A NOTE ON THE INTERACTION OF A SHOCK-WAVE WITH A CURVED BOUNDARY

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The analytical expression for the pressure field behind an arbitrary plane shock-wave which encounters a curved boundary at nearly glancing incidence has been obtained. The pressure distribution on the surface for different shocks is calculated and compared with that of a wedge.

When a shock wave advancing over a plane surface meets a bend, the phenomenon of diffraction takes place. The non-uniform region behind the diffracted shock is of interest. Bergmann[1] has considered the interaction of normal shock with a small bend. He has linearized the basic equations by assuming that the flow in the non-uniform region is irrotational. Lighthill[2] has investigated the problem of diffraction of normal blasts by a wedge. He has taken into account the rotationality of flow in the non-uniform region which is physically more consistent. Ting & Ludloff[3] have dealt with a blast passing over a surface of an arbitrary flat structure. Their method is more general than that of Lighthill. Smyrl[4] has considered the impact of a shock wave on a supersonic aerofoil utilizing Lighthill's[2] theory. Pack [5] has given a detailed review on problems relating to diffraction and reflection of shock waves. Srivastava[6,7] has extended Lighthill's theory to the case of oblique shock wave. In this work the theory of Ting & Ludloff[3] has been applied to obtain the pressure field behind a normal shock wave of arbitrary strength which has encountered a curved boundary (parabolic shape) at nearly glancing incidence.

Let a plane shock of any strength be advanced (Fig. 1) over the surface of a two-dimensional curved boundary [$y = f(x)$]. The small perturbation theory applied to the non-uniform region enclosed by the rigid boundary, diffracted shock and Mach circle leads to the wave equation[3]:

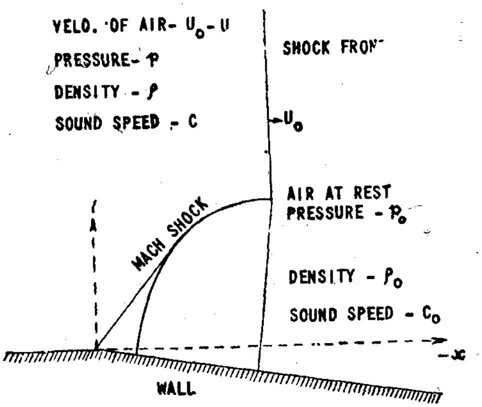

Fig. 1—Simplified model of shock-curved boundary interaction.

$$\overset{(1)}{p}_{xx} + \overset{(1)}{p}_{yy} - \frac{1}{c^2}\, \overset{(1)}{p}_{tt} = 0 \qquad (1)$$

The problem now is to solve equation (1) under the given initial and boundary conditions.

Boundary and Initial Condition in $p^{(1)}$

(*i*) *On the incident shock front*:

$$D_{x,t}\, p^{(1)}\, (x = Ut, y > 0, t) = 0 \qquad (2)$$

where

$$D_{x,t} = (\Omega_1 + M + \Omega_2 M) \frac{\partial^2}{\partial t^2} + (1 + M^2 + 2M\Omega_1)\, c.\, \frac{\partial^2}{\partial x\, \partial t}$$

$$+ (\Omega_1 M^2 + M - M\Omega_2)\, c^2 \frac{\partial^2}{\partial x^2}\, , M = \frac{U}{c}\, ,$$

Ω_1 and Ω_2 are functions of M and γ as defined in an earlier paper[8].

(*ii*) *On the wall*: The shape of the curved boundary is given by

$$y = -\epsilon f(x_1) = -\epsilon \frac{x_1^2}{2} \tag{3}$$

where ϵ is small and $x_1 = x + (U_0 - U)\, t$. Equation (3) represents a parabola of very large latus rectum $(2/\epsilon)$ with vertex at the origin.

Then

$$\left.\begin{aligned} f'(x_1) &= x_1 \\ f''(x_1) &= 1 \end{aligned}\right\} \text{ for } x_1 > 0 \tag{4}$$

and

$$f''(x_1) = 0 \qquad \text{for } x_1 < 0 \tag{5}$$

where the prime means the differentiation with respect to the whole argument. Therefore,

$$v^{(1)}(x < Ut, y = 0, t) = -(U_0 - U) f'(x_1) \tag{6}$$

Using equation of motion in $v^{(1)}$ we get

$$p_y^{(1)}(x < Ut, y = 0, t) = \rho\,(U_0 - U)^2\, \bar{H}[\, x + (U_0 - U)\, t\,] \tag{7}$$

where $\bar{H}$ is the Heaviside unit step function.

(*iii*) *On the mach shock*:

$$p^{(1)} = 0 \quad \text{as} \quad \sqrt{x^2 + y^2} \to \infty \tag{8}$$

(*iv*) *Initial conditions*:

$$p^{(1)}(x \leqslant Ut, y \geqslant 0, t \leqslant 0) = 0 \tag{9}$$

$$p_t^{(1)}(x \leqslant Ut, y \geqslant 0, t \leqslant 0) = 0 \tag{10}$$

ANALYTIC SOLUTION

Using the method given in reference 3 we get the perturbed pressure in the non-uniform region as

$$p^{(1)}(\bar{x}, \bar{y}, \bar{t}) = \frac{\rho c^2 A_0}{\pi} \int_0^{\bar{t} - \bar{y}} d\tau \int_{\bar{x} - [(\bar{t} - \tau)^2 - \bar{y}^2]^{\frac{1}{2}}}^{0} \frac{\bar{H}[\, \bar{a}\,(\tau + \bar{\lambda}_0 \xi)\,]}{K(\tau, \xi)}\, d\xi$$

$$+ \sum_{i=1}^{2,3} \frac{\rho c^2 A_i}{\pi} \int_0^{\bar{t} - [\bar{x}^2 + \bar{y}^2]^{\frac{1}{2}}} d\tau \int_0^{\bar{x} + [(\bar{t} - \tau)^2 - \bar{y}^2]^{\frac{1}{2}}} \frac{\bar{H}[\, \bar{a}(\tau - \bar{\lambda}_i\, \xi)\,]}{K(\tau, \xi)}\, d\xi \tag{11}$$

where $A_0, A_1, A_2, A_3, \bar{\lambda}_0, \bar{\lambda}_1, \bar{\lambda}_2, \bar{\lambda}_3$, and $\bar{a}$ are known functions of U_0, U, c and γ as given in reference 8, $\bar{x}, \bar{y}, \bar{t}$ are Lorentz coordinates and

$$K(\tau, \xi) = [(\bar{t} - \tau)^2 - (\bar{x} - \xi)^2 - \bar{y}^2]^{\frac{1}{2}} .$$

On integration, eqn. (11) gives

$$p^{(1)}(\bar{x}, \bar{y}, \bar{t}) = \frac{\rho c^2}{\pi} \sum_{i=1}^{2,3,4} A_i \Bigg[\bar{x} \cosh^{-1} \frac{\bar{t}}{[\bar{x}^2 + \bar{y}^2]^{\frac{1}{2}}}$$

$$- \frac{\bar{y}}{2} \cos^{-1} \left\{ 1 - \frac{2\bar{x}^2 \bar{t}^2}{(\bar{t}^2 - \bar{y}^2)(\bar{x}^2 + \bar{y}^2)} \right\} - M_i - yN_i \Bigg] \tag{12}$$

where

$$A_4 = -A_0, \quad \bar{\lambda}_4 = -\bar{\lambda}_0$$

$$M_i = \frac{\bar{r}_i}{\sqrt{1 - \bar{\lambda}_i^2}} \cos^{-1} \frac{|\bar{x} - \bar{\lambda}_i \bar{t}|}{[\bar{r}_i^2 + (\bar{\lambda}_i^2 - 1)\ \bar{y}^2]^{\frac{1}{2}}} \quad \text{for } \bar{\lambda}_i < 1$$

$$= \frac{\bar{r}_i}{\sqrt{\bar{\lambda}_i^2 - 1}} \cosh^{-1} \frac{|\bar{x} - \bar{\lambda}_i \bar{t}|}{[\bar{r}_i^2 + (\bar{\lambda}_i^2 - 1)]^{\frac{1}{2}}} \quad \text{for } \bar{\lambda}_i > 1,$$

$$N_i = [\bar{r}_i^2 - \bar{y}^2]^{\frac{1}{2}} \cos^{-1} Q \quad \text{for } \bar{r}_i > \bar{y}$$

$$= [\bar{y}^2 - \bar{r}_i^2] \cosh^{-1} Q \quad \text{for } \bar{y} > \bar{r}_i,$$

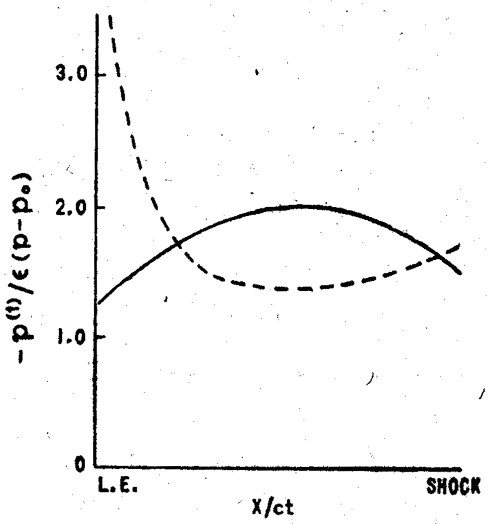

Fig. 2—The pressure distribution on the wall for $p/p_0 = 3$. The solid curve is for the curved boundary and the broken curve is for the wedge.

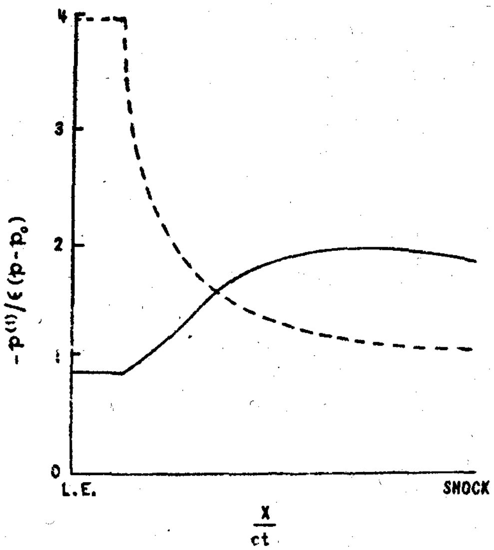

Fig. 3—The pressure distribution on the wall for $p/p_0 = 7 \cdot 3036$, The solid curve is for the curved boundary and the broken curve is for the wedge.

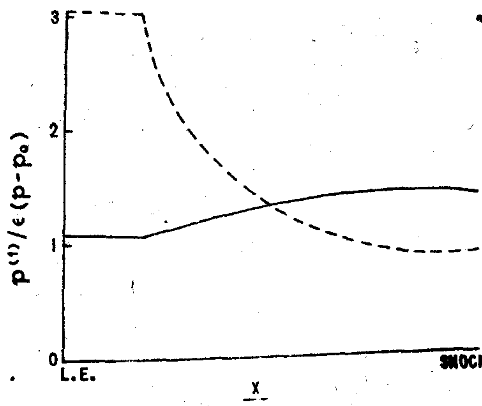

Fig. 4—The pressure distribution on the wall for $p/p_0 = 10$. The solid curve is for the curved boundary and the broken curve s for the wedge.

$$Q = \frac{[(\bar{\lambda}_i^2 + 1)\bar{y}^2 - \bar{r}_i^2](\bar{t}^2 - \bar{r}_i^2) - 2\bar{\lambda}_i^2 \bar{x}^2 \bar{y}^2}{(\bar{t}^2 - \bar{r}_i^2)[(\bar{\lambda}_i^2 - 1)\bar{y}^2 + \bar{r}_i^2]}$$

and

$$\bar{r}_i = \bar{t} - \bar{\lambda}_i \bar{x}.$$

NUMERICAL RESULTS

The pressure distribution on the surface of the boundary has been calculated for different shocks ($p/p_0 = 3, 7 \cdot 3076, 10$) and the results are illustrated by Figs. 2, 3 and 4. The curves showing pressure distribution on wedge for corresponding shocks are given for comparison.

It has been found that the perturbed pressure at the leading edge of the curved boundary is finite when the relative airflow behind the shock is subsonic. This is due to the fact that there is zero slope at the point. This has been confirmed by Prof. L. Ting of New York University in a personal communication.

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REFERENCES

1. BERGMANN, V., N.D.R.C., Report AMP 108-2R, 1945.
2. LIGHTHILL, M. J., *Proc. Roy. Soc.*, **198** A (1948), 454.
3. TING, L. & LUDLOFF, H. F., *J. Aero. Sci.*, **19** (1952), 317.
4. SMYRL, J. L., *J. Fluid Mech.*, **15** (1963), 223.
5. PACK, D. C., *J. Fluid Mech.*, **18** (1964), 549.
6. SRIVASTAVA, R. S., "Diffraction of Oblique Shock Waves". Aeronautical Research Council, U.K.,Current Paper No. 612, 1962.
7. SRIVASTAVA, R. S., "Diffraction of Blast Wave for the Oblique Case". Aeronautical Research Council, U.K. Current Paper No. 1008, 1968.
8. BISWAS, R. K. *Def. Sci. J.*, **15** (1965), 79.