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A Typical Magnetohydrodynamic Flow of a Viscous Incompressible Fluid Between a Parallel Flat Wall and a Wavy Wall : Constant Suction Through the Former Wall

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Abstract. Amagnetohydrodynamic flow of a viscous, incompressible and slightly conducting fluid is developed between a parallel flat wall and a wavy wall whereas at the same time fluid is continuously sucked through the flat wall with a constant suction velocity. The velocity and temperature distribution are determined along-with the pressure gradient and co-efficient of skin friction.

1. Introduction

Bhaskara Reddy[1] & Bathaiah' investigated magnetohydrodynamic flow of a viscous, incompressible and conducting heated fluid between a parallel flat wall and a wavy wall where the fluid is also sucked through the former wall with a uniform suction velocity normal to its surface. The present author tackles herein the same problem totally discarding authors" assumption that the flat wall is to be infinitely long and that a constant temperature can be maintained on the wavy surface, and thereby modifies the boundary conditions so as to evince physical significance of such a typical two dimensional flow.

The equations of momentum, continuity and energy as obtained by Reddy[1] and his co-author are also used here but with different boundary' conditions.

In course of the solutions to the afore-said differential Eqns. (1) to (4) in paper[1], authors' obtained the x-component of the fluid velocity at any point (x, η) :

$$u = \frac{C}{M}\left[\frac{\sinh \{a (\epsilon \cos \lambda x - \eta)\} + \sinh a\eta}{\sinh (a \epsilon \cos \lambda x)} - 1\right]$$

which is a function of both x and η (non-dimensional co-ordinates) contrary to their own assumption $\frac{\partial u}{\partial X} = 0$. Moreover, they have used the same symbol K for both 'thermal conductivity' and 'frequency parameter' of the wall. Similarly, assuming the temperature gradient along the X-axis to be negligibly small i.e., $\left(\frac{\partial T}{\partial X} = 0\right)$, the evaluation of the temperature distribution by them as a function of x and η is again paradoxal. Besides, they have not made any attempt to find the value of the constant pressure gradient C appearing in the expressions for velocity, temperature distributions and skin friction. Hence, without knowing the value of C which depends on the boundary conditions as well as the values of suction, magnetic and frequency parameters, the effects of the latter parameters on the velocity and temperature distributions cannot be ascertained. However, their[1] assumption of the same symbol for the 'pressure' and 'Prandtl no' does not affect the analysis'.

2. Modified Boundary Conditions

In view of the foregoing situations discussed, the boundary conditions' have been amended in the present paper :-

(1) The X-axis is taken parallel to the flat wall, and intersects the wavy wall at the origin; (the Y-axis is perpendicular to the X-axis).

(2) The equations of the flat and wavy walls are, respectively;

$$Y = d^* \tag{1}$$

$$Y = \epsilon^* \sin K^* X \tag{2}$$

(3) Because of Eqns. (1) and (2), the width of the system of the flat wall and the wavy wall at the inlet is d^*. The length of the flat wall = that of the system = L, which is much less than one-fourth of the wavelength of the latter wall, as shown in the Fig. 1. The breadth of the flat wall = b^*

(4) A salient feature of the flow is that there will be no flow across the wavy wall, i.e., there exists a velocity of flow tangent to the wavy surface consistent with the condition (3).

(5) Two-dimensional flow is so developed that it does not evolve velocity and temperature gradients along the X-axis, or in other words they are too small to be accounted for so that

$$\frac{\partial u}{\partial x} = \frac{\partial T}{\partial x} = 0. \tag{3}$$

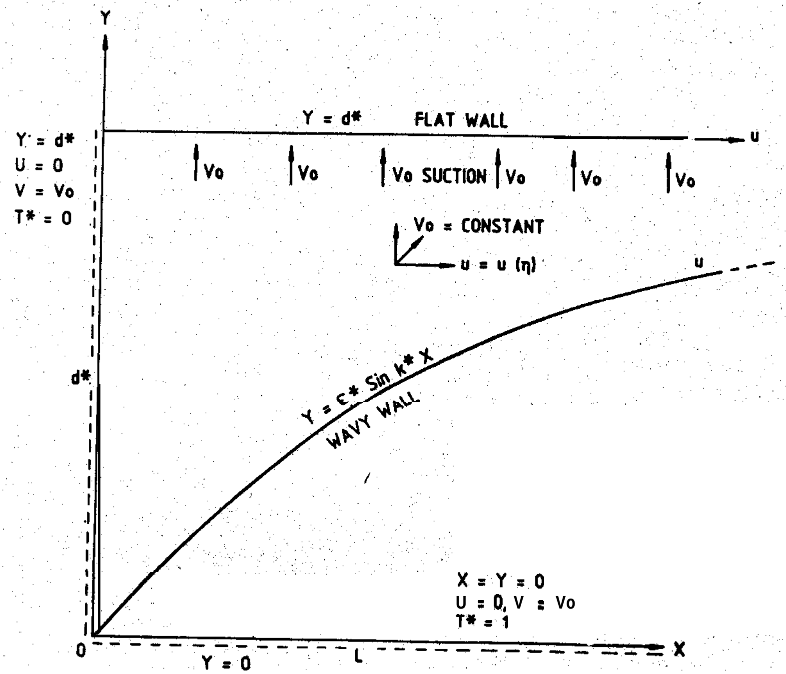

Figure 1. Flow configuration.

(6) To make up the deficiencies of paper' the undermentioned boundary conditions are incorporated in the present quest :

when

$$X = Y = 0;\ U = 0,\ V = V_0 \text{ and } T = T_1$$

and when

$$\left.\begin{array}{l} X = 0 \text{ and } Y = d^*;\ U = 0,\ V = V_0 \text{ and } T = T_0 \\ V_0 = \text{constant suction velocity through the flat wall} \end{array}\right\} \quad (4)$$

For a velocity of flow tangent to the wavy surface (2) and for the two-dimensional flow,

$$\frac{dY}{dX} = \epsilon^* K^* \cos K^* X = \frac{V_0}{U} \quad (5)$$

Hence the X-component of the velocity at any point (X, Y) on the wavy surface is

$$U = \frac{V_0}{\epsilon^* K^* \cos K^* X} \quad \text{(5a)}$$

Now since $\frac{dY}{dX}$ at (0, 0) = ∞, the Y-axis is to be chosen as the tangent to the wavy surface at the origin situated at the inlet of the system, and that too is quite consistent with conditions (4) and (5).

3. Solution to the Problem

Prior to an attempt for solutions to the foregoing differential equations by the use of the boundary conditions (1) to (5), the non-dimensional quantities and other magnetohydrodynamic parameters as employed in paper[1] are here-under re-introduced :

$$u = \frac{U}{V_0}, \eta = \frac{Y}{h}, P = \frac{P^*h}{\mu_0 V_0}, v = \frac{V}{V_0}$$

$$x = \frac{X}{h}, T^* = \frac{T - T_0}{T_1 - T_0}, d = \frac{d^*}{h}, l = \frac{L}{h}, b = \frac{b^*}{h} \tag{6}$$

where $M = \sigma\mu_e^2 H_0^2 h^2$ (Magnetic parameter)

$P_r = \mu C_P/K$ (Prandtl number)

$E = V_0^2 /C_P (T_1 - T_0)$ (Eckert number)

$\epsilon = \epsilon^*/h$ (Non-dimensional amplitude parameter)

$-R = \frac{V_0 h}{\nu}$ (Suction parameter)

$k = k^*h$ (Non-dimensional frequency parameter) (7)

As a consequence of Eqns. (6) and (7), the boundary conditions (3), (4) and (5) reduce to the non-dimensional form;

$$\frac{\partial u}{\partial x} = \frac{\partial T^*}{\partial x} = 0 \tag{8a}$$

$$\left.\begin{array}{l} u = 0, v = 1, T^* = 1 \text{ when } x = \eta = 0, \\ u = 0, v = 1, T^* = 0 \text{ when } x = 0, \eta = d \end{array}\right\} \tag{8b}$$

$v = 1$ on the flat wall

$$\frac{1}{u} = \epsilon K \cos Kx \quad \text{on the wavy wall} \tag{8c}$$

In view of the relationships in Eqns. (6) to (8), the partial differential Eqns. (1) to (4) of the authors' are converted to the following dimensionless ordinary linear and non-linear equations with constant co-efficients :

By Eqns. (3) and (8), V is a function of x only. But by Eqn. (9) $V(x, d) = V_0$. Hence $V(x, \eta) = V_0$, i.e., the velocity along the Y-axis is always V_0.

$$\frac{d^2u}{d\eta} + R\frac{du}{d\eta} - Mu = \frac{\partial P}{\partial x} \tag{9}$$

$$\frac{\partial P}{\partial \eta} = 0 \tag{10}$$

$$\frac{d^2T^*}{d\eta^2} + P_r R \frac{dT^*}{d\eta^2} = -P_r E\left(\frac{du}{d\eta}\right)^2. \tag{11}$$

Owing to Eqns. (8a) and (9), $\frac{\partial P}{\partial x}$ is a function of η while Eqn. (10) implies P as a function of x so that

$\frac{\partial^2 P}{\partial x^2} = 0$ and consequently $\frac{dP}{dx}$ is a constant, say, C. Then the integrals of Eqns. (9) to (1 I) can be brought about in the following manner :

$$u = e^{\frac{-R}{2}}(C_1 \cosh a? + C_2 \sinh a\eta) - \frac{C}{M}.$$

Making us: of the boundary conditions (8b) the values of the constants C_1 and C_2 are evaluated as

$$C_1 = \frac{C}{M} e^{R/2}, \; C_2 = \frac{C}{M} e^{R/2} \frac{1 - \cosh ad}{\sinh ad},$$

as such the velocity distribution in the region represented by $\epsilon \sin kx < \eta \leqslant d$ and $0 \leqslant x \leqslant 1$ and also at the inlet and outlet is given by

$$u(x, \eta) = -\frac{2C \sinh a\left(\frac{d-\eta}{2}\right)\sinh\frac{a\eta}{2}}{M\cosh\frac{ad}{2}}$$

$$= -\frac{C}{M}\left[1 - \frac{\cosh a\left(\eta - \frac{d}{2}\right)}{\cosh\frac{ad}{2}}\right] \tag{12}$$

where

$$a = \frac{1}{2}\left(R^2 + 4M\right)^{\frac{1}{2}} \tag{13}$$

At the outlet where $x = l,\ \eta_l = \eta_l = \epsilon \sin Kx = \epsilon \sin Kl$; the x-component of the velocity, in consequence of Eqn. (12), is given by

$$u_l = -\frac{2C \sinh a}{M\cosh \frac{ad}{2}}\left(\frac{d - \epsilon \sin Kl}{2}\right) \sinh a\left(\frac{\epsilon \sin Kl}{2}\right).$$

Again from the boundary condition (8c)

$\frac{1}{u_l} = \epsilon K \cos Kl.$ Hence the pressure gradient C can be determined as

$$C = -\frac{M\cosh \frac{ad}{2}}{2K\epsilon \cos Kl \sinh a\left(\frac{d - \epsilon \sin Kl}{2}\right) \sinh a\left(\frac{\epsilon \sin Kl}{2}\right)} \tag{14}$$

$= a$ negative quantity.

If P_0 and P_l be the pressures of the fluid at the inlet and outlet respectively, then

$$P_l = P_0 - \frac{Ml \cosh \frac{ad}{2}}{2K\epsilon \cos Kl \sinh a\left\{\frac{d - \epsilon \sin Kl}{2}\right\} \sinh a\left(\frac{\epsilon \sin Kl}{2}\right)}, \tag{15}$$

revealing that fluid pressure decreases along the x-direction. Eqn. (12) gives

$$\frac{du}{d\eta} = \frac{Ca \sinh a\left(\eta - \frac{d}{2}\right)}{M\cosh \frac{ad}{2}} \tag{16}$$

which is substituted into Eqn. (I 1) to yield the solution of the latter as

$$T^* = -\frac{P_r EB}{2a}\left[\frac{\cosh (2\eta - d \cdot \delta)}{\sqrt{4a^2 - P_r^2 R^2}} - \frac{2a\eta}{P_r R}\right] + C_1^1 \frac{e^{-P_r R\eta}}{P_r R} + C_2^1$$

where

$$B = \frac{C^2 a^2 \sinh^2 a\left(\eta - \frac{d}{2}\right)}{M^2 \cosh^2 \frac{ad}{2}} \tag{17}$$

By virtue of Eqns. (8a) and (8b), the values of the constants C_1^1 and C_2^1 are found out ;

$$C_1^1 = \frac{P_r R}{1 - e^{-P_r Rd}} \left[\frac{P_r EB}{a} \left\{ \frac{\sinh ad \sinh a\delta}{\sqrt{4a^2 - P_r^2 R^2}} + \frac{ad}{P_r R} \right\} + 1 \right]$$

$$C_2^1 = \frac{1}{1 - e^{-P_r Rd}} \left[\frac{P_r EB}{2a} \left\{ \frac{\cosh a(d - \delta) - e^{-P_r Rd} \cosh a(d + \delta)}{\sqrt{4a^2 - P_r^2 R^2}} - \frac{2ad}{P_r R} \right\} - e^{-P_r Rd} \right]$$

where

$$\tanh \delta = \overline{2a} \tag{18}$$

Subsequently the temperature distribution can, therefore, be expressed as

$$T^* (x, \eta) = - \frac{P_r EB}{2a} \left(\frac{\cosh a (2\eta - d - \delta)}{\sqrt{4a^2 - P_r^2 R^2}} - \frac{2a\eta}{P_r R} \right)$$

$$+ \frac{e^{-P_r R\eta}}{1 - e^{-P_r Rd}} \left[\left\{ \frac{\cosh a (d + \delta) - \cosh a (d - \delta)}{\sqrt{4a^2 - P_r^2 R^2}} + \frac{2ad}{P_r R} \right\} \times \frac{P_r EB}{2a} + 1 \right]$$

$$+ \frac{1}{1 - e^{-P_r Rd}} \left[\frac{P_r EB}{2a} \left\{ \frac{\cosh a (d - \delta) - e^{-P_r Rd} \cosh a (d + \delta)}{\sqrt{4a^2 - P_r^2 R^2}} - \frac{2ad}{P_r R} \right\} - e^{-P_r Rd} \right] \tag{19}$$

However, the temperature distribution on the wavy surface is obtained by taking $\eta = \epsilon \sin kx$ in Eqn. (19) in view of, Eqn. (3), and has to be maintained as another necessary boundary condition besides (8a) to (8c) for such a contrived flow of fluid.

4. Diseussion and Conclusion

Combining Eqns. (14) and (15), the co-efficient of skin friction on the flat plate is given by

$$C_f = \frac{2}{R_e}\left(\frac{\partial u}{\partial \eta}\right)_{\eta = d}$$

$$= \frac{-a \sinh a \frac{d}{2}}{R_e \epsilon K \cos Kl \sinh a \left(\frac{d - \epsilon \sin Kl}{2}\right) \sinh a \left(\frac{\epsilon \sin 2 Kl}{}\right)} \qquad (20a)$$

or,

$$C_f = \frac{-2a \tanh \frac{ad}{2}}{R_e \epsilon K \cos Kl \left\{ 1 - \frac{\cosh a \left(\frac{d}{2} - \epsilon \sin Kl\right)}{\cosh \frac{ad}{2}} \right\}} \qquad (20b)$$

Eqn. (20b) clearly reveals that the magnitude of the skin friction of the flat wall, because of Eqn. (13), increases as the magnetic or suction parameter increases; but no such observation can be made as regards the variation of this friction with frequency parameter

$$\left(\lambda = \frac{2\pi}{K}\right).$$

It can be realised from Eqn. (16) that the x-component of velocity is maximum at $\eta = \frac{d}{2}$ such that

$$u_{max} = -\frac{C}{M}\left(\frac{\cosh \frac{ad}{2} - 1}{\cosh \frac{ad}{2}}\right). \qquad (21)$$

The expression for u obtained upon substitution of Eqn. (14) into Eqn. (12) indicates that increase in the x-component of velocity can also be produced by increasing the suction or magnetic parameter, whereas a similar kind ofco nclusion cannot be arrived at in the case of temperature distribution as is clear from Eqn. (19). However, by Eqn. (19) the temperature gradient is acquired as

$$\frac{dT^*}{d\eta} = -P_r EB \left[\frac{\sinh a (2\eta - d - \delta)}{\sqrt{4a^2 - P_r^2 R^2}} \quad \frac{2}{P_r R}\right.$$

$$- \frac{P_r R\, e^{-P_r R \eta}}{1 - e^{-P_r R d}} \left[\frac{P_r E B}{2a} \left\{ \frac{\cosh a\,(d+\delta) - \cosh (d-\delta)}{\sqrt{4a^2 - P_r^2 R^2}} + \frac{2ad}{P_r R} \right\} + 1 \right]. \tag{22}$$

The maximum temperature can be computed by solving the transcendental equation obtained from Eqn. (22) by $\frac{dT^*}{d\eta} = 0$, and that unlike u does not occur at $\eta = \frac{d}{2}$.

In closing, for such a flow an important aspect to be considered is that the quantity of liquid flowing into the system through the inlet per unit time is equal to that flowing out of the system (including the suction). This principle of conservation of fluid mass is obviously applicable where the flat and wavy walls are chosen of finite length, and calls for the following integral equation :

$$b \int_0^d u\,(0, \eta)\, d\eta = vbl + b \int_{\eta = \epsilon \sin Kl}^{d} u\,(l, \eta)\, d\eta$$

Because $v = 1$

$$u\,(0, \eta) = u\,(l, \eta),$$

$$\int_0^{\epsilon \sin Kl} u\,(\eta)\, d\eta = l. \tag{23}$$

As a result of Eqn. (12), Eqn. (23) leads to

$$- C \left[\epsilon \sin Kl - 2 \cosh a \left(\frac{d - \epsilon \sin Kl}{2} \right) \tanh a \left(\frac{\epsilon \sin Kl}{2} \right) \right] = Mla. \tag{24}$$

Hence this relationship combined with Eqn. (14) must be satisfied in regard to the choice of the aforementioned parameters.

References

1. **Reddy, N. Bhaskara & Bathaiah, D.,** *Def. Sci. J.*, 31 **(1981),** 315-322.
2. **Shames, I. H., 'Mechanics of Fluid' (McGraw-Hill Book Company, Inc., New York), pp.** 457-460.