Def Sci J, Vol 41, No 1, January 1991, pp 27-38

A Computer Code for Parametric Optimisation of Section Cartridge for Small Arms

P.K. Roy and D.A. Mishra

Armament Research & Development Establishment, Pune–411 021

ABSTRACT

A computer code (SACOP) has been devised for evaluation of optimum parameters of the cartridge (ammunition) for small arms used by the infantry.

NOMENCLATURE

C_0	standard ballistic coefficient (carrying power) of bullet
D	bullet diameter or calibre, mm
Er	acceptable recoil energy, J
Es	required remaining/striking energy at desired maximum effective range, J
Hv	vertex height of bullet trajectory, m
Je	mechanical equivalent of heat
K_0	shape and steadiness or form factor for calculation of C_0
K_1	a constant relating to the efficiency of the system for recoil velocity calculation
K_2	a constant, depending on propellant characteristics for propellant gas pressure calculation
K_3	a shape sensitive coefficient used to compute twist of rifling
Lb	bullet length, mm
Mb	bullet mass, kg
Mc	propellant charge mass, kg
Mw	allowable weapon (rifle) mass, kg
N	bullet CRH, i.e., radius of curvature of bullet ogive in terms of its calibre

Received 13 April 1989, revised 23 April 1990.

Prp	maximum gas pressure due to burning of propellant charge, MPa
Q	heat liberated by burning unit mass of propellant charge, J/kg
$R_{(j)}$	retardation of bullet between any two successive points on its flight trajectory, m/s^2
Sf	bullet shape factor used in computation of stopping power index
Spi	stopping power index of bullet
T	time of flight of bullet, s
Twr	twist of rifling for spin stabilisation of bullet, mm
$V_{(j)}$	bullet velocity at any point *(j)* of the flight trajectory, m/s
Vm	muzzle velocity of bullet, m/s
Vr	recoil velocity of weapon, m/s
Vs	remaining or striking velocity of bullet at maximum effective range, m/s
Vw	velocity of wind, m/s
Volc	cartridge case internal volume, cm^3
Wd	deflection of bullet due to cross-wind, m
Xm	maximum effective range desired, m
Zh	efficiency in converting heat energy of propellant charge into mechanical/kinetic energy
ρ_b	bullet material density, g/cm^3
ρ_c	propellant charge density, g/cm^3

1. INTRODUCTION

A section cartridge, as shown in Fig. 1, is a complete round of ammunition fired by small arms (weapons) of the infantry section and consists of a cartridge case containing a suitable propellant charge, a primer at the base end to ignite the propellant, and a bullet fitted at the mouth of the cartridge case. The cartridge which causes the desired damage at the target is the most basic and important element in

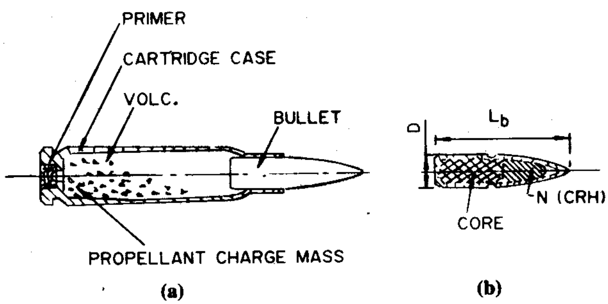

Figure 1. **Elements of a section cartridge, (a) complete cartridge, and (b) bullet.**

any weapon system. Weapons have no reason to exist except to fire an ammunition or cartridge at a specified target. Therefore, the first step in the design of a small arms system is to design the cartridge around which the weapon is then designed.

What is an ideal section cartridge for small arms? Before answering this question, it is important to look at the role of an infantry section of the army and its deployment in offence and defence. A section is the smallest combat unit of an infantry in which the principal small arms used are rifles, light machine guns (LMGs) and carbines. The maximum effective range is normally 800 m in case of LMG while that for rifles does not exceed 400 m. There is every possibility that apart from usual clothing the enemy soldier may be protected by wearing helmet and some sort of synthetic (nylon) body armour on the battlefield so that the incapacitation requirement is often coupled with a demand for some armour piercing ability. Hence, in the context of present day warfare, the ideal section cartridge is one that meets the following main requirements :–

(a) It should be designed to have optimum values of its parameters (calibre, shape and size of bullet, charge mass, etc.) so that lighter and effective weapons could be developed to produce high volume of fire;

(b) It should be of lesser overall weight so that more number of cartridges can be carried by the soldier;

(c) It should have good incapacitation or stopping power;

(d) It should follow a flat trajectory, i.e., minimum vertex height in flight to increase hit probability;

(e) It must have required striking or remaining energy at the maximum range desired to defeat the protected enemy soldier;

(f) It should produce minimum possible recoil of the weapon;

(g) It should cause least possible wear in weapon barrel by ensuring slower rifling twist; and

(h) It should be made of indigenously available materials.

2. FUNCTIONING PRINCIPLES AND IMPORTANT PARAMETERS

2.1 Cartridge Functioning

The first action of the firing cycle of all conventional small arms is the loading of a cartridge (ammunition/round) into the rear (breech) end of the weapon barrel (called chamber). When the cartridge is fired inside the barrel chamber of the weapon, the propellant charge contained in the cartridge case burns and is almost completely transformed into gaseous product which expands to some 14,000 times the volume of the original charge. The temperature inside the barrel rises as high as 3000°C whereas the high pressure generated is of the order of 350 MPa or even more. This high propellant gas pressure drives the bullet out of the cartridge case and then acts on the base of the bullet causing it to move up the bore as shown in Fig. 2. The work done by the propellant gases is converted into kinetic energy of the bullet. Along with its forward motion, the bullet is also given a rotary motion provided by the rifling in the barrel bore which assures its stability in flight.

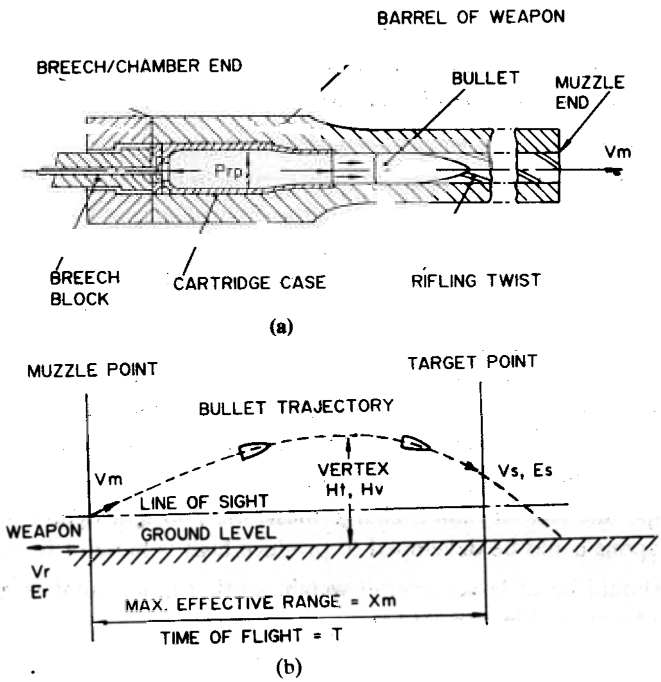

Figure 2. (a) Functioning principles of a cartridge inside a barrel bore. (b) various performance parameters of a bullet.

Another phenomenon of importance is recoil. The propellant gas exerts pressure on the barrel chamber in every direction. This pressure, while forcing the bullet forward along the barrel length, also drives backwards the breech block which is secured to the barrel and body of the weapon. As a result, the whole weapon is pushed back causing what is known as recoil when the bullet and gases emerge out of barrel. The weapon's backward momentum depends on the forward momentum of the bullet and the emerging gases. For better control of automatic firing and providing more comfort to the firer, the recoil energy should be as less as possible.

The bullet leaves the muzzle of the weapon with a velocity called muzzle velocity which is principally dependant on the amount and characteristics of the propellant charge, in addition to the bullet mass and weapon barrel parameters. The significant factors that govern the maximum range of the bullet (i.e., the maximum distance over which the bullet can be fired) and the remaining energy with which it can strike the target at desired range are the muzzle velocity and the carrying power of the bullet. The carrying power is strongly influenced by the ballistic coefficient determined by bullet shape and size which affects the resistance to its motion or drag forces slowing down the velocity of the bullet.

2.2 Important Cartridge Parameters

Based on the foregoing, the important cartridge parameters may be categorised into two groups, namely, the design parameters and the performance parameters.

2.2.1 Cartridge Design Parameters

Design parameters include cartridge case volume; mass and characteristics of propellant charge, bullet parameters like its calibre, mass, section density, ogive profile, etc., and muzzle velocity of bullet.

2.2.2 Cartridge Performance Parameters

Performance parameters are propellant gas pressure generated in the weapon chamber, recoil energy of the weapon firing the cartridge, remaining or striking energy of the bullet at the desired range, vertex height of flight trajectory, twist of rifling required in barrel for spin stabilisation of the bullet, deflection of the bullet from its true trajectory due to crosswind, and relative stopping power index of the bullet.

2.3 Criteria for Selection of Optimum Calibre

The following criteria with respect to some of the above parameters may be considered for selecting the optimum calibre.

(a) Pressure generated due to the burning of the propellant charge inside the barrel should not exceed 345–350 MPa in order to keep the barrel wall thickness and the weapon mass within reasonable limits;

(b) Vertex height of the trajectory should be lower for greater hit probability;

(c) Crosswind deflection should be minimum possible to increase the accuracy of firing; and

(d) Twist of the rifling should not be very tight to avoid overstressing of the bullet jacket and damage in the barrel bore. Therefore, the length required for one complete turn of the bullet (rifling) for stabilisation should be more.

The above criteria are applicable after the evaluation of other parameters of the cartridge for different calibres meeting the requirements of desired effective range and remaining energy of the bullet as well as acceptable mass and recoil of the weapon.

3. FORMULAE USED FOR THE CODE

Various design as well as performance parameters of the cartridge are interrelated by some well-established relationships which have been utilised in the subject code. However, most of the formulae, particularly the empirical ones, are originally in FPS units. These formulae, wherever required, have been converted into SI units while incorporating in the code and are outlined in the succeeding paragraphs.

3.1 Bullet Mass

Initially the bullet section density (*Sd*), which is the ratio of bullet mass (*Mb*) to the square of its diameter or calibre (*D*), is assumed in the range of 8.75×10^{-5} to 21×10^{-5} kg/mm^2 from which the bullet mass is calculated as under

$$Mb = Sd \times D^2 \qquad (1)$$

3.2 Remaining or Striking Velocity of Bullet

If the remaining energy at the time of striking the target at the desired maximum range (Es) is decided and specified, then the corresponding remaining velocity (Vs) with which the bullet will strike the target can be calculated from the basic kinetic energy equation as follows:

$$Es = (1/2) Mb Vs^2$$

Hence

$$Vs = (2Es/Mb)^{0.5} \tag{2}$$

3.3 Muzzle Velocity

The magnitude of the retardation of bullet velocity at any point on its flight trajectory is influenced by the instantaneous bullet velocity at that point and the bullet ballistic coefficient (C_0). An empirical formula has been established long ago by Siacci and converted to FPS units by Ingal's[1] in calculation of the retardation acting on the bullet at the instant when its velocity is V. The same formula has been suitably converted to SI units and utilised in the subject code. Knowing the required remaining velocity (Vs) at desired maximum effective range, the muzzle velocity (Vm) can be obtained by calculating the back increase in velocity at different consecutive points progressively till the muzzle point is reached.

For this purpose, the desired maximum range (Xm) is divided into y number of small equal parts, each part measuring a small distance X. Thus there will be $y + 1$ number of points over the entire length of the range (Xm) at every interval of X distance. The first point is taken as the point at the maximum effective range while the last or $(y + 1)$th point is taken as the point at the muzzle of the weapon. Let $R_{(j)}$ be the retardation of bullet velocity during its flight over a distance X from the $(j + 1)$th point to the jth point at which its velocities are $V_{(j+1)}$ and $V_{(j)}$ respectively.

Now we can write the formulae as follows:

Bullet velocity at the first point, $V_{(1)} = Vs$

For $j = 1, 2, 3, \ldots\ldots\ldots$ up to y

$$R_{(j)} = 0.273[F1_{(j)} + F2_{(j)} + F3_{(j)}]/C_0 \tag{3}$$

in which
$$F1_{(j)} = [(0.768819 V_{(j)} + 223.754)^2 + 209.043]^{0.5} \tag{4}$$

$$F2_{(j)} = \frac{0.062848 V_{(j)}(3.28 V_{(j)} - 984.261)}{371 + [V_{(j)}/200.053]^{10}} \tag{5}$$

$$F3_{(j)} = 0.933967 V_{(j)} - 224.221 \tag{6}$$

and
$$C_0 = Mb/(0.0007 K_0 D^2 \tag{7}$$

where
$$K_0 = (2/N)[(4N - 1)/7]^{0.5} \tag{8}$$

Further
$$X = Xm/y \tag{9}$$

and $V_{(j+1)} = \sqrt{V^2_{(j)} + 2 \times R_{(j)}}$ (10)

The muzzle velocity is obtained by putting $j = y$, when $Vm = V_{(y+1)}$ (i.e., the velocity at the $(y+1)$th point of the range.)

3.4 Time of Flight

Total time of flight (T) of the bullet is also calculated by summing up the times required to travel X distances between different points of the range as follows :

$T_{(j)} = [V_{(j+1)} - V_{(j)}]/R_{(j)}$ (12)

where $j = 1,2,3,........,y$; and $T = T_{(j)}$ (13)

3.5 Propellant Charge Mass

The heat energy of the propellant charge converted to mechanical work may be equated to the kinetic energy of the bullet and propellant charge at the muzzle point in order to obtain the mass of the propellant charge (Mc) as follows[2] :

$$McQJeZh = (1/2)(Mb + 0.5\ Mc)\ V^2_m$$

from which $Mc = (MbV^2_m)/(2QJeZh - 0.5V^2_m)$ (14)

3.6 Recoil Velocity and Recoil Energy

Recoil energy (Er) is calculated from the weapon mass (Mw) and the velocity of recoil (Vr) as under :

$Er = (1/2)\ MwVr^2$ (15)

The recoil velocity[3] can be found out by equating the forward momentum of the bullet and propellant charge to the backward momentum of the weapon at the time of bullet leaving the muzzle.

Thus, mathematically

$$MbVm + Mc(Vm \times K_1) = MwVr$$

in which K_1 lies between 1.0 and 2.0.

Therefore $Vr = (Mb + K_1Mc)\ Vm/Mw$ (16)

If the same cartridge is used for both rifles and LMG's, the muzzle velocity in case of rifle firing may be taken as 90 per cent of that obtained in LMG firing because of normally shorter effective range and thereby shorter barrel in the case of rifles as compared to LMGs.

3.7 Propellant Gas Pressure in the Weapon Barrel

The work done by propellant gases is converted into the kinetic energy of the bullet. Therefore

$$(Prp)_{mean} \times \text{Area} \times \text{shot travel} = (1/2)MbVm^2$$

and for small arms, normally

$$(Prp)_{max} = 2 \times (Prp)_{mean}$$

However, Sarrau's formula[2] gives an empirical relationship relating the maximum propellant gas pressure with the grain length, web thickness and external circumference as well as mass and loading density of the propellant charge. This formula has finally been transformed by the authors, for simplicity into

$$Prp = K_2(McMb)^{0.5}/D^2 \tag{17}$$

where K_2 is a constant which depends on the type and characteristics of the propellant charge, and may be evaluated from the experimental firing results. In this code, the value of K_2 has been taken as 3838632.8 for ball powder propellant when the units of pressure, mass and diameter were in MPa, kg and mm respectively.

3.8 Vertex Height

If air resistance is ignored and gravity is considered to be the only force acting on a bullet during its flight, the drop or distance it falls can be expressed as

$$S = (1/2)gT^2$$

where S is the drop, g is the acceleration due to gravity and T is the time of flight.

Weapon sight setting takes this into account so that the bullet is fired at an upward angle (angle of elevation) instead of parallel to the earth. Vertex height is the distance between the ground and the apogee of the trajectory which can be expressed as[4]

$$Hv = 1.22\ T^2 \tag{18}$$

3.9 Wind Deflection

Crosswinds deflect a bullet from its true trajectory. The amount of deflection depends on wind speed and direction, the velocity, time of flight and range of the bullet. The amount of deflection of the bullet for maximum range is given by[4]

$$Wd = Vw\left(T - \frac{Xm}{Vm}\right)$$

3.10 Twist of Rifling

Bullets are made stable in flight by the spin imparted by the rifling inside the barrel. The twist of rifling (Twr) is the length required for one complete turn of the bullet and its value changes with the change in calibre and section density of the bullet for the purpose of its spin stabilisation in flight. The Green Hills formula[4] which is stated in a simple form, after complex mathematical analysis, gives a good idea. The formula states that the twist required (in calibre) equals to 150 divided by the bullet length (in calibre). The formula is stated for bullet specific gravity of 10.9 and the units of twist, length and calibre are in inches. By suitably modifying the formula for the units used in the subject code, the twist of rifling required to stabilise the bullet can be expressed as

$$Twr = \frac{K_3, D^2}{Lb}$$

For good stability the value of K_3 lies between 150 and 200, usually 175.

Since the shape of the bullet is cylindro-ogival, the length of bullet (*Lb*) can be calculated from its mass, material density and calibre using standard formula as follows :

$$Lb = \left[\left(\frac{Mb \times 1000}{\rho_b \times 10^{-3}}\right) \Big/ \left(\frac{\pi}{4} D^2\right)\right] + 1.1051D \quad (21)$$

3.11 Stopping Power Index

Stopping power of the bullets fired from small arms may be defined as its capability to stop the opponent and prevent him from carrying out his task (assault or defence), the main contributing factor being the mass, cross-sectional area, striking energy of the bullet. The stopping power index (*Spi*) which would indicate the relative stopping power of a bullet may be expressed as

$$Spi = MbVsD^2 \frac{\pi}{4} Sf$$

The value of the shape factor *Sf* is normally 1.25 for blunt nose bullet and 0.9 for sharp nose bullet.

3.12 Cartridge Case Volume

Knowing the mass and density of the propellant charge, the inside volume of cartridge case (*Volc*) required to contain the charge can easily be computed as under

$$Volc = 1000Mc/\rho_c \quad (23)$$

4. COMPUTER CODE (SACOP)

Flow chart or logic diagram for the computer code SACOP devised for parametric optimisation of an infantry section cartridge for small arms is shown in Fig. 3. The formulae outlined in the preceeding sections have been suitably employed at various stages in the code. Once the input data on allowable weapon mass, acceptable recoil energy, desired maximum range, required remaining energy at maximum range, propellant charge characteristics, approximate range of bullet section density, etc. are fed to the computer, the subject code yields various design parameters such as cartridge case volume, bullet mass and exact section density, charge mass as well as important performance parameters like muzzle velocity of bullet, propellant gas pressure in the barrel, trajectory vertex height and crosswind deflection, required twist of rifling on the barrel bore, relative stopping power index, etc. for different calibres in a systematic tabular form. Based on the selection criteria for various characteristics parameters, the optimum calibre can then be chosen and the corresponding parameters may be adopted as optimum cartridge parameters. A sample computer printout from the code listing, the input data as well as parametric output is shown in Table 1. The interrelationships between various parameters are graphically represented in Figs. 4 and 5.

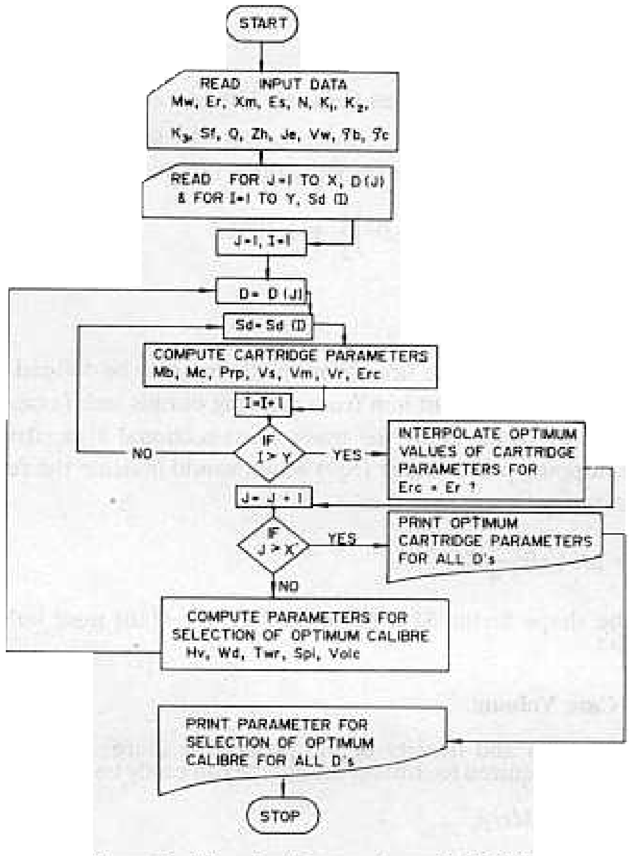

Figure 3. Flow chart for computer code SACOP.

Table 1. A sample print-out from computer code SACOP

Data Input

Mw = 3.5 kg, Zh = 0.414, Er = 5.4 J, Es = 488 J, K_1 = 1.5, K_2 = 3838633, N = 10, Q = 3516912 J/kg, ρc = 0.998 g/cc, ρb = 8.99 g/cc, Vw = 2 m/s, Je = 1, Sf = 0.9, K_3 = 175, Xm = 800 m, Y = 80

(a) Optimum cartridge parameters for different calibre

D (mm)	$Sd\times10^{-5}$ (kg/mm^2)	$Mb\times10^{-3}$ (kg)	$Mc\times10^{-3}$ (kg)	Prp (MPa)	Vm (m/s)	Vs (m/s)	Vr (m/s)	T (s)
4.0	13.6	2.18	1.87	485.8	1323	668	1.2	0.86
4.5	13.8	2.78	1.88	434.0	1212	592	1.2	0.95
5.0	14.3	3.56	1.82	390.9	1087	523	1.3	1.08
5.5	15.0	4.53	1.71	353.8	962	464	1.4	1.22
6.0	15.8	5.68	1.59	320.8	846	414	1.4	1.39
6.5	16.5	6.96	1.47	290.6	745	374	1.5	1.57
7.0	16.9	8.29	1.35	262.2	662	343	1.5	1.75

(b) Parameters for selection of optimum calibre

D (mm)	*Volc* (cc)	*Hv* (m)	*Wd* (m)	*Spi*	*Twr* (mm)	*Prp* (MPa)
4.0	1.881	0.90	0.51	16.5	118	485.8
4.5	1.885	1.11	0.59	23.6	145	434.0
5.0	1.820	1.42	0.68	32.9	170	390.9
5.5	1.717	1.82	0.78	45.0	194	353.8
6.0	1.597	2.35	0.88	59.9	217	320.8
6.5	1.472	2.99	0.98	77.9	242	290.6
7.0	1.353	3.74	1.08	98.6	270	262.2

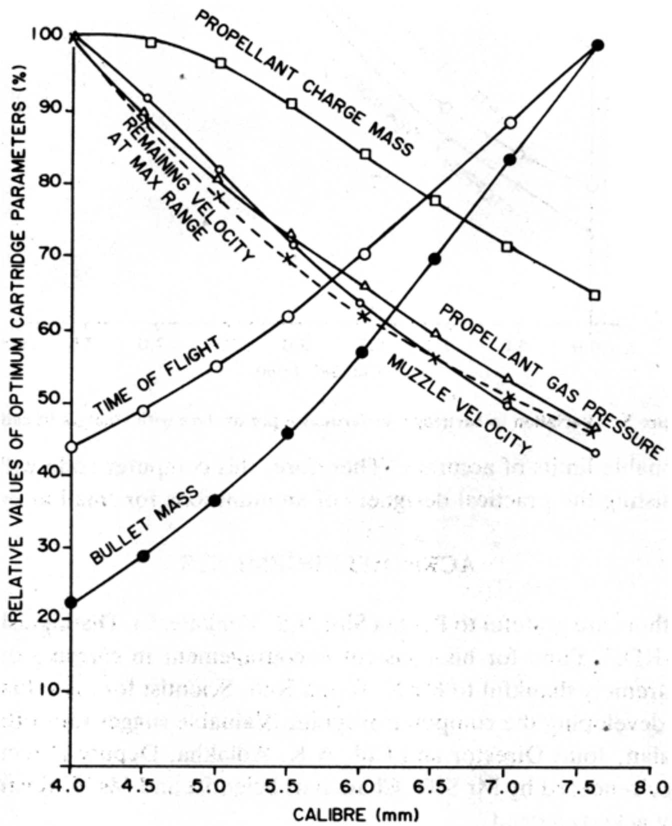

Figure 4. Change in optimum cartridge parameters with changes in calibre.

5. CONCLUSION

The computer code SACOP developed aids not only in the evaluation of optimum small arms cartridge parameters but also in the performance analysis of an existing cartridge design. The code is fairly simple to use and at the same time quite reliable

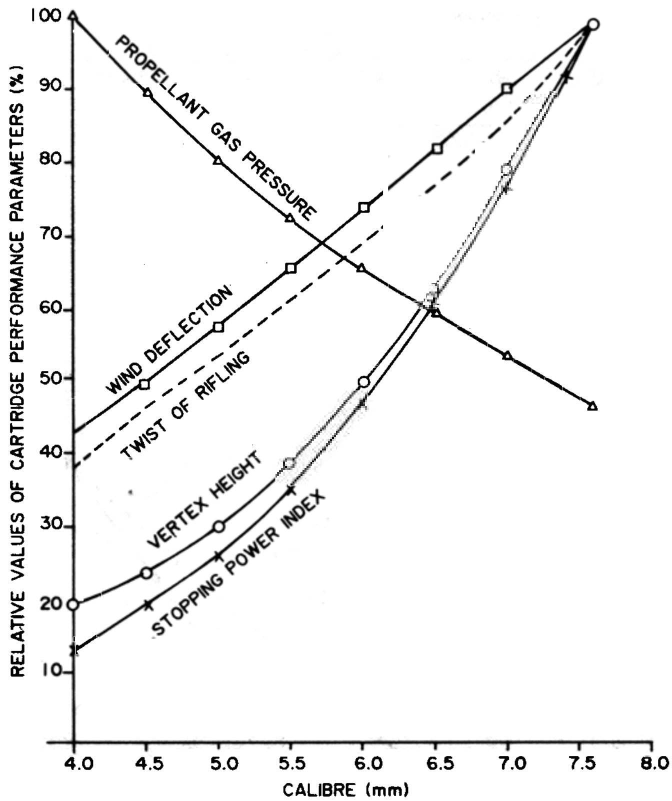

Figure 5. Variation in cartridge performance parameters with changes in calibre.

within reasonable limits of accuracy. Therefore, this computer code will certainly be useful in assisting the practical designers of ammunitions for small arms system.

ACKNOWLEDGEMENTS

The authors are grateful to Padma Shri N.S. Venkatesan, Distinguished Scientist, Director, ARDE, Pune for his constant encouragement in carrying out this work. They are extremely thankful to Mr K. Rama Rao, Scientist for his valuable guidance and help in developing the computer program. Valuable suggestions offered by Brig. S. Rajagopalan, Joint Director and Col. A.K. Adlakha, Deputy Director as well as the assistance rendered by Mr S.D. Chaudhari Scientist and Ms N. Kasturi of ARDE are sincerely acknowledged.

REFERENCES

Ingal's Ballistic Tables, (Govt. Printing Office, Washington), 1971.

2. Orlikon Machine Tool Works, Orlikon Pocket Book, (Orlikon, Zurich), 1960.
3. HMSO, Text Book of Small Arms, (HMSO, London), 1929.
4. War Office, Text Book of Ballistics and Gunnery, (War Office, London), 1954.