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A Study of Sectionally Related Dispersion and Chemical Reaction Effects

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ABSTRACT

This theoretical investigation is aimed at finding the influence of the cross-section on the equivalent dispersion coefficient of a solute in a non-Newtonian medium flowing through a channel by considering the Power law, Bingham plastic, Casson models of fluids and it has been noted that in the case of the first two, there is a steep rise with the increase of the width of the channel, but in the case of Casson model, equivalent dispersion coefficient attains a maximum at $R/a = 1.7$. Some explanation is offered for this behaviour of the fluids.

1. NOMENCLATURE

c	concentration of the solute in the fluid
dp/dx	pressure gradient
D	molecular diffusion coefficient
D^*	equivalent dispersion coefficient
J^*	flux used in Fick's law of diffusion
R/a	width of the channel
k	homogeneous chemical reaction rate constant
m	consistency of the power law fluid
n	flow behaviour index (power law index)
$\bar{Q}$	average flux of the solute across a section of the channel
a	characteristic length
(x, y)	coordinate system

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t	time,
	velocity component of the fluid along x-direction,
$\bar{v}$	mean velocity of the fluid along x-direction,
τ	shear stress,
τ_0	yield stress,
μ	consistency of the fluid (in Bingham and Casson models)

1. INTRODUCTION

Inspired by the work of Griffith[1], Taylor[2] observed that one way of investigating the dispersion through a fluid is by considering the dispersion of a solute in a solvent flowing through a tube. He has observed that the solute diffused with an effective dispersion coefficient, relative to a plane moving with the average flow speed, which depends upon the radius of the tube, mean velocity and the molecular diffusion coefficient. He[2,3] also stated the conditions under which dispersion of a solute in a solvent can be utilized to measure the molecular diffusion coefficient. However, in a subsequent work Aris[4] removed the conditions imposed by Taylor. Wageningen[5] suggested a more generalized approach, which was found to be of a particular interest in physiological systems. Since then the studies of dispersion of a solute in a fluid flowing through channels and pipes have assumed growing importance in applications in various chemical and biological systems. Katz[6], Walker[7], Soloman *et al.*[8], Gill *et al.*[9,10], Gupta *et al.*[11] and Scherer *et al.*[12] are some of the workers, who have investigated the dispersion through Newtonian fluids by taking into account various aspects such as steady and unsteady conditions for homogeneous and heterogeneous chemical reactions.

Fan and Hwang[13] expanded the frontiers of the studies into realm of non-Newtonian fluids by considering a power law fluid following Taylor's approach. Later, Fan and Wang[14], Govier and Aziz[15] studied the dispersion through Bingham plastic and Ellis model fluids. Ghoshal[16] and Shah *et al.*[17] considered dispersion through Reiner-Philippoff model and Eyring model fluids respectively. Following Aris[4], Prenosil *et al.*[18] have investigated the dispersion through a power law fluid. Nigam and Vasudeva[19] have studied the diffusion and reaction with non-Newtonian laminar flow in tubular reactor. Recently Shukla *et al.*[20] have studied dispersion through various non-Newtonian fluids. In this paper we have studied the influence of width of the channel and that of the core (if present) on the diffusion and chemical reaction in non-Newtonian fluids, by considering (a) Power law, (b) Bingham plastic, and (c) Casson models.

Amongst the various industrial applications of investigations of this paper mention may be made of jet aircrafts and defence equipments using diffusers. Moreover recent studies have revealed that these investigations have significant bearing on such biological studies as are connected with hemodialyser and molecular transport of oxygen from blood plasma to the living tissues of lungs and brain. All these models are known to explain the behaviour of blood under various conditions of flow.

However, Bingham plastic behaviour is of historical importance though real fluids rarely follow this law. Besides biofluids which come very close to Casson model, the present investigation may be significantly relevant to industrial effluents with time dependent characteristic of visco-elastic systems.

2. BASIC EQUATIONS

Consider the symmetrical flow of a non-Newtonian fluid flowing under a constant pressure gradient in a channel. The x-axis is taken along the flow in the channel and y-axis is at right angles to it. The functions are independent of z.

The constitutive equation for non-Newtonian fluids may be written as Govier *et al.*[15] and Copley[21].

$$\left(\frac{dv}{dy}\right) = f(\tau)$$

where τ is the shear stress and $-\frac{dv}{dy}$ is the shear rate and $f(\tau)$ is a general function prescribed for a given fluid model.

Considering a simple force balance in the fluid in the channel we have following Govier *et al.*[15] and Copley[21].

$$\tau = y\left(-\frac{dp}{dx}\right)$$

and

$$\tau_R = R\left(-\frac{dp}{dx}\right)$$

which gives

$$\frac{\tau}{\tau_R} = \frac{y}{R}$$

where τ_R is the shear stress at the wall.

Integrating Eqn. (1) and using $v = 0$ at $y = R$ on the boundary.

$$v = \int_y^R f(\tau)\, dy = a\int_{y'}^{R/a} f(\tau)\, dy'$$

where $y' = y/a$, $R/a = mx + 1$ and $f(\tau)$ is a function of y'

The average velocity $\bar{v}$ is defined as

$$\bar{v} = \frac{1}{R}\int_0^R v \cdot dy = \frac{a}{R}\int_0^{R/a} v(y')\, dy'$$

which on using Eqn. (5) gives

$$\bar{v} = \frac{a^2}{R}\int_0^{R/a} y' f(\tau)\, dy' \tag{7}$$

$$v - \bar{v} = \bar{v}\, g_0(y') \tag{8}$$

$$g_0(y') = \frac{\displaystyle\int_{y'}^{R/a} f(\tau)\, dy'}{\displaystyle\frac{a}{R}\int_{-}^{R/a} y' f(\tau)\, dy'} \tag{9}$$

Assuming that the solute while diffusing, undergoes an irreversible chemical reaction in the fluid, the equation governing the concentration c of the solute is given by

$$\frac{\partial c}{\partial t} + v\frac{\partial c}{dx} = D\frac{\partial}{\partial y}\left(\frac{\partial c}{\partial y}\right) - kc \tag{10}$$

where D is the constant molecular diffusion coefficient and k is the homogeneous chemical reaction rate constant.

Following Taylor[2], Eqn. (10), relative to a plane moving with the mean speed of the flow, can be written as $(\bar{x} = x - \bar{v}t)$.

$$\frac{\partial c}{\partial t} + (v - \bar{v})\frac{\partial c}{\partial \bar{x}} = D\frac{\partial}{\partial y}\left(\frac{\partial c}{\partial y}\right) - kc \tag{11}$$

Introducing the following non-dimensional variables

$$\theta = t/t', \quad t' = x_0/\bar{v}, \quad x' = x/a, \quad y' = y/a \tag{12}$$

Eqn. (11) can be transformed, after using Eqns. (8) and (12)

$$\frac{}{t'}\frac{\partial c}{\partial \theta} + \bar{v}g_0(y') + \frac{1}{a}\frac{\partial c}{\partial x'} = \frac{D}{a^2}\frac{\partial}{\partial y'}\left(\frac{\partial c}{\partial y'}\right) - kc \tag{13}$$

Dropping the primes over the variables of space and time for convenience and assuming that the Taylor's[2] limiting condition is valid, i.e.,the partial equilibrium over any cross-section of the channel is established, the Eqn. (13) may be approximated as

$$\frac{\partial}{\partial y}\left(\frac{\partial c}{\partial y}\right) - \alpha^2 c = \frac{a}{D}\,\bar{v}\,\frac{\partial c}{\partial x}\, g_0(y) \tag{14}$$

where

$$\alpha^2 = \frac{ka^2}{D}$$

Since there is no chemical reaction at the wall the conditions for concentration c can be written as

$$c_0(0) = 0, \quad \frac{\partial c}{\partial y} = 0 \quad \text{along the wall} \tag{15}$$

Eqn. (14) is a generalized Bessel equation and its solution under conditions (15) gives the concentration profile as follows:

$$c = \frac{a\bar{u}}{D}\frac{\partial c}{\partial x}\, c_0(y)$$

$$c_0(y) = A_0 \cosh \alpha y + \frac{1}{\alpha}\int_0^y \sinh \alpha(y - t)\, g_0(t)\, dt \tag{17}$$

$$A_0 = -\frac{1}{\alpha \sinh \alpha}\int_0^{R/a} \cosh \alpha\left(\frac{R}{a} - t\right) g_0(t)\, dt$$

where $g_0(t)$ is the function defined by Eqn. (9).

Now the average solute flux $\bar{Q}$, across the plane moving with the mean speed of the flow can be written as

$$\bar{Q} = \frac{a\bar{u}^2}{D}\left(\frac{\partial c}{\partial x}\right)\int_0^{R/a} c_0(y)\, g_0(y)\, dy$$

Comparing Eqn. (19), with Fick's law of diffusion

$$J^* = -\frac{D^*}{a}\left(\frac{\partial c}{\partial x}\right)$$

after deleting primes, the equivalent dispersion coefficient D^* is given by

$$D^* = -\frac{a^2\bar{u}^2}{D}\int_0^{R/a} c_0(y)\, g_0(y)\, dy$$

Now we shall take up the cases of three different models of fluid.

3. PARTICULAR CASES

3.1 Power Law Model

In this case we consider the dispersion in a power law fluid flowing through the channel. The function $f(\tau)$ in this case is given by Govier *et al.*[15] and Copley[21]

$$f(\tau) = \left(\frac{\tau}{m}\right)^{1/n} \tag{21}$$

where m is the consistency and n is the flow behaviour index of the fluid.

Using Eqn. (21) for $f(\tau)$ in Eqns. (9), (17), (18), and (20), we get the expression for $g_0(y)$, concentration profile and effective dispersion coefficient as follows:

$$g_0(y) = \frac{2n+1}{n+1}\left\{ - \left(\frac{ay}{R}\right)^{1+(1/n)}\right\} - 1 \tag{22}$$

$$c_0(y) = A_0 \cosh \alpha y + \frac{1}{\alpha}\int_0^y \sinh \alpha(y - t)\left[\frac{2n+1}{n+1}\left\{1 - \left(\frac{at}{R}\right)^{1+(1/n)}\right\} - 1\right] dt \tag{23}$$

$$A_0 = -\frac{1}{\alpha \sinh \alpha}\int_0^{R/a} \cosh \alpha\left(\frac{R}{a} - t\right)\left[\frac{2n+1}{n+1}\left\{1 - \left(\frac{at}{R}\right)^{1+(1/n)}\right\} - 1\right] dt \tag{24}$$

$$D^* = \frac{a^2\bar{u}^2}{D} M_0(\alpha, n) \tag{25}$$

where

$$M_0(\alpha, n) = -\int_0^{R/a} c_0(y)\left[\frac{2n+1}{n+1}\left\{1 - \left(\frac{ay}{R}\right)^{1+(1/n)}\right\} - 1\right] dy \tag{26}$$

Taking $n = R/a = 1$ in Eqn. (26), we can get the result of Gupta *et al.*[11]. Taking the limit when $a \to 0$ in Eqn. (26), we have

$$M_0(0, n) = -\left(\frac{n}{n+1}\right)^2 \frac{(R/a)^3}{6} + \frac{n^2}{(4n+1)(n+1)^2}\left\{\frac{8n^2+5n+1}{2(3n+1)}\right\}\left(\frac{R}{a}\right)^3$$
$$\frac{n^3(2n+1)}{(n+1)^2(3n+1)(5n+2)}\left(\frac{R}{a}\right)^3 \tag{27}$$

Taking $n = R/a = 1$ in Eqn. (27) and substituting the resulting value of $M_0(0,1)$ in Eqn. (25), we get Wooding[22] result.

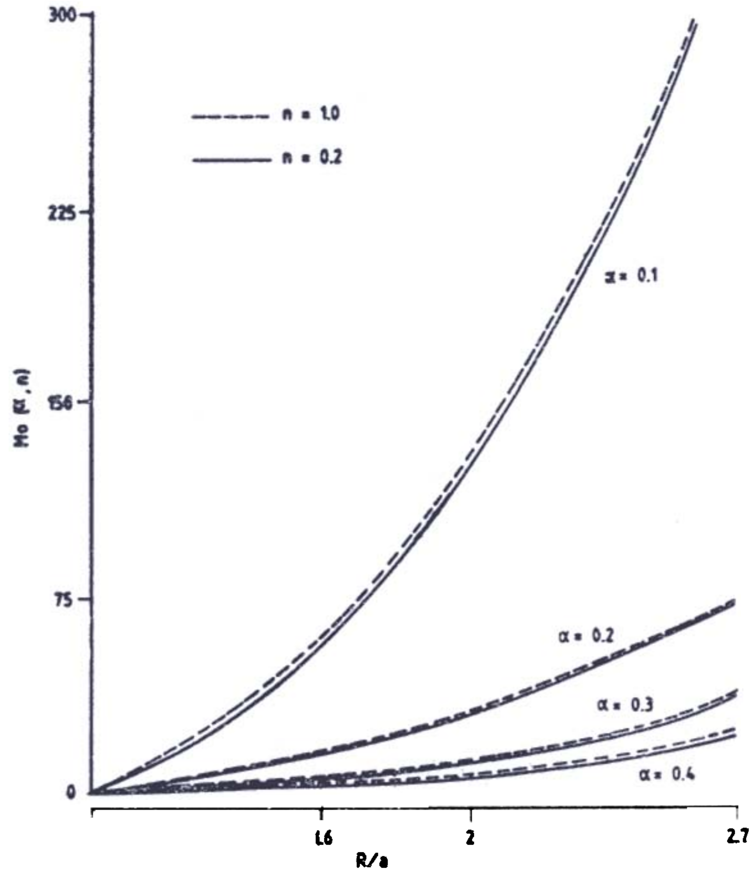

Figure 1. **Variation between equivalent dispersion coefficient and the width of the channel in power law model.**

The evaluation of the integral in Eqn. (26) for general values of n is not possible analytically for all a. It can be seen from the graphs that, when a increases, the rate of chemical reaction becomes prominent and when a decreases molecular diffusion of the solvent becomes dominent. Fig. (1) shows that decrease in the values of a, increases the dispersion effects of the solvent. This is due to the physical property of the power law fluids having the characteristic of higher order of molecular diffusion. This means the power law fluids which have greater diffusion coefficient will show significantly larger dispersion of the solute and this is in consonence with the normal behaviour of the fluids. Moreover as the section tends to increase, the effect of dispersion also becomes equally prominent which is probably due to increased space for mobility of the molecules of the solute. This observation gets added support from the graphs for small values of a (=0.1) showing a steep rise in the values of $M_0(a,n)$ as section is increased. Under the same conditions the influence of n, i.e.,index of the power law is only marginal, though for smaller values, we get corresponding smaller values for $M_0(a,n)$. These observations hold for constant values of mean velocity in case of naturally occuring solvents.

3.2 Bingham Plastic Model

Consider the case of dispersion through a Bingham plastic model fluid flowing through the channel. The function $f(\tau)$, in this case, is given by Govier *et al.*[15] and Copley[21] as

$$f(\tau) = \frac{1}{\mu}(\tau - \tau_0)$$

for $\quad \tau \geqslant \tau_0, \quad y_0 \leqslant y \leqslant R/a$

and $\quad f(\tau) \quad 0$

for $\quad \tau \leqslant \tau_0, \quad 0 \leqslant y \leqslant y_0$

where

$$\tau \quad ay\frac{\tau_R}{R} \qquad \tau_0 \quad ay_0\tau_R$$

As before, the function $g_0(y)$, concentration profile and effective dispersion coefficient can be calculated as follows by using appropriate values of $f(\tau)$ from Eqns. (28) and (29) in Eqns. (9), (17), (18) and (20). Here

$$g_0(y) \quad \left(\frac{1}{m_0} \quad\right), \qquad 0 \leqslant y \leqslant y_0$$

$$g_0(y) = \left(\frac{1}{m_0} - 1\right) \quad \frac{(y - y_0)^2}{m_0\left(\frac{R}{a} \quad y_0\right)^2}, \qquad y_0 \leqslant y \leqslant R/a$$

where

$$\frac{2+y_0}{3}R \qquad c_0(y) \qquad \cosh \qquad \frac{R}{3m_0\alpha^2} \quad 0 \leqslant \quad \leqslant \tag{31}$$

$$c_0(y) \quad B_0 \cosh \quad + E_0 \sinh \quad + \; m_0\alpha^2 \left\{ \frac{(y-y_0)^2}{\left(\frac{R}{a} - y_0\right)^2} \right.$$

$$\left. + \frac{}{\alpha^2 \left(\frac{R}{a} - y_0\right)^2} \qquad \frac{y_0 \frac{-}{} R}{3} \right\}, \quad \leqslant y \leqslant R/a \tag{32}$$

where

$$A_0 \quad \frac{2}{m_0\alpha^2 \left(\frac{R}{} \quad y_0\right)^2 \sinh} \quad \frac{(1-m_0) \; \sinh \alpha \frac{R}{a} \left(\frac{R}{a} - y_0\right)^2}{}$$

$$\left(\frac{R}{a} \quad y_0\right) + \frac{\sinh \alpha \left(\frac{R}{a} - y_0\right)}{\alpha^2} + \frac{(1-m_0)\left(\frac{R}{a} - y_0\right)^2 \sinh \alpha}{} \Bigg\}$$

$$B_0 \quad \frac{2}{\alpha^2\left(\frac{R}{a} - y_0\right)^2 \; m_0 \sinh} \left\{ \frac{(1-m_0)\left(\frac{R}{a} - y_0\right)^2 \; \sinh \alpha \cdot \frac{R}{a}}{2} \right.$$

$$+ \left(\frac{R}{a} \quad y_0\right. + \frac{\sinh \alpha y_0 \cosh \alpha \quad R}{\alpha^2} \quad \frac{\cosh \alpha y_0 \sinh \quad R}{\alpha^2}$$

$$\frac{(1-m_0)\left(\frac{R}{a} - y_0\right)^2 \; \sinh}{2} + \frac{\cosh \alpha y_0 \sinh}{\alpha^2} \tag{33}$$

$$E_0 \quad \frac{2 \sinh \alpha y_0}{\alpha^4 m_0 \left(\frac{R}{a} - y_0\right)^2}$$

$$D^* \quad \frac{a^2 \beta^2}{D} M_0(\alpha \;\; y_0) \tag{34}$$

where

$$M_0(\alpha \;\; y_0) \qquad \int_0^1 c_0(y) g_0(y)\, dy \qquad \int^{R/a} c_0(y) g_0(y)\, dy \tag{35}$$

When y get D^* from Eqn (34)

$$D^* = \frac{a^2\bar{v}^2}{D}\Bigg[-\frac{\left(3-2\frac{R}{a}\right)\left(\sinh\alpha\frac{R}{a}\right)^2}{2\alpha^3(R/a)\sinh\alpha} - \frac{3\sinh\alpha\frac{R}{a}}{\alpha^4(R/a)^2\sinh\alpha} + \frac{3\left(\sinh\alpha\frac{R}{a}\right)^2}{\alpha^5(R/a)^3\sinh\alpha}$$

$$+\frac{\left(3-2\frac{R}{a}\right)\sinh\alpha\frac{R}{a}}{2\alpha^3(R/a)^3} \quad \frac{3\sinh\alpha\frac{R}{a}}{\alpha^5(R/a)^3} + \frac{3\left(3-2\frac{R}{a}\right)\cosh\alpha\frac{R}{a}\sinh\alpha\frac{R}{a}}{2\alpha^4(R/a)^2\sinh\alpha}$$

$$+\frac{9\cosh\alpha\frac{R}{a}}{\alpha^5(R/a)^3\sinh\alpha} - \frac{9\sinh\alpha\frac{R}{a}\cosh\alpha\frac{R}{a}}{\alpha^6(R/a)^4\sinh\alpha} - \frac{3\left(3-2\frac{R}{a}\right)\cosh\alpha\frac{R}{a}}{2\alpha^4(R/a)^4}$$

$$+\frac{9\cosh\alpha\frac{R}{a}}{\alpha^6(R/a)^4} - \frac{3\left(3-2\frac{R}{a}\right)\left(\sinh\alpha\frac{R}{a}\right)^2}{2\alpha^5(R/a)^3\sinh\alpha} - \frac{9\sinh\alpha\frac{R}{a}}{\alpha^6(R/a)^4\sinh\alpha}$$

$$+\frac{9\left(\sinh\alpha\frac{R}{a}\right)^2}{\alpha^7(R/a)^5\sinh\alpha} \quad \frac{3\left(3-2\frac{R}{a}\right)\sinh\alpha\frac{R}{a}}{2\alpha^5(R/a)^5} - \frac{9\sinh\alpha\frac{R}{a}}{\alpha^7(R/a)^5}$$

$$-\frac{R/a}{4\alpha^2} - \frac{6-\alpha^2(R/a)^2}{4\alpha^4(R/a)} + \frac{9(R/a)^3}{20\alpha^2} + \frac{6-\alpha^2(R/a)^2}{4\alpha^4(R/a)}$$

When $R/a = 1$, Eqn. (36) coincides with the result obtained by Gupta *et al.*[11]. When $a = 0$, i.e. there is no homogeneous reaction, then from Eqn. (34), D^* reduces to

$$D^* = \frac{a^2\bar{v}^2}{D}\Bigg[\frac{1}{6}\left(\frac{1-y_0\frac{a}{R}}{2+y_0\frac{a}{R}}\right)^2 (R/a) + \frac{\left(\frac{R}{a}-y_0\right)^3\left(1-y_0\frac{a}{R}\right)}{20\left(2+y_0\frac{a}{R}\right)^2}$$

$$+\frac{\left(\frac{R}{a}-y_0\right)\left(1-y_0\frac{a}{R}\right)}{20\left(2+y_0\frac{a}{R}\right)^2}\{6(R/a)^2+3(R/a)\,y_0+y_0^2\}$$

$$\frac{3\left(\frac{R}{a}-y_0\right)^3}{28\left(2+y_0\frac{a}{R}\right)^2}\Bigg]$$

When $R/a = 1$, Eqn. (37) coincides with the result of Fan and Wang[14]

It can be seen from the profiles depicted in Fig. 2 that with increase of cross-section (given by R/a), $M_0\ (a,y_0)$ increases as a increases and the rate of increase depends on a and the gradient. This is in contrast to the trend for power law fluids. This is due to the fact that, beyond the core, the rate of shear in Bingham plastic fluid is reduced considerably and in the case of power law model it is not, and with the given

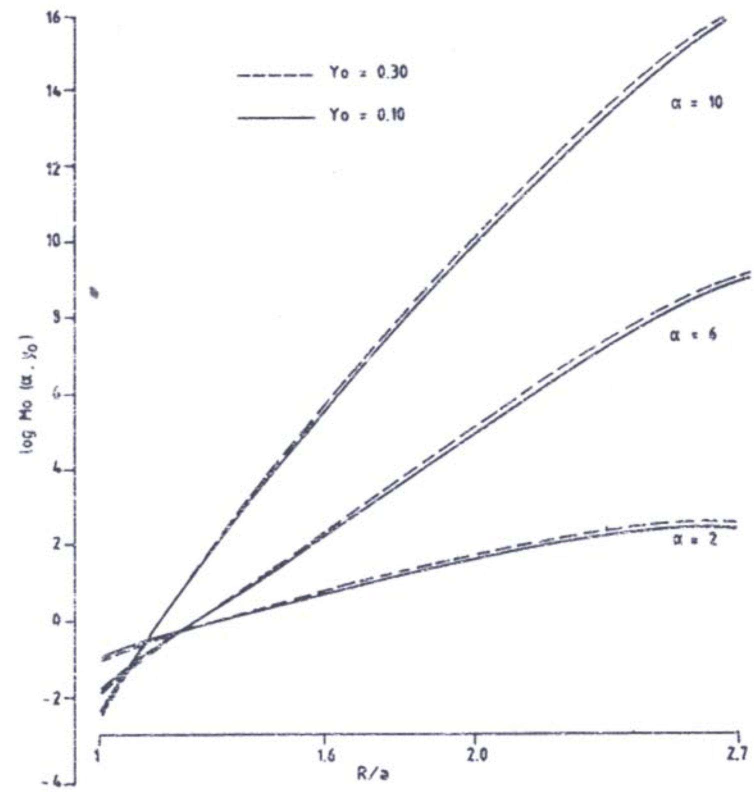

Figure 2. Variation between equivalent dispersion coefficient and the width of the channel in Bingham plastic model.

cross-section of the core, it tends to influence the mobility of the solute which will be larger with lesser shear rate. Whereas in case of R/a $(=1)$ influence of the thickness of the core is everywhere uniform, it is a which governs $M_0\ (a,y_0)$ in the normal manner i.e. increase of a decreases $M_0\ (a,y_0)$. It is further born by the fact that dotted graphs for higher values of y_0 show higher values of $M_0\ (a,y_0)$ for larger cross-section.

3.3 Casson Model

Consider the case of dispersion in a Casson model fluid flowing through a channel The function $f\,(\tau)$ in this case is given by

$$\frac{1}{\mu}\{\tau + \tau_0 \quad 2\tau^{1/2}\tau_0^{1/2}\} \tag{38}$$

for $\tau \geqslant \tau_0,\ y_0 \leqslant y \leqslant R/a$

and

$$0 \tag{39}$$

for

$$\tau \leqslant \tau_0, \qquad 0 \leqslant y \leqslant y_c$$

As before, the expression for $g_0(y)$, concentration profile and equivalent dispersion coefficient are calculated as follows by using appropriate value of $f\,(\tau)$ from Eqns. (38) and (39) in the Eqns. (9), (17), (18) and (20).

$$g_0(y) = \left(\frac{1}{m_0} \qquad \right), \qquad 0 \leqslant y \leqslant y_0$$

$$g_0(y) = \left(\frac{1}{m_0} \qquad \right) \qquad \frac{y^2 + 2yy_0 \quad \frac{8}{3}y^{3/2}y_0^{1/2} \quad \frac{1}{3}y_0^2}{\qquad}, \quad y_0 \leqslant y \leqslant R/a$$

where

$$m_0 = \frac{1}{h_0}\left\{\frac{2}{3}\left(\frac{R}{a}\right)^3 + \left(\frac{R}{a}\right)^2 y_0 \quad \frac{8}{5}\left(\frac{R}{a}\right)^{5/2} y_0^{1/2} \quad \frac{1}{15}y_0^3\right\}$$

$$h_0 \quad \left(\frac{R}{a}\right)^3 + 2\left(\frac{R}{a}\right)^2 y_0 - \frac{8}{3}\left(\frac{R}{a}\right)^{5/2} y_0^{1/2} \quad \frac{1}{3}\left(\frac{R}{a}\right) y_0^2$$

$$c_0(y) \quad A_0 \cosh \alpha y \quad \frac{1}{\alpha^2}\left(\frac{1}{m_0} \quad 1\right), \qquad 0 \leqslant y \leqslant y_0$$

$$c_0(y) = B_0 \cosh \alpha y + E_0 \sinh \alpha y + \frac{1}{3m_0h_0\alpha^4}\{3\alpha^2(y + y_0)^2 \quad 4\alpha^2 y_0^2 + 6\}$$

$$\frac{1}{\alpha^2}\left(\frac{1}{m_0} \qquad \right) + \frac{8y_0^{1/2}}{3m_0h_0\alpha}\int_{y_0}^{y} t^{3/2} \sinh \alpha(y \quad t)\, dt, \qquad y_0 \leqslant y \leqslant R/a$$

where

$$A_0 = \left(\frac{1 - m_0}{\alpha^2 m_0}\right) \qquad \frac{(1 - m_0)\sinh \alpha \dfrac{R}{a}}{\alpha^2 m_0 \sinh \alpha} + G \operatorname{cosec} \alpha$$

$$+\left(\frac{8y_0^2}{3m_0h_0\alpha^2} + \frac{2}{m_0h_0\alpha^4}\right)\left\{\frac{\sinh \alpha \dfrac{R}{a} \cosh \alpha y_0 \quad \cosh \alpha \dfrac{R}{a} \sinh \alpha y_0}{\sinh \alpha}\right.$$

$$+\left(\frac{4y_0}{m_0h_0\alpha^3}\right)\left\{\frac{\cosh \alpha \dfrac{R}{a} \cosh \alpha y_0 - \sinh \alpha \dfrac{R}{a} \sinh \alpha y_0}{\sinh \alpha}\right\}$$

$$B_0 = \quad \left(\frac{8y_0^2}{3m_0h_0\alpha^2} + \frac{2}{m_0h_0\alpha^4}\right)\cosh \alpha\, y_0 + \left(\frac{4y_0}{m_0h_0\alpha^3}\right)\sinh \alpha y_0$$

$$+ 2\left(\frac{1 - m_0}{\alpha^2 m_0}\right) \qquad \frac{(1 - m_0)\sinh \alpha \dfrac{R}{a}}{\alpha^2 m_0 \sinh \alpha} + G \operatorname{cosec} \alpha$$

$$E_0 \quad \frac{4y_0}{m_0 h_0 \alpha^3} \cosh \alpha y_0 + \left(\frac{8y_0^2}{m_0 h_0 \alpha^2} + \frac{2}{m_0 h_0 \alpha^4}\right) \sinh \alpha y_0 \tag{45}$$

$$G \quad \frac{8y_0^{1/2}}{3m_0 h_0 \alpha} \int^{R/\alpha} t^{3/2} \cosh \quad R \quad t) \quad \frac{2\left(\frac{R}{a} +\right.}{m_0 h_0 \alpha^3} \tag{46}$$

$$D^* \quad \frac{a^2 \bar{u}^2}{D} M_0(\alpha, y_0) \tag{47}$$

where

$$M_0(\alpha \quad y_0) \quad \int co(y) g_0(y)\, dy \quad \int co(y)\, g_0(y)\, dy \tag{48}$$

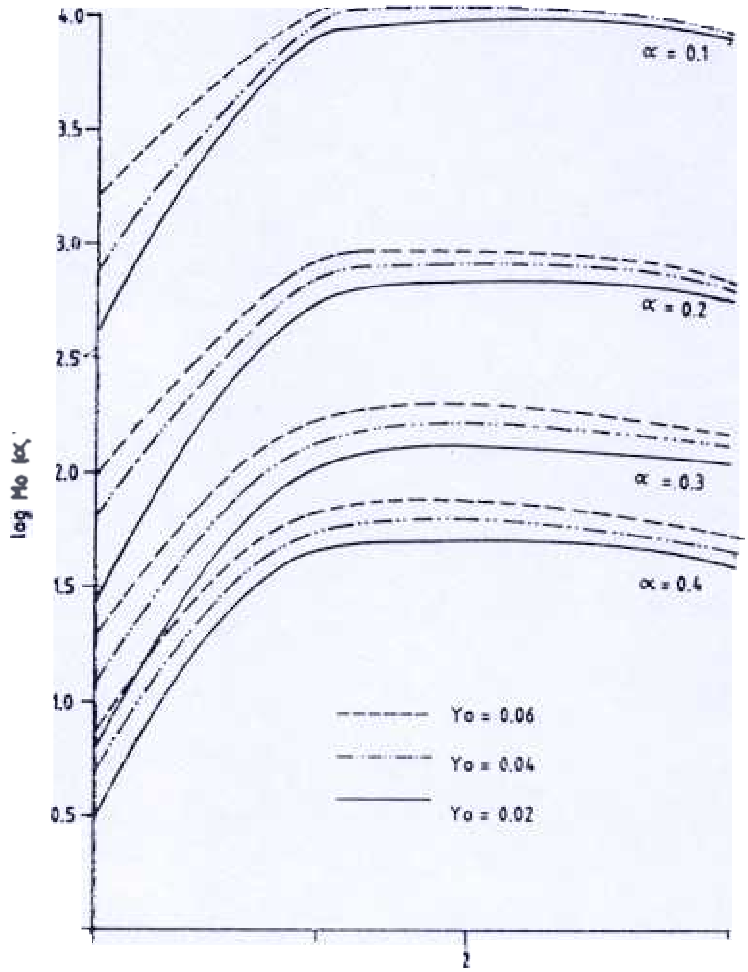

Figure Variation between equivalent dispersion coefficient and the width channel

In particular when $a \to 0$ in Eqn. (47), we get

$$D^* \quad 2.25\left\{1 + 4.8\sqrt{\frac{y_0}{R/a}}\right\}[0.01(R/a)^{-1} + 0.02(R/a) + 0.27(R/a)^2$$

$$0.04(R/a)^3 + \sqrt{y_0} \quad - 0.33(R/a)^{1/2} + 0.16(R/a)^{3/2} + 0.12(R/a)^{5/2}\}]$$

which gives the dispersion coefficient when there is no reaction. The Eqn. (49) gives the Wooding[22] result when $R/a = 1$ and $y_0 = 0$.

When $R/a = 1$ and $y_0 = 0$ in Eqn. (47), then D^* is same as obtained by Gupta *et al.*[11].

To study the effect of homogeneous reaction on dispersion, values are evaluated for small values of y_0 and a. It can be seen from the graphs for Casson model that there is remarkable deviation from the behaviour exhibited in case of other two models (Fig.3). Here, for all combinations between a and y_0 considered, $M_0\ (a,y_0)$ attains a maximum near R/a (=1.7) and as a increases equivalent dispersion coefficient decreases for $R/a > 1.7$. This gives, in a way, the most efficient cross-section for the model. $M_0\ (a,y_0)$ tends to decrease but the changes are small. However, this decrease is enhanced further by increasing y_0. It is further noted from the graph that a (=0.4) and y_0 (=0.06) gives the same value of $M_0\ (a,y_0)$ as that for a (=0.3) and y_0 (=0.02) near R/a (=1.05). Thus by changing y_0 and a we may get the same value of equivalent dispersion coefficient at the same value of R/a.

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