Def Sci J, Vol 40, No. 2, April 1990, pp 183–189

Unsteady Flow of Two Immiscible Viscous Fluids Over a Naturally Permeable Bed

K.S. Sai

SVH College of Engineering, Machilipatnam–521 002

ABSTRACT

The unsteady flow of two viscous, incompressible and immiscible **fluids in a long parallel channel of which the upper one is impervious** and lower one is porous of infinite thickness is considered by taking a pressure gradient of the form Pe^{ct}, where Pand care constants. Beavers **and Joseph's slip condition at the permeable interface and the generalised Darcy's law in the porous region have been used. The** analysis reveals that the flow depends upon the Reynolds numbers for **the upper and lower fluids, slip parameter and the porous parameter. The effect of slip parameter a and the porous parameter σ on the flow are** studied in detail.

1. INTRODUCTION

The analysis of fluid flows of immiscible liquids has been a popular area of research since several years. Russell and Charles' have examined the effect of a less viscous liquid such as water on the laminar flow of a high viscous liquid. They have shown that the pressure gradient required for the flow of the high viscous liquid can be reduced if water is injected into the channel. Considering the flow of a tighter fluid with less viscosity over a heavier fluid with high viscosity in a parallel plate channel, Bird et *al.*[2] have shown that the fluid having less viscosity flows more rapidly than that with high viscosity. Kapur and Shukla[3] have extended the analysis of the flow of two immiscible liquids discussed by Bird et *al.*[2] to the case of timedependent pressure gradient. They have noticed that as the Reynolds numbers for the two flows increase the interface velocity, the flux and skin friction at the plates decrease. In a recent

Received 3 March 1986, revised 18 August 1989

paper Sai and Agarwal[4] have investigated the flow of two immiscible fluids with different densities and viscosities under constant pressure gradient in a parallel plate channel bounded by a rigid wall at the top and a permeable bed of infinite thickness at the bottom. Using Darcy's law for the flow in the permeable medium and Beavers and Joseph's slip condition[5] at the permeable interface it has been shown that the fluid velocity and mass flux increase with permeability of the bed.

The objective of the present paper is to discuss the effect of time dependent pressure gradient on the rectilinear flow of a two-layer Newtonian fluid in a channel with an impermeable upper wall $y = h$ and a permeable lower wall $y = -h$; the fluid interface is at $y = 0$. The flow in the porous medium $y \leq -b$ it-taken to be spatially uniform. It is assumed that the fluids are viscous, incompressible and immiscible and the flow in the three zones is in the x-direction and is driven by a common time dependent pressure gradient, say, Pe^{ct} where c is a quantity with dimension t^{-1} and the porous medium is assumed to be homogeneous and isotropic so that its permeability is constant. The positive value for c is taken for a mathematical convenience. However, such a positive constant could model a non-autonomous system in which, time increasing pressure gradient can be maintained by a source. The two layer flow problems are of wide industrial importance, and examples of their application include in-tube condensers, (for example, air cooled condensers in process plants), petroleum industry, certain types of waste-heat boilers, in the area of ground water technology and many others.

2. FORMULATION AND SOLUTION OF THE PROBLEM

Consider the fully developed laminar flow of two viscous, incompressible, immiscible fluids between two parallel plates subject to a pressure gradient of the form Pe^{ct} where P and care constants and t is the time. The lower plate of the channel is permeable and is of infinite thickness while the upper plate is rigid.. Choosing the origin midway between the plates, taking y-axis perpendicular to the plates, and x-axis in the direction of the flow, the equations governing the flow can be written as

$$\frac{\partial^2 u_i}{\partial y^2} - \frac{\rho_i}{\mu_i}\frac{\partial u_i}{\partial t} = \frac{1}{\mu_i}\frac{\partial p}{\partial x} \tag{1}$$

where $u_i(y, t)$ is the velocity, μ_i the coefficient of viscosity and ρ_i is the density of the two fluids. The subscripts $i = 1, 2$ denote the two fluids where $i = 1$ and 2 correspond to the lighter and heavier fluids respectively.

The generalised Darcy's law[6] governing the flow in the porous medium is given by

$$\frac{\rho_2}{\epsilon}\frac{\partial u_3}{\partial t} = -\frac{\partial p}{\partial x} - \frac{\mu_2}{k} u_3 \tag{2}$$

where ε and k are the porosity and permeability of the medium.

Now introducing the relations $u_i = v_i e^{ct}$ and $\partial p / \partial x = -Pe^{ct}$, the Eqns. (1) and (2) can be written as

$$\frac{d^2v_l}{dy^2} - \frac{M_l^2}{h^2}v_l = \frac{-P}{\mu_l} \tag{3}$$

and

$$\frac{v_3\mu_2}{Ph^2} = \frac{?}{M_2^2 + \sigma^2\epsilon} \tag{4}$$

where the Reynolds number $M_l^2 = \frac{ch^2}{\nu_l}$, $\nu_l = \frac{\mu_l}{\rho_l}$ and $\sigma = \frac{h}{\sqrt{k}}$

The boundary conditions appropriate for the problem are

$$v_1 = 0 \quad \text{at} \quad y=h \tag{5}$$

$$v_1 = v_2 = v_0 \text{ at } y = 0$$

and

$$\mu_1\left(\frac{\partial v_1}{\partial y}\right) = \mu_2\left(\frac{\partial v_2}{\partial y}\right) \text{ at } y = 0 \quad , \tag{6}$$

$$\frac{\partial v_2}{\partial y} = \frac{\alpha}{\sqrt{k}}(v_B - v_3) \text{ at } y = -h$$

where

$$v_2 = v_B \text{ at } y = -h \tag{7}$$

The relation (7) is the Joseph Beavers' condition where a is a non-dimensional quantity which depends upon the structure of the porous material.

Solving Eqn. (1) subject to the conditions of Eqns. (5) to (7) we get

$$V_1 = \frac{V_0}{\lambda}\frac{\sinh M_1(1-\bar{y})}{\sinh M_1} + \frac{1}{M_1^2}\left[1 - \frac{\sinh M_1(1-\bar{y}) + \sinh M_1\bar{y}}{\sinh M_1}\right] \tag{8}$$

$$V_2 = V_0\frac{\cosh M_2(1+\bar{y})}{\cosh M_2} + \frac{1}{M_2^2}\left[-\frac{\cosh M_2(1+\bar{y})}{\cosh M_2} 4 + \alpha\sigma(v_B - v_3)\frac{\sinh M_2\bar{y}}{M_2\cosh M_2}\right. \tag{9}$$

where

$$V_1 = \frac{v_1\mu_1}{Ph^2},\; V_2 = \frac{v_2\mu_2}{Ph^2},\; V_0 = \frac{v_0\mu_2}{Ph^2},\; \bar{y} = \frac{y}{h}$$

$$\lambda = \frac{\mu_2}{PI'},\; V_B = \frac{v_B\mu_2}{Ph^2} \text{ and } V_3 = \frac{v_3\mu_2}{Ph^2}$$

in which the non-dimensional interface velocity

$$V_0 = \frac{M_2}{Z}\left[\alpha\sigma\left\{\frac{1-\cosh M_2}{M_2^2} - V_3\alpha\sigma\frac{\sinh M_2}{M_2}\right\} + \frac{X}{M_2}\left\{\frac{\sinh M_2}{M_2} + \frac{\cosh M_2 X \tanh (M_1/2)}{MI} + \alpha\sigma v_3\right\}\right] \quad (10)$$

Where

$$X = M_2 \cosh M_2 + \alpha\sigma \sinh M_2$$

$$Z = X\left(M_2 \sinh M_2 + \frac{M_1 \cosh M_2}{\lambda \tanh M_1}\right) + \alpha\sigma M_2$$

and the velocity at the porous surface

$$V_B = \frac{M_2}{X}\left[V_0 + \alpha\sigma V_3 \frac{\sinh M_2}{M_2} - \frac{1-\cosh M_2}{M_2^2}\right] \quad (11)$$

Now it can be readily shown that the velocities V , and V_2 have their maximum values at

$$\bar{y}_1 = \frac{1}{2M_1}\log\left[-\left(\frac{1+\psi_1 e^{M_1}}{1+\psi_1 e^{-M_1}}\right)\right] \quad (12)$$

and

$$\bar{y}_2 = \frac{1}{2M_2}\log\left[-\left(\frac{1-\psi_2 e^{-M_2}}{1+\psi_2 e^{M_2}}\right)\right] \text{ respectively.} \quad (13)$$

In these relations $\bar{y}_1 = \frac{y_1}{h}$, $\bar{y}_2 = \frac{y_2}{h}$, $\psi_1 = \frac{M_1^2 V_0 - \lambda}{\lambda}$

and $\psi_2 = \frac{M_2^2 V_0 - 1}{\alpha\sigma M_2(V_B - V_3)}$

The mass flow rate (non-dimensional) G is given by

$$G = \frac{M_2 \mu_2}{Ph^3\rho_2} = V_0\left[\frac{\tanh(M_1/2)}{M_1\gamma} + \frac{\tanh M_2}{M_2}\right] + \frac{\lambda}{M_1^2\gamma}\left[1 - \frac{\tanh(M_1/2)}{(M_1/2)}\right] + \frac{1}{M_2^2}\left[1 - \frac{\tanh M_2}{M_2}\right] + \frac{\alpha\sigma}{M_2^2}[V_3 - V_B][\tanh M_2][\tanh(M_2/2)] \quad (14)$$

Where

$$\gamma = \frac{\rho_2}{\rho_1} \quad \text{and} \quad M_2^2 = \frac{M_1^2\gamma}{\lambda}$$

The fractional increase of mass flow rate ϕ_1 due to the permeability is given by

$$\phi_1 = \frac{G - G_c}{G_c} \quad (15)$$

Where G_c the limiting value of G as $\sigma \to \infty$ and it denotes the mass flow rate in the case of channel with no permeable bed.

The fractional increase in the mass flow rate ϕ_2 due to unsteady pressure gradient is obtained as

$$\phi_2 = \frac{G - G_B}{G_B} \tag{16}$$

where G_B is the limiting value of G as M_1 or M_2 tends to zero.

The non-dimensional skin friction coefficients T_1 and T_2 respectively at the permeable bed and at the upper wall are obtained at

$$T_1 = \frac{\tau_1}{Ph} = \alpha\sigma(V_B - V_3) \tag{17}$$

and

$$T_2 = \frac{\tau_2}{Ph} = \frac{1 - \cosh M_1}{M_1 \sinh M_1} - \frac{V_0 M_1}{\lambda \sinh M_1} \tag{18}$$

3. DISCUSSION

The ordinates $\bar{y}_1$ and $\bar{y}_2$ at which the velocity of the fluids in the upper and lower halves assume maximum values are calculated for different values of α and σ. These results are graphically presented in Fig. 1. The slip velocity V_B at the interface of the

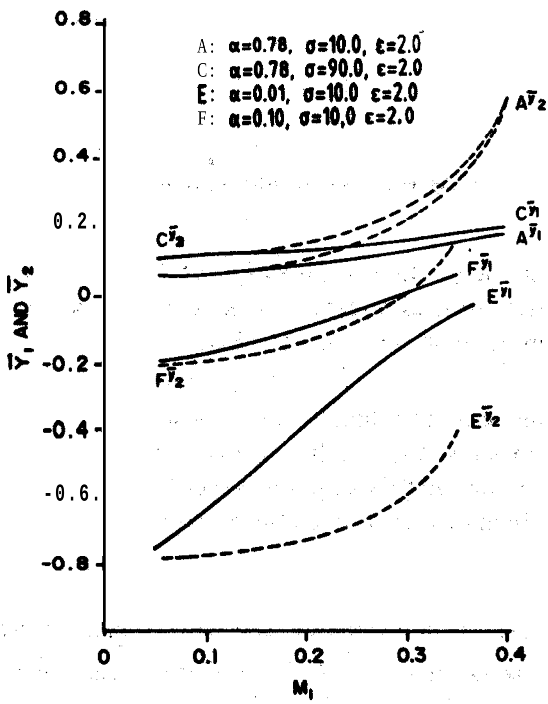

Figure 1. Plots of $\bar{y}_1$ and $\bar{y}_2$ against M_1 for different values of α and σ.

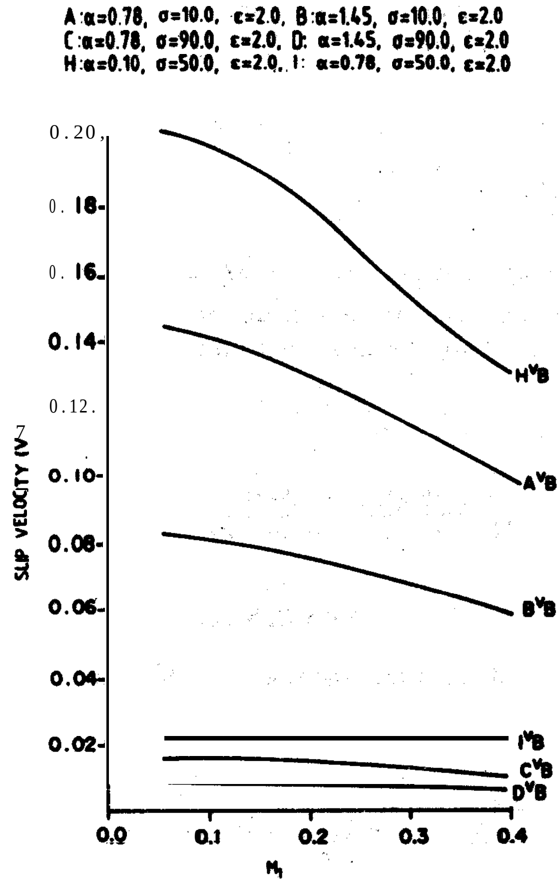

Figure 2. Slip velocity V_B versus M_1.

permeable bed and the lower fluid is given in Fig. 2 for various values of a and σ. The flow characteristics such as interface velocity V_0, mass flow rate G, fractional increase of mass flow rates ϕ_1 and ϕ_2 and skin friction coefficients T_1 and T_2 are displayed in Fig. 3.

From Fig. 1 it can be noticed that $\bar{y}_1$, $\bar{y}_2$ are positive or negative depending upon a and σ. The negative values of $\bar{y}_1$ and the positive values of $\bar{y}_2$ are inadmissible. From this, it is interesting to note that the upper or lower fluid will have maximum velocity depending upon a is high or low. It can also be noticed that $\bar{y}_1$ and $\bar{y}_2$ increase as σ or M_1 increases.

From Fig. 2 it can be found that the velocity at the porous surface V_B decreases as a increases. This effect is more pronounced when σ takes higher values.

Figure 3, clearly shows that V_0, G, ϕ_1, ϕ_2 decrease as a or σ increases. Further it is also seen that skin friction coefficients T_1 at the permeable bed increases with a or σ while the skin friction coefficient T_2 decreases as a or σ increases.

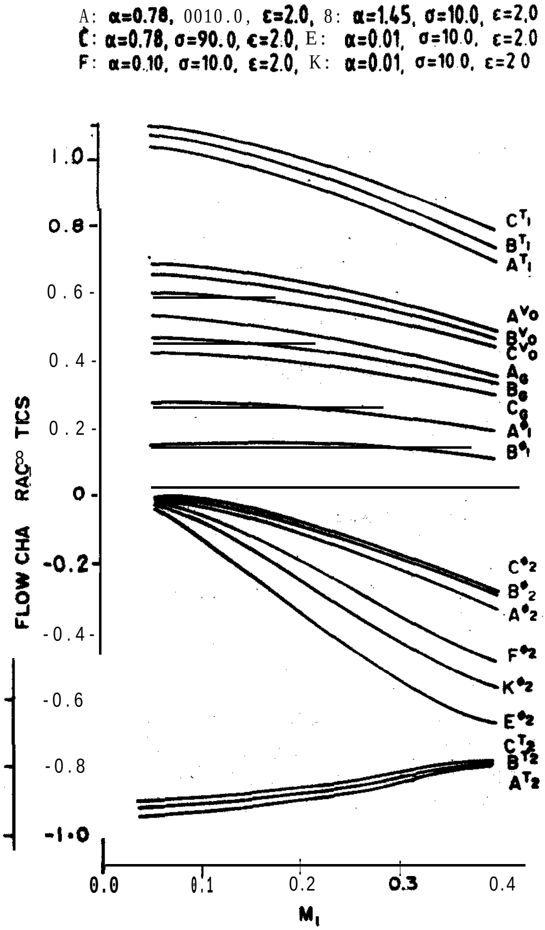

Figure 3. Flow characteristics versus M_1.

REFERENCE!3

1. Russell, T.W.F. & Charles, M.E., ***Canad. J. Chem. Engg.*, (1959),** 18.
2. Bird, R.B., Stewart, W.E. & Lightfoot, E.N., Transport Phenomena, (John Willey & Sons, New York), **1960.**
3. **Kapur,** J.N. & Shukla, J.B., ***ZAAM*,** 44 **(1964),** 268.
4. Sai, **K.S. &** Agarwal, J.P., I. Proceedings of the International Symposium on Nonlinear Continuum Mechanics, Kharagpur, 1980.
5. **Beavers,** S.G. & Joseph, D.D., ***J. Fluid Mech.*, *30* (1967),** 197.
6. Yih, C.S., Dynamics of Non-homogeneous Fluids, (Macmillan Company, New York), **1965,** p.197.