Def Sci J, Vol 35, No 3, July 1985, pp 281–285

Frequency Zooming Techniques for High Resolution Spectrum Analysis

A. V. PRABHU, V. K. AATRE, T. SOUMINI & S. A. KARIPEL

Naval Physical & Oceanographic Laboratory, Cochin-682004

Abstract. Zooming or high resolution spectrum analysis over narrow spectral bands commonly employs Fast Fourier Transform. In this paper, various Zooming techniques are compared from the point of view of hardware implementation and complexity of computation.

1. Introduction

Many of the modern signal processing algorithms employ 'spectrum estimation' for signal detection and classification. In many instances, fine resolution spectral analysis is needed. But very often important information is clustered and thus fine resolution is required only over a selected band.

Techniques similar to zooming in optical lens systems are usually employed for achieving high resolution. These are called Zoom techniques. Here the frequency band of interest is recognised by performing a low resolution FFT (Fast Fourier Transform) on wide band signal and a high resolution FFT is performed on the selected band. Thus a frequency interval is magnified and hence the name 'Zooming'.

2. Spectral Estimation

Estimation of a spectrum is generally achieved through Fast Fourier Transform (FFT) techniques. The frequency resolution in FFT based spectral estimators depends on the data length T. For an M point FFT, the frequency resolution is $fr = \frac{f_s}{M} = \frac{1}{T}$, where f_s = sampling frequency. If the entire basic band is to be analysed with high resolution, the data length is to be increased. This in turn calls for large storage, N point FFT, where $N >> M$ and a display capable of accomodating N points. Zoom techniques are used to avoid these limitations.

The different zooming techniques use various approaches for bringing the signal to baseband before zooming. The following sections discuss and compare the complexities involved in terms of real multiplications and in storage requirements for the different methods.

3. Complex Demodulation

This is similar to the analog double modulation method employed in many commercial equipments. Here incoming sampled signals are hetrodyned to the required frequency f_0 by multiplying the input samples by $\exp\left(\frac{-j2\pi f_0}{f_s}\right)$. These inphase and quadrature samples are passed through a lowpass filter (LPF) of required bandwidth. In this process, the bandwidth about f_0 gets translated to the baseband with f_0 getting translated to D.C. The output of the LPF is decimated by K, where K is the zoom-factor given by $K = \frac{\text{Basic bandwidth}}{\text{Zoom bandwidth}}$. The LPF is to have a fall off of 110 to 120 db/octave beyond its cut off to reduce aliasing. The M points after decimation correspond to a data length of $M \times K \times \Delta t$. The M point FFT of the filtered output will have a resolution $f_r = \frac{1}{MK}$, which is 'K' times that of the initial resolution.

In all the above, it is assumed that the signal is complex at the input and output of the complex demodulator. In case the signal at the input of complex demodulator is real, with bandwidth f_u and sampling frequency f_s, the resolution can be increased by a factor of two, by mixing the real signal with the frequency $f_u/2$ and decimating by 2. The resulting complex signal now represents a bandwidth $f_u/2$.

The process described above requires a total of $2MK \times 2F + 4MK + 2M \log_2 M$ real multiplications where K = Zoom factor and F = Order of the filter.

4. Time Concatenation

Another method used for zooming is called 'Time Concatenation' to indicate the manner in which filtering is achieved to bring the signal to the baseband. The process is best explained by a diagram (Fig. 1).

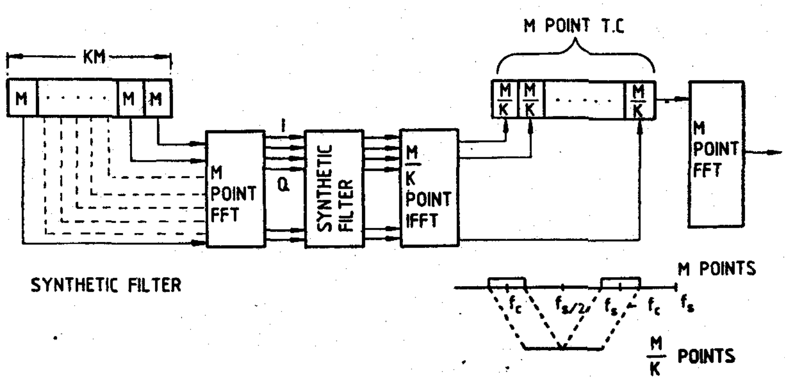

Figure 1. Time concatenation method.

For a zoom factor of K,

(a) K FFTs of M points each are carried out

(b) $\frac{M}{2K}$ points around f_0 and $f_s - f_0$ from each M point FFT are retained.

(c) $\frac{M}{K}$ point inverse FFT of these retained outputs is taken and the K time-domain outputs are concatenated.

(d) Finally, an M point FFT is done on this concatenated output. Zoomband-width and final resolution are given by the relations Zoom bandwidth $= \frac{f_s}{2K}$

and $$f_r = \frac{f_s}{MK}$$

The number of multiplications for this process is given by the expression

$$2(K+2)\, M \log_2 M - 2M \log_2 K.$$

The storage required for this method depends on the way FFT-IFFT is achieved. If a flexible FFT processor is available, the storage requirement is

$$2M + \frac{2M}{K} + 2M = 2M\left(2 + \frac{1}{K}\right) \text{ words}$$

5. Vernier Fast Fourier Transform

The method called Vernier FFT popularly used in constant Q spectral analysis is similar to the time concatenation method. Here a short-duration FFT is performed on the input data to separate it into coarse spectral bins. After collecting successive outputs of each coarse spectral bin, a vernier FFT is done to obtain any desired resolution within that bin. The steps involved in the vernier FFT are as follows (Fig. 2) :

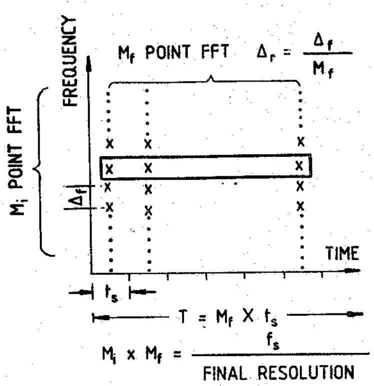

Figure 2. Vernier FFT method.

(a) An initial M_i point FFT is performed on input data and the frequency sample corresponding to the frequency of interest is retained.

(b) This process is repeated M_f times and finally an M_f point FFT is done. Zoom bandwidth and final resolution are given by the relations

$$\text{Zoom bandwidth} = \frac{f_s}{M_i}$$

$$\text{and } f_r = \frac{f_s}{M_i M_f}$$

The total number of real multiplication is given by $2M_f M_i \log_2 M_i + 2M_f \log_2 M_f$. Generally this requires less memory than the concatenation method since both M_i & M_f are shorter in length than M and M/K and intermediate storage is required only for M_f points. Reconfigurable FFT is required for this method also.

6. Discussion

A 7th order recursive lowpass filter was assumed for computing multiplication required for the complex demodulation method. The number of multiplications required for the complex demodulation and Time concatenation methods for different zoom factors (K) and FFT lengths (M) are shown in Fig. 3. It can be seen that except for $M > 2048$

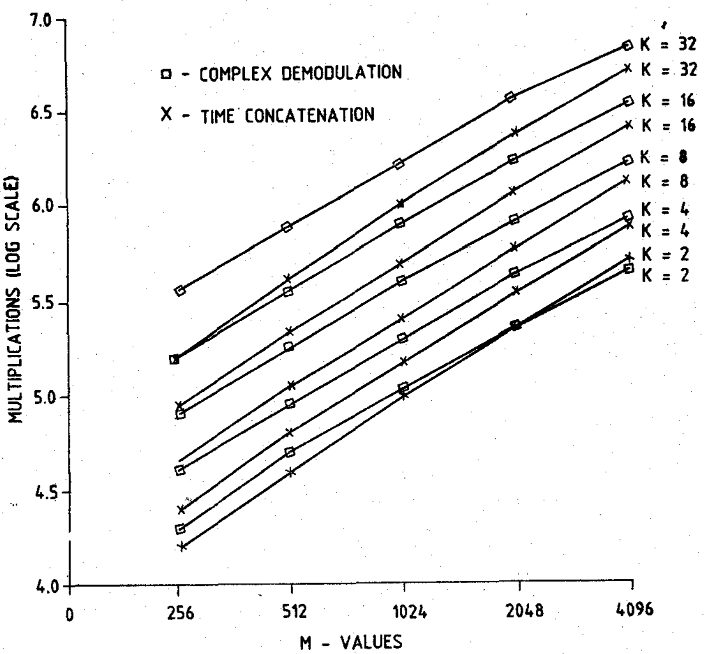

Figure 3. Comparison of zooming techniques for computation complexity.

and $K = 2$, the time concatenation requires less number of multiplications. Table 1 shows the number of multiplications required for the three methods to achieve the same resolution over a given bandwidth and a data lengdth of 1024 points. It is clear that the vernier FFT has much less multiplications than the other two. However, there is no freedom of chosing any desired centre frequency.

Table 1. Comparison of high resolution spectral analysis methods.

M = 1024								
Time Concatenation/ Complex Demodulation			No. of Multiplications		Vernier FFT			
^fR_f (H_z)	BW (KH_z)	K	C.D	T.C	^fR_i (KH_z)	FFT M_i	FFT M_f	No. of Multiplication
4	1.024	2	86016	79872	1.024	8	256	16384
2	0.512	4	151552	118784	0.512	16	256	36864
1	0.256	8	282624	198656	0.256	32	256	86016
0.5	0.128	16	544768	360448	0.128	64	256	200704
0.25	0.064	32	1069056	686079	0.064	128	256	462848

The methods that have been described are mostly 'On Line'. Methods which require initial storge of long data and work based on 'K' FFTs followed by phase corrections and accumulation have not been considered.

7. Conclusion

The discussions above indicate that if one has a reconfigurable FFT processor, Time concatenation method is best suited for signal processing applications requiring minimum time and minimum storage. It also allows a free choice of any centre frequency within the base band.

References

1. Otnes, R. K. & Enochson, L., 'Digital Time Series Analysis'. (John Wiley & Sons, New York), 1972.
2. Bonfield, D. J. *et al.*, (Ed) 'Signal Processing Operations', in 'On Board Signal Processing in Technology & Systems'. Vol. I, Dec 1977, pp. 79–80.
3. Michael, W. F., Newman, H. S. & Nuttall, A. H., An approximate FFT Technique for Constant-Q Spectral Analysis, EASCON-75, 1975, pp. 37A-D.