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Hall Effect in the Viscous Incompressible Flow Through a Rotating Channel Between Two Porous Walls

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ABSTRACT

Exact solutions for the velocity and induced magnetic field distributions, accounting for Hall currents have been obtained for the flow of a conducting liquid, maintained between two parallel non-conducting porous walls under the action of a constant pressure gradient and in the presence of a uniform magnetic field transversely applied to the flow. Further, the channel is rotated with constant angular velocity about an axis perpendicular to the walls. For the purpose of mathematical simplicity, the magnetic Prandtl number is assumed to be negligible. An expression for the boundary layer thickness dependent on Taylor, Hartmann, suction Reynolds numbers and Hall parameter has been obtained.

1. INTRODUCTION

It has been shown by Vidyanidhi and Nigam[1] that secondary motion is set up when a straight channel formed by two parallel walls through which liquid is flowing under a constant pressure gradient, is rotated about an axis, perpendicular to the walls with an angular velocity Ω'. They have shown that when $\Omega' \to \infty$ such that the pressure gradient remains finite, there exists in the vicinity of the walls a boundary layer, the thickness of which is of the order $(\Omega/\nu)^{-1/2}$. Vidyanidhi[2] studied the effect of a uniform magnetic field H_0 applied transverse to the flow, on the above problem who has shown in that the thickness of the boundary layer is of the order :

$$\left\{\frac{\Omega'}{\nu} + \frac{\mu_e^2 H_0^2 \sigma}{2\rho\nu}\right\}^{-1/2} \tag{1}$$

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Vidyanidhi *et al.*[3] studied the application of uniform suction at one wall and an equal rate of injection at the other wall on the flow investigated by Vidyanidhi and Nigam[1]. It has been shown that for a given u_0' (u_0' < or > 0 according as suction or injection), the thickness of the boundary layer near the suction wall is of the order $(u_0'/2\nu + \sqrt{\Omega'/\nu})^{-1}$, while its thickness near the injection wall is of the order $(-u_0'/2\nu + \sqrt{\Omega'/\nu})^{-1}$. This problem has been extended in the frame-work of hydromagnetics for a weakly conducting liquid by Bala Prasad and Ramana Rao[4] who neglected Hall currents. It has been shown that the boundary layer near the suction wall is of the order:

$$\left(\frac{u_0'}{2\nu} + \sqrt{\frac{\Omega'}{\nu} + \frac{\mu_e^2 H_0^2 \sigma}{2\rho\nu}}\right)^{-1} \tag{2}$$

while near the injection wall is of the order

$$\left(-\frac{u_0'}{2\nu} + \sqrt{\frac{\Omega'}{\nu} + \frac{\mu_e^2 H_0^2 \sigma}{2\rho\nu}}\right)^{-1} \tag{3}$$

In this paper the boundary layer thickness near the walls has been estimated for any conducting liquid taking Hall currents also into account. The analysis is however, subject to the limitation that the magnetic Prandtl number is negligible.

2. GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

The equations of motion and continuity for the steady state in a rotating frame of reference o', x', y', z' as given by Squire[5] for an incompressible liquid are

$$\rho(\vec{u}'\cdot\nabla')\vec{u}' - 2\rho(\vec{u}'\times\vec{\Omega}') = -\nabla'\pi + \rho\nu\nabla'^2\vec{u}' + \mu_e(\nabla'\times\vec{H}')\times\vec{H}', \tag{4}$$

$$\nabla'\cdot\vec{u}' = 0 \tag{5}$$

where $\Pi' = p' - \tfrac{1}{2}\rho\,|\vec{\Omega}' \times \vec{r}'|^2$, $\vec{u}'$, $\vec{\Omega}'$ and $\vec{r}'$ are the modified pressure, velocity, angular velocity and position vector of the liquid particle, respectively. Also ρ, ν, μ_e, σ and $\vec{H}'$ respectively stand for the density, kinematic viscosity, permeability, electrical conductivity of the liquid and the magnetic field vector.

In the steady state, Maxwell's equations are

$$\nabla'\cdot\vec{H}' = 0 \tag{6}$$

$$\nabla'\times\vec{E}' = \vec{0}' \tag{7}$$

$$\nabla'\times\vec{H}' = \vec{J}' \tag{8}$$

$$\nabla'\ \vec{E}' = \rho_e/\epsilon \tag{9}$$

where ρ_e is the charge density and ε is a constant.

Here the generalised Ohm's law in which ion-slip and pressure diffusion are neglected (Sutton and Sherman[6]) is taken as

$$\vec{J}' = \sigma[\vec{E}' + \mu_e \vec{u}' \times \vec{H}' - \beta_e \mu_e (\vec{J}' \times \vec{H}')] \tag{10}$$

where $\beta_e = 1/n_e\, e$; $\sigma = n_e\, e^2\, \tau/m_e$; e, n_e, τ, m_e being the charge of an electron, number of electrons per unit volume, the mean time between successive collisions of an electron with ions and mass of an electron, respectively.

Choose a right-handed Cartesian system such that z'-axis is perpendicular to the motion of the liquid under the action of a constant pressure gradient $(-\partial\pi'/\partial x')$ in the direction of x'-axis between two parallel porous walls $z' = \pm L$ (stationary relative to o', x', y', z').

Assuming that it is independent of y' and z'; π is given by

$$\left(\frac{p_1' - p_2'}{D}\right)x' - p_1' \tag{11}$$

where p_1' and p_2' stand for the pressure on the planes $x' = 0$ and $x' = D$, respectively.

Suppose that the normal velocity at the wall $z' = -L$ is u_0' $(u_0' > 0)$ so that this represents a porous wall[7], through which liquid is forced into the channel with a uniform velocity. It is further assumed that this rate of injection at the lower wall is equal to the suction rate at the upper wall. The liquid velocity is then represented by

$$\vec{u}' = [u_{x'}'(z'),\ u_{y'}'(z'),\ u_0']$$

Assuming further,

$$\vec{H}' \quad [h_{x'}'(z'),\ h_{y'}'(z'),\ H_0]$$

$$\vec{J}' = [j_{x'}'(z'),\ j_{y'}'(z'),\ 0] \tag{14}$$

$$\vec{E}' = [E_{x'}' = C_1',\ E_{y'}' \quad C_2',\ E_{z'}'(z')]$$

and expanding the generalized Ohm's law given by Eqn. (10) into its three components, we get

$$j_{x'}' = \sigma(E_{x'}' + \mu_e u_{y'}' H_0 - \mu_e \beta_e j_{y'}' H_0 - \mu_e u_0' h_{y'}') \tag{16}$$

$$j_{y'}' = \sigma(E_{y'}' - \mu_e u_{x'}' H_0 + \mu_e \beta_e j_{x'}' H_0 + \mu_e u_0' h_{x'}') \tag{17}$$

$$0 = \sigma[E_{z'}' + \mu_e u_{x'}' h_{y'}' - \mu_e u_{y'}' h_{x'}' - \beta_e \mu_e (j_{x'}' h_{y'}' \quad j_{y'}' h_{x'}')] \tag{18}$$

solving the first two for $j_{x'}'$ and $j_{y'}'$, it is found that

$$j_{x'}' \quad \bar{\sigma}[E_{x'}' + \mu_e u_{y'}' H_0 - \omega_0\tau(E_{y'}' - \mu_e u_{x'}' H_0)\,. \quad \mu_e u_0'(\omega_0\tau h_{x'}' + h_{y'}')]$$

$$j'_{y'} \quad \bar{\sigma}[E'_{y'} - \mu_e u'_{x'} H_0 + \omega_0\tau(E'_{x'} + \mu_e u'_{y'} H_0) + \mu_e u'_0(h'_{x'} - \omega_0\tau h'_{y'})]$$

where

$$\bar{\sigma} = \frac{\sigma}{1+\omega_0^2\tau^2}, \quad \omega_0 = \frac{\mu_e e H_0}{m_e}$$

Eqn. (18) determines $E'_{z'}$.

Further assuming,

$$\vec{\Omega}' = (0, 0, \Omega'), \tag{22}$$

$$\omega_0\tau = m(\text{Hall parameter}), \tag{23}$$

$$C'_1 = C'_2 = 0, \tag{24}$$

and in terms of the following non-dimensional variables,

$$\vec{r}' = \vec{r}L, \; \vec{u}' - \frac{\vec{u}Q}{\rho L}, \; \Omega' - \frac{\Omega Q}{\rho L^2}, \; \pi = \frac{Q^2}{\rho L^2}, \vec{H}' = H_0\vec{H}$$

β (suction Reynolds number) = $u'_0\, L/\nu$
M (Hartmann number) = $\mu_e\, H_0\, L\, (\sigma/\rho\nu)^{1/2}$
R_m (magnetic Reynolds number) = $\mu_e\sigma Q\, /\, \rho$
a (Taylor number) = $L\sqrt{\Omega'/\nu}$
$P = -L/Q\Omega' \cdot \partial\pi'/\partial x'$

magnetic Prandtl number, $P_m = \mu_e\sigma\nu$ (25)

masss flux, $Q = (Q'_{x'} + Q'_{y'})^{1/2}$

$$Q'_{x'} = \int_{L}^{L} \rho u'_{x'}\, dz', \quad Q'_{y'} = \int_{-L}^{L} \rho u'_{y'}\, dz'$$

Eqn. (4) gives

$$\frac{\beta}{\alpha^2}\frac{du_x}{dz} - 2u_y = P + \frac{1}{\alpha^2}\frac{d^2u_x}{dz^2} + \frac{M^2}{\alpha^2 R_m}\frac{dh_x}{dz}$$

$$\frac{\beta}{\alpha^2}\frac{du_y}{dz} + 2u_x = \frac{1}{\alpha^2}\frac{d^2u_y}{dz^2} + \frac{M^2}{\alpha^2 R_m}\frac{dh_y}{dz}$$

Further from Eqns. (8) and (10), we get

$$\frac{-\beta P_m}{}\left(\frac{dh_x}{} \quad m\frac{dh_y}{dz}\right) + \frac{d^2h_x}{dz^2} \quad \frac{R_m}{1+m^2}\left(\frac{du_x}{dz} \quad \frac{du_y}{dz}\right) = 0$$

$$\frac{\beta P_m}{1+m^2}\left(\frac{dh_y}{dz} + m\frac{dh_x}{dz}\right) + \frac{d^2h_y}{dz^2} + \frac{R_m}{}\left(\frac{du_y}{dz} + m\frac{du_x}{dz}\right) = 0$$

In terms of complex notation as given by Cramer and Pai[8],

$q = u_x + iu_y;\ h = h_x + ih_y;$

R_m^* (complex magnetic Reynolds number)

$$R_m^* = \frac{R_m}{1 - im} \tag{32}$$

Eqns. (28) to (31) reduce to

$$\frac{\beta}{\alpha^2}\frac{dq}{dz} + 2iq = P + \frac{1}{\alpha^2}\frac{d^2q}{dz^2} + \frac{M^2}{\alpha^2 R_m}\frac{dh}{dz}$$

$$-\frac{\beta P_m}{R_m}\frac{dh}{dz} + \frac{1}{R_m^*}\frac{d^2h}{dz^2} + \frac{dq}{dz} = 0$$

If the walls are non-conducting, we have

$$h = 0 \text{ at } z = \pm 1$$

Also, the condition of no-slip at the walls gives

$$q = 0 \text{ at } z = \pm \tag{36}$$

3. SOLUTION OF THE EQUATIONS

Assuming that $p_m << 1$, the solution of Eqns. (33) and (34) subject to the boundary conditions in Eqns. (35) and (36)

$$u_x = -C_1 e^{\beta z/2} \sinh \theta z \sin \phi z + C_2 e^{\beta z/2} \cosh \theta z \cos \phi z$$
$$+ \frac{\left(\theta^2 - \phi^2 - \frac{\beta^2}{4}\right)}{\left(\theta^2 - \phi^2 - \frac{\beta^2}{4}\right)^2 + 4\theta^2\phi^2}\left\{(C_3 - mC_4)\left(\theta^2 - \phi^2 - \frac{\beta^2}{4}\right)\right.$$
$$\left. + 2(C_4 + mC_3)\theta\phi + 2P_0\right\}$$

$$u_y = C_1 e^{\beta z/2} \cosh \theta z \cos \phi z + C_2 e^{\beta z/2} \sinh \theta z \sin \phi z$$
$$+ \frac{1}{\left(\theta^2 - \phi^2 - \frac{\beta^2}{4}\right)^2 + 4\theta^2\phi^2}\left[-4P_0\theta\phi\right.$$
$$+ \left(\theta^2 - \phi^2 - \frac{\beta^2}{4}\right)\left\{(C_4 + mC_3)\left(\theta^2 - \phi^2 - \frac{\beta^2}{4}\right)\right.$$
$$\left.\left. - 2(C_3 - mC_4)\theta\phi\right\}\right]$$

$$\frac{R_m e^{\beta z/2}\{C_2(f - mg) - C_1(g + mf)\}}{+ m^2)\left\{\left(\theta^2 - \phi^2 - \frac{\beta^2}{4}\right)^2 + 4\theta^2\phi^2\right.}$$

$$\frac{R_m z}{m^2)\left\{\left(\theta^2 - \phi^2 - \frac{\beta^2}{4}\right)^2 + 4\theta^2\phi^2\right\}}$$

$$C_3 \quad \phi^2 \quad \frac{\beta^2}{4} \quad 1m\theta\phi\Big(\quad \phi^2 \quad \frac{\beta^2}{4}$$

$$4\theta^2\phi^2 \; + \quad \theta^2 \quad \phi^2 \quad \frac{\beta^2}{4}$$

$$^2)\theta \quad \phi^2 \quad \frac{\beta^2}{4} \; + \; m\theta^2\phi^2$$

$$\Gamma_0 \quad \frac{\beta^2}{4} + 2m$$

$$\frac{R_m e^{\beta z/2}\{C_1(f \quad mg) + C_2(g + m}{m^2)\left\{\left(\theta^2 \quad \phi^2 - \frac{\beta^2}{4}\right)^2 + 4\theta^2\phi^2\right\}}$$

$$\frac{R_m z}{(1 + m^2)\left\{\left(\theta^2 - \phi^2 - \frac{\beta^2}{4}\right) + 4\theta^2\phi^2\right\}}$$

$$\Big[C_3\Big\{m\Big(\quad \phi^2 \quad \frac{\beta^2}{4}\Big)^2 \quad 2(\quad m^2)\theta\phi \; \theta^2 \quad \phi^2 \quad \frac{\beta^2}{4}$$

$$^2\phi^2 \quad m^2\Big(\theta^2 \quad \phi^2 \quad \frac{\beta^2}{4}$$

$$4m\theta\phi\Big(\theta^2 \quad \phi^2 \quad \frac{\beta^2}{4} \quad 4\theta^2\phi^2$$

$$P_0 \quad 2\theta\phi \quad \phi^2 \quad \frac{\beta^2}{4} \tag{40}$$

h

$$\phi^2 \quad \frac{\beta^2}{4} \qquad \phi^2 \quad \frac{\beta^2}{4}$$

$$\frac{\beta}{2}\Big(\frac{\beta^2}{4} \quad \phi^2 \quad \text{co} \quad \phi\beta \quad \theta z \quad \phi$$

$$\theta^2 + \phi^2 \quad \frac{\beta^2}{4} \quad \phi z \quad \theta^2 + \phi^2 \quad \frac{\beta^2}{4} \quad \theta z \quad \phi z$$

$$\frac{\beta}{2}\;\frac{\beta^2}{4} \quad \phi^2 \quad \phi$$

and

$$\theta^2 - \phi^2 = \frac{M^2}{1 + m^2} + \frac{\beta^2}{4}, \quad \theta\phi = \alpha^2 + \frac{M^2 m}{2(1 + m^2)}, \quad P_0 = \frac{1}{2}\alpha^2 P \tag{42}$$

and C_1, C_2, C_3, and C_4, are constants of integration to be determined by solving the following system of linear equations for a given P:

$$\begin{aligned} & C_1 \sinh\theta \cosh\left(\frac{\beta}{2}\right) \sin\phi - C_2 \cosh\theta \cosh\left(\frac{\beta}{2}\right)\cos\phi \\ & - \frac{\left(\theta^2 - \phi^2 - \frac{\beta^2}{4}\right)}{\left(\theta^2 - \phi^2 - \frac{\beta^2}{4}\right)^2 + 4\theta^2\phi^2} \left[\left(\theta^2 - \phi^2 - \frac{\beta^2}{4} + 2m\theta\phi\right) C_3 \right. \\ & \left. - \left\{ m\left(\theta^2 - \phi^2 - \frac{\beta^2}{4}\right) - 2\theta\phi \right\} C_4 \right] = \frac{2P_0\left(\theta^2 - \phi^2 - \frac{\beta^2}{4}\right)}{\left(\theta^2 - \phi^2 - \frac{\beta^2}{4}\right)^2 + 4\theta^2\phi^2} \end{aligned} \tag{43}$$

$$\begin{aligned} & C_1 \cosh\theta \cosh\left(\frac{\beta}{2}\right)\cos\phi + C_2 \sinh\theta \cosh\left(\frac{\beta}{2}\right)\sin\phi \\ & + \frac{\left(\theta^2 - \phi^2 - \frac{\beta^2}{4}\right)}{\left(\theta^2 - \phi^2 - \frac{\beta^2}{4}\right)^2 + 4\theta^2\phi^2} \left[\left\{ m\left(\theta^2 - \phi^2 - \frac{\beta^2}{4}\right) - 2\theta\phi \right\} C_3 \right. \\ & \left. + \left\{\left(\theta^2 - \phi^2 - \frac{\beta^2}{4}\right) + 2m\theta\phi \right\} C_4 \right] = \frac{4P_0\theta\phi}{\left(\theta^2 - \phi^2 - \frac{\beta^2}{4}\right)^2 + 4\theta^2\phi^2} \end{aligned} \tag{44}$$

$$\begin{aligned} & C_1 \left[-\{m(f_1 - f_{-1}) + (g_1 - g_{-1})\} \cosh\left(\frac{\beta}{2}\right) \right. \\ & \left. - \{m(f_1 + f_{-1}) + (g_1 + g_{-1})\} \sinh\left(\frac{\beta}{2}\right) \right] \\ & + C_2 \left[\{(f_1 - f_{-1}) - m(g_1 - g_{-1})\} \cosh\left(\frac{\beta}{2}\right) \right. \\ & \left. + \{(f_1 + f_{-1}) - m(g_1 + g_{-1})\} \sinh\left(\frac{\beta}{2}\right) \right] \\ & \qquad \left\{ -m^2\left(\theta^2 - \phi^2 - \frac{\beta^2}{4}\right)^2 + 4m\theta\phi\left(\theta^2 - \phi^2 - \frac{\beta^2}{4}\right) - 4\theta^2\phi^2 \right\} \\ & + 2C_4 \left\{ -m\left(\theta^2 - \phi^2 - \frac{\beta^2}{4}\right)^2 + 2(1 - m^2)\left(\theta^2 - \phi^2 - \frac{\beta^2}{4}\right)\theta\phi \right. \\ & \left. + 4m\theta^2\phi^2 \right\} = -4P_0\left\{\theta^2 - \phi^2 - \frac{\beta^2}{4} + 2m\theta\phi\right\} \end{aligned} \tag{45}$$

$$C_1\left[\{(f_1 - f_{-1}) - m(g_1 - g_{-1})\}\cosh\left(\frac{\beta}{2}\right)\right.$$
$$\left. + \{(f_1 + f_{-1}) - m(g_1 + g_{-1})\}\sinh\left(\frac{\beta}{2}\right)\right]$$
$$+ C_2\left[\{(g_1 - g_{-1}) + m(f_1 - f_{-1})\}\cosh\left(\frac{\beta}{2}\right)\right.$$
$$\left. + \{(g_1 + g_{-1}) + m(f_1 + f_{-1})\}\sinh\left(\frac{\beta}{2}\right)\right]$$
$$+ 2C_3\left\{m\left(\theta^2 - \phi^2 - \frac{\beta^2}{4}\right)^2 - 2(1 - m^2)\theta\phi\left(\theta^2 - \phi^2 - \frac{\beta^2}{4}\right)\right.$$
$$\left. - 4m\theta^2\phi^2\right\} + 2C_4\left(-m^2\left(\theta^2 - \phi^2 - \frac{\beta^2}{4}\right)^2\right.$$
$$\left. + 4m\theta\phi\left(\theta^2 - \phi^2 - \frac{\beta^2}{4}\right) - 4\theta^2\phi^2\right\}$$
$$= -4P_0\left\{-2\theta\phi + m\left(\theta^2 - \phi^2 - \frac{\beta^2}{4}\right)\right\} \tag{46}$$

where

$$f_1 = \theta\left(\theta^2 + \phi^2 - \frac{\beta^2}{4}\right)\sinh\theta\cos\phi + \phi\left(\theta^2 + \phi^2 + \frac{\beta^2}{4}\right)\cosh\theta\sin\phi$$
$$+ \frac{\beta}{2}\left(\frac{\beta^2}{4} - \theta^2 + \phi^2\right)\cosh\theta\cos\phi - \theta\phi\beta\sinh\theta\sin\phi$$
$$f_{-1} = -\theta\left(\theta^2 + \phi^2 - \frac{\beta^2}{4}\right)\sinh\theta\cos\phi - \phi\left(\theta^2 + \phi^2 + \frac{\beta^2}{4}\right)\cosh\theta\sin\phi$$
$$+ \frac{\beta}{2}\left(\frac{\beta^2}{4} - \theta^2 + \phi^2\right)\cosh\theta\cos\phi - \theta\phi\beta\sinh\theta\sin\phi$$
$$g_1 = \theta\left(\theta^2 + \phi^2 - \frac{\beta^2}{4}\right)\cosh\theta\sin\phi - \phi\left(\theta^2 + \phi^2 + \frac{\beta^2}{4}\right)\sinh\theta\cos\phi$$
$$+ \frac{\beta}{2}\left(\frac{\beta^2}{4} - \theta^2 + \phi^2\right)\sinh\theta\sin\phi + \theta\phi\beta\cosh\theta\cos\phi$$
$$g_{-1} = -\theta\left(\theta^2 + \phi^2 - \frac{\beta^2}{4}\right)\cosh\theta\sin\phi + \phi\left(\theta^2 + \phi^2 + \frac{\beta^2}{4}\right)\sinh\theta\cos\phi$$
$$+ \frac{\beta}{2}\left(\frac{\beta^2}{4} - \theta^2 + \phi^2\right)\sinh\theta\sin\phi + \theta\phi\beta\cosh\theta\cos\phi$$

putting $m = 0$ and $\beta = 0$ in Eqns. (37) to (47) we recover the solution arrived by Vidyanidhi[2].

When $a \to \infty$ such that P_0/a^2 is finite, for $0 < z < 1$, it is obtained

$$u_x \simeq \frac{P_0}{\alpha^2}\{e^{(\theta+\beta/2)(z-1)}\sin\phi(1 - z)\} \tag{48}$$

$$u_y \simeq \frac{P_0}{\alpha^2}\{e^{(\theta+\beta/2)(z-1)}\cos\phi(1 - z) - 1\} \tag{49}$$

similar expressions can be written for $0 \geqslant z \geqslant -1$

4. DISCUSSION AND CONCLUSIONS

If the Suction Reynolds number β is replaced by its negative value and z by $-z$, the expression for both the primary and secondary velocity distributions as given by Eqns. (37) and (38) respectively do not change, while h_x and h_y as given by Eqns. (39) and (40) respectively change in sign. This symmetry in u_x and u_y ; h_x and h_y can also be seen from Eqns. (28) to (31) which remain invariant under the transformation $z \to -z$, $\beta \to -\beta$, $u_x \to -u_x$, $u_y \to -u_y$, $h_x \to -h_x$ and $h_y \to -h_y$, for any P_m. This shows that when there is uniform injection at the lower wall, the primary and secondary flow distributions in the lower half are the same as in the upper half for the case of uniform suction at the upper wall and vice-versa. Similarly, when there is uniform injection at the lower wall, h_x and h_y in the lower half and these in the upper half for the case of uniform suction at the upper wall are also the same but of opposite sign and vice-versa. It is, therefore, concluded for any P_m that u_x, u_y, h_x and h_y deviate more and more from the solutions obtained by Vidyanidhi[2] when $\beta = 0$.

When the side walls are made of conducting material and short-circuited by an external conductor, the induced electric current flows out of the channel. In this case no electric potential exists between side walls. If we assume the zero electric field also in z'-direction, we have

$$E'_{x'} = 0, \quad E'_{y'} \quad 0 \tag{50}$$

These conditions are realised, for instance, for the flow between two concentric cylinders under the radial magnetic field with the pressure gradient parallel to the axis of cylinder.

Calculations reveal that as the Hall parameter m increases, the primary velocity u_x changes from the characteristic Hartmann profile (square shape) to the typical Poiseuille profile. Also the cross flow given by u_y which is non-existent when $m = 0$ and $a = 0$, increases to a maximum value and then returns to zero as m increases in value.

By defining the viscous drags in x-direction at the lower and upper walls as $du_x/dz \mid z = -1$ and $- du_x/dz \mid z = 1$ respectively, then each can be obtained from the other on replacing β by $-\beta$. This also holds true for the drags in the y-direction at the walls. For such a replacement of β, the magnetic drags at these walls in the x-direction are just the opposite and similarly in the y-direction. These results have been concluded from Eqns (28) to (31) and therefore, hold true for any P_m.

It is noted from Eqns. (48) and (49) that the amplitudes of u_x and u_y are positive and the functions $\sin \phi\ (1-z)$ and $\cos \phi\ (1-z)$ can take positive or negative values. For $a \to \infty$, such that P_0/a^2 is finite, the disturbance is confined to regions of the order $L/\ (\theta + \beta/2)$ in the vicinity of the suction wall and $L/(\theta - \beta/2)$ in the vicinity of the injection wall, the thicknesses of the boundary layers being of the order $(u'_0/2\nu + \theta)^{-1}$ and $(- u'_0/2\nu + \theta)^{-1}$ respectively, where

$$\theta = \left(\alpha^2 + \frac{M^2}{2(1+m^2)} + \frac{\beta^2}{8}\right)^{1/2}$$

$$= \left[L^2\left\{\frac{\Omega'}{\nu} + \frac{\mu_e^2 H_0^2 \sigma}{2\rho\nu(1+\omega_0^2\tau^2)} + \frac{u_0'^2}{8\nu^2}\right\}\right]^{1/2}$$

The thickness of the boundary layers near the suction and injection walls which are respectively given by

$$\left(\frac{u_0'}{2\nu} + \sqrt{\frac{\Omega'}{\nu} + \frac{\mu_e^2 H_0^2 \sigma}{2\rho\nu}}\right)^{-1} \text{ and } \left(-\frac{u_0'}{2\nu} + \sqrt{\frac{\Omega'}{\nu} + \frac{\mu_e^2 H_0^2 \sigma}{2\rho\nu}}\right)^{-1}$$

as in the study of Bala Prasad and Ramana Rao[4] are now modified as :

$$\left\{\frac{u_0'}{2\nu} + \sqrt{\frac{\Omega'}{\nu} + \frac{\mu_e^2 H_0^2 \sigma}{2\rho\nu(1+\omega_0^2\tau^2)} + \frac{u_0'^2}{8\nu^2}}\right\}^{-1} \text{ and}$$

$$\left\{-\frac{u_0'}{2\nu} + \sqrt{\frac{\Omega'}{\nu} + \frac{\mu_e^2 H_0^2 \sigma}{2\rho\nu(1+\omega_0^2\tau^2)} + \frac{u_0'^2}{8\nu^2}}\right\}^{-1}$$

From Eqn. (52) it is concluded that as the Hall parameter $\omega_0\tau$ increases, the boundary layer thickness increases.

For large M, the boundary layer thicknesses at suction and injection wall are given by

$$\left\{\frac{u_0'}{2\nu} + \sqrt{\frac{\mu_e^2 H_0^2 \sigma}{\rho\nu(1+\omega_0^2\tau^2)}}\right\}^{-1} \text{ and } \left\{-\frac{u_0'}{2\nu} + \sqrt{\frac{\mu_e^2 H_0^2 \sigma}{\rho\nu(1+\omega_0^2\tau^2)}}\right\}^{-1}$$

respectively, while for large β, they are respectively given by

$$\left(\frac{u_0'}{\nu}\right)^{-1} \text{ and } \left(-\frac{u_0'}{\nu}\right)^{-1}.$$

It is concluded that the magnetic field and suction cause thinning, while injection and Hall parameter cause thickening of the boundary layer.

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