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On the Propagation of Shock Waves Produced by Explosion of a Spherical Charge in Deep Sea

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ABSTRACT

The propagation of spherical shock waves, produced by explosion of a spherical charge in deep sea, have been studied using the energy hypothesis of T.Y. Thomas. The energy release from the charge is supposed to be time dependent and the effects of earth's gravitation is taken into account. It is found that the shock compression varies very slowly with variation in the direction of shock propagation. The effects of earth's gravitation is to decrease the shock compression and its decay with shock radius. A comparison has also been made between the results of time dependent and instantaneous energy release.

1. INTRODUCTION

The study of propagation and attenuation of shock waves produced by explosions in deep sea are of great importance in Anti Submarine Warfare. The shock wave propagation in water has been studied by Bhatnagar[1] *et al.*, Ranga Rao and Ramana[2], Singh[3] *et al.*, Singh[4], and Singh and Srivastava[5] without taking the effects of earth's gravitation.

Singh[3] *et al.* and Singh[4] have studied shock propagation theoretically by Energy Hypothesis[6]. They have compared their results with those obtained by experiments and found that the results by energy hypothesis is quite agreeable with experimental

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data. In most of the works done on underwater explosions the energy release has been assumed to be instantaneous. But the energy release, though very rapid, should be considered to be time-dependent[7,8].

In the present paper, the propagation of spherical shock wave in all directions produced by explosion of a spherical charge in deep sea has been studied. The energy release from the charge is supposed to be time-dependent and the effects of earth's gravitation is taken into account. The variation of shock compression (density ratio across the shock) with shock radius has been obtained by energy hypothesis. It is found that the shock wave decays as shock radius increases. This decay is smaller for time-dependent energy release in comparison with that of instantaneous energy release. The shock compression varies very slowly with the variation in the direction of shock propagation. Also, due to the earth's gravitational force there is a decrease in the shock compression and its decay with shock radius.

2. FORMULATION OF THE PROBLEM

Let us assume that a spherical charge of radius R_0 is fired in sea at depth Z_E from the surface of water. We assume that the shock wave produced, is perfectly spherical and propagate in all directions. Taking the point of explosion (centre of charge) as origin, let R be the radius of shock front at any time t. If θ be the angle made by a shock radius with the vertical direction, then depth z of the point (corresponding to above shock radius) on the shock front, from the water surface is

$$z = z_E - R\cos\theta \tag{1}$$

Equation of state of sea water is given[9] as

$$P = B(s)\left[(\rho/\rho_0)^\gamma - 1\right] \tag{2}$$

where B is a function of entropy s, γ a constant and ρ_0 the density of water at zero pressure. Since the entropy variations in water are small even when very strong shocks are present, B is taken as a constant. If p_z be the pressure and ρ_z be the density at depth z from the water surface, then equation (2) can be written as

$$p - p_z = (B + p_z)\left[(\rho/\rho_z)^\gamma - 1\right] \tag{3}$$

Similarly,

$$p - p_l = (B + p_l)\left[(\rho/\rho_1)^\gamma - 1\right] \tag{4}$$

where p_1 and ρ_1 are the pressure and density on the water surface. The equations of conservation of mass and momentum for a flow of sea water can be written as

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial r}(\rho u) + \frac{1}{r}\frac{\partial}{\partial\theta}(\rho v) + \frac{2\rho u}{r} + \frac{\cot\theta}{r}\rho v = 0 \tag{5}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \theta} + \frac{1}{\rho}\frac{\partial p}{\partial r} + g\cos\theta = 0 \tag{6}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \theta} + \frac{1}{\rho r}\frac{\partial p}{\partial \theta} - g\sin\theta = 0 \tag{7}$$

where u and v are the radial and transversal fluid velocity respectively, and g, the acceleration due to gravity is given[10] by

$$g = g_s\{R_e/R_e - z\}^2 \tag{8}$$

R_e and g_s being the radius of earth and the value of g at the water surface, respectively.

In the stationary conditions, we have from Eqn. (6),

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = -g\cos\theta \tag{9}$$

This equation when combined with Eqns. (4), (8) and (1), gives density and pressure at depth z as

$$\rho_z = \rho_1\left[1 + \frac{KZ}{R_e - z}\right]^{1/\gamma-1} \tag{10}$$

$$P_z = (B + p_1)\left[\left(1 + \frac{KZ}{R_e - z}\right)^{\gamma/\gamma-1} - 1\right] + p_1 \tag{11}$$

where

$$K = \frac{(\gamma - 1)g_s R_e \rho_1}{\gamma(B + p_1)} \tag{12}$$

We assume that the energy release from the charge is not instantaneous, but it depends on time. If Q be the total energy released from the charge, at any time t during the process of energy release, we suppose that

$$Q = Q_0\left[(\beta - 1)\frac{R - R_0}{R' - R_0} + 1\right] \tag{13}$$

where Q_0 is the energy released at $t = 0$, βQ_0 ($\beta \geqslant 1$) the total energy of the charge, R' the shock radius just at the instant when Q becomes equal to βQ_0, and R (the shock radius) a function of time t.

As the shock travels some distance, it deviates from its spherical shape but the deviation is sufficiently small. Here, we assume that the deviation of the shock wave from its spherical shape is negligible. Hence, the transversal velocity v at the shock surface is always tangential to it. This implies that jump in v across the shock surface is zero. The other jump conditions may be written[11] as

$$u_2 = \left[\frac{A_z(\delta^\gamma - 1)(\delta - 1)}{\rho_z\delta}\right]^{1/2} \tag{14}$$

$$U = \left[\frac{A_z(\delta^\gamma - 1)\delta}{\rho_z(\delta - 1)}\right]^{1/2} \tag{15}$$

$$E_2 - E_z = \frac{p_2 u_2}{\rho_z U} - \frac{1}{2}u_2^2 \tag{16}$$

where E is internal energy, U the shock velocity, $A_z = (B+p_z)$ and $\delta = (\rho_2/\rho_z)$. Subscripts 2 and z respectively denote values behind and in front of the shock. At the shock, the Eqn. (3) of state becomes

$$p_2 - p_z = A_z\,(\delta^\gamma - 1) \tag{17}$$

Eqns. (14) to (17) are four relations in five unknowns E_2, δ, p_2, u_2, U. The aim is to find the variation of the shock compression δ with the shock radius R.

3. SOLUTION OF THE PROBLEM

Energy hypothesis is used[6,12,13] to find the relation between δ and R. If $T = E + \frac{1}{2}u^2$ be the total energy per unit mass of water, then

$$T_2 - T_z = \frac{3\alpha Q}{4\pi\rho_2 R^3} \tag{18}$$

where a is constant of proportionality given by the relation

$$\alpha = \lim_{R\to R_0} \frac{4\pi R^3\rho_2}{3Q}(T_2 - T_z) \tag{19}$$

Combining Eqns. (16) and (18), we have

$$\frac{p_2 u_2}{\rho_z U} = \frac{3\alpha Q}{4\pi\rho_2 R^3} \tag{20}$$

Using Eqns. (14), (15) and (17) in the Eqn. (20), we obtain

$$\{A_z(\delta^\gamma - 1) + p_z\}(\delta - 1) = \frac{3\alpha Q}{4\pi R^3} \tag{21}$$

Since at $t = 0$, $R = R_0$ and $Q = Q_0$, the above equation when combined with Eqn. (19) gives,

$$\frac{[A_z(\delta^\gamma - 1) + p_z](\delta - 1)}{[A_z^*(\delta^{*\gamma} - 1) + p_z^*](\delta^* - 1)} = \frac{Q/Q_0}{(R/R_0)^3} \tag{22}$$

where a quantity with superscript '*' denotes its value at $R = R_0$.

To find δ^* we use the mismatch method due to Buchanan and James[14], i.e. at the water explosive boundary we have

$$\frac{p_2}{p_D} = \frac{2\rho_z U}{\rho_D U_D + \rho_z U} \tag{23}$$

where p_D, ρ_D, U_D are detonation pressure, density of explosive charge and detonation velocity, respectively. p_2 and U are functions of δ and p_D, ρ_D, U_D are known quantities for a given explosive. Thus value of δ^* can be evaluated from Eqn. (23). Now, combining Eqn. (22) with Eqn. (13) we have a relation between δ and R for the values of R in the range $R_0 \leq R \leq R'$ as

$$\frac{\left[\frac{A_z}{B}(\delta^\gamma - 1) + \frac{p_z}{B}\right](\delta - 1)}{\left[\frac{A_z^*}{B}(\delta^{*\gamma} - 1) + \frac{p_z^*}{B}\right](\delta^* - 1)} = \frac{\left[(\beta - 1)\frac{\bar{R} - 1}{\bar{R}' - 1} + 1\right]}{(\bar{R})^3}, \tag{24}$$

where $\bar{R} = (R/R_0)$ and $\bar{R}' = (R'/R_0)$. For values of R greater than R' we have the following relation

$$\frac{\left[\frac{A_z}{B}(\delta^\gamma - 1) + \frac{p_z}{B}\right](\delta - 1)}{\left[\frac{A_z^*}{B}(\delta^{*\gamma} - 1) + \frac{p_z^*}{B}\right](\delta^* - 1)} = \frac{\beta}{(\bar{R})^3} \tag{25}$$

Eqns. (24) and (25) give the variation of shock compression δ with shock radius R in a given direction

If energy release is assumed to be instantaneous, the relation between δ and R takes the following form

$$\frac{\left[\frac{A_z}{B}(\delta^\gamma - 1) + \frac{p_z}{B}\right](\delta - 1)}{\left[\frac{A_z^*}{B}(\delta^{*\gamma} - 1) + \frac{p_z^*}{B}\right](\delta^* - 1)} = \frac{1}{(\bar{R})^3} \tag{26}$$

4. RESULTS AND DISCUSSION

For numerical calculations, the values of the parameters used are given[11] as

z_E	= 1000 m		p_D	=	256.78 kb
R'	= 10 m		U_D	=	7.8×10^3 m/sec
R_0	= 0.25 m		ρ_D	=	1.68 g/cm^3
R_e	= 6371.23×10^3 m		ρ_1	=	1 g/cm^3
g_s	= 9.81 m/sec^2		γ	=	7.25
B	= 2.94 kb		β	=	2
p_1	= 1 bar	Therfore	K	=	183.6115

$K = 0$ corresponds to the case when effects of gravity are neglected. Variation of shock compression δ with reduced shock radius $\bar{R}$ is shown in Tables 1 and 2 for $K = 0$, 183.6115 and $\theta = 0$, $\pi/2$, π. Table 1 corresponds to time-dependent energy

Table 1. Variation of shock compression δ with $\bar{R}$ when energy release is time-dependent

$\bar{R}$	δ			
		$K = 183.6115$		
	$K=0$	$\theta=0$	$\theta=\pi/2$	$\theta=\pi$
1.0	1.74680	1.70467	1.70466	1.70465
1.25	1.63335	1.62325	1.62324	1.62323
1.5	1.54906	1.53976	1.53974	1.53974
1.75	1.48361	1.47492	1.47491	1.47491
2.00	1.43114	1.42295	1.42294	1.42293
2.5	1.35199	1.34515	1.34514	1.34513
3.0	1.29516	1.28848	1.28847	1.28847
3.5	1.25240	1.24704	1.24703	1.24702
4.0	1.21934	1.21361	1.21359	1.21358
4.5	1.19280	1.18853	1.18852	1.18851
5.0	1.17154	1.16645	1.16645	1.16644
10	1.07487	1.07128	1.07126	1.07125
15	1.04492	1.04186	1.04184	1.04184
20	1.03118	1.02841	1.02839	1.02838
25	1.02353	1.02093	1.02091	1.02090
30	1.01874	1.01625	1.01623	1.01622
35	1.01547	1.01309	1.01307	1.01306
40	1.01314	1.01083	1.01081	1.01079
45	1.01105	1.00879	1.00879	1.00878
50	1.00944	1.00728	1.00727	1.00726

Table 2. Variation of shock compression δ with $\bar{R}$ when energy release is instantaneous

$\bar{R}$	δ			
		$K = 183.6115$		
	$K=0$	$\theta=0$	$\theta=\pi/2$	$\theta=\pi$
1.00	1.74680	1.70467	1.70466	1.70465
1.25	1.63224	1.62218	1.62217	1.62216
1.5	1.54716	1.53786	1.53785	1.53785
1.75	1.48098	1.47231	1.47230	1.47230
2.00	1.42788	1.41972	1.41971	1.41970
2.5	1.34780	1.34096	1.34095	1.34094
3.0	1.29018	1.28361	1.28360	1.28358
3.5	1.24710	1.24162	1.24161	1.24160
4.0	1.21342	1.20771	1.20771	1.20770
4.5	1.18713	1.18239	1.18238	1.18236
5.0	1.16560	1.16009	1.16008	1.16007
10	1.06821	1.06472	1.06471	1.06471
15	1.03896	1.03593	1.03592	1.03591
20	1.02583	1.02311	1.02311	1.02310
25	1.01862	1.01621	1.01619	1.01617
30	1.01433	1.01191	1.01190	1.01190
35	1.01144	1.00913	1.00912	1.00911
40	1.00935	1.00718	1.00716	1.00715
45	1.00782	1.00576	1.00575	1.00573
50	1.00673	1.00471	1.00469	1.00467

release and Table 2 to instantaneous energy release. It is found that shock compression decreases (i.e. shock wave decays) as shock radius increases. They decay of shock compression is smaller for time-dependent energy release in comparison with that of instantaneous energy release. Also, shock compression decreases very slowly with the increase of θ. It is also found that the earth's gravity has significant effects on the shock propagation. The shock compression and its decay with $\bar{R}$, both are reduced due to consideration of gravity.

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