moves with uniform velocity $2C_2\alpha$ (i.e. velocity u given by Eqn. (9) is independent of position and time which can be obtained by setting $\gamma = \alpha + \beta$), the differential equation (11) reduces to

$$\frac{1}{C_2}\frac{\partial\theta}{\partial t} = (1-x^2)\frac{\partial^2\theta}{\partial x^2} - 2(x+\alpha)\frac{\partial\theta}{\partial x}\ , \tag{30}$$

and the solution in this case is given by

$$\theta(x, t) = 2^{\rho+\sigma}\sum_{\nu=0}^{\infty}\sum_{N=0}^{\nu}\frac{\Gamma(2\nu+2)(-\nu)_N(\nu+1)_N}{\Gamma(2\nu+1)\,\Gamma(\nu-\alpha+1)\,\Gamma(-\nu)\,N!\,(\alpha+1)_N}$$

$$.\exp\{-\nu C_2(\nu+1)\,t\}G^{1,2}_{2,2}\left[\tfrac{1}{2}(x-1)\,\middle|\,\begin{matrix}\nu+1, & -\nu\\ 0, & -\alpha\end{matrix}\right]F(\rho+\alpha, \sigma+\beta; N), \tag{31}$$

provided for $\gamma = \alpha + \beta$, the conditions given in Eqn. (29) are satisfied.

(*ii*) When the medium is stationary (i.e. u given by Eqn. (9) is zero which can be achieved by setting $\alpha = 0$ and $\beta = \gamma$), the differential equation (11) reduces[5] to

$$\frac{1}{C_2}\frac{\partial\theta}{\partial t} = (1-x^2)\frac{\partial^2\theta}{\partial x^2} - 2x\frac{\partial\theta}{\partial x}, \tag{32}$$

and the solution in this case is given by

$$\theta(x, t) = 2^{\rho+\sigma}\sum_{\nu=0}^{\infty}\sum_{N=0}^{\nu}\frac{\Gamma(2\nu+2)(-\nu)_N(\nu+1)_N}{\Gamma(2\nu+1)\,\Gamma(\nu+1)\,\Gamma(-\nu)\,N!\,(1)_N}$$

$$.\exp\{-\nu C_2(\nu+1)\,t\}\ G^{1,2}_{2,2}\left[\tfrac{1}{2}(x-1)\,\middle|\,\begin{matrix}\nu+1, & -\nu\\ 0, & 0\end{matrix}\right]F(\rho, \sigma; N), \tag{33}$$

provided that for $\gamma = \beta$, $\alpha = 0$, the conditions given in Eqn. (29) hold.

(*iii*) If we let $\gamma = 0$, Eqn. (11) reduces to

$$\frac{1}{C_2}\frac{\partial\theta}{\partial t} = (1-x^2)\frac{\partial^2\theta}{\partial x^2} + [\beta-\alpha-(\alpha+\beta+2)\,x]\frac{\partial\theta}{\partial x} - (\alpha+\beta)\,\theta, \tag{34}$$

and the solution (29), then, leads to

$$\theta(x, t) = 2^{\rho+\sigma}\sum_{\nu=0}^{\infty}\sum_{N=0}^{\nu}\frac{\nu!\,\Gamma(\alpha+\beta+2\nu+2)\,\Gamma(\alpha+\beta+\nu+1)\,(-\nu)_N\,(\alpha+\beta+\nu+1)_N}{N!\,(\alpha+1)_N\,\Gamma(\alpha+\nu+1)\,\Gamma(\beta+\nu+1)\,\Gamma(\alpha+\beta+2\nu+1)}$$

$$.\exp\{-C_2(\nu+1)(\nu+\alpha+\beta)\,t\}\ P^{(\alpha,\beta)}_{\nu}(x)\,F(\rho+\alpha, \sigma+\beta; N), \tag{35}$$

provided that for $\gamma = 0$, the conditions given in Eqn. (29) are true.

(*iv*) Also, for various initial temperature distributions given in terms of elementary functions, e.g.

$$\theta(x, 0) = (1 - x)^{\rho} (1 + x)^{\sigma} \exp (x^2 - 1),$$

$$\theta(x, 0) = (1 - x)^{\rho} (1 + x)^{\sigma} \sin \left(\sqrt{1 - x^2}\right),$$

$$\theta(x, 0) = (1 - x)^{\rho} (1 + x)^{\sigma} \log \left(\sqrt{1 - x^2} + \sqrt{2 - x^2}\right),$$

$$\theta(x, 0) = (1 - x)^{\rho} (1 + x)^{\sigma} (1 - x^2)^{1/2} J_m \left(\sqrt{1 - x^2}\right) \text{ etc. },$$

solution to the corresponding problem may be obtained from Eqn. (29) by suitable specializations of parameters involved.

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