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Relationships between Explosive Performance Potential and Detonation Properties—Application of Dimensional Analysis and G-Inverses

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ABSTRACT

An approach based on dimensional analysis has been used to obtain relationships between the detonation properties of a high explosive and its performance potential(X_{pp}) as measured by strength and brisance tests. A matrix is set up wherein each physical quantity is expressed in terms of basic dimensions *(M, L, T)* and solved using the method of generalised inverses. A correlation analysis has been carried out between X_{pp}s calculated from property data for 29 military explosives using these expressions and strength/brisance values. The paper discusses the results obtained and their utility. New expressions relating *P* and other detonation properties have also been derived and tested for the degree of correlation.

1. INTRODUCTION

Two terms 'strength' and 'brisance' are commonly used in literature while referring to the performance characteristics of an explosive. However, no clear and quantitative definition based on physico-chemical properties is available. Most authors have obtained relations which provide information concerning the empirical dependency of brisance and strength test data on detonation properties.[1-4] In general, these relations

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have no physical meaning and are not valid dimensionally. An attempt has been made to derive a physically meaningful relation by combining the various detonation properties such as density ρ, volume of gases released upon detonation GV, heat of detonation Q, and velocity of detonation V, in such a manner that their product has the dimensions of power density.[4] It has been proposed that explosive performance potential, X_{pp} is given by

$$X_{pp} = (\rho.GV)^{1/3} (\rho.Q).V \tag{1}$$

where ρ is expressed in kgl^{-1}, GV in $l\,kg^{-1}$, Q in kcal kg^{-1} and V in ms^{-1}. X_{pp} has been found to correlate with strength and brisance data from several representative tests. One criticism which may be extended regarding the above work is that relation (1) is arrived at intuitively. The present work has been carried out with the objective to circumventing this by employing the method of dimensional analysis to examine the possible ways of grouping the various detonation properties. The correlative ability of relations thus found.out with experimental test data was then checked.

2. NATURE OF DIMENSIONAL ANALYSIS

Dimensional analysis is a study of the restrictions placed on the form of an algebraic function by the requirement of dimensional homogeneity. This requirement ensures that any expression in physical algebra will have physical significance. Thus this analysis helps in deducing valid ways of combining different physical properties of importance for the phenomenon under consideration. While dimensional analysis alone can never give a complete solution of a problem, it does, however, usually permit considerable simplifications in investigating complex phenomena and may show the effect of particular variables. The complexity in any given problem may result from a large number of independent variables affecting the phenomenon, or from a mathematically complicated form of expressions relating these variables. In the present case the former is not true since the obvious factors of importance such as ρ, GV, Q, V and P (detonation pressure) are limited in number. On the other hand, the nature of dependence of the measured performance data on these independent variables is not known. Thus it is possible that a dimensional analysis exercise could provide a valuable insight into this problem.

3. PROCESS OF DIMENSIONAL ANALYSIS

The problem is to find a relation of the form

$$X_{pp} = \phi(\rho, GV, Q, V, P) \tag{2}$$

Each physical quantity expressed in terms of the basic dimensions such as mass M, length L, and time T, forms a column matrix and the different columns for the various quantities make the matrix A, where A will be a $m \times n$ matrix corresponding to m

basic units and n physical quantities. In the present case A is given by

$$
\begin{array}{c} \\ M \\ L \\ \\ \end{array}
\left[\begin{array}{cccccc}
\rho & GV & Q & V & P & X_{pp} \\
1 & -1 & 0 & 0 & 1 & 1 \\
-3 & 3 & 2 & 1 & -1 & 0 \\
0 & 0 & -2 & -1 & 2 & -3
\end{array}\right]
$$

We need to solve the equation

$$A \propto = 0 \tag{3}$$

where $\propto$ is a solution with integral or rational components. For this equation to have non-trivial solutions, A has to be singular. For such matrices, the conventional methods of inversion fail and it is here that the method of generalised inverses is being sought. For any $(m \times n)$ matrix A, its reflexive generalised inverse A^- is defined as follows

$$
\begin{array}{l}
A \quad A^- A = A \\
A^- A \quad A^- = A^-
\end{array} \tag{4}
$$

The general solution of the Eqn. (3) is given by

$$\propto = (I - A^- A)Z \tag{5}$$

where I is an identity matrix and Z is an arbitrary vector. The computation of A^- involves firstly a series of row column transformations of A with the aid of a transformation matrix E to reduce A to its row echelon form (Hermite-Normal Form – HNF), A_1.

Another transformation matrix F operates on $A^T{}_1$ (transpose of A_1 to give its HNF, C. Then

$$A^- = F^T C E \tag{6}$$

The details of the above procedure are given in references 6 and 7. It can be shown that for the matrix A given earlier

$$
(I - A^- A) =
\left[\begin{array}{ccccccc}
\rho & 0 & 0 & 0 & & & 1 \\
GV & 0 & 0 & 0 & & 0 & 0 \\
Q & 0 & 0 & '2 & 0 & -3/2 & -1 \\
V & 0 & 0 & & 0 & 0 & 0 \\
P & 0 & 0 & 0 & 0 & 0 & 1 \\
X_{pp} & 0 & 0 & 0 & 0 & & 0
\end{array}\right] \tag{7}
$$

Accordingly there are four linearly independent solutions and these are obtained by setting the arbitrary vector Z^T to the following four linearly independent vectors

$$
\begin{array}{l}
Z^T = (0, 0, 1, 0, 0, 0) \\
Z^T = (0, 0, 0, 1, 0, 0) \\
Z^T = (0, 0, 0, 0, 1, 0) \\
Z^T = (0, 0, 0, 0, 0, 1)
\end{array} \tag{8}
$$

This leads to the following four dimensionless expressions

$$Q^{-1/2}.V$$
$$\rho.GV$$
$$\rho^{-1}.Q^{-3/2}.X_{pp}$$
$$\rho^{-1}.Q^{-1}.P. \qquad (9)$$

Using these, we can form other solutions by substitution.

4. CORRELATIVE ANALYSIS

The following four relations were chosen for examining their predictive ability of performance data from different tests:

$$X_{pp}(1) = \rho.Q^{3/2};\ X_{pp}(2) = \rho^2.GV.Q.V;$$
$$X_{pp}(3) = \rho.Q.V;\ X_{pp}(4)=P.V$$

In addition the following relations for detonation pressure P have been obtained:

$$P_c(1) = \rho.Q \quad ; \quad P_c(2) = \rho.Q^{1/2}.V \qquad (1$$

The degree of correlation between the computed pressures P_c, and the detonation pressure data, theoretical and experimentlal taken from literature has also been checked.

In this study twenty-nine high explosives have been included. Their detonation properties, strength and brisance data taken from literature are listed in Tables 1 and 2. Reference[5] gives details of the origin of the data. In the first part of the analysis, the groups of dependent variables numbered six : lead block expansion volumes, ballistic mortar strengths, explosive energies as measured in cylinder test in both the head-on and tangential geometries, plate dent depths and detonation impulse scales. The independent variables were eight in number; $X_{pp}(1)$ to $X_{pp}(4)$ with both theoretical and experimental values. The analysis has been carried out with a computer program incorporating the linear regression subroutine GO2 CCF from the N A G library. In the second part, correlation between Ps and P_cs as the independent and the dependent variables, respectively has been tested.

5. RESULTS AND DISCUSSION

A detailed examination of the correlation data presented in Table 3 for various explosive performance potential relations leads to the following conclusions. Among the dependent variables, lead block volume relates poorly to any of the expression given in Eqn. (10) while the results are, in general, satisfactory for the other 5 groups of test data, irrespective of the nature of the test. From the statistical test results,

Table 1. Detonation properties of explosives

	Explosive	Density/kg l^{-1}	Gas volume/l kg^{-1}	Heat of detonation/k cal kg^{-1}		Detonation velocity/ms^{-1}		C–J pressure/k bar	
				Theoretical	Experimental	Theoretical	Experimental	Theoretical	Experimental
1.	PETN	1.76	780	1650	1490	8421	8260	332	335
2.	TNT	1.63	620	1410	1090	6950	6930	223	210
3.	RDX	1.80	908	1620	1510	8754	8700	348	338
4.	HMX	1.96	782	1620	1480	9159	9110	394	390
5.	Tetryl	1.71	672	1510	1140	7629	7850	260	–
6.	BTF	1.87	800	1690	1410	8156	8490	309	360
7.	NM	1.13	1092	1620	1230	6463	6350	144	125
8.	TATB	1.88	651	1200	–	8411	7760	291	–
9.	Pentolite(PETN/TNT 50/50)	1.69	700	1530	1290	7740	7530	255	–
10.	Pentolite (PETN/TNT 55/45)	1.65	708	1542	1310	–	7515	–	266
11.	Cyclotol (RDX/TNT 75/25)	1.75	836	1567	1405	8311	8300	305	316
12.	Octol (HMX/TNT 75/25)	1.81	741	1567	1382	8555	8480	333	342
13.	Comp B–3 (RDX/TNT 60/40)	1.72	792	1536	1342	–	7890	304	287
14.	Comp B (RDX/TNT/WAX/ 63/36/1)	1.71	795	1528	1200	8084	7920	284	295
15.	Comp A–3 (RDX/WAX 91/9)	1.64	826	1474	1374	–	8470	317	287
16.	LX–07 (HMX/VITON A 90/10)	1.86	704	1458	1332	8805	8640	346	–
17.	LX–10 (HMX/VITON A 95/5)	1.86	743	1539	1406	8890	8820	360	375
18.	LX–15 (HNS/KEL F 95/5)	1.58	709	1349			6840	188	
19.	LX–17.0 (TATB/KEL F 92.5/7.5)	1.91	602	1310			7630		
20.	PBX (RDX/KEL F 90/10)	1.79	817	1458	1359	8371	8370	313	328
21.	TNT/DNT (60.8/39.2)	1.58	612	1272	–	–	6735	–	178
22.	Trinitrobenzene	1.76	600	1275	–	–	7300	–	–
23.	Picric acid	1.76	610	1200	–	–	7350	265	–
24.	Nitroguanidine	1.55	895	1060	–	–	7650	–	–
25.	Ammonium picrate	1.50	685	1021	–	–	7150	–	–
26.	EGDN	1.48	737	1630	–	–	7300	–	–
27.	Hydrazine nitrate	1.64	1001	924	–	–	8690	–	–
28.	Nitroglycerine	1.59	715	1590	–	7700	7580	251	253
29.	Cyclotol (RDX/TNT 75.2/24.8)	1.20	836	1568	1405	–	6490	–	124

Table 2. Strength and brisance values for explosives from different tests.

	Explosive	Lead block volume/ ml per 10 g of explosive	Ballistic mortar strength/ % TNT	Cylinder test		Plate dent depth/ % TNT	Detonation impulse/ % RDX–TNT 50:50
				Head-on 1/2 mm^2	Tangential s^{-2} $MJkg^{-1}$		
1.	PETN	523	145	1.255	1.575	146[1]	119
2.	TNT	300	100	0.735	0.975	100	85
3.	RDX	480	150	–	1.6	157[2]	121
4.	HMX	480	150	1.410	1.745	–	130
5.	Tetryl	410	130	–	–	121	103
6.	BTF	–	–	1.305	1.680	–	–
7.	NM	400	–	0.560	0.745	62[3]	69
8.	TATB	–	–	0.874	1.079	–	–
9.	Pentolite (PETN/TNT 50/50)	–	126	0.960	1.260	–	–
10.	Pentolite (PETN/TNT 55/45)	–	–	–	–	117	–
11.	Cyclotol (RDX/TNT 75/25)	–	–	1.140	1.445	140	–
12.	Octol (HMX/TNT 75/25)	–	–	1.215	1.535	150	116
13.	Comp B–3 (RDX/TNT 60/40)	–	–	1.010	1.322	129	–
14.	Comp B (RDX/TNT/WAX 63/36/1)	–	133	1.035	1.330	–	–
15.	Comp A–3 (RDX/WAX 91/9)	–	135	–	1.2	122	–
16.	LX–07 (HMX/VITON A 90/10)	–	–	1.250	1.575	–	–
17.	LX–10 (HMX/VITON A 95/5)	–	–	1.315	1.670	–	–
18.	LX–15 (HNS/KEL F 95/5)	–	–	0.700	0.929	–	–
19.	LX–17.0 (TATB/KEL F 92.5/7.5)	–	–	0.870	1.070	–	–
20.	PBX 9010 (RDX/KEL F 90/10)	–	–	1.160	1.470	–	–
21.	TNT/DNT (60.8/39.2)	–	–	–	–	86	–
22.	Trinitrobenzene	325	–	–	–	–	–
23.	Picric acid	315	112	–	–	–	–
24.	Nitroguanidine	305	104	–	–	–	–
25.	Ammonium picrate	280	99	–	–	–	–
26.	EGDN	620	–	–	–	–	–
27.	Hydrazine nitrate	408	–	–	–	–	–
28.	Nitroglycerene	520	–	–	–	–	–
29.	Cyclotol (RDX/TNT 75.2/24.8)	–	–	–	–	80	–

Value corresponds to the following explosive properties (experimental) : (1) density 1.67 kg l^{-1}; detonation velocity 7985 ms^{-1}; C-J pressure 310 k bar. (2) density 1.76 kg l^{-1}; detonation velocity 8540 ms^{-1}; C-J pressure 338 k bar. (3) density 1.13 kg l^{-1}; detonation velocity 6245 ms^{-1}; C-J pressure 139 K bar.

'Table 3) it is clear that the two sets of X_{pp} variables, calculated using theoretical and :xperimental values, exhibit different trends. For experimental values, the relation

$$X_{pp} = P.V$$

;ives the best fit. Relation X_{pp} (3) ranks a close second for all the five tests excluding ead block volumes and it is to be preferred over X_{pp} (4) since experimental letermination of P is difficult. On the other hand, for theoretical values the relation

$$X_{pp} = \rho.Q.V$$

ields the best correlation, because P is sensitive to the equation of state. For the ead block expansion volume data, X_{pp}(1) gives the maximum correlation, which could e indicative of the lesser importance of properties such as V and P in this test. From 'able 3, it is also apparent that $\rho.Q.V$ (theoretical) relates better than $\rho.Q.V$ experimental). This is desirable since Q and V can be computed easily employing a :omputer code such as TIGER while experimental determination of Q is difficult.

The pressure relations given in Eqn. (11) have also been tested to examine the extent of correlation with the six dependent varaibles and the results are summarised in Table 4. On the basis of results given in Tables 3 and 4 it appears that while $\rho.Q.V$ (theoretical) gives better correlation than $\rho.Q^{1/2}.V$ (theoretical)), the opposite is true in the case of the corresponding experimental quantities. An important point to be noted that pressure (basic dimensions $ML^{-1}\,T^{-2}$) alone gives as good a correlation as explosive performance potential (basic dimensions MT^{-3}). It has been found earlier that experimental detonation pressure gave the best correlation among several independent variables checked.[4]

The regression constants and coefficients for the three 'near optimum' independent variables are given in Table 5. Using these, one could estimate the strength/brisance value for a particular explosive in any given test. Figs. 1 and 2 are typical regression plots obtained from the correlation analysis.

$P_c s$ given by expressions in Eqn. (11) have also been correlated with Ps both experimental and theoretical. An examination of the 4 × 4 correlation matrix (Table 6) shows that $P_c s$ calculated using the relations derived from dimensional analysis are in close agreement with Ps calculated using TIGER code or those determined experimentally. The closeness of fit can also be judged from Fig. 3. Since measurement of P is not easily accomplished, it is advantageous to compute this property using relations derived here. Thus experimental Chapman-Jouguet pressure can be computed using an expression of the form

$$P = 0.54947.10^{-3}\rho.Q^{0.5}.V\text{-}35.36 \qquad (14)$$

where the detonation properties on the right hand side are theoretical quantities.

Table 3. Correlation data for explosive performance potential relations.

Independent variable \ Dependent variable	Lead block volume			Ballistic mortar strength			Cylinder test Head-on			Cylinder test Tangential			Plate dent depth			Detonation impulse		
$\rho.Q^{3/2}$(Theo)	13	0.67	44.6	11	0.91	82.2	16	0.92	85.1	18	0.94	88.1	12	0.96	93.2	8	0.99	99.3
$\rho.^2GV.Q.V$(Theo)	7	0.61	25.2	7	0.92	84.5	13	0.96	92.4	14	0.94	88.1	5	0.97	92.0	8	0.93	86.8
$\rho..Q.V)$(Theo)	7	0.63	39.2	7	0.93	86.0	13	0.99	97.3	14	0.98	96.2	7	0.99	98.8	8	0.99	98.2
$P.V$(Theo)	7	0.59	35.1	7	0.90	80.9	13	0.93	87.2	14	0.91	83.5	7	0.99	98.0	8	0.98	95.4
$\rho.Q^{3/2}$(Exp)	6	0.81	65.6	[illegible]	0.88	77.0	13	0.97	93.8	15	0.95	90.2	11	0.93	87.1	7	0.96	92.3
$\rho.^2GV.Q.V$(Exp)	8	0.66	34.2	[illegible]	0.90	80.7	13	0.96	91.7	15	0.92	84.2	13	0.93	85.6	7	0.93	87.1
$\rho.Q.V$(Exp)	6	0.76	58.4	[illegible]	0.90	81.4	13	0.99	97.6	15	0.96	93.0	11	0.97	95.1	7	0.98	96.6
$P.V$(Exp)	6	0.60	36.1	[illegible]	0.93	87.2	13	0.99	97.9	13	0.98	95.6	11	0.98	96.8	7	0.99	97.3

Figures represent the number of data points (n), correlation coefficient (R), and the percentage variance accounted for, (% VAR) respectively.

Table 4. Goodness of fit results (n, R, % VAR) for pressure relations versus performance data.

Independent variable \ Dependent variable	Lead block volume			Ballistic mortar strength			Cylinder test Head-on			Cylinder test Tangential			Plate dent depth			Detonation impulse		
$\rho.Q$(Theo)	13	0.61	36.8	11	0.90	81.5	16	0.95	90.3	18	0.96	91.4	12	0.98	96.5	8	0.99	98.7
$\rho.Q^{1/2}.V$(Theo)	7	0.58	33.2	7	0.92	83.6	13	0.96	92.3	14	0.95	92.3	7	0.99	99.4	8	0.99	97.7
$\rho.Q$(Exp)	6	0.76	58.2	8	0.88	77.6	13	0.98	95.8	15	0.97	93.7	11	0.97	94.7	7	0.98	96.3
$\rho.Q^{1/2}.V$(Exp)	6	0.71	50.1	8	0.90	81.8	13	0.99	97.7	15	0.97	94.1	11	0.99	97.9	7	0.99	98.3

Table 5. Regression data (intercept C, slope m)[a] for the most suited performance potential and pressure relations.

Independent variable \ Dependent variable	Lead block volume/ml	Ballistic mortar strength % TNT	Cylinder test Head-on $^1/2mm^2$	Cylinder test Tangential $s^{-2}\,MJ\,kg^{-1}$	Plate dent depth % TNT	Detonation impulse % RDX-TNT
$P.V$ (Exp)/10^4	338.1, 0.502 (81.0, 0.334)	70.7, 0.221 (12.2, 0.048)	0.229, 0.00318 (0.034, 0.00013)	0.425, 0.00384 (0.066, 0.00025)	41.3, 0.376 (4.9, 0.023)	54.2, 0.222 (4.1, 0.016)
$\rho.Q.V$(Theo)/10^5	268.4, 0.845 (101.5, 0.471)	50.0, 0.375 (15.3, 0.067)	0 ,0.00499 (– ,0.00027)	0 ,0.00639 (– ,0.00035)	–11.4, 0.661 (6.8, 0.032)	29.4, 0.359 (4.3, 0.019)
$\rho.Q^{1/2}.V$(Theo)/10^3	274.8, 0.0324 (111.0, 0.205)	40.4, 0.165 (18.6, 0.032)	0 ,0.00195 (– ,0.00015)	0 ,0.00248) (– ,0.00021)	-18.1, 0.275 (1.6, 0.031)	26.3, 0.148 (5.1, 0.009)

(a) . $Y = mx + C$

(b) Values in parenthesis are standard errors for C and m.

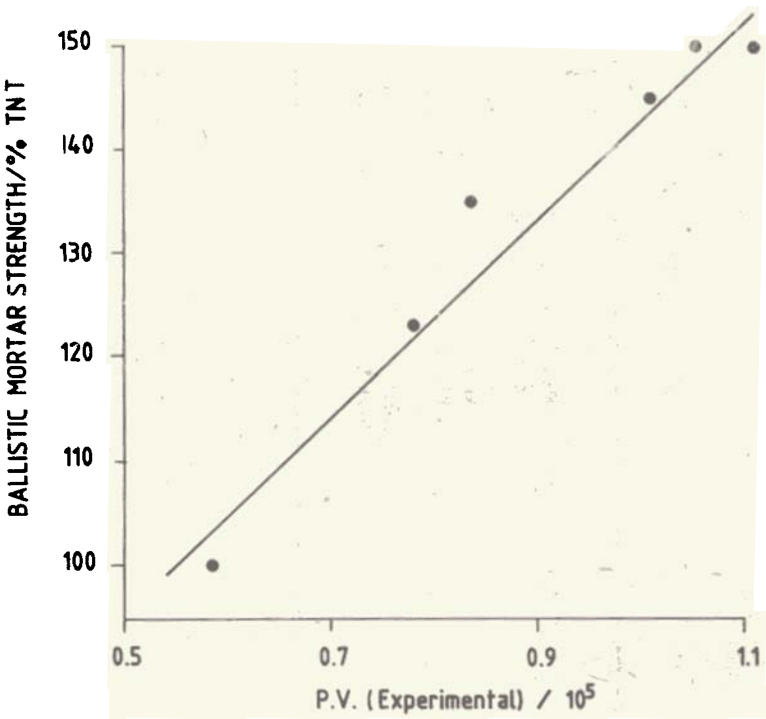

Figure 1 Regression plot of X_{pp} (4) vs. ballistic mortar strength data.

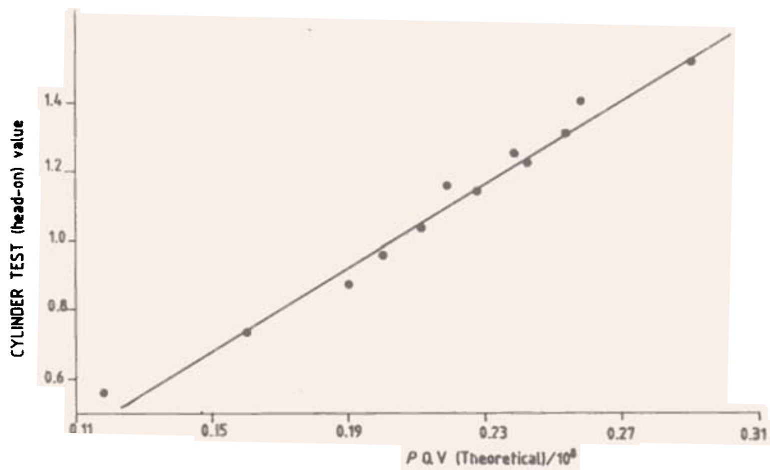

Figure 2. Plot of ρQV (theoretical) values vs. strength data from cylinder test (head on) configuration.

Table 6. Test results (*n, R, % VAR*) for theoretical and experimental detonation pressures versus pressures computed using relations obtained from dimensional analysis.

Independent variable \ Dependent variable	Detonation pressure: Theoretical			Experimental		
$\rho.Q$(Theo)	17	0.96	93.0	20	0.83	69.6
$\rho.Q^{1/2}.V$(Theo)	15	0.99	97.9	12	0.99	97.3
$\rho.Q$(Exp)	15	0.96	91.4	16	0.95	90.1
$\rho.Q^{1/2}.V$(Exp)	15	0.98	96.8	16	0.98	95.5

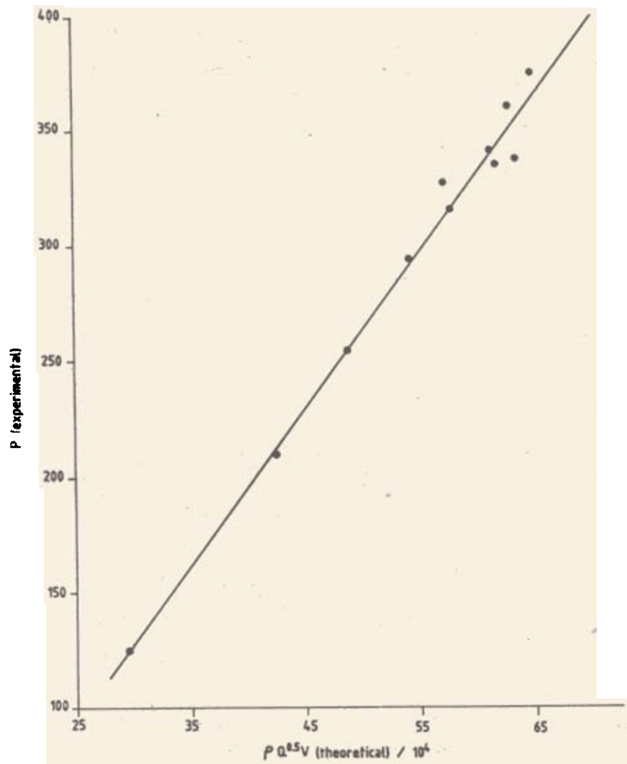

Figure 3. Regression plot of theoretical $\rho Q^{0.5}V$ values vs. experimental Chapman Jouguet pressure data.

6. CONCLUSIONS

It is evident from the above discussion that application of dimensional analysis technique to the problem of detonation properties/explosive performance potential relationship has yielded expressions which exhibit a high degree of correlation with strength/brisance data. In fact the most suited relation obtained in this work

$$X_{pp} = \rho.Q.V \tag{15}$$

is found to correlate better than the earlier defined[5] one for performance potential and as good as the experimental detonation pressure. Further this approach has given new expressions relating *P* and other detonation properties. All these could be of immense value in an apriori estimation of the performance potential of high explosives. Although this work has been carried out with data obtained for military explosives which possess ideal detonation behaviour, it is quite possible that the relations derived will be valid even in the case of commercial explosives which detonate in a non-ideal manner. For the latter, a new set of regression constants and coefficients have to be determined. Finally, it should be mentioned that the method of dimensional analysis employing generalised inverses could find applications in other areas of detonation physics, rock fragmentation by explosives etc.

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