

# Equivalent Stress Approach to Life Prediction of Creeping Solids with Initial Defect

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## ABSTRACT

High temperature components contain many crack like defects introduced during the fabrication processes. These defects will grow during service under the action of applied stress and elevated temperature. An equivalent stress approach has been developed in this paper which can be effectively used to predict the life of defective materials under creep conditions.

## 1. INTRODUCTION

Many engineering components are fabricated by welding and are used in high temperature conditions like the boiler power plants. These components are likely to have initial defects or cracks introduced during the fabrication processes which will grow in size under the creep conditions during service.

The crack growth under creep has been studied by a number of research workers<sup>1-5</sup>. Different parameters like stress intensity factor,  $K$ , net section stress, and energy rate line integral  $C^*$  have been used to characterise the creep crack growth (CCG) rate. Each method has its own limitations. However, the energy rate line integral is widely accepted and hence this parameter is used in this analysis also.

## 2. THE $C^*$ INTEGRAL AND THE CCG RATE

The energy rate line integral represents the energy supplied to the material as the crack is growing. Hence it is assumed that the CCG rate will be a function of the line integral. The line integral is dependent on the applied load ( $P$ ) and the load point deflection or deformation rate ( $\dot{\Delta}_c$ ), in addition to the crack length ( $a$ ), and the geometry of the specimen.

In a simple case where a plate specimen of width  $W$ , thickness ( $B$ ) and having an edge crack of length ( $a$ ), the energy rate-line integral  $C^*$  is given by<sup>6</sup>

$$C^* = \frac{n}{n+1} \frac{P \dot{\Delta}_c}{B(W-a)} \quad (1)$$

where  $n$  is creep stress exponent the value of which will be in the range of 3-10. Typical relations between the CCG rate and the energy rate line integral for some technical alloys are shown in Fig. 1 (taken from Koterazawa and Mori<sup>7</sup>). The CCG rate can be related to the  $C^*$  by

$$da/dt = A(C^*)^r \quad (2)$$

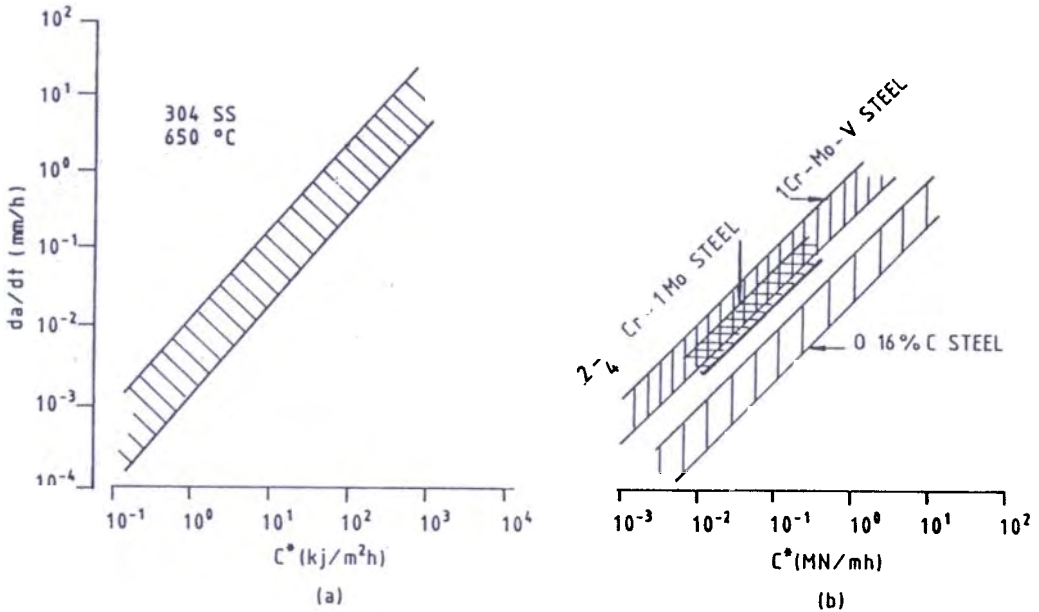


Figure 1. Typical correlation of crack growth data with the energy rate line integral  $C^*$  for some technical alloys<sup>7</sup>.

where  $A$  is a constant. The exponent  $r$  is given as  $r = n/(n+1)$ , the value of which, in some cases, has been found to be different<sup>8</sup>. But, in general, the exponent  $r$  is given by the above relation.

### 3. LIFE EVALUATION

Normally the creep crack starts from a small initial value  $a_i$  where the ratio  $a_i/W = 0.1$  or  $0.2$  and grows slowly till it reaches the critical length  $a_c$  when the failure occurs. At this time the ratio  $a_c/W$  will be of the order of  $0.6$  or  $0.7$  depending on the load. Thus, knowing the initial crack length  $a_i$  from non-destructive testing (NDT) examinations and assuming the final crack length to be of the order of  $0.6$  times the width, the Eqn (2) can be suitably integrated to obtain the failure time  $t_r$  of the material. Thus we have

$$\int_{a_i}^{a_c} \frac{da}{(C^*)^{n/(n+1)}} = A t_r \quad (3)$$

The relation for  $C^*$  depends on the crack length and can be in general given as

$$C^* = \frac{n}{n+1} \frac{P \dot{\Delta}_c}{BW} f(a/W) \quad (4)$$

where  $f(a)$  is a function of crack length  $a$ .

The load point deflection rate is related to the applied load in the form

$$\Delta_c = GP^n \quad (5)$$

where  $G$  is the compliance factor dependent on the crack length. Thus, Eqn (4) can be rewritten as

$$C^* = \frac{n}{n+1} \frac{G(a)P^{n+1}}{BW} f(a) \quad (6)$$

or

$$C^* = A_1 \sigma^{n+1} f'(a) \quad (7)$$

where the function  $f'(a)$  accounts for the other two geometry functions and is also dependent on the creep stress exponent  $n$ . The constant  $A_1 = (BW)^n$ . Substituting the above relation in Eqn (3) we get

$$W \int_{a_i/W}^{a_c/W} \frac{da/W}{(f'(a))^{n/(n+1)}} = A_2 \sigma^n t_r \quad (8)$$

where

$$A_2 = (A_1)^{n/(n+1)} \quad (9)$$

Webster<sup>6</sup> and Ernst<sup>9</sup> have given the function  $f'(a)$  which is dependent on the crack length and the width of the specimen. Thus we have the relations (after Webster and Ernst) as

$$f'(a) = \frac{n}{n+1} \frac{1}{(1 - (a/W))} \quad (10)$$

$$f'(a) = \frac{n}{n+1} \frac{1}{(1 - (a/W))} (R - (\beta/n)) \quad (11)$$

The values  $R$  and  $\beta$  can be expressed in the form

$$R = 2.0 + 0.9717(a/W) - 0.9756(a/W)^2 - 0.637(a/W)^3 + 0.6352(a/W)^4$$

and

$$\beta = 0.2392 - 0.7261(a/W) + 0.4899(a/W)^2 + 0.0369(a/W)^3 - 0.9324(a/W)^4$$

#### 4. CONCEPT OF EQUIVALENT STRESS ( $\sigma^*$ )

The left hand side of the integral in Eqn (8) can be evaluated from the initial crack length to a final critical value and the integral can be denoted by the symbol  $I$ . Thus we have

$$\frac{\sigma^n}{i} t_r = \frac{W}{A_2} = A_3 \quad (12)$$

Thus, the integral  $I$  has been evaluated from an initial crack length of  $a_i/W$  to a final value  $a_c/W$  and the relation between the integral values and the normalised crack length  $a/W$  is shown in Fig. 2. The first one (Fig. 2(a)) is based on the Webster's equation and the second one (Fig. 2(b)) on Ernst's equation. The initial crack length

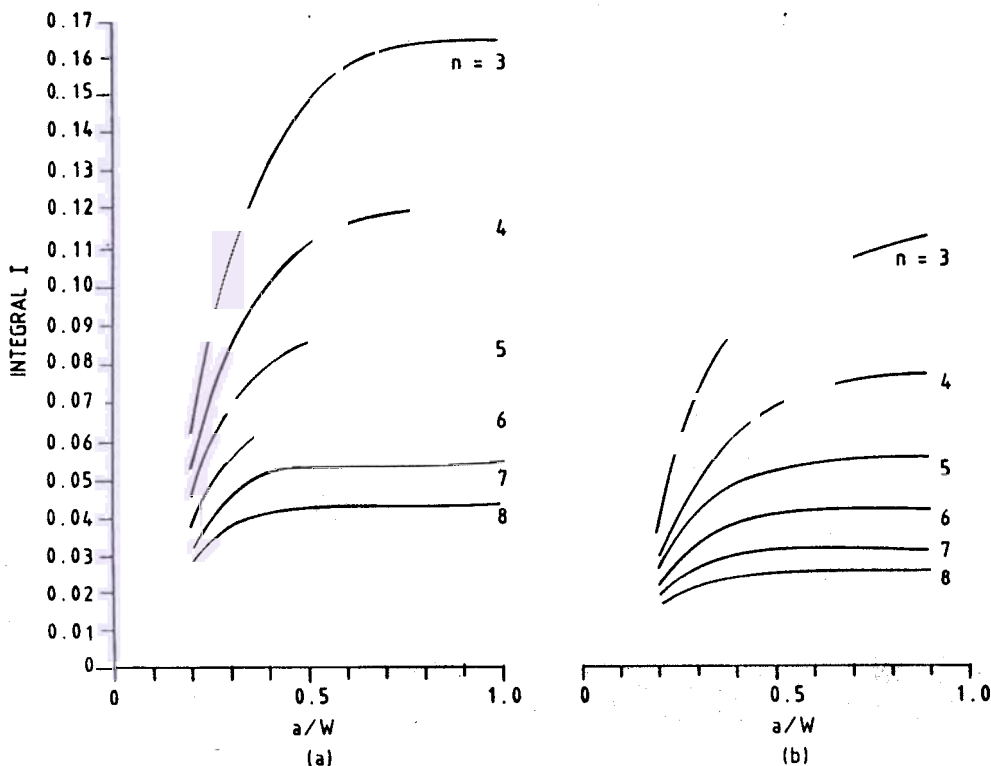


Figure 2. Relation between the integral  $I$  and the crack length  $a/W$  for  $a_i/W = 0.1$ , (a) based on Webster's equation, and (b) based on Ernst's equation.

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$a_i/W = 0.1$ . Figure 3 shows the relations, for the initial crack length of  $a_i/W = 0.2$ . It can be seen that the integral value depends on the creep stress exponent  $n$ .

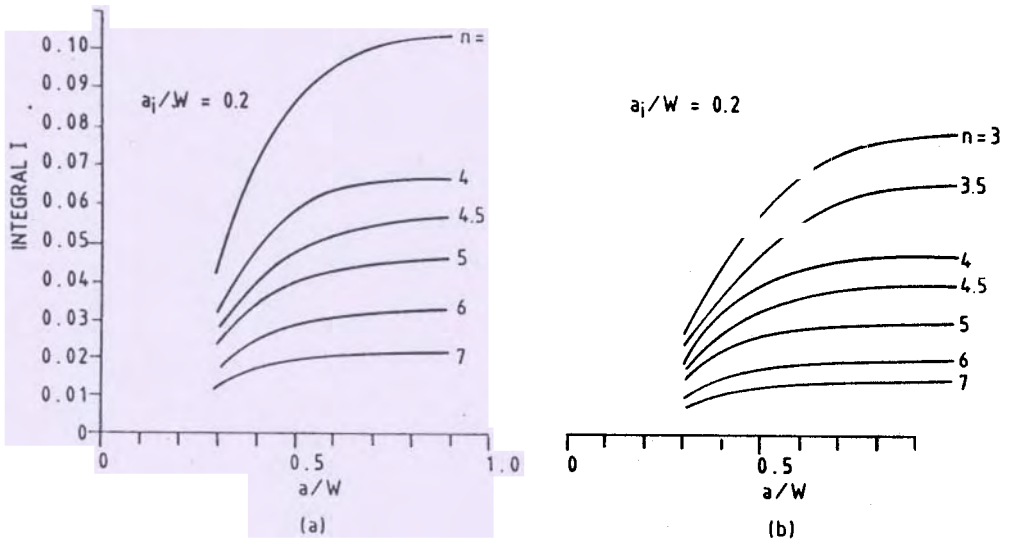


Figure 3. Relation between the integral  $I$  and the crack length  $a/W$  for  $a_i/W = 0.2$ , (a) based on Webster's equation, and (b) based on Ernst's equation.

Equivalent stress  $\sigma^*$  has been defined as

$$\sigma^* = \frac{\sigma}{\sqrt[3]{I}}$$

so that the rupture time relation can be given by

$$\sigma^{*n} t_r = A_3 = \text{constant}$$

which is similar to the normal stress-rupture time relation under ordinary creep.

### 5. EXPERIMENTAL STUDIES

Experimental investigations have been carried out on type-316 stainless steel with and without welded joints<sup>8</sup>. The chemical composition of the material is  $C = 0.069$  per cent,  $Cr = 19.4$  per cent,  $Ni = 11.6$  per cent,  $Si = 0.66$  per cent,  $Mn = 1.9$  per cent,  $S = 0.005$  per cent, and balance =  $Fe$ . For experiments with welded joints, the plates were welded autogenously by electron beam welding. CCG experiments were carried out under constant load condition. The specimens used were thin plates 150 mm long, 15 mm wide and 0.5 mm thick, with mid-section welds. A sharp starter notch has been introduced at the edge in the middle region of the specimen to initiate the crack. The depth of the notch was 3.0 mm. The experiments were carried out at specimen temperatures of 600, 700 and 800°C. The temperature was measured and controlled within  $\pm 3^\circ\text{C}$  by a chromel-alumel thermocouple. The crack length and the load point deflection were measured by means of a travelling microscope and a

dial gauge respectively with an accuracy of 0.01 mm. The experiments were repeated under different loads.

## 6. DATA ANALYSIS

Creep experiments carried out<sup>8</sup> have indicated that the values of the creep exponent  $n$  are 6.8, 5.0 and 4.5 for the three temperatures of 600, 700 and 800 °C respectively. The value of energy rate line integral  $C^*$  has been calculated using the relation proposed by Webster<sup>6</sup>. For type-316 stainless steel, the relation between the crack growth rate  $da/dt$  and the energy rate line integral  $C^*$ , is shown in Fig. 4. All the data points fall within a scatter band and the relation can be given, in general as

$$\frac{da}{dt} = A(C^*)^r$$

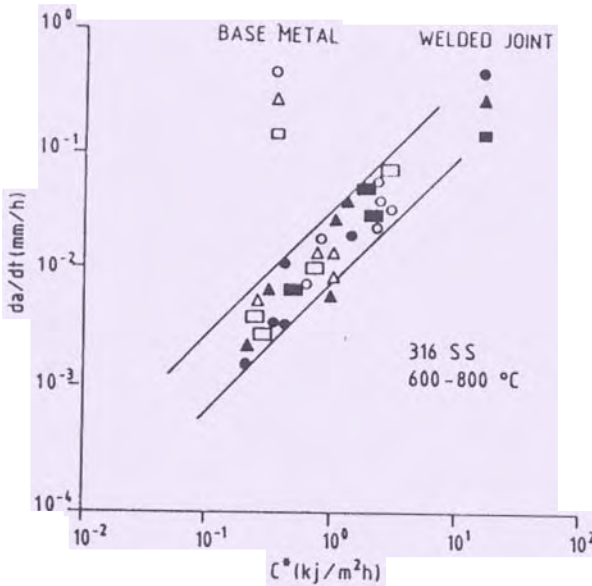


Figure 4. Relation between the crack growth rate and the energy rate line integral  $C^*$  for type-316 stainless steel.

where  $A$  is a constant and the exponent  $r = 0.85$ . With the scatter band, this may be taken to represent the value  $n/(n+1)$  for practical applications.

Based on the integral  $I$  given in Fig. 3(a) for  $a_i/W = 0.2$ , the equivalent stress  $\sigma^*$  has been computed using the relation

$$\sigma^* = \frac{\sigma}{\sqrt{I}}$$

and the relation between the equivalent stress and the rupture time  $t_r$  is shown in Fig. 5. The correlation appears to be good. Thus, the approach by the equivalent stress can be effectively used for the prediction of the creep life of the material which contains an initial defect.

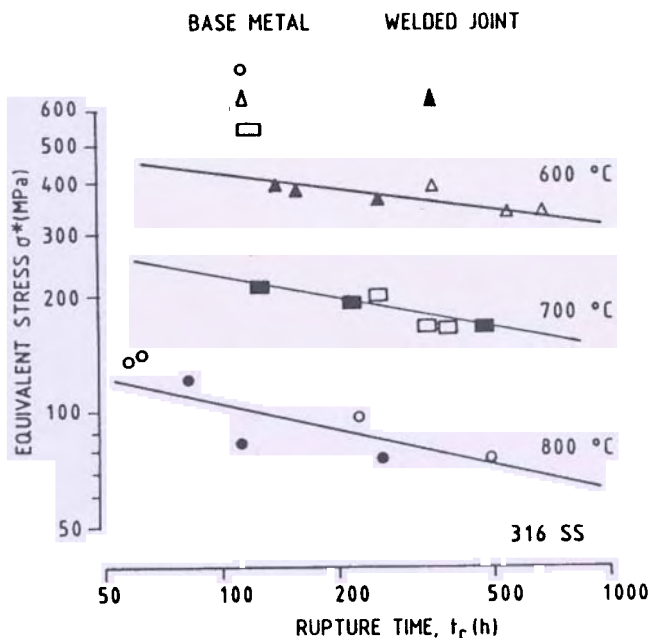


Figure 5. Relation between the equivalent stress and the rupture time.

The concept of equivalent stress is widely applied to fatigue problems of welded joints<sup>10</sup>. In this study the concept has been extended to creep failure of specimens which may contain initial crack-like defects. The experimental investigations and the data analysis clearly show that the approach is applicable to creep conditions also.

## CONCLUSIONS

From the investigations carried out to study the CCG characterisation, the following conclusions are arrived.

- Under plane stress condition in tensile mode of loading, the energy rate line integral  $C^*$  appears to give a good correlation with the CCG rate.
- The rupture time of the material can be obtained by suitable integration of the relation from the initial crack length to final critical crack length. The integral has been evaluated for different creep stress exponents.
- Based on the integral, the concept of equivalent stress is proposed which appears to give a good correlation with the rupture time of the specimens.
- The approach can be extended to creeping components having known initial defect size if the applied stress and the creep stress exponent value are known for the material.

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