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Criteria for Selection of Frequency of Electromagnetic Radiation for Underwater Proximity Fuzes

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ABSTRACT

Electromagnetic proximity fuzes play important role in underwater weapons. The characteristics of the electromagnetic field, i.e., the wavelength, penetration, propagation constant, velocity, etc in sea water are different from those in air. The conducting properties of the medium are strongly dependent on the frequency of the electromagnetic waves. This paper highlights the basis for selection of frequency of electromagnetic propagation for such applications as proximity fuze for underwater weapons.

1. INTRODUCTION

Electromagnetic proximity fuzes are in use in almost all the navies of world. These are active proximity fuzes, where an alternating magnetic field is generated around the rear end of the torpedo[1]. The fuze gets triggered when the source magnetic field is disturbed by the presence of the target ship[2].

2. DISPLACEMENT AND CONDUCTION CURRENTS IN SEA WATER

Maxwell's first curl equation can be written as

$$\nabla \times H = J + \frac{\partial D}{\partial t}^{3} \tag{1}$$

where H is magnetic field, J is conduction current density, and D is electric flux density.

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In a non-conducting medium $J=0$, but in a conducting medium J may not be negligible. It is given by

$$J=\sigma E \tag{2}$$

where σ is conductivity, and E is electric field. Substituting Eqn. (2) in Eqn. (1), the result is

$$\nabla \times H = \sigma E + \frac{\delta D}{\delta t} \tag{3}$$

For underwater weapons, linearly polarised wave is used. Assuming the wave is travelling in the x direction with E in the y direction as shown in Fig. 1, the vector Eqn. (3) is reduced to

$$\frac{\delta H_z}{\delta x} = \sigma E_y + \varepsilon \frac{\delta E_y}{\delta t} \tag{4}$$

where σE_y is conduction current density, and $j\omega\varepsilon E_y$ is displacement current density.

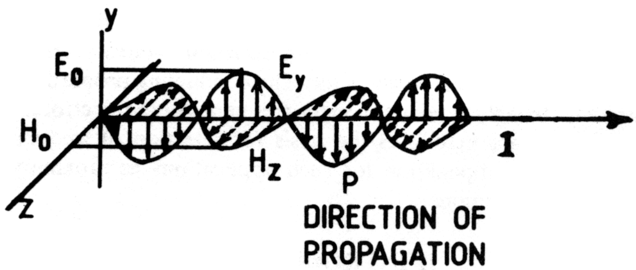

Figure 1. The directions of E_y and H_z[4].

It is clear from Eqn. (4) that the space rate of the z component of magnetic field, H_z equals the sum of the conduction and displacement current densities. For the conducting medium, conductivity is not zero and the conduction current term does not vanish.

According to the values of ratio $\sigma/\omega\varepsilon$, the medium behaves as conductor and non-conductor[4],

For dielectric	$(\sigma/\omega\varepsilon) < (1/100)$	
For quasi-conductor	$(1/100) < (\sigma/\omega\varepsilon) < 100$	
For conductor	$100 < (\sigma/\omega\varepsilon)$	(5)

3. FREQUENCY CRITERIA

It is clear from Eqn. (5) that the ratio of $\sigma/\omega\varepsilon$ will be much more than 100 for a conducting medium as in the case of sea water. The relation between these various characteristics is shown[4] in Fig. 2. For sea water $\varepsilon_r = 80$ and σ is $4\Omega\ \mathrm{m}^{-1}$. The ratio of $\sigma/\omega\varepsilon$ for sea water is calculated as follows.

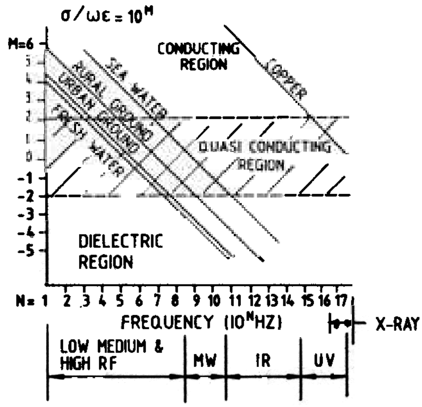

Figure 2. Ratio of $\sigma/\omega\varepsilon$ as a function of frequency[4].

$$\omega\varepsilon = 2\pi f \varepsilon_r \varepsilon_0 \tag{6}$$

$$\frac{\sigma}{\ } = \frac{\sigma}{\ } \tag{7}$$

where f is the frequency of the operation. The value of $\sigma/\omega\varepsilon$ at various frequencies is shown in Table 1. It is assumed that the constants maintain their low frequency values at all frequencies.

Table 1. Value of the ratio of $\sigma/\omega\varepsilon$ with respect to frequency

$\varepsilon_r = 80$ $\sigma = 4\Omega^{-1}\mathrm{m}^{-1}$ $\varepsilon_0 = 8.85\times10^{-12}$ F/m

Frequency (Hz)	$\sigma/\omega\varepsilon$ ($\times10^4$)
200	445
400	222
500	178
600	148
700	127
2000	
10000	8
100000	0.8

4. PROPAGATION CONSTANT

The familiar equation[5-7] relating electrical and magnetic fields in a medium is

$$\frac{\partial E_y}{\partial x} = -\mu \frac{\partial H_z}{\partial t} \quad (8)$$

Assuming linear harmonic variation with reference to t in E_y, the expression for E_y can be written as[3]

$$E_y = E_0 \, e^{j\omega t} \quad (9)$$

Differentiating E_y with reference to x and substituting in Eqn. (8), the result is

$$E_y = E_0 \exp \left\{ -(1+j) \sqrt{\frac{\omega \mu \sigma}{2}} \; x \right\}$$

Equation (10) can be expanded as follows

$$E_y = E_0 \{\exp \left[\sqrt{\frac{\omega \mu \sigma}{2}} \; x \exp(-j \sqrt{\frac{\omega \mu \sigma}{2}} \; x) \right] \}$$

Let $V^2 = j\omega\mu\sigma$

where V is called propagation constant

$$V = \sqrt{j} \qquad \sqrt{\omega \mu \sigma}$$

$$V = \sqrt{\omega \mu \sigma} \qquad \sqrt{45} \text{ degrees}$$

$$\frac{(1+j)}{2} \qquad \sqrt{\frac{\omega \mu \sigma}{2}}$$

The Eqn. (10) gives the variation of E_y in both magnitude and phase with respect to x. The electromagnetic field gets attenuated exponentially and is retarted linearly with increasing x.

5. DEPTH OF PENETRATION

The depth to which an electromagnetic field can penetrate is a very important parameter in the design of proximity fuze system. One of the specifications of the proximity fuze is the operating depth. This is the depth upto which the reflected electromagnetic field (from the target hull) is sufficient for signal processing to activate the fuze.

$$\text{Let } \delta = \sqrt{\frac{2}{\omega\mu\sigma}}$$

where δ is the depth of penetration. At the starting point $x = 0$, $E_y = E_0$

At the point $x = \delta$ $\quad E_y = E_0\, e^{-1}\, e^{-j}$

$$|E_y| = \frac{E_0}{e} = E_0 \times 0.368 \tag{16}$$

That is, amplitude of E_y reduces to 36.8 per cent of its initial value, while the wave penetrates to a depth of δ. The penetration depth δ, for various frequencies is given in Table 2.

Table 2. Penetration depth

Frequency (Hz)	Penetration depth (m)
200	17.8
400	12.6
600	10.2
700	9.48
2000	5.61
10000	2.51
100000	0.8

From Table 2, it is clear that the depth of penetration decreases as frequency increases. These results, read in conjunction with those given in Table 1, make it clear that the electromagnetic frequency for proximity fuze application in sea water should be less than 700 Hz.

6. VELOCITY AND WAVELENGTH OF ELECTROMAGNETIC WAVES IN SEA WATER

Phase velocity of an electromagnetic wave in sea water is given by

$$V = \sqrt{\frac{2\,\omega}{\sigma\,\mu}}$$

and the wavelength is given by $2\,\pi\,\delta = 2\,\pi\sqrt{\frac{2}{\omega\,\mu\,\sigma}}$

The values of velocity and wavelengths of electromagnetic wave in sea water for various frequencies are shown in Table 3. It can be observed from Table 3 that as frequency decreases, the velocity decreases and wavelength increases.

Table 3.

Frequency (Hz)	Velocity (m/s)$\times 10^4$	Wavelength (m)
200	2.23	111.90
300	2.74	91.35
400	3.16	79.113
500	3.53	70.76
600	3.87	64.59
700	4.18	59.80

7. CHARACTERISTIC IMPEDANCE OF SEA WATER

Taking time into consideration, the expression for E_y can be rewritten as

$$E_y = E_0\, e^{j\omega t - V_x}$$

The magnetic field is $H_z = H_0\, e^{j(\omega t - \xi) - V_x}$

Where ξ is the lead angle of E_y with reference to H_z. The characteristic impedance of the medium is given by

$$Z_c = \frac{E_y}{H_z}$$

$$Z_c = \frac{E_0\, e^{j(\omega t) - V_x}}{H_0\, e^{j\omega t - j\xi - V_x}}$$

$$Z_0\, e^{j(\omega\, t) - V_x - j\omega\, t + j\xi + V_x}$$

$$Z_c = Z_0 e^{j\xi} \tag{22}$$

Maxwell's equation derived from Faraday's Law for a plane wave with components E_y and H_z is[4]

$$\frac{\partial E_y}{\partial x} = -\mu \frac{\partial H_z}{\partial t} \tag{23}$$

Taking x derivative of Eqn. (19), time derivative of Eqn. (20) and substituting in Eqn. (23), we get

$$VE_y = j\mu\omega H_z \tag{24}$$

$$\frac{E_y}{H_z} = \frac{j\mu\omega}{V} \tag{25}$$

$$= \frac{j\,\mu\,\omega}{(1 + j \sqrt{\frac{\mu\,\omega\,\sigma}{2}}\,\}}$$

$$Z_c = \left\{ \frac{(1+j)}{2} \right\} \sqrt{\frac{\mu\,\omega}{\sigma}}$$

$$\sqrt{\frac{\mu\,\omega}{\sigma}} \qquad \sqrt{\;} 45 \text{ degrees} \tag{26}$$

It shows that the magnitude of the characteristic impedance of a conductor is given by $\sqrt{\mu\omega/\sigma}$ with a phase of 45 degrees (magnetic field lags the electric field by 45 degrees). Multiplying and dividing the Eqn. (21) with ε, ε_0 and μ_0 gives

$$Z_c = \frac{1+j}{2} \sqrt{\frac{\mu_0}{\varepsilon_0}} \sqrt{\frac{\mu_r}{\varepsilon_r}} \sqrt{\frac{\omega\,\varepsilon}{\sigma}}$$

$$Z_c = 376.7 \sqrt{\frac{\mu_r}{\varepsilon_r}} \sqrt{\frac{\omega\,\varepsilon}{\sigma}} \tag{27}$$

The characteristic impedance of sea water for various frequencies is shown in Table 4. It is observed from Table 4 that the conducting medium behaves like a short circuit for the electric field and in sea water only the magnetic field component of the electromagnetic field remains.

Table 4.

Frequency (Hz)	Characteristic impedance ($\times 10^{-2}$ ohms)
200	1.98
300	2.43
400	2.80
500	3.13
600	3.43
700	3.71

8. CONCLUSION

Choice of frequency of electromagnetic field for proximity fuze applications in sea is determined by the various characteristics of sea water. The choice of frequency also limits the operating distance around the torpedo in proximity fuze system. The above analysis enables the system designer to choose optimum frequency for a specific application.

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