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Discrete Deterministic Modelling of Autonomous Missiles Salvos

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ABSTRACT

The paper deals with mathematical models of sequent salvos battle, of autonomous flight missiles (AFM) organized in the groups of combatants. Tactical integration of AFM system distance-controlled weapon is considered by performances of simultaneous approaches on targets, and continual battle models of guerilla and direct fire, are redesigned to the discrete-continual mixed model, for checking missiles sudden, and further salvos, attack effects. Superiority parameters, as well as losses and strengths of full, or the part of salvo battle, for the missiles groups as technology sub-systems combatants', is expressed by mathematical and simulation examples. Targets engagements capacities of the missiles battle unit, is conducted through designed scenarios and mathematically derived in the research. Model orientated on answers about employment of rapid reaction defending tactics, by distance missiles attacks.

Keywords: Combat modelling, discrete shots, salvo missiles, Lanchester square laws, guerilla warfare model, superiority coefficients, autonomous flight missiles

NOMENCLATURE

$M_{B\max}$	Maximum number of blue combatants
N_B	Number of blue missiles
ω	Number of missiles in one combatant group
M_B	Number of blue combatants
$M_R = m_{R0}$	Number of red combatants
M_{Bk}	Number of blue combatants in each particular salvo
$m_B(\tau), m_R(\tau)$	Remained number of blue (red) combatants in the observed moment of time
$m_{Bk-1}, (m_{Rk-1})$	Remained number of blue (red) combatants after previous salvos
$\mu_B = m_B(\tau)/M_B$	No dimensional strengths of the blue combatants
$\mu_R = m_R(\tau)/M_R$	No dimensional strengths of the red combatants
$\mu_{B(k)}(\mu_{Bk})$	No dimensional strengths of blue combatants, time dimensional value (instant value) for each salvo in continual modelling
$\mu_{R(k)}(\mu_{Rk})$	No dimensional strengths of the red combatants, time dimensional value (instant value) for each salvo in continual modelling
$\mu'_{B(k)}(\mu'_{Bk})$	No dimensional strengths of the blue combatants, time dimensional value (instant value) in mixed modelling
$\mu'_{R(k)}(\mu'_{Rk})$	No dimensional strengths of the red combatants, time dimensional value (instant value) in mixed modelling
$\alpha_B(\alpha_R)$	Blue (red) combatant's attritions rates coefficient
t	Battle run time
$\tau = t \cdot \sqrt{\alpha_B \cdot \alpha_R}$	No dimensional battle time as continual variable
$\Delta\tau$	No dimensional battle time interval of one salvo
Δt^*	Infinitesimal discrete time interval of final shot
ε	Infinitesimal discrete value of no dimensional time interval of final shot
$\phi_0 = \dfrac{M_B}{M_R}\sqrt{\dfrac{\alpha_B}{\alpha_R}}$	Superiority coefficient of forces
$\sum_{k=1}^{n+1} M_{bk}$	Total number of consumed missiles combatants (groups) in the multi-layer salvo battle.

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i	Number of salvo from 1 to n
j	Number of intermediate salvo from 1(2) to i
k	Number of any salvo from 1 to n+1
(k)	Index k in the brake means that value is variable in time
B	Blue combatants
R	Red combatants
’	First phase of each salvo in mixed model

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1. INTRODUCTION AND RELEVANCE OF MODELLING

The modern strategy of the attacker usually has intention to weaken or degrade the combat potential of the defender by means of undermining operations and ground combat operations, as it was stated in modern battle classification given by Menq[1], *et al.* This manner of battle seems as the part of the 'decapitation strategy', and appropriate preventive guard forces have to be most modern designed for the rapid reaction answers[1]. The concept of defenders battle tactics orientates engagement of protected forces, to use simultaneous attacks on concentrated groups of suddenly-appeared targets, from the distance positions, in a rapid reaction manner. They can use smart weapon, with joint detection and engagement capabilities integrated on a single autonomous flight missile (hereafter AFM), referred in the earlier paper of Kalam[2] as new visions, and also in the last modern developed, tactical and technology integrated requirements, stated by Liu[3,4], *et al.* In these suppositions weapon performances offer new tactical forces capabilities and the basic deterministic Lanchester and mixed developed models, as the continual modelling approaches require to be modified. The basic idea of the research was to derive, check, and test a model where AFM missiles are the blue combatants of one side opposed to the enemy combatants distance concentrated as the small group of point targets.

2. GENERAL SCENARIO

The battle theater considers as grouped shooting event of the blue missiles on the red concentrated group of targets, where one missile's salvo assigned targets immediately at the moment of approaching in the target area, and dive toward targets simultaneously. The situation when the enemy's red forces are attacked suddenly, by blue's AFM missiles salvo, seems tactically, at the initial stage of the battle, as an ambush for the red's combatants. It could be expected that sudden effect of missiles appearing above the targets, confuses enemy, and consequently, have maximum, defending battle effect of engaged forces (combatants). Research suppositions of AFM launching and flight toward targets area, have not been considered as the influencing on the rapid reaction effects of blues', because these have been assumed as the masked part of salvos. Battle time of one salvo is considered only by final diving sequences of AFM approach. Blue combatants task, that could be, to destroy all red combatants is, or isn't satisfied in the first, sudden salvo attack, where defender achieved surprising effect. Consequently, this scenario requires, also, that salvos could be repeated, in the synchronized time delay intervals, but not as the replenishment by new unit capacity, then using available missiles on the launcher disposed on the firing pod. Missiles are organized in groups considered as the particular combatants in one salvo, and one group assigns and shoots one target with the maximum probability. Number of groups is number of defenders blue combatants M_{BK} and is implemented in the battle superiority considerations. In the scenario of single-layer salvo, enemy red forces reciprocate by random, unprepared fire, on the missiles attack. In the multi-layer salvos, after the first, red forces reciprocate on the next, by the prepared fire, as an expected attack. Some missiles groups, in one salvo, have been hit by mentioned red's continual counterfire in salvos. Missile groups survived in one salvo have been consumed on the targets shooting in the final sequence of salvo approach when missile groups hit some red targets. This corresponds to the model of prepared direct fire on the reds, where blue forces always consumed all combatants (all missiles, or groups of missiles), engaged in one salvo, but battle unit consumes all or the part of available missiles to destroy all targets in several salvos. The battle is ended when new, missiles salvo annihilate last remained targets, survived from the previous salvo, or, if the total number of available missiles for the salvo launching are consumed in the blue combat unit.

In this scenario based on blue weapon performances, it offers new tactical forces capabilities, and the basic deterministic continual modelling approaches by Lanchester and mixed developed models, so require to be modified. In the scenario, a situation appears where composed continual and discrete shooting events have to be designed as integral approach. In the paper of MacKay[5], tactical battle models are classified into two main headings: mathematical and simulation bases. Lanchester equations of warfare in the new approaches are all considered as basically deterministic models as in Jau-yeu Menq[1], *et al.*, which refers to the list of all derived modern models based on Lanchester equations. Also, Ozdemirel[6], *et al.*, McIntosh[7], *et al.*, Kaup[8], *et al.*, McNaught[9], have used these models with simulations of particular events which influence the battle outcome. In the paper of Lauren[10], new fractals determinations of sequent influences have been developed. Breton[11], *et al.* used equations for closed- and open-loops of the battle. In the studies of Chen[12,13], so called square law regulated battle gradients to converge calculations with the aim to destroy allowable targets. Square law model is effectively used for the smallest tactical questions as well as for conceptual strategic modelling at the same time, as shown by Hillestad[14], *et al.* González[15], *et al.*, presented a spatial modelling and Armstrong[16] presented salvo firing duels, where one side can count, at the end of battle, zero remained combatants. In different approaches of salvo attacks, this paper has considered both sides zero remained forces.

Constrains of the above-mentioned derived models have orientated this paper on the answer, under which conditions missiles in salvos could be organised in the smart ammunition groups considered as combatants. This raises the questions about determined battle superiority coefficients as a measure in the battle efficiency, not only as the planed forces potential, but at a first place, as the requirement for the engagement weapon capacity, for the rapid reaction in the counter decapitation missions. Second constrain is mixed continual and discrete shooting events modelling and employment in the variable salvos model. Third is the question of multi-layer salvo superiority parameter and its variation for continual and mixed continual-discrete manner of shoots, and question looses considering for blue and red combatants.

Using Przemieniecky's[17] conventional models and old-fashioned Schaffer's[18] and Milinovic[19], *et al.* of Lanchester's and mixed laws, singular shooting events were included.

3. MATHEMATICAL MODEL OF BATTLE BY SUCCESSIVE SALVOS

Discrete shooting of the reds and continual shooting of the blue forces, during sequent salvo battle, separate considerations of no dimensional combatants' strengths in two different mathematical models applied in the same salvo.

First, model is fully continual deterministic approach and coming from the assumption that manner which AFM shooting red targets could be the kind 'direct fire' and red side could only be able to reciprocate in the form of 'area fire', as stated by Przemieniecky[17], because the precise location of the blue forces AFM missiles in the air would be unknown, which corresponds to the continual model of 'guerilla warfare', for the first salvo. Time-dependent, rearranged equations of forces strengths, are as follows:

$$\frac{d\mu_{R(1)}}{d\tau} = -\mu_{B(1)}\phi_{01} \tag{1}$$

for the red, and

$$\frac{d\mu_{B(1)}}{d\tau} = -\frac{\mu_{R(1)}\mu_{B(1)}}{\phi_{01}}, \quad 0 \le \tau \le \Delta\tau \tag{2}$$

for the blue. In the model of multi-layer salvos, red forces, reciprocate by model of continual direct fire, and new, repeated Eqns (1) and (2), become

$$\frac{d\mu_{R(i+1)}}{d\tau} = -\mu_{B(i+1)}\phi_{0i+1} \tag{3}$$

$$\frac{d\mu_{B(i+1)}}{d\tau} = -\frac{\mu_{R(i+1)}}{\phi_{0i+1}}, \quad i = 1,2,3...n, \quad 0 \le \tau \le \Delta\tau \tag{4}$$

Based on general superiority parameter of forces, ϕ_0 J.S. Przemieniecky[17], for all salvo battle this value is

$$\phi_0 = \frac{\sum_{k=1}^{n+1} M_{Bk}}{M_R}\sqrt{\frac{\alpha_B}{\alpha_R}} \tag{5}$$

Superiority parameter of salvo sequence in the first, as well as in each subsequent salvo to the *n*+1, is given by expression

$$\phi_{0k} = \frac{M_{Bk}}{m_{Rk-1}}\sqrt{\frac{\alpha_B}{\alpha_R}}, \qquad k = 1,2,3,...,n+1 \tag{6}$$

Continual differential models, expressed in the first and in the next salvos, for the initial conditions $\mu_{R(k)}(\tau = 0) = 1$ and $\mu_{B(k)}(\tau = 0) = 1$ gives

$$\mu_{B(1)} - \frac{1}{2\phi_{01}^2}\mu_{R(1)}^2 = \frac{2-\phi_{01}^{-2}}{2} \tag{7}$$

for the first, and square law, in the next particular salvos, Przemieniecky[17], as

$$\phi_{0i+1}^2\mu_{(Bi+1)}^2 - \mu_{R(i+1)}^2 = \phi_{0i+1}^2 - 1 \tag{8}$$

Salvo battle consumed all blue forces missiles organized in combatant groups $\mu_{Bk} = \mu_{B(k)}(\Delta\tau) = 0$, in each particular salvo. Particular superiority parameters ϕ_{01} and ϕ_{0i+1} and expected red targets remaining strengths $\mu_{Rk} = \mu_{R(k)}(\Delta\tau)$, after salvos, using Eqns (7) and (8), are expressed as

$$\phi_{01} = \sqrt{\frac{1-\mu_{R1}^2}{2}} \tag{9}$$

for the first salvo, and

$$\phi_{0i+1} = \sqrt{1-\mu_{Ri+1}^2} \tag{10}$$

for sequent salvos.

Continual guerilla warfare model shows that total consumption of all red and blue forces in the first salvo corresponds to superiority parameter $\phi_{01} = \sqrt{2}/2 = 0.705$ from Eqn. (9). If in the first salvo, value is $\phi_{01} < \sqrt{2}/2$, new sequent salvos are required. From, Eqn. (10), battle is ended when some of subsequent salvos satisfied $\phi_{0i+1} = 1$. More sequent salvos appear with remained reds as μ_{Ri+1}, if the $\phi_{0i+1} < 1$. Equation derived from Eqn. (6), for sequent salvos is

$$\phi_{0i+1} = \phi_{01}\frac{M_{Bi+1}}{M_{B1}}\frac{1}{\prod_{j=1}^{i}\mu_{Rj}}, \text{ for the i=1,2,3....n} \tag{11}$$

Using continual laws conditions, (Eqns (9), (10) and (11)), joint salvo battle superiority coefficient become

$$\phi_0 = \phi_{01}\left[1+\sqrt{\frac{2\mu_{R1}^2}{1-\mu_{R1}^2}}\sum_{i=1}^{n}\left(\sqrt{1-\mu_{Ri+1}^2}\prod_{j=2}^{i}\sqrt{1-\phi_{0j}^2}\right)\right] i < j \text{ for } \prod_{j=2}^{i=1}\mu_{Rj} = 1, \tag{12}$$

The number of missile groups, in each subsequent salvo, determines expression derived from Eqns. (6) and (11), as

$$M_{Bi+1} = M_{B1}\sqrt{\frac{2\mu_{R1}^2}{1-\mu_{R1}^2}}\left[\left(\sqrt{1-\mu_{Ri+1}^2}\right)\left(\prod_{j=2}^{i}\mu_{Rj}\right)\right] \tag{13}$$

Second battle model is mixed continual-discrete, also respects Launcester equations. In the time interval $0 \le \tau \le \Delta\tau - \varepsilon$, blue forces strength is described also as the sudden attack, similar as in Eqns (2) and (4), in the form

$$\frac{d\mu'_{B(1)}}{d\tau} = -\frac{\mu'_{R(1)}\mu'_{B(1)}}{\phi_{01}} \tag{14}$$

$$\frac{d\mu'_{B(i+1)}}{d\tau} = -\frac{\mu'_{R(i+1)}}{\phi_{0i+1}} \tag{15}$$

for conditions $\mu'_{B(k)}(\tau = 0) = 1$, $\mu'_{R(k)}(\tau) = 1$, $\mu'_{Bk} = \mu'_{B(k)}(\tau \approx \Delta\tau)$, in the continual part of model. Time interval for the discrete shot event, Δt^*, is connected with no dimensional time, ε, by a relation

$$\Delta t^* = \frac{\varepsilon}{\sqrt{\alpha_B\alpha_R}} \tag{16}$$

Remained strength of red targets is derived as the integral of Eqns. (1) and (3) for the infinitesimal value of no dimensional time ε and $\mu_{Bk} = 0$ as the expression

$$\mu_{Rk} = 1 - \mu'_{Bk}\phi_{0k}\varepsilon, \text{ valid for the } \Delta\tau - \varepsilon < \tau \le \Delta\tau \tag{17}$$

This model is derived for the maximum effect of remained blue forces.

4. SOLUTION OF MIXED CONTINUAL-DISCRETE MODELS OF SUCCESSIVE SALVOS BATTLE

For the discrete shooting events, solutions of Eqns. (14) and (15) are given in the form

$$\mu'_{B1} = e^{-\frac{\Delta\tau}{\phi_{01}}}, \qquad \mu'_{B1} \neq 0 \tag{18}$$

$$\mu'_{Bi+1} = 1 - \frac{\Delta\tau}{\phi_{0i+1}}, \; (\mu'_{Bi+1} \neq 0, \; \Delta\tau < \phi_{0i+1}) \tag{19}$$

From the solutions, Eqns (18) and (19), by successive replacment, in to Eqn. (17), and after elimination of time interval $\Delta\tau$, gives expression of forces strengths during the salvo battle is given as

$$\frac{1-\mu_{Ri+1}}{\phi_{01}\ln\mu'_{B1} + \phi_{0i+1}} = \frac{1-\mu_{R1}}{\phi_{01}\mu'_{B1}} \tag{20}$$

Superiority of each *i+1* salvo ϕ_{0i+1}, as the function of the first salvo superiority ϕ_{01} is

$$\phi_{0i+1} = \phi_{01}\left[(1-\mu_{Ri+1})\frac{\mu'_{B1}}{1-\mu_{R1}} - \ln\mu'_{B1}\right] \tag{21}$$

Also, the number of missile groups in subsequent salvos is similar with Eqn (13) and is given in the form

$$M_{Bi+1} = M_{B1}\left[(1-\mu_{Ri+1})\frac{\mu'_{B1}}{1-\mu_{R1}} - \ln\mu'_{B1}\right]\left(\prod_{j=2}^{i}\mu_{Rj}\right) \tag{22}$$

Total superiority of discrete singular model of multi-salvo battle is extended from Eqn. (5), also as for the continual battle Eqn. (12), and is given in the form

$$\phi_0 = \phi_{01}\left\{1 + \sum_{i=1}^{n}\left[(1-\mu_{Ri+1})\frac{\mu'_{B1}}{1-\mu_{R1}} - \ln\mu'_{B1}\right]\prod_{j=2}^{i}\mu_{Rj}\right\} \tag{23}$$

Particular salvo superiority ϕ_{0n+1} where all red forces vanished is

$$\phi_{0n+1} = \phi_{01}\left(\frac{\mu'_{B1}}{1-\mu_{R1}} - \ln\mu'_{B1}\right), \; \mu_{Rn+1} = 0 \text{ for the n+1} > 1 \tag{24}$$

Simultaneous shots in the second part of discrete events during battle, where circular error probably (CEP) of one missile is less than target silhouette means that the group of two missiles, taken as one blue combatant, assign and shot one target maximum probability. The consequence of this is that product of blue attrition rate and instant of infinitesimal time value Δt^* satisfies $\alpha_B \Delta t^* = 1$. Using Eqn. (16) and substituting in to Eqn. (17), with this condition, superiority, in fact, disappeared, and only the forces number ratio in salvo decides about final remained strength of the reds. This equation is

$$\mu_{Rk} = 1 - \mu'_{Bk}\frac{M_{Bk}}{m_{Rk-1}} \tag{25}$$

5. SIMULATION TEST SUPPOSITIONS AND RESULTS

Simulation model is realised according to the flow chart given in Fig. 1, and follows up built scenario, and mathematical model derived for the mixed model of continual and discrete shooting events in one and in multi-layer salvos. The number of the blue missiles was constrained by the battle unit capacity and is supposed to be M_{Bmax}*=12* groups with 2 AFM in each group, attacked on the M_{Rmax}*=10* concentrated point red targets. The maximum number of missiles groups M_{BK} organised in each repeated K^{th} salvo were constrained to 4, regarding expected guidance and navigation capacity of combat unit less number of groups as combatants (8 missiles total as the maximum value in one salvo), depends on remained number of missiles and targets. A simulation test is also considered, hypothetically, more missile groups in the first salvo, to test, possible number of destroyed targets and is the optimal superiority relation for the continual battle of guerilla warfare valid.

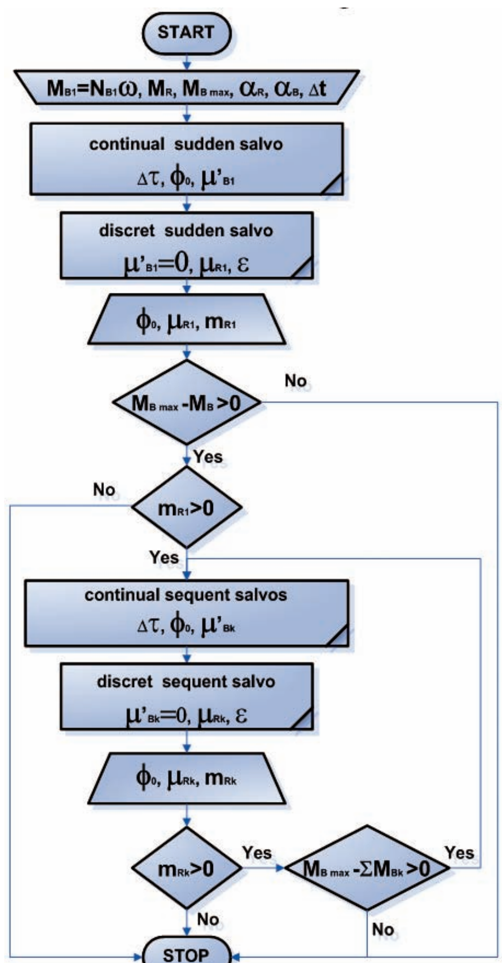

Figure 1. Flow chart of salvo shouting simulation.

The results of calculations in simulated model are obtained using standard software packages, MathCAD and Mat Lab Simulations, and expressed by Excel standard software package. Some of the tests examples, for remained strength of blues and reds on time, by the mixed continual-discrete shooting, are shown in Fig. 2 for one, sudden salvo (as referred scenarios S1, S2, S3 on the graphs) and in Fig. 3, also, by referred scenarios S4, S5, S6, S7 on the graphs for the multi-layer salvos. In both the models, reds do not suffer looses during their continual fire by anti-aircraft weapons (AA), but have suffered in discrete shoots of remained blue missiles. Battles vary number of targets, missile groups and salvos on the figures.

Battle salvo no dimensional time, $\Delta\tau$ was varied between 0.125-0.3, in the particular single- and multi-salvos launching

Table 1. Simulation data

Value / Forces	Fire frequency [λ]	Number of pcs./miss. [ω]	Probabilities p per req. number	Shooting distance [m]	Velocity of missiles [m/s]	Number of dimensional battle time Δτ	Attrition rate α	Number of forces variations $N_B(M_B)$, M_R
Blue forces (group of missiles) M_B	1.25	2	0.247/2	1200	80	0.125	0.31	N_{Bi}=4,8,., 24 (M_{Bi}=2-12)
Red forces (targets) M_R	0.885	3	0.015/3	1200	1000	0.125	0.57	M_R=6, 8, 10

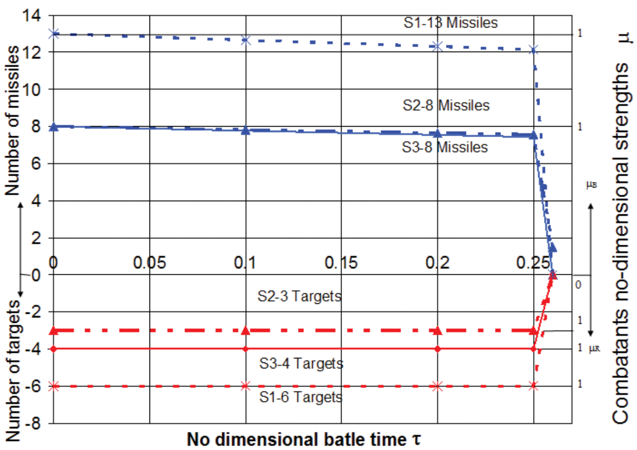

Figure 2. Sudden attacks with different numbers of missiles in the salvo.

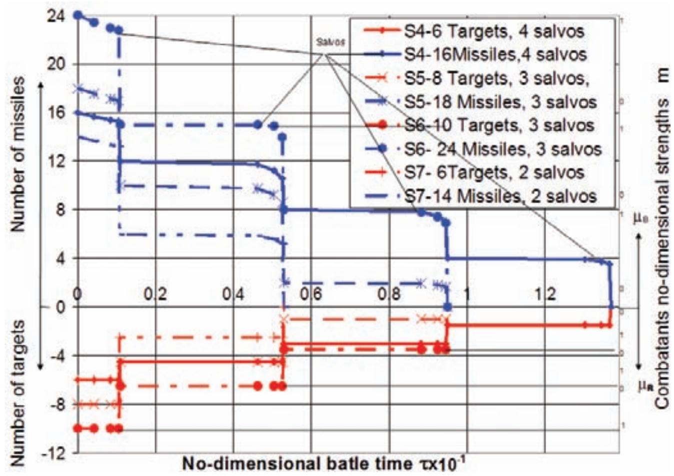

Figure 3. Multi-salvo attacks with different numbers of missiles in the salvo.

scenarios depends on approaching suppositions of the missiles groups. Test was also designed to vary initial number of targets given in Figs 2 and 3 and shown on the negative part of ordinate axes. Data used in simulations are determined in the Table 1 and these are taken from the group of authors[20] as the literature example for the similar unmanned aerial systems.

6. CONCLUSIONS AND DISCUSSION

Mathematical model for the continual battle is derived for the blue forces designed in defending guerilla (ambush) attack. The model is redesigned in to the discrete shooting respecting performances of AFM, and by eliminating infinitesimal singular time which represents their simultaneous approaching on the targets.

Multi-layer salvo attack is derived and tested by superiority parameters and combatants strengths, based on, expected influence of sudden attacks effect. Simulation test was given, which showed, as the example, how many repeated salvos and combatants could be required to annihilate on one side by consumption of other side combatants' in different particular cases, of their initial strengths.

Respecting performances of the combatants, model of salvo losses and consumptions is successfully mathematically derived and tested for the mixed continual and discrete shooting events.

Continual modelling of salvo events shows increasing superiority of particular shots in each of the subsequent salvos but it missed explanation of sudden effect realised by condition $\alpha_B \Delta t^* = 1$ happened as possible in final discrete shots.

In the mixed discrete-continual model, the superiority parameter for the final salvo, at the end of battle, depends only on the initial, first sudden salvo strengths and superiority in the first salvo derived by Eqn. (24). That means that sudden effect in the first salvo exposes maximum influence on the last salvo.

Supposed weapon superiority of the blue forces seems less important, which is not appropriate by assumed conditions, but number of missiles in the group as one combatant, determines this item as the important.

Theoretical approach of continual modelling is inappropriate for the salvo attack by simultaneously approaching missiles, where the number of red combatants' has to be less than the number of blue combatants' missiles groups.

Superiority parameters do not have the same meanings and values applied for the same conventional models if these are mathematically expressed for the discrete and continual battles.

Research has led to the designing of battle units capacities regarding their engagement number of targets. Salvo considerations could determine employed force's tactics in using AFM weapon.

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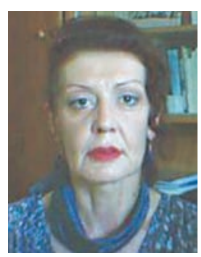

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