Defence Science Journal, Vol. 65, No. 1, January 2015, pp. 70-76, DOI : 10.14429/dsj.65.5474

Range-only Target Localisation using Geometrically Constrained Optimisation

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ABSTRACT

The problem of optimal range-only localisation of a single target is of considerable interest in two-dimensional search radar networking. For coping with this problem, a range-only target localisation method using synchronous measurements from radars is presented in the real ellipsoidal earth model. In the relevant radar localisation scenario, geometric relationships between the target and three radars were formed. A set of localisation equations was derived on range error in such a scenario. Using these equations, the localisation task has been formulated as a nonlinear weighted least squares problem that can be performed using the Levenberg- Marquardt (LM) algorithm to provide the optimal estimate of the target's position. To avoid the double value solutions and to accelerate the convergence speed for the LM algorithm, the initial value was approximately given according to observations from two radars. In addition, the relative validity has been defined to evaluate the performance of the proposed method. The performance of the proposed approach is evaluated using two simulation experiments and a real-test experiment, and it has been found to possess higher localisation accuracy than the other conventional method.

Keywords: Range-only target localisation, target localisation, nonlinear optimisation, Cramer-Rao lower bound, Levenberg-Marquardt algorithm, radar networking, two-dimensional search radar

1. INTRODUCTION

Two-dimensional (2-D) search radar networking, which is generally utilised in the first line of air defence, has been widely applied in modern airspace control[1-3]. This fact generally has two reasons: on one hand, 2-D radars are cheaper than three-dimensional (3-D) radars; on the other, single 2-D radar can't directly determine a target's position since it is merely able to obtain the range and azimuth without the height of the target. Consequently, it is a valuable task-how to utilise the resources and measurements from multiple 2-D radars to estimate the position of a target. To simplify the description, all radars below denote two-dimensional search radars.

According to the type and working mode of sensors, the target localisation methods mainly form three categories: active localisation using range[4-7], passive localisation using azimuth[8-11], and combined localisation using both range and azimuth[12-15]. Moreover, three factors are often taken into consideration in radar networking[4,12]:

(a) a radar possesses higher precision in range than in azimuth since its emission lobes approximating to spindles have certain widths in the horizontal plane;
(b) range error has less influence on localisation results than azimuth error in propagation; and
(c) a localisation dead area exists near the baseline between two radars while only two radars are employed to locate a target. For the above reasons, this research utilises three radars and their range for target localisation.

In the traditional localisation methods, only one assumption is given for simplifying the design procedures as one such assumption is, all radars are placed in the same horizontal plane and their north direction is consistent or parallel to each other by ignoring the earth's curvature. However, these assumptions are often unsatisfied in big scenes due to the wide coverage of radar networks[15]. The other important information such as the types, performances, working modes, number and geometric distribution of radars are omitted, which is the important consideration in real localisation process. In addition, the least squares estimator and its modified approaches are often applied in various applications for target localisation problems[5,7,11,13]. Based on the above facts, a range-only target localisation method is presented in this paper. In the proposed method, the localisation model is established using range from three radars in the real ellipsoidal earth model, and the target localisation problem is formulated as a nonlinear weighted least squares problem through a set of localisation equations driven on range error. Then Levenberg-Marquardt (LM) algorithm is applied to solve the localisation equations. At last, two simulation experiments and a real-date experiment were used to show both the validity and feasibility of the proposed method.

Received 16 December 2013, revised 19 December 2014, online published 27 February 2015

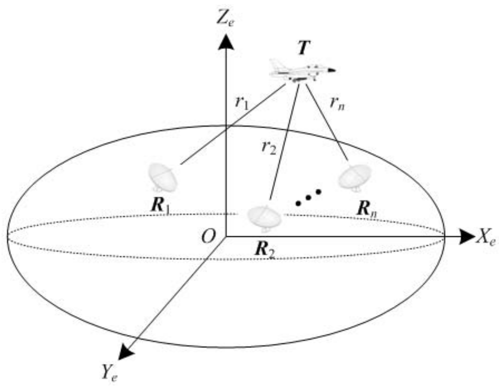


Figure 1. Target localisation model.

2. TARGET LOCATISATION MODEL

As shown in Fig. 1, n radars $\boldsymbol{R}_i$ $(i = 1, 2, \cdots, n)$ are placed at the corresponding geographical coordinates (B_i, L_i, H_i) in the localisation model, where B_i, L_i and H_i denote the longitude, latitude, and height coordinate respectively; and their observations $\boldsymbol{z}_i = (r_i, \theta_i)$ are received simultaneously, where r_i and θ_i denote the range and azimuth of the target $\boldsymbol{T}$. Theoretically, n range spheres with each corresponding radius r_i and its centre at the position $\boldsymbol{R}_i$ intersect at the point, the target's position. Hence, at least three radars are required for range-only target localisation in this case. Therefore, three radars and their range information are applied to locate a target in this research.

Taking the earth curvature into consideration, the geographical coordinate (B_i, L_i, H_i) of each radar should be converted into the geocentric coordinate (X_i, Y_i, Z_i):

$$\begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} = \begin{bmatrix} (N_i + H_i)\cos L_i \cos B_i \\ (N_i + H_i)\cos L_i \sin B_i \\ \left[N_i(1-\rho^2) + H_i\right]\sin L_i \end{bmatrix} \tag{1}$$

where R_e and r_e represent the long and short radii of the earth respectively, $\rho = \sqrt{1 - \frac{r_e^2}{R_e^2}}$ is the earth eccentricity ratio, and

$$N_i = R_e \Big/ \sqrt{1 - \rho^2 \sin^2 L_i}$$

Under the assume of radars with range error, a set of geometrical equations are derived according to the geometrical relationships between the target and three radars in the localisation model as follow:

$$r_i^2 = \left(X - X_i\right)^2 + \left(Y - Y_i\right)^2 + \left(Z - Z_i\right)^2, i = 1, 2, 3 \tag{2}$$

where $\boldsymbol{X} = (X, Y, Z)$ is the true geocentric coordinate of the target. In consideration of range error, Eqn (2) can be rewritten in the following formula:

$$r_i^2 = \left(X - X_i\right)^2 + \left(Y - Y_i\right)^2 + \left(Z - Z_i\right)^2 + 2r_i v_i - v_i^2, i = 1, 2, 3 \tag{3}$$

where v_i is range error of the i th radar. Based on Eqn (3), the localisation equations are derived as follow:

$$d_i(\boldsymbol{X}) = r_i - \sqrt{\left(X - X_i\right)^2 + \left(Y - Y_i\right)^2 + \left(Z - Z_i\right)^2}, i = 1, 2, 3 \tag{4}$$

Furthermore, one can define the objective function in consideration of different influence of each radar's range error on localisation results as follow:

$$F(\boldsymbol{X}) = \sum_{i=1}^{3} \left(d_i(\boldsymbol{X})/\sigma_i\right)^2 \tag{5}$$

where σ_i denotes the standard deviation of range error on the i^{th} radar. As a result, the localisation task can be formulated as a nonlinear weighted least squares problem to provide the optimal estimate such as the minimum value for $F(\boldsymbol{X})$.

3. PROPOSED METHOD AND ITS VALIDITY ANALYSIS

To overcome a nonlinear least square problem, many methods have been developed such as the Newton, Gauss-Newton, and LM algorithm. The LM algorithm has the advantages on both the local convergence property of the Gauss-Newton algorithm and the global convergence property of the gradient descent algorithm, and it is one of the most effective algorithms to solve a nonlinear least square problem with small margins[13,16]. As a result, the LM algorithm is applied to solve target localisation problems in the following.

3.1 LM Algorithm and the Initial Value of the Target's Position

3.1.1 The LM algorithm for Solving the Localisation Equations

The optimal estimate for the target's position is the value $\boldsymbol{X}$ while $F(\boldsymbol{X})$ obtains its minimum value. To minimise $F(\boldsymbol{X})$, the vector is defined as:

$$\boldsymbol{d}(\boldsymbol{X}) = \left(d_1(\boldsymbol{X}), d_2(\boldsymbol{X}), d_3(\boldsymbol{X})\right)^{\mathrm{T}} \tag{6}$$

Moreover, introducing the diagonal matrix Λ, whose main diagonal elements are σ_1^2, σ_2^2 and σ_3^2 respectively, the objective function (Eqn(5)) can be rewritten as:

$$F(\boldsymbol{X}) = \boldsymbol{d}(\boldsymbol{X})^{\mathrm{T}} \Lambda^{-1} \boldsymbol{d}(\boldsymbol{X}) \tag{7}$$

Then define the Jacobi matrix of $d(\boldsymbol{X})$ at the point $\boldsymbol{X} = (X', Y', Z')$:

$$\boldsymbol{J}(\boldsymbol{X}) = \begin{bmatrix} \frac{\partial d_1(\boldsymbol{X})}{\partial X} & \frac{\partial d_1(\boldsymbol{X})}{\partial Y} & \frac{\partial d_1(\boldsymbol{X})}{\partial Z} \\ \frac{\partial d_2(\boldsymbol{X})}{\partial X} & \frac{\partial d_2(\boldsymbol{X})}{\partial Y} & \frac{\partial d_2(\boldsymbol{X})}{\partial Z} \\ \frac{\partial d_3(\boldsymbol{X})}{\partial X} & \frac{\partial d_3(\boldsymbol{X})}{\partial Y} & \frac{\partial d_3(\boldsymbol{X})}{\partial Z} \end{bmatrix} \tag{8}$$

Given $\boldsymbol{f}_i(\boldsymbol{X}) = d_i(\boldsymbol{X})/\sigma_i$, $i = 1, 2, 3$, and $\boldsymbol{f}(\boldsymbol{X}) = (f_1(\boldsymbol{X}), f_2(\boldsymbol{X}), f_3(\boldsymbol{X}))^{\mathrm{T}}$, the gradient vector for $F(\boldsymbol{X})$ can be obtained as:

$$\boldsymbol{g}(\boldsymbol{X}) = 2\boldsymbol{J}(\boldsymbol{X})^{\mathrm{T}} \boldsymbol{f}(\boldsymbol{X}) \tag{9}$$

The second derivative matrix (Hesse matrix) for $F(\boldsymbol{X})$ is:

$$\boldsymbol{G}(\boldsymbol{X}) = 2\boldsymbol{J}(\boldsymbol{X})^{\mathrm{T}} \boldsymbol{J}(\boldsymbol{X}) + 2\sum_{i=1}^{3} f_i(\boldsymbol{X}) \nabla^2 f_i(\boldsymbol{X}) \tag{10}$$

Setting

$$\boldsymbol{S}(\boldsymbol{X}) = \sum_{i=1}^{3} f_i(\boldsymbol{X}) \nabla^2 f_i(\boldsymbol{X}) \tag{11}$$

And substituting $\boldsymbol{S}(\boldsymbol{X})$ in Eqn (10), yields the following expression:

$$\boldsymbol{G}(\boldsymbol{X}) = 2\boldsymbol{J}(\boldsymbol{X})^{\mathrm{T}}\boldsymbol{J}(\boldsymbol{X}) + 2\boldsymbol{S}(\boldsymbol{X}) \tag{12}$$

To solve Eqn (12) for $\boldsymbol{X}$, the recursive formularies of the Newton algorithm[11] is introduced:

$$\left(\boldsymbol{J}_k^{\mathrm{T}}\boldsymbol{J}_k + \boldsymbol{S}_k\right)\delta_k = -\boldsymbol{J}_k^{\mathrm{T}}\boldsymbol{f}_k \tag{13}$$

$$\boldsymbol{X}_{k+1} = \boldsymbol{X}_k + \delta_k \tag{14}$$

where δ_k is the k^{th} iterative step length. Due to the large computational complexity on $\boldsymbol{S}_k$ and the possible singularity on $\boldsymbol{J}_k^{\mathrm{T}}\boldsymbol{J}_k$ in the Newton algorithm, Levenberg and Marquardt utilised the following Equ (15) to replace Eqn (13) for calculating δ_k [14]:

$$\left(\boldsymbol{J}_k^{\mathrm{T}}\boldsymbol{J}_k + \mu_k\boldsymbol{I}\right)\delta_k = -\boldsymbol{J}_k^{\mathrm{T}}\boldsymbol{f}_k, \tag{15}$$

where $\boldsymbol{I}$ stands for the unit matrix, and $\mu_k \ge 0$ refers to the control parameter in the iterative process. Consequently, Eqn (14) together with Eqn (15) is called the LM algorithm. The LM algorithm is applied in the proposed method to solve the localisation equations. As a result, the main design procedure of the proposed method is illustrated as Table 1.

Table 1. Design procedure by the proposed method

1: **Input:** Initialise the target's position $\boldsymbol{X}_0$, the allowable error $\varepsilon > 0$, the suitable control parameter μ.
2: **Output:** the target's position $\boldsymbol{X}_k$ at time $k = 1, 2, \cdots$.
3: Initialise the localisation model: the geocentric coordinate (X_i, Y_i, Z_i), range r_i and range error σ_i of each radar $\boldsymbol{R}_i$, the diagonal matrix Λ, and the unit matrix $\boldsymbol{I}$.
4: **while** $\|\delta_k\| < \varepsilon$ do
5: Calculate the value of the functions d_i^k and $\boldsymbol{f}_k$, and the Jacobi matrix $\boldsymbol{J}_k$.
6: Solve δ_k according to Eqn (15).
7: Update $\boldsymbol{X}_k = \boldsymbol{X}_k + \delta_k$ and $k = k + 1$;
8: **end**

3.1.2 The Initial Value of the Target's Position

To avoid double value solutions[16] and accelerate the convergence speed for the LM algorithm, the initial value $\boldsymbol{X}_0$ for the proposed method, which is the approximate position of the target $\boldsymbol{T}$, can be calculated according to the observations $\boldsymbol{z}_i = (r_i, \theta_i)$ of two radars selected from three radars. Referring to Fig. 2, a local Cartesian coordinate system was constructed, where the position of the radar $\boldsymbol{R}_1$ is as the origin $\boldsymbol{O}$ and x axis is coincident or parallel to the north direction. The position of the radar $\boldsymbol{R}_2$ is denoted as $\boldsymbol{A}$. In the coordinate system, $\boldsymbol{T}'$ is the projection of the target $\boldsymbol{T}$ onto the plane $x\boldsymbol{O}y$. Finally, the coordinate $\boldsymbol{x}_0 = (x_0, y_0, z_0)$ of the target $\boldsymbol{T}$ in the local Cartesian coordinate system can be calculated as:

$$\begin{cases} x_0 = r_0\sin(\theta_2-\theta_0)\cos\theta_1 / \sin(\theta_2-\theta_1) \\ y_0 = r_0\sin(\theta_2-\theta_0)\sin\theta_1 / \sin(\theta_2-\theta_1) \\ z_0 = \left[r_1^2 - r_0^2\sin^2(\theta_2-\theta_0)/\sin^2(\theta_2-\theta_1)\right]^{1/2} \end{cases} \tag{16}$$

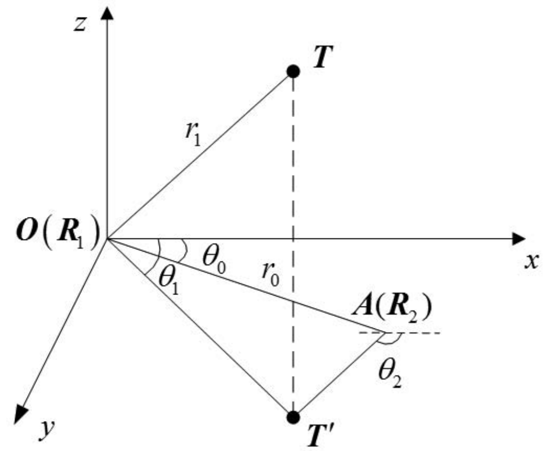


Figure 2. Approximate geometric relations of the target and two radars.

where r_0 and θ_0 are the magnitude and angle of the vector $\boldsymbol{OA}$ respectively. Consequently, the approximate geocentric coordinate $\boldsymbol{X}_0$ can be given by transforming $\boldsymbol{x}_0$ from the local coordinate system to the geocentric coordinate system.

3.2 Validity Analysis of the Proposed Method

The Cramer-Rao lower bound (CRLB)[6,7,12] is the minimum mean square error (MMSE), which the unbiased estimators can reach, and it is normally used as a performance metric for localisation methods. In this paper, the localisation CRLB is driven, and the relative validity is defined as the ratio of MMSE of the proposed method to the CRLB for estimating its localisation results as below.

3.2.1 The CRLB on Localisation Results

Express $\boldsymbol{Z} = (X, Y, Z)^{\mathrm{T}}$ as the estimate of the target's position and $\Lambda\boldsymbol{Z}$ as the likelihood function of the observation set $\boldsymbol{Z} = \{r_i, i = 1, 2, 3\}$, which is given as follow:

$$\Lambda\boldsymbol{Z} = p(\boldsymbol{Z}|\boldsymbol{X}) = \prod_{i=1}^{3} p(r_i \mid \boldsymbol{X}) \tag{17}$$

The CRLB on localisation results is determined in the following formula:

$$\boldsymbol{P}_{CRLB}^{-1} = E\left\{[\nabla_{\boldsymbol{X}} \ln(\Lambda\boldsymbol{Z})]^{\mathrm{T}}[\nabla_{\boldsymbol{X}} \ln(\Lambda\boldsymbol{Z})]\right\}_{\boldsymbol{X}=\boldsymbol{X}_{true}} \tag{18}$$

where $\boldsymbol{X}_{true}$ is the true position of the target. The Gaussian density of the range r_i is obtained by the following equation:

$$p(r_i \mid \boldsymbol{X}) = \frac{1}{\sqrt{2\pi\sigma_i^2}}\exp\left\{-\frac{1}{2}\left[\frac{(r_i - f_i(\boldsymbol{X}))^2}{\sigma_i^2}\right]\right\} \tag{19}$$

Then the logarithm likelihood function of the observation set $\boldsymbol{Z}$ can be expressed as:

$$\ln(\Lambda\boldsymbol{Z}) = -\frac{1}{2}\sum_{i=1}^{3}\left[\ln(2\pi\sigma_i^2) + \frac{(r_i - f_i(\boldsymbol{X}))^2}{\sigma_i^2}\right] \tag{20}$$

By substitution of Eqn. (20) into Eqn. (18), $\boldsymbol{P}_{\mathrm{CLRB}}(\boldsymbol{X})$ is simplified as:

$$\boldsymbol{P}_{CRLB}(\boldsymbol{X}) = \left[\left(\boldsymbol{J}^{\mathrm{T}}\Lambda^{-1}\boldsymbol{J}\right)^{-1}\right]_{\boldsymbol{X}=\boldsymbol{X}_{true}} \tag{21}$$

Consequently, $\boldsymbol{P}_{\mathrm{CLRB}}(\boldsymbol{X})$ is used to calculate the relative validity of the proposed method.

3.2.2 *Relative Validity Analysis of the Proposed Method*

The relative validity of the proposed method can be defined as:

$$RE^{CE} = E(\hat{\boldsymbol{X}} - \boldsymbol{X})^2 / \boldsymbol{P}_{CRLB}(\boldsymbol{X}) \tag{22}$$

Here, $E(\hat{\boldsymbol{X}} - \boldsymbol{X})^2$ acts as the mean square error of the proposed method, and it can be approximately expressed as:

$$E(\hat{\boldsymbol{X}} - \boldsymbol{X}_{true})^2 \approx \frac{1}{M}\sum_{i=1}^{N} E(\hat{\boldsymbol{X}}_i - \boldsymbol{X}_{true})^2 \tag{23}$$

where M is the number of the Monte Carlo simulation runs, and $\hat{\boldsymbol{X}}_i$ is the estimate of the target's position for the i^{th} simulation.

4. EXPERIMENT RESULTS AND ANALYSIS

Two simulation experiments and a real-test experiment were carried to estimate the performance of the proposed method in comparison with the least squared target localisation (LSTL) method in the paper[7]. All experiments were conducted using a computer with a dual-core CPU of Intel Pentium 4 2.93 GHz, 1 GB RAM. The program for each experiment was performed using the MathWorks MATLAB 2009a version software.

4.1 Two Simulation Experiments

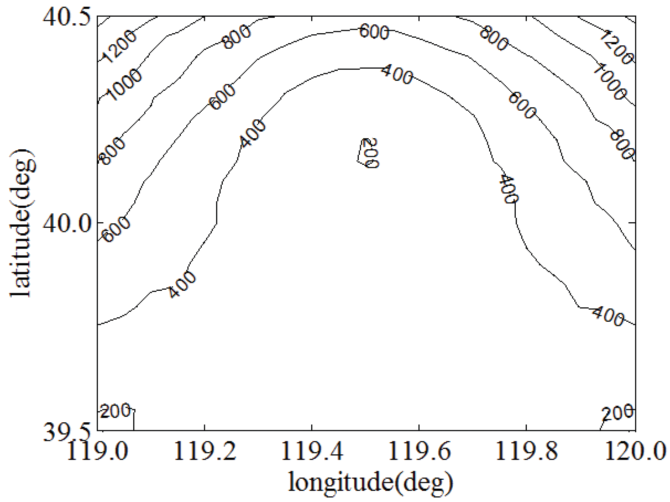
The detection area, its longitude ranging from 119.0° to 120.0° and its latitude from 39.5° to 40.5°, is equally divided into 21*21 grids. Three radars locate at (119.0°, 39.5°, 0.0 km), (120.0°, 39.5°, 0.0 km) and (119.0°, 40.2°, 0.0 km) respectively. The standard deviation σ_r of each radar's range error is 100 m. To intuitively reflect the localisation performance, geometrical dilution of precision (GDOP)[17] was applied to describe the 3D geometric distribution of the localisation error :

$$\mathrm{GDOP} = \sqrt{\sigma_X^2 + \sigma_Y^2 + \sigma_Z^2}\ , \tag{24}$$

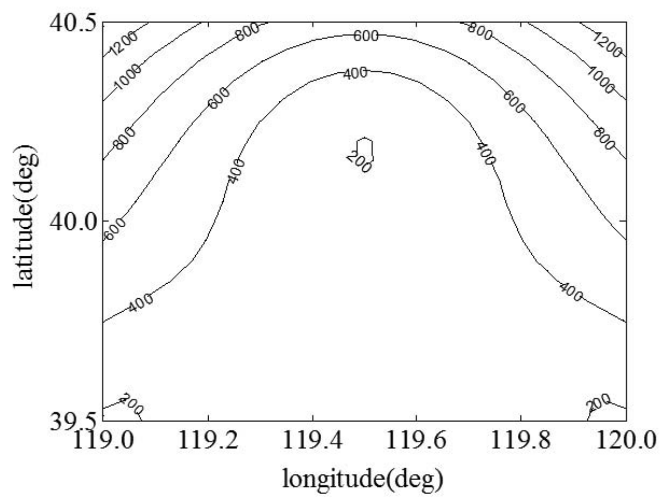
where σ_X, σ_Y and σ_Z are the standard deviations of location errors in direction of X, Y ,and Z axes, respectively in the geocentric coordinate system.

Figures 3(a) and 3(b) provide the GDOP of both the proposed method and the CRLB, respectively. Fig. 4 shows the relative validity RE^{CE} of the proposed method according to Eqn. (22). As can be seen from Fig. 3, the GDOP of the proposed method is much closed to that of the CRLB. It can be seen from Fig. 4, the values of RE^{CE} range from 0.9 to 1.1. Based on the above facts, it verified that, the proposed method can approximate the localisation accuracy of the CRLB.

To compare the proposed method with the LSTL method, six target positions were located at the same longitude 119.5° and latitude 40.0° but for different heights ranging from 6



(a)



(b)

Figure 3. GDOP contours by two methods: (a) the proposed method and (b) the CRLB.

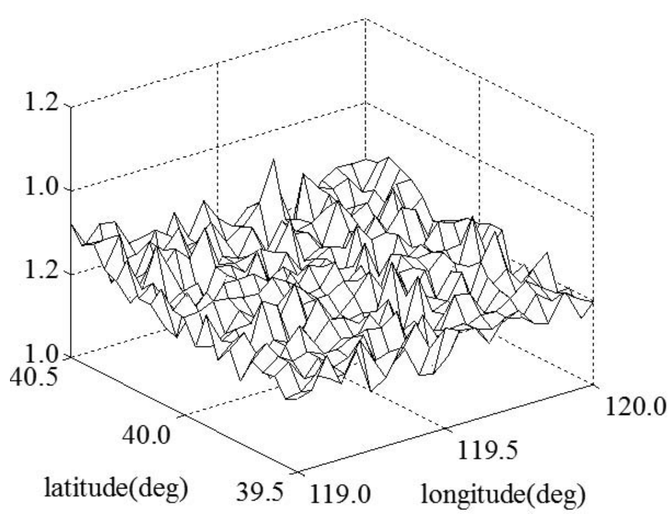


Figure 4. Relative validity RE^{CE} of the proposed method.

km to 16 km with equal intervals. Figure 5 shows the root mean-squared errors (RMSE) for the target's position using two methods for 100 Monte Carlo simulation runs. As is seen from Fig. 5, the RMSE of the proposed method are smaller than the LSTL method. Therefore, due to range with higher precision than azimuth, the LM algorithm can provide good estimates for the target's position through the establishment of the localisation equations using range in the real ellipsoidal earth model.

4.2 Real-test Experiment

The real-test experiment was evaluated using real-tracking data, which is generated from of three radars (ID=RI, RII, and RIII) listed in Table 2. The radars are stationed at (119.5150°, 31.75428°, 0.0 km), (119.9560°, 31.96631°, 0.0 km) and (119.5010°, 27.99498°, 0.0 km), respectively. Due to the restricted test condition, the true positions of the targets are

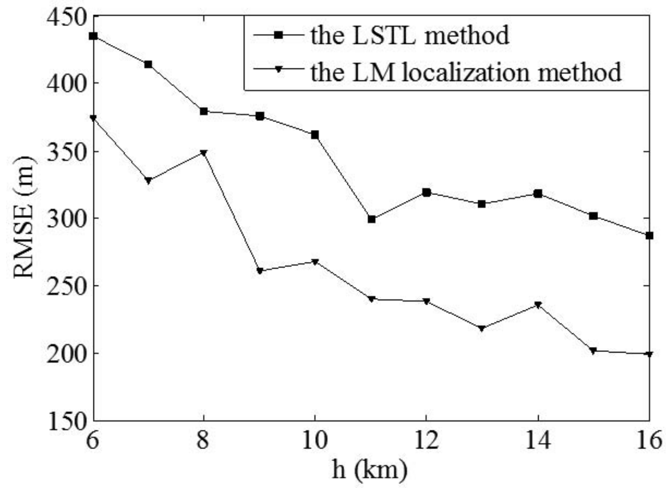


Figure 5. Comparison of localisation error by two methods.

Table 2. Localisation results by the proposed method

No.	RI	RII	RIII	Localisation results
Row1	(127.4 km, 321.1º)	(171.2 km, 295.8º)	(142.2 km, 307.7º)	(119.5975º, 30.10883º, 3555 m)
	(122.0 km, 321.8º)	(165.6 km, 295.5º)	(136.5 km, 307.8º)	(119.5999º, 30.12459º, 3553 m)
	(119.3 km, 322.1º)	(162.9 km, 295.2º)	(133.8 km, 307.7º)	(119.6010º, 30.13263º, 3551 m)
	(116.6 km, 322.6º)	(160.1 km, 295.0º)	(131.0 km, 307.8º)	(119.6021º, 30.14066º, 3549 m)
	(113.9 km, 323.0º)	(157.4 km, 294.8º)	(128.3 km, 307.9º)	(119.6031º, 30.14870º, 3547 m)
	(111.2 km, 323.4º)	(154.8 km, 294.6º)	(125.3 km, 307.8º)	(119.6038º, 30.15554º, 3545 m)
	(108.6 km, 323.8º)	(151.9 km, 294.4º)	(122.6 km, 307.9º)	(119.6050º, 30.16387º, 3543 m)
	(105.9 km, 324.2º)	(149.1 km, 294.1º)	(129.9 km, 308.0º)	(119.6064º, 30.20248º, 3522 m)
	(103.2 km, 324.8º)	(146.4 km, 293.9º)	(116.9 km, 307.9º)	(119.6068º, 30.17934º, 3538 m)
	(100.6 km, 325.3º)	(143.6 km, 293.7º)	(114.3 km, 308.0º)	(119.6078º, 30.18768º, 3534 m)
Row2	(207.0 km, 145.9º)	(131.1 km, 178.2º)	(169.7 km, 123.7º)	(120.2639º, 30.33694º, 4466 m)
	(204.4 km, 146.1º)	(128.4 km, 177.9º)	(166.7 km, 123.8º)	(120.2172º, 30.34018º, 4409 m)
	(199.1 km, 146.6º)	(123.0 km, 177.2º)	(161.1 km, 123.7º)	(120.1298º, 30.34829º, 4289 m)
	(196.5 km, 146.9º)	(120.4 km, 176.9º)	(158.3 km, 123.6º)	(120.0927º, 30.35384º, 4233 m)
	(193.8 km, 147.2º)	(117.7 km, 176.6º)	(155.5 km, 123.6º)	(120.0565º, 30.36031º, 4178 m)
	(191.2 km, 147.5º)	(114.9 km, 176.2º)	(152.7 km, 123.6º)	(120.0256º, 30.36825º, 4128 m)
	(188.6 km, 147.9º)	(112.2 km, 175.8º)	(149.9 km, 123.5º)	(119.9952º, 30.37357º, 4079 m)
	(186.1 km, 148.1º)	(109.5 km, 175.3º)	(147.1 km, 123.5º)	(119.9723º, 30.37970º, 4034 m)
	(183.5 km, 148.4º)	(106.8 km, 174.9º)	(144.4 km, 123.4º)	(119.9498º, 30.38749º, 3993 m)
	(180.8 km, 148.8º)	(104.2 km, 174.4º)	(141.5 km, 123.3º)	(119.9308º, 30.39474º, 3954 m)
Row3	(97.9 km, 295.3º)	(139.8 km, 307.0º)	(86.5 km, 320.9º)	(119.6078º, 30.12378º, 3570 m)
	(95.2 km, 294.7º)	(137.0 km, 306.9º)	(83.7 km, 321.1º)	(119.6084º, 30.13151º, 3567 m)
	(92.5 km, 294.2º)	(134.1 km, 306.6º)	(80.9 km, 321.4º)	(119.6093º, 30.13901º, 3565 m)
	(89.8 km, 293.5º)	(131.4 km, 306.3º)	(78.1 km, 321.7º)	(119.6100º, 30.14754º, 3562 m)
	(84.4 km, 292.3º)	(125.8 km, 306.2º)	(72.4 km, 322.3º)	(119.6113º, 30.16232º, 3557 m)
	(81.8 km, 291.8º)	(122.9 km, 306.0º)	(69.9 km, 322.9º)	(119.6122º, 30.17198º, 3553 m)
	(79.2 km, 291.2º)	(120.1 km, 306.0º)	(67.0 km, 323.4º)	(119.6127º, 30.17870º, 3550 m)
	(76.6 km, 290.5º)	(117.4 km, 306.0º)	(64.5 km, 324.0º)	(119.6136º, 30.18804º, 3546 m)
	(74.0 km, 289.9º)	(114.7 km, 306.0º)	(61.7 km, 324.6º)	(119.6141°, 30.19589°, 3542 m)
	(97.9 km, 295.3º)	(139.8 km, 307.0º)	(86.5 km, 320.9º)	(119.6078º, 30.12378º, 3570 m)
	(95.2 km, 294.7º)	(137.0 km, 306.9º)	(83.7 km, 321.1º)	(119.6084º, 30.13151º, 3567 m)

unknown. The radar performance parameters are represented as follow: the maximum detection distance 230.0 km, range error 200.0 m and azimuth error 1°. Table 2 lists the localisation results by the proposed method. As can be seen from Table.2, the propose method has good performance in estimate accuracy on the targets' positions. Therefore, it is verified that the proposed method is feasible to locate a target in real case.

5. CONCLUSIONS

In this paper, the characteristics and shortcomings of the traditional target localisation methods in 2-D search radar networking have been analysed. Due to range with higher precision than azimuth for two-dimensional radars, a range-only target localisation method is presented using synchronous measurements from three radars. Considering the real ellipsoid earth model, the equivalent geometric model is introduced in the proposed method. A set of localisation equations is derived on range error in such a scenario. Consequently, the target localisation problem is converted into a nonlinear weighted least squares problem. The LM algorithm is applied to solve the localisation equations and to estimate the target's position. By giving the initial value approximately, the proposed method is able to avoid the value solutions and accelerate the convergence speed. Furthermore, it can approximate the localisation accuracy of the CRLB through the analysis of the relative validity defined. The simulation results show that the proposed method is effective and has high accuracy, while the real-test result illustrates that the proposed method is feasible.

ACKNOWLEDGEMENTS

This work was supported by the National Natural Science Foundation of China (61271107, 61301074), Key Project in the National Science & Technology Pillar Programme (2011BAH24B12), Specialized Research Fund for the Doctoral Programme of Higher Education (20104408120001), Natural Science Foundation of Ministry of Education (S20120100 09417), Shenzhen Basic Research Project (JCYJ20140418095735618), and Defense Advance Research Fund Project (91400C8005001140C80340).

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