

Assessment of Rotor Critical Speeds*. - A Note

RAHUL BASU

Gas Turbine Research Establishment, Bangalore-560075

Received 22 January 1986; revised 22 August 1986

Abstract. In this paper, the evaluation of an aircraft engine rotor critical speeds has been studied and the comparison of calculated results with computer and transfer matrix **method** is described.

Symbols

E Modulus of elasticity

I **Moment** of inertia.

1. Introduction

Evaluation of rotor critical speeds is a standardised procedure nowadays and commercial computer programmes are available, usually at considerable cost, (for instance the NASTRAN developed by NASA which includes structural analysis). Other programmes have been developed in the past decade and are readily available in the technical literature¹.

Any amount of theoretical analysis can be done to explain the observation of resonant frequencies and critical speeds in **practise**. A modern aircraft engine is a tremendously delicate as well as **complicated** piece of machinery and hence any analysis of the critical speeds would have to be simplified in order to keep computational time and effort within reasonable **bounds**. One must not lose sight of the fact that the mathematical scheme used may not always give a valid result, due in part to computational convergence or to lack of accurately **modelling** the physical and engineering variables.

* Current address : 204 Harelson Hall NCSU, Raleigh, NC 27695.

There are various degrees of simplification

- (i) The rotor is idealised with a point mass situated suitably on a uniform beam supported by two identical and isotropic bearings. The critical speed of this approximation is then readily calculated from classical elastic beam theory.
- (ii) The number and value of the weights on the beam changes.
- (iii) A change is made in the cross section, that is **different** portions of the beam have different cross section.
- (iv) Additional bearings are placed along the span of the shaft or beam, and flexible couplings could also be added.
- (v) The bearings are made anisotropic, i.e., having different **elastic and damping** coefficients in the x and y directions.
- (vi) Gyroscopic and aerodynamic effects are included.
- (vii) Additional spools (concentric rotors) are added, and interaction with the engine casing and aircraft wing, and other external support structure and superstructure could be included.

2. Addition of Fluid Film Supported Bearings

The use of 'soft mounted' rotors in aircraft engines is very common today. Almost all manufacturers use fluid film bearings ('squeeze film') and claim success. A discussion of the design considerations of such soft mounting of aircraft rotors is given in the **literature**,² and the interested reader is referred to **it**. **However it is to be added** as a matter of interest that rigidly mounted rotors can also be stabilised by appropriate adjustment of rotor material and aerodynamic **parameters**^{3,4}.

Since it is practically very difficult to manufacture a large rotor (shaft) with a defect free **structure** or with perfect tolerance, this possibility is not likely to be encountered in the near future unless new manufacturing processes are developed.

From (ref. 2) it is concluded that it is possible to raise the limiting speed of an aircraft engine using fluid film mounted supports, but that the upper limit is about 2.27 times the first critical speed of the rotor.

Hence the estimation of the first critical speed is of primary interest and concern for any evaluation of the upper limit of speed for an aircraft engine. Other references expounding on this **theme**^{5,6} are where support and fluid film effects are expounded upon.

3. Computer Programmes

As mentioned earlier, there are several computer programmes one could use. A well known programme available in the literature], was adapted for use on a PDP 11

minicomputer. It uses finite sectioning of the rotor and can accommodate upto 200 sections, and includes effects of change of cross-section and incorporation of couplings and flexible bearings.

The theoretical basis of this method is well explained in the reference and need not be gone into here.

Another equivalent method is that of the 'transfer matrix'. It is basically the same as that used in the computer method, except that the equations are expressed in matrix form and one can proceed from section to section and easily keep track of the various elastic equations used.

The method is well described' and a number of examples worked to illustrate the method.

An example given here which is worked out for the transfer matrix method was applied to the computer method above. The results were compared for the first three critical speeds and also for inclusion of gyroscopic effects.

4. Comparison of Calculated Results with Computer and Transfer Matrix Method

The computer programme was applied to a rotor as described in ref (7). This rotor consists of a 1.27 cm. diameter steel shaft with two discs located 8.89 cm. from the left hand end and 7.62 cm. from the right hand end. The mass of the left hand disc is 1.27 kg and a transverse moment of inertia 0.00128 kg M^2 , whereas the mass of the other disc is 0.87 kg., with negligible moment of inertia. There are two bearings placed near each other at the left hand end amounting to what is equivalent to a rigid cantilever type support with corresponding deflection and slope zero over there. A coupling occurs 7.62 cm. from disc 1 having a mass of 0.7 kg., and a third bearing occurs 7.62 cm. from the coupling, with the second disc placed 7.62 cm. away to the right of this (overhung disc).

This rotor is sectioned into five segments for analysis. It is observed that there are four segments of 7.62 cm. each and one of 8.89 cm. We take the thickness of the first disc as 1.27 cm. for lack of any other information and similarly that of the coupling. Since the second disc is observed to be thin, a different thickness of 0.632 cm. was chosen. The input parameters for the programme are as in Table 1.

The gyroscopic effects of disc I only were included by taking I (transverse) = 0.00128 kgM^2 , and I (polar) = 0.00256 kgM^2 , (valid for thin discs). The results obtained from the computer programme were compared to those given in ref (7) and are given in Table 2.

A slight difference is observed in the second critical speed. This appears to be due to the shaft mass and shear deflection included in the computer calculation.

Table 1. Data input to computer programme pertaining to rotor sections

X cm	od. cm	id. cm	E kg/cm ²	density kg sec ² /cm ⁴	extra weight kg sec ² /cm ²
8.89	1.27	0	2.1 x 10 ⁶	7.96 x 10 ⁶	0.00102
7.62	”	0	”	”	0.00056
7.62	”	0	”	”	0.0
7.62	”	0	”	”	0.0
7.62	”	0	”	”	0.00129

Table 2. Comparison of computer output with ref (7)

Critical speed r.p.m	Computer result		Reference (7)	
	without gyro	gyro	without gyro	two
1	8948	8964	8546	8851
2	20141	20145	22427	22427
3	26282	26288	26346	26430

Another run was made to include gyroscopic effects due to the second disc. The thickness of this disc was changed to 1.27 cm. The entry in Table 1 under EW in row 5 then changes to 0.00070. The results are given in Table 3.

Table 3. Comparison of computer output with ref (7)-disc 2 thickness 1.27 cm.

Critical speed r.p.m.	Computer result		Reference (7)	
	without gyro	gyro	without gyro	gyro
1	9211	9246	8546	8851
2	24222	24222	22427	22427
3	28201	28216	26346	26430

References

1. Trivisonno, R., NASA TND 7385, (1973).
2. Magge, N. *J. Aircraft*, 12, (1975), 3318.
3. Rasu, R. *Tribology* 17, 6, (1984), 348.
4. Tondl, A., 'Some Problems of Rotor Dynamics', (Chapman and Hall, London), 1965, p. 21.
5. Gunter, E.J., NASA SP-113, Wash. D.C., (1966).
6. Kirk, R. G. & Gunter, E.J., NASA CR-2083, (1972).
7. Rao, J.S., 'Rotor Dynamics'. (J. Wiley (Eastern), N. Delhi), 1984.