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Measures of Performance for a Flexible Manufacturing System

VINOD KUMAK
School of Business Carleton University Ottawa, Ontario

J. N. KAPUR
Indian Institute of Technology, Kanpur-208016

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Abstract. A number of measures of performance of flexible manufacturing system, represented by a closed queueing network model are given and the relationships among them have been developed.

1. Introduction

Flexible manufacturing systems (FMS) are of great importance to Defence manufao turing. With the exception of some ammunition products, defence manufacturing in general has two important characteristics viz. relatively low production and high variety. and high technology products with high rate of technical changes. FMS (sometimes known as Integrated Manufacturing Systems and Computerized Manufacturing Systems) provides answers to these two problems due to their inherent flexibility of quick change over to new type of part or of adopting the product design changes by simply some adjustments in the computer software which controls the operation of programmable machine tools and material handling devices. These systems have been proved quite efficient in simultaneously manufacturing many different type of parts in small batches. However these systems are capital intensive and thus their behaviour should be studied thoroughly to fully exploit their capabilities.

These flexible manufacturing systems are modelled mathematically by closed network of queues (CNQ). The development of the theory of CNQ models over a period of twenty-five years has been reviewed by Koenigesberg[1] who has also referred to the application of these models to a wide variety of fields such as communication detworks and tele-traffic, computer time-sharing and multi-programming systems, transportation networks etc. All these applications are of interest to defence organisations. The theory of these models has been developed by Jackson[2], Schweitzer[3], Gordon and Newell[4] and others. An interesting application of these models to study aircraft availability and spares management has been recently given by Mani and Sarma[5].

The CNQ models for FMS have been studied by Solberg[6,7], Stecke[8,9], Stecke and Morin[10], Stecke and Solberg[11], Kapur and Kumar", Kumar and Kapur[13], Kapur, Hawaleshka and Kumar[14,15], Suri[16,17] and others.

A number of measures of performance for flexible manufacturing system have been proposed which are :

(i) Expected values of proportions of busy machines in each machine-group and in the system as a whole

(ii) Probability of a machine chosen at random from a group (on the system) being found busy

(iii) The ratio of the expected value of the actual production rate to the ideal production rate

(iv) The probability of all the machines being found busy

(v) The variance of the proportion of busy machines in each group and in the system as a whole

(vi) Mean queue lengths at various machine-groups

(vii) 'Entropy of the probability distribution of the number of parts in each machine group.

Expressions for some of these measures are already available and expressions for **some** others are given here. Further the relations between the various measures are discussed here.

2. Expected Proportion of Busy Machines

If y_i is the number of busy machines in the ith machine-group which has s_i machines, it has been shown (Solberg[7] Kapur[12,14] er *al* that

$$E\,(Y_i) = x_i \, \frac{G\,(M,\ N-1)}{G\ \ (M,N)}\,, \tag{1}$$

where x_i is the scaled work-load on the ith machine-group given by

$$x_i = \frac{q_i\ t_i}{\sum\limits_{i=1}^{M} q_i\ t_i} \sum_{i=1}^{M} s_i = \frac{q_i\ t_i}{\sum\limits_{i=1}^{M} q_i\ t_i}\, m \tag{2}$$

Here M is the total number of machine-groups, m is the total number of machines, t_i is the average time for an operation in the ith machine-group and q_i is the average number of times a part visits the ith machine-group. Also

$$G(M, N) = \sum_{S(M,N)} g_1(n_1)\, g_2(n)_2 \ldots g_M(n_M), \tag{3}$$

where n_i is the number of parts being processed or waiting to be processed in the ith machine-group so that $n_1 + n_2 + \ldots + n_M = N,$ the total number of parts in the system. Also

$$(g_i\, n_i) = \frac{x_i^{n_i}}{n_i!} \quad \text{if } n_i \leqslant s_i$$

$$\frac{\quad}{s_i!\; s_i} \quad \text{if } n_i > s_i \tag{4}$$

and S (M, N) is the set of all non-negative integers $n_1, n_2, \ldots n_M$ *whose* sum is N. Now from Eqn. (1)

$$E\left(\frac{Y_i}{s_i}\right) = \frac{x_i}{s_i}\,\frac{G(M, N-1)}{G(M, N)}, \tag{5}$$

This gives the expected proportion of busy machines in the ith machine-group. The expected proportion of busy machines in the whole system is given by

$$E\left(\frac{Y_1 + Y_2 + \ldots\; Y_M}{m}\right) = \frac{x_1 + x_2 + \ldots + x_M}{m}\,\frac{G(M, N-1)}{G(M, N)}$$

$$= \frac{G(M, N-1)}{G(M, N)} \tag{6}$$

Thus the expected proportion of busy machines in the first, second, . . . , Mth machine-group are in the ratio

$$\frac{x_1}{s_1} : \frac{x_1}{s_2} : \ldots : \frac{x_M}{s_M} \tag{7}$$

All these proportions will be equal if and only if

$$\frac{x_1}{s_1} = \frac{x_2}{s_2} = \ldots = \frac{x_M}{s_M} = 1 \tag{8}$$

i. e. if the system is balanced.

3. The Probability of a Machine Chosen at Random being Found Busy

(a) Let p_i be the probability of a machine chosen at random from the ith machine-group being found busy. Now we define random variables

$$x_{ij} = 1 \text{ if the jth machine of the ith group is busy} = 0 \text{ otherwise} \tag{9}$$

so that

$$E(x_{ij}) = 1 \,.\, p_i + 0.(1 - p_i) = p_i \tag{10}$$

$$E(x_{i1} + x_{i2} + \ldots + x_{is_i}) = p_i s_i \tag{11}$$

but $x_{i1} + x_{i2} + \ldots + x_{is_i}$ gives the number of busy machines in the ith machine-group, so that this **sum is** equal to y_i. **From** Eqn. (1) and Eqn. (11)

$$E(y_i) = p_i s_i = x_i \frac{G(M, N-1)}{G(M, N)}, \tag{12}$$

so that

$$p_i = \frac{x_i}{s_i} \frac{G(M, N-1)}{G(M, N)} = E\left(\frac{y_i}{s_i}\right) \tag{13}$$

(b) Let p be the probability of a machine chosen at random from the whole system being found **busy** so that

$p = \sum_{i=1}^{M}$ [(Probability that the machine belongs to the ith group) x (probability that a machine chosen at random from ith group **isbusy)]**

$$= \sum_{i=1}^{M} \frac{s_i}{m} \frac{x_i}{s_i} \frac{G(M, N-1)}{G(M, N)} = \frac{G(M, N-1)}{G(M, N)} \tag{14}$$

(c) Alternatively let random variables $z_1, z_2, \ldots z_m$ be defined by

$$z_j = 1, \text{ if jth machine is busy}, j = 1, 2, \ldots m \quad = 0 \text{ otherwise} \tag{15}$$

then

$$E(z_j) = 1.p + 0(1-p) = p$$

and

$$E(z_1 + z_2 + \ldots + z_m) = pm \tag{16}$$

but $z_1 + z_2 + \ldots + z_m$ is the number of **busy** machines, so that

$$p = E \text{ (proportion of busy machines)} = \frac{G(M, N-1)}{G(M, N)} \qquad (17)$$

4. An Inequality for Expected Proportion of Busy Machines

From Eqn. (I 3)

$$\frac{G(M, N-1)}{G(M, N)} \leqslant \frac{s_i}{x_i}, \; i = 1, 2, \ldots M \qquad (18)$$

or

$$\frac{G(M, N-1)}{G(M, N)} \leqslant \min \left(\frac{s_1}{x_1}, \frac{s_2}{x_2}, \ldots, \frac{s_M}{x_M} \right) \qquad (19)$$

However from Eqn. (2) $s_1 + s_2 + \ldots + s_M = x_1 + x_2 + \ldots + x_M = m,$ (20)

so that $s_1/x_1, s_2/x_2, \ldots, s_M/x_M$ caanot all be greater than unity and if one of these is greater than unity, than another must he less than unity and in this case, the expected proportion of **busy** machine (EPBM) would be less than unity. The best situation arises when the system is balanced.

5. Equality of Probability of all States

(a) Let $\vec{n} = (n_1, n_2, \ldots n_M)$ (21)

represent a state of the system. Since $n_1 \geqslant 0, n_2 \geqslant 0, \ldots, n_M \geqslant 0, n_1 + n_2 + \ldots + n_M = N$, the total number of states is

$$\binom{N+M-1}{M-1} \text{ and EPBM} = \Sigma\, p(\vec{n})\, u(\vec{n}) \qquad (22)$$

where $p(\vec{n})$ is the probability of the state $\vec{n}$ and $u(\vec{n})$ is the proportion of busy machines in the state; and the summation is over the $\binom{N+M-1}{M-1}$ states.

The case when all the probabilities are equal is of special interest.

(b) ***For single-machine machine-groups***

$$p(\vec{n}) = p(n_1, n_2, \ldots n_M) = \frac{x_1^{n_1} x_2^{n_2} \ldots x_M^{n_M}}{\sum_{S(M,N)} x_1^{n_1} x_2^{n_2} \ldots x_M^{n_M}} \qquad (23)$$

If $x_1 = x_2 = \ldots = x_M = 1$, then each of the probabilities is the same and each is equal to $\binom{N+M-1}{M-1}^{-1}$. Conversely if all the probabilities are known to be equal, we get

$$\frac{x_1^{n_1} x_2^{n_2} \ldots x_M^{n_M}}{\sum_{S(M,N)} x_1^{n_1} x_2^{n_2} \ldots x_M^{n_M}} = \frac{1}{\binom{N+M-1}{M-1}} \tag{24}$$

We have then $\binom{N+M-1}{M-1}$ equations to determine M unknowns $x_1, x_2, \ldots x_M$. We have also the equation

$$x_2 + x_2 + \ldots + x_M = M \tag{25}$$

These equations may or may not be consistent. However let us use M of these for the states $(N, 0, 0, \ldots 0), (0, N, 0, \ldots 0), \ldots, (0, 0, \ldots N)$ to get

$$x_1^N = x_2^N = \ldots = x_M^N = \frac{G(M, N)}{\binom{N+M-1}{M-1}} \tag{26}$$

This gives

$$x_1 = x_2 = \ldots = x_M \tag{27}$$

Equations (25) and (27) together give

$$x_1 = x_2 = \ldots = x_M = 1 \tag{28}$$

We find that these values satisfy all the equations (24). Thus Eqns. (24) and (25) give a consistent system of equations with solution Eqn. (28).

For single-machine machine-groups, if the system is balanced all the state probabilities are equal and conversely if all the state probabilities are equal, the system is balanced.

It may be noted here that Stecke's[7] argument that Eqn. (26) gives

$$x_i^N = \text{constant} \tag{29}$$

which has only one real root and all its other roots are infeasible (negative or complex) and so

$$x_i = \left[G(M, N) \Big/ \binom{N+M-1}{M-1}\right]^{1/N} = C \tag{30}$$

independent of i and $x_1 = x_2 = \ldots = x_M$, is incorrect, since G (M, N) is not constant and is in fact a function of $x_1, x_2, \ldots, x_M$. However it is a symmetric function of $x_1, x_2, \ldots, x_M$ and as such the result obtained is correct.

(c) For multiple-machine machine-groups

Here $p(n_1, n_2, \ldots, n_M) = g_1(n_1), g_2(n_2), \ldots g_M(n_M)/G(M,N)$, (31) where $g_i(n_i)$ and G (M,N) are given by Eqns. (3) and (4). Now for a balanced system.

$$g_i(n_i) = s_i^{n_i}/n_i! \text{ for } n_i = 0, 1, 2, \ldots, s_i - 1$$

$$= s_i^{s_i}/s_i! \text{ for } n_i = s_i, s_i + 1, \ldots, N \tag{32}$$

It is obvious that $g_1(n_1), g_2(n_2), \ldots, g_M(n_M)$ will not be same for all possible states. Thus unlike the case of single-machine machine-groups, the probabilities of all states will not be equal for the balanced system. However

$$g_1(n_1)\, g_2(n_2) \ldots g_M(n_M) = \frac{s_1^{s_1}}{s_1!} \frac{s_2^{s_2}}{s_2!} \cdots \frac{s_N^{s_M}}{s_M!} \tag{33}$$

for all those states for which

$$n_1 > s_1 - 1,\ n_2 > s_2 - 1, \ldots\ n_M > s_M - 1$$

$$n_1 + n_2 + \ldots + n_M = N \tag{34}$$

Thus the probabilities of all those states in which either all the machines are busy or at most one machine in a group is not busy, are equal.

The number of such probabilities is the number of non-negative integer solutions of the equation

$$z_1 + z_2 + \ldots + z_M = (n_1 - s_1 + 1) + (n_2 - s_2 - 1) + \ldots$$
$$+ (n_M - s_M + 1) = N - m + M \tag{35}$$

The number of solutions is given by $\binom{N-m+2M-1}{M-1}$, assuming that $N - m + M > 0$.

This number of equations given by (33) together with $x_1 + x_2 + \ldots + x_M = m$ gives in general more than M equations to determine $x_1, x_2, \ldots, x_M$.

In particular, the probabilities of the following states are equal

$$s_1 - 1,\ s_2 - 1, \ldots\ s_{M-1} - 1,\ N - (m - s_M) + (M - 1)$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \tag{36}$$
$$N - m + M + s_1 - 1,\quad s_2 - 1,\ S_M - 1$$

The (M − 1) equations expressing the equality of the probabilities of these states and $x_1 + x_2 + \ldots + x_M = m$ are satisfied by

$$\frac{x_1}{s_1} = \frac{x_2}{s_2} = \ldots = \frac{x_M}{s_M} = 1 \tag{37}$$

These values also make all the other probabilities equal. Thus for multiple-machine machine-groups, for a balanced system, the probabilities of all these states in which all machines are busy or at most one machine per group is not busy, will be equal. Conversely if all these probabilities are equal, the system will be balanced.

(d) For single-machine machine-groups Eqn. (22) gives, when

$$x_1 = x_2 = \ldots = x_M = 1$$

$$\sum_{S(M,N)} u(\vec{n}) = \binom{N+M-1}{M-1} \frac{G(M, N-1)}{G(M, N)} = \binom{N+M-1}{M-1} \frac{\binom{N+M-2}{M-1}}{\binom{N+M-1}{M-1}}$$

$$= \binom{N+M-2}{M-1} \tag{38}$$

Thus

$$\frac{G(M, N-1)}{G(M, N)} = \frac{\sum_{S(M,N)} u(\vec{n})}{\binom{N+M-1}{M-1}} \tag{39}$$

Thus in this case, the expected production function can be interpreted as the average proportionate utilization of machines per state.

6. Actual and Ideal Production Rates

$\sum_{i=1}^{M} q_i t_i / \sum_{i=1}^{M} s_i$ gives the average work load per machine in machine-time per part when all the machines are busy. Its reciprocal which gives the production in parts per unit time is called the ideal production rate so that

$$Pr(I) = \sum_{i=1}^{M} s_i / \sum_{i=1}^{M} q_i t_i \tag{40}$$

In any state $\vec{n}$, the actual production rate is obtained by multiplying the ideal production rate by the proportionate utilization of machines in that state so that

$$Pr(A)\,[\vec{n}]\;\; Pr\,(I)\, u\,(\vec{n}) \tag{41}$$

The actual production rate *Pr* (A) is obtained by averaging the actual production rate over all the states, so that

$$\begin{aligned} Pr\,(A) &= \Sigma\, p\,(\vec{n})\; pr\,(A)\;[\vec{n}] \\ &= \Sigma\, p\,(\vec{n})\, u\,(\vec{n})\, pr\,(I) \end{aligned} \tag{42}$$

or

$$\frac{Pr\,(A)}{Pr\,(I)} = \Sigma\; p\;(\vec{n})\, u(\vec{n}) = EPF$$

so that *EPF* is the ratio of the actual production rate to the ideal production rate when all the machines are busy. Thus *Pr* (*A*)/*Pr* (I) does not provide a really distinct measure of performance of an *FMS.*

7. Probability of all Machines being Busy

For single machine machine-groups

P [all machines being busy]

$$\frac{\displaystyle\sum_{\substack{n_1 \geqslant 1,\; n_2 \geqslant 1,\; \ldots\; n_M \geqslant 1 \\ n_1 + n_2 + \ldots + n_M = N}} x_1^{n_1} x_2^{n_2} \ldots x_M^{n_M}}{\displaystyle\sum_{\substack{n_1 \geqslant 0,\; n_2 \geqslant 0,\; \ldots\; n_M \geqslant 0 \\ n_1 + n_2 + \ldots + n_M = N}} x_1^{n_1} x_2^{n_2} \ldots x_M^{n_M}} = x_1 x_2 \ldots x_M \frac{G\,(M,\, N-M)}{G\,(M,\, N)}$$

$$= x_1 x_2 \ldots x_M \;\frac{G\,(M,\, N-M,\, X)}{G\,(M,\, N-M+1,\, X)}\;\frac{G\,(M,\, N-M+1,\, X)}{G\,(M,\, N-M+2,\, X)} \;\cdots\; \frac{(G,\, N-1,\, X)}{G\,(M,\, N,\, X)}$$

$$= x_1 x_2 \ldots x_M\; Pr\,(M,\, N-M+1,\, X)\;\; Pr\,(M,\, N-M+2,\, X) \ldots \ldots Pr\,(M,\, N,\, X) \tag{43}$$

Here G (M, N, X) is same as G (M, N).

This probability depends on the expected production function *Pr* (M, *N*) but is not identical with it. In fact since each production function $\leqslant 1$ and $x_1 x_2 \ldots x_M \leqslant 1$, the probability of all machines being busy $\leqslant$ expected production function.

Since each production function is maximum when $x_1 = x_2 = \ldots = x_M = 1$ and $x_1 x_2 \ldots x_M$ is also maximum in this case, the probability of all machines being busy is maximum when the expected production is maximum i. e. for the balanced system. Also

$$[P \text{ (all machines being busy)}]_{max} = 1 \cdot \frac{\binom{N-M+M-1}{M-1}}{\binom{N+M-1}{M-1}} = \frac{(N-1) \ldots (N-M+1)}{(N+M-1) \ldots (N+1)} \tag{44}$$

and this approaches unity as $N \to \infty$ and decreases as M increases. *P* (all machines being busy)

$$= \frac{x_1^{s_1}}{s_1!} \frac{x_2^{s_2}}{s_2!} \cdots \frac{x_M^{s_M}}{s_M!} \frac{\sum\limits_{S(M,N\ M)} \left(\frac{x_1}{s_1}\right)^{n_1} \left(\frac{x_2}{s_2}\right)^{n_2} \cdots \left(\frac{x_M}{s_M}\right)^{n_M}}{G(M, N, X)} \tag{45}$$

If $s_1 = s_2 = \ldots = s_M = s$, then this probability

$$= \frac{(x_1 x_2 \ldots x_M)^s}{(s!)^M} \frac{1}{(s)^{N-M}} \frac{\sum\limits_{S(M,N-M)} x_1^{n_1} x_2^{n_2} \ldots x_M^{n_M}}{G(M, N, X)} \tag{46}$$

This is a symmetric function of $x_1, x_2, \ldots x_M$ and is maximum when $x_1 = x_2 = \ldots = x_M = s$. When $s_1, s_2, \ldots s_M$ are not all equal, the maximum of the probabilities in Eqn. (45) will occur for an unbalanced system. The load distribution for maximizing this probability will in general have to be found numerically.

8. Variance as a Criterion of System Performance

Since the production function (the proportion of busy machines) is a random variable, we are interested in choosing $x_1, x_2, \ldots x_M$ so as to maximize the expected value of this random variable. We would also like to choose $x_1, x_2, \ldots x_M$ so as to minimize the variance or the standard deviation.

For every X, we can find

$$E = EPF = G(M, N-1, X)/G(M, N, X) \tag{47}$$

$$V = \text{Variance} = \frac{G(M,N-1,X)}{G(M,N,X)}\left[\frac{G(M,N-2,X)}{G(M,N-1,X)}\right.$$

$$- \frac{G(M,N-1,X)}{G(M,N,X)} + \frac{1}{m}$$

$$\left. - \frac{1}{m^2}\sum_{i=1}^{M} x_i \sum_{k=s}^{N-1} g_i(k)\, \frac{G(M-1,N-1-k,X)}{G(M,N-1,X)}\right] \quad (48)$$

and find a point (E, V) in the positive quadrant of the E-V plane (Fig. 1) corresponding to all X, we may get a set of points bounded by a closed curve. Out of all these points the set of points on the shaded arc AB form a preferred set in the sense that if p is any point inside the feasible region, the points on the arc QR give better solutions than P since these give greater expected production function and smaller

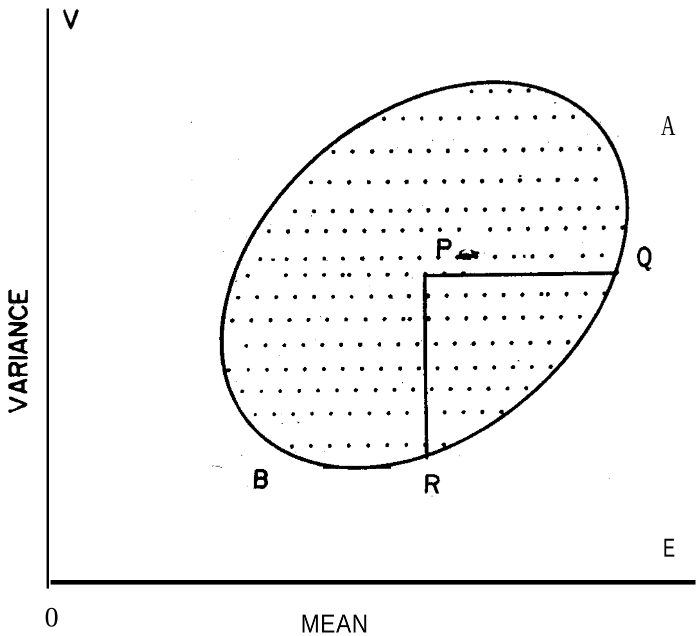

Figure 1.

variance than the solution corresponding to P. In the language of Markowitz, the arc AB gives a mean-variance efficient function. The manufacturer has to choose a solution between A and B depending on bis attitude to risk.

If he is prepared to take risks, he can choose the solution corresponding to B i. e. he can maximize the expected production function and need not worry about the variance. If he is cautious and is completely averse to risk-taking, he can choose the solution corresponding to A by minimizing the variance without worrying about the value of the expected production function. In general, he will choose a point between A and B on the arc $A \quad B$. ,

The object can be achieved by choosing $x_1, x_2, \ldots, x_M$ so as to maximize $E\text{-}\lambda V$ for a fixed λ. As λ varies from 0 to ∞, we get solutions corresponding to points between B and A. The choice of λ is the privilege of the manufacturer.

Alternatively, we may choose out of all these pareto-optimal solutions that solution for which the entropy- $\sum_{i=1} x_i$ in x_i is maximum. In this way we can get EPF-variance, EPF — ME, ME-variance and EPF-variance-ME frontiers.

9. Mean Queue Length Criterion

For each given $(x_1, x_2, \ldots . x_M)$, we can find the expected queue length for the ith machine-group by using

$$E(n_i) = \bar{n}_i = \frac{\sum\limits_{S(M,N)} n_i\, g_1(n_1)\, g_2(n_2) \ldots g_M(n_M)}{\sum\limits_{S(M,N)} g_1(n_1)\, g_2(n_2) \ldots g_M(n_M)} \tag{49}$$

Our criterion requires us to choose $x_1, x_2, \ldots x_M$ in such a way that $E(n_1), E(n_2), \ldots,$ $E(n_M)$ are as nearly proportional to $s_1, s_2, \ldots, s_M$ as possible i. e. the proportions $E(n_1)/N$, $E(n_2)/N, \ldots,$ $E(n_M)/N$ should be as nearly equal as possible to the proportions $s_1/m, s_2/m, \ldots . s_M/m$. In fact equating these proportions, we get

$$\frac{\sum\limits_{S(M,N)} \frac{n_i}{N}\, g_1(n_1)\, g_2(n_2) \ldots g_M(n_M)}{\sum\limits_{S(M,N)} g_1(n_1)\, g_2(n_2) \ldots g_M(n_M)} = \frac{s_i}{m} \quad i = 1, 2, \ldots, M \tag{50}$$

so that we get M equations to determine $x_1, x_2, \ldots . x_M$. However only $M-1$ of these equations are independent, but we have the additional equation $x_1 + x_2 + \ldots + x_M = m$, so that we can determine $x_1, x_2, , x_M$. If $s_1 = s_2 = \ldots s_M = s_M = s$, then $x_1 = x_2 = \ldots = x_M = s$, obviously gives a solution i. e. the mean queue lengths for the balanced system are equal. However when $s_1, s_2, \ldots, s_M$ are not equal, we will have to solve Eqn. (50) for $x_1, x_2, \ldots, x_M$ numerically

10. Maximum Entropy Criterion

Using Eqns (23) and (31), we can find $p(\vec{n}) = p(n_1, n_2, \ldots . n_M)$ and then we can find

$$H = -\Sigma p(\vec{n}) \ln p(\vec{n}) = -\sum_{S(M,N)} p(n_1, n_2, \ldots n_M) \ln p(n_1, n_2, \ldots n_M), \tag{51}$$

the entropy of this probability distribution. Our maximum entropy criteria'* requires us to maximize H subject to

$$x_1 + x_2 + \ldots + x_M = m, \quad x_i \geqslant 0 \text{ for each } i. \tag{52}$$

For $s_1 = s_2 = \ldots = s_M = s$, H is a symmetric function of $x_1, x_2, \ldots x_M$ and is maximum when $x_1 = x_2 = \ldots = x_M$. When $s_1, s_2, \ldots, s_M$ are not all equal, H will not be a symmetric function of $x_1, x_2, \ldots, x_M$, but we can find analytically or numerically the values of $x_1, x_2, \ldots, x_M$ for which H will be maximum.

11. Conclusions

We have seen that maximization of expected production function i. e. the expected proportion of busy machines or the probabilities of a machine chosen at random being found busy or the ratio of the expected value of the actual production rate to the ideal production rate gives equivalent criteria. However maximizing the probabilities of all the machines being found busy or equating the ratios of expected queue lengths to the number of machines in each group or maximizing the entropy of the probability distribution of the number of parts in different machine-groups, give different criteria.

The implications of the first set of criteria have been exhaustively worked out by Solberg[7] and Stecke[8]. The corresponding implications of the other criteria are being worked out by us.

However when $s_1 = s_2 = s_M = s$, all criteria give the same optimal load distribution viz. $x_1 = x_2 = \ldots = x_M = s$. The differences arise when the number of machines in different groups are not all equal. In this case also, the numerical and analytical work done so far shows that the deviations from the balanced system are in the same direction by all the criteria. The magnitude of the deviations are also nearly equal, though they are not identical.

A third category of criteria of system performance include the following:

(i) minimizing the variance of the proportion of busy machines in the system

(ii) minimizing the semi-variance of this proportion

(iii) minimizing the variance of the mean queue lengths i. e.

$$\sum_{i=1}^{M} \left(\bar{n}_i - \frac{N}{M} \right) \tag{35}$$

Problems on multidecisions arise when two or more criteria are desired to be used simultaneously e. g. when we may like to maximize EPF and entropy and minimize variance.

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